Linear Programming

Graphical Method · Class 12 Board and MHT CET

Feasible region · corner points · unbounded region · word problems

Questions on Linear Programming

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13 questions · CBSE 2020–2024 · NCERT Exemplar

Graph of a Linear Inequality

Region of $ax + by \leq c$ (or $\geq c$)

Draw the line $ax + by = c$: solid for $\leq$ and $\geq$, dotted for $<$ and $>$.

Put (0, 0) in the inequality. If it is true, shade the origin side; if false, shade the other side.

Feasible region

The common region of all the constraints. Because $x \geq 0$ and $y \geq 0$, it stays in the first quadrant.

Corner points

The vertices of the feasible region. Solve two boundary lines at a time and keep only the points that satisfy every constraint.

1

Class Question

Ans: B

The region represented by $30x + 20y \geq 120$, $10x + 30y \geq 110$, $x \geq 0$, $y \geq 0$ is

A. bounded, on the origin side

B. unbounded, on the non-origin side

C. bounded, on the non-origin side

D. unbounded, on the origin side

Q.1 · Solution

Test (0, 0)	30(0) + 20(0) = 0 ≥ 120 is false, so the region lies away from the origin.
Intercepts	30x + 20y = 120 meets the axes at (4, 0) and (0, 6). 10x + 30y = 110 meets them at (11, 0) and (0, 11/3).
Shape	Both inequalities are ≥ and nothing stops the region going up and to the right, so it is unbounded.

Corner point
(0, 6)
(2, 3)
(11, 0)

Answer: (B) unbounded, non-origin side

Linear Programming Problem

Terms

Objective function: $Z = ax + by$, the quantity to maximise or minimise.

Constraints: linear inequalities in x and y, together with $x \geq 0$, $y \geq 0$.

Optimal solution: the feasible point that gives the maximum or minimum Z.

Corner point method

Step 1. Graph all the constraints and shade the feasible region.

Step 2. Find every corner point of the region.

Step 3. Find Z at each corner point. The greatest value is the maximum and the smallest is the minimum.

2

CBSE 2024

Ans: C

The common region determined by all the constraints of a linear programming problem is called

A. an optimal region **B.** a bounded region **C.** a feasible region

D. an infeasible region

Q.2 · Solution

Definition	The common region of all the constraints (including $x \geq 0$, $y \geq 0$) is the feasible region. Every point in it satisfies every constraint.
Trap	“Optimal” describes the point that gives the maximum or minimum Z. “Bounded” describes the shape of a region, not its name.

Answer: (C) a feasible region

3

CBSE 2024

Ans: C

The number of corner points of the feasible region determined by $x \geq 0$, $y \geq 0$, $x + y \geq 4$ is

A. 0 **B.** 1 **C.** 2 **D.** 3

Q.3 · Solution

Boundary	x + y = 4 meets the axes at (4, 0) and (0, 4).
Region	(0, 0) gives 0 ≥ 4, which is false, so we shade above the line in the first quadrant. The region is unbounded.
Corners	Only the two points where the line meets the axes.

Corner point
(4, 0)
(0, 4)

Answer: (C) 2

4

Class Question

Ans: Max Z = 120

Maximise $Z = 4x + y$ subject to

$x + y \leq 50, \quad 3x + y \leq 90, \quad x \geq 0, y \geq 0$

Q.4 · Solution

Corners Lines x + y = 50 and 3x + y = 90 meet where 2x = 40, so x = 20, y = 30. The 3x + y = 90 line meets the x-axis at (30, 0). The region is bounded.

Evaluate Z = 4x + y at each corner (table).

Compare Greatest value is 120 at (30, 0).

Corner point	Z = 4x + y
(0, 0)	0
(30, 0)	120
(20, 30)	110
(0, 50)	50

Maximum Z = 120 at (30, 0)

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Class Question

Ans: Min 60, Max 180

Minimise and maximise $Z = 3x + 9y$ subject to

$x + 3y \leq 60, \quad x + y \geq 10, \quad x \leq y, \quad x \geq 0, y \geq 0$

Q.5 · Solution

Corners	x = y meets x + y = 10 at (5, 5) and x + 3y = 60 at (15, 15). On the y-axis the region runs from (0, 10) up to (0, 20). Bounded.
Evaluate	Z = 3x + 9y (table).
Note	Max 180 occurs at two corners, so it holds at every point of the segment joining (0, 20) and (15, 15).

Corner point	Z = 3x + 9y
(5, 5)	60
(0, 10)	90
(0, 20)	180
(15, 15)	180

Min 60 at (5, 5); Max 180 on the segment (0, 20)–(15, 15)

6

CBSE 2024 · 3 Marks

Ans: Max Z = 3200

Maximise $Z = 500x + 300y$ subject to

$x + 2y \leq 12, \quad 2x + y \leq 12, \quad 4x + 5y \geq 20, \quad x \geq 0, y \geq 0$

Q.6 · Solution

Corners — x + 2y = 12 and 2x + y = 12 meet at (4, 4). 4x + 5y = 20 cuts the axes at (5, 0) and (0, 4). The region is bounded.

Evaluate — Z = 500x + 300y (table).

Compare — Greatest value is 3200 at (4, 4).

Corner point	Z = 500x + 300y
(5, 0)	2500
(6, 0)	3000
(4, 4)	3200
(0, 6)	1800
(0, 4)	1200

Maximum Z = 3200 at (4, 4)

Unbounded Region and Multiple Optimum

Bounded region

Both the maximum and the minimum exist, and they occur at corner points.

Unbounded region: does the optimum exist?

Let m be the smallest corner value. Draw the half-plane $Z < m$. If it has no point in common with the feasible region, m is the minimum. Otherwise there is no minimum.

For the maximum, use the largest corner value M and the half-plane $Z > M$.

More than one optimal point

If Z has the same optimum value at two corner points, it has that value at every point of the segment joining them. To find an unknown in Z, equate its values at those two corners.

7

CBSE 2020

Ans: Min Z = 38

Minimise $Z = 5x + 7y$ subject to

$2x + y \geq 8, \quad x + 2y \geq 10, \quad x, y \geq 0$

Q.7 · Solution

Corners — 2x + y = 8 and x + 2y = 10 meet at (2, 4). Other corners (0, 8) and (10, 0). The region is unbounded.

Evaluate — Smallest corner value is m = 38.

Half-plane — Draw 5x + 7y < 38. It has no point in common with the feasible region, so m is the minimum.

Corner point	Z = 5x + 7y
(0, 8)	56
(2, 4)	38
(10, 0)	50

Minimum Z = 38 at (2, 4)

8

Class Question

Ans: No minimum

Minimise $Z = -50x + 20y$ subject to

$2x - y \geq -5, \quad 3x + y \geq 3, \quad 2x - 3y \leq 12, \quad x, y \geq 0$

Q.8 · Solution

Corners	(0, 3), (1, 0), (6, 0), (0, 5). The region is unbounded.
Evaluate	Smallest corner value is m = −300 at (6, 0).
Half-plane	Draw −50x + 20y < −300, i.e. −5x + 2y < −30. It has points in common with the unbounded region (for example (10, 4): −500 + 80 = −420 < −300), so Z can go below −300.

Corner point	Z = −50x + 20y
(0, 3)	60
(1, 0)	−50
(6, 0)	−300
(0, 5)	100

Z has no minimum value

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CBSE 2020

Ans: B

The corner points of the feasible region are (0, 0), (4, 0), (2, 4), (0, 5). If the maximum value of z = ax + by (a, b > 0) occurs at both (2, 4) and (4, 0), then

A. a = b **B.** a = 2b **C.** 2a = b **D.** a = 4b

Q.9 · Solution

Idea	Same maximum at two corners, so equate z at those corners.
Equate	z(4, 0) = 4a and z(2, 4) = 2a + 4b.
Solve	4a = 2a + 4b gives 2a = 4b, so a = 2b.

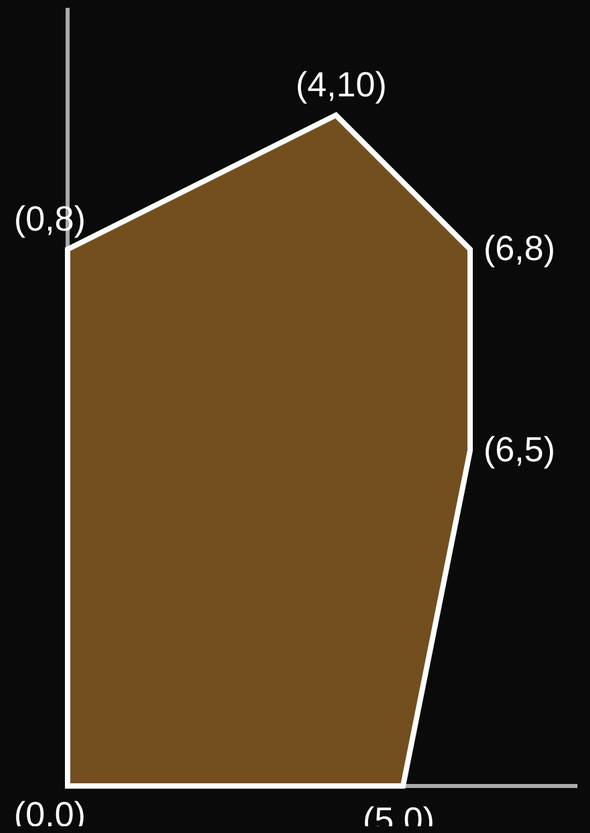
Answer: (B) a = 2b

10

NCERT Exemplar

Ans: B

For the feasible region shown in the figure, the minimum value of z = 3x − 4y occurs at



A. (0, 0) **B.** (0, 8) **C.** (5, 0) **D.** (4, 10)

Q.10 · Solution

Method Minimum of a bounded region lies at a corner point, so evaluate z = 3x − 4y at each of the six vertices.

Result The smallest value is −32, at (0, 8).

Corner point	z = 3x − 4y
(0, 0)	0
(5, 0)	15
(6, 5)	−2
(6, 8)	−14
(4, 10)	−28
(0, 8)	−32

Answer: (B) (0, 8), minimum −32

Word Problems

Formulate first, then solve

1. Let x and y be the two quantities asked for.

2. Write Z from the cost or profit: minimise cost, maximise profit.

3. One constraint per resource or requirement. “At least” gives ≥ and “at most” gives ≤.

4. Add $x \geq 0$, $y \geq 0$, then use the corner point method.

Limitations of linear programming

It works only when the objective function and constraints are linear.

The data must be accurate, and items such as machines or packets are whole numbers.

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Class Question · Diet Problem

Ans: 15 and 20; 150 units

A dietician uses two foods P and Q. One packet of P has 12 units calcium, 4 iron, 6 cholesterol, 6 vitamin A. One packet of Q has 3 calcium, 20 iron, 4 cholesterol, 3 vitamin A. The diet needs at least 240 calcium, at least 460 iron and at most 300 cholesterol. How many packets of each food minimise the vitamin A, and what is that minimum?

Q.11 · Solution

Formulate	x packets of P, y of Q. Minimise Z = 6x + 3y.
Constraints	12x + 3y ≥ 240, 4x + 20y ≥ 460, 6x + 4y ≤ 300, x, y ≥ 0.
Corners	Region is bounded (the cholesterol line closes it): (15, 20), (2, 72), (40, 15).
Compare	Smallest Z is 150 at (15, 20).

Corner point	Z = 6x + 3y
(15, 20)	150
(2, 72)	228
(40, 15)	285

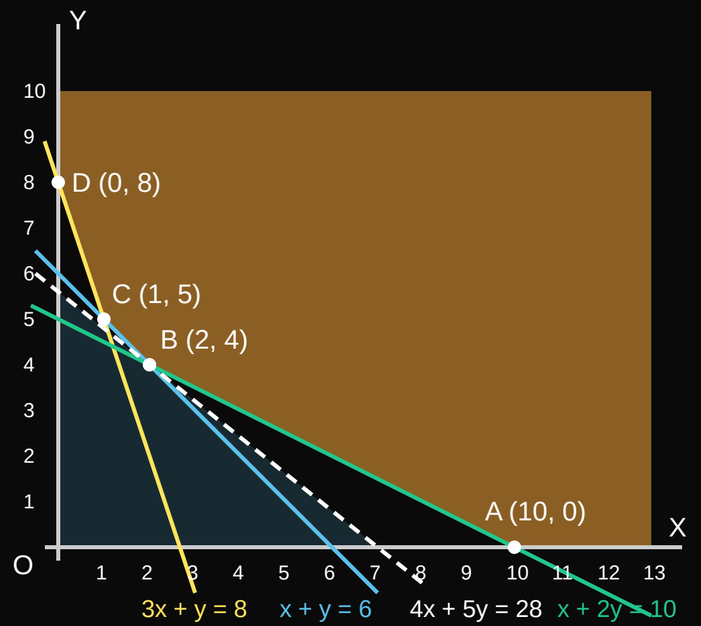
15 packets of P, 20 of Q; minimum vitamin A = 150 units

12

CBSE 2024 · Case Study · 4 Marks

Ans: x+2y≥10, x+y≥6, 3x+y≥8; Rs 112

A dietician minimises the cost of a diet using food X (x kg, Rs 16/kg) and food Y (y kg, Rs 20/kg). The feasible region is shown. (i) Write all the constraints that determine it. (ii) Minimise Z = 16x + 20y; find x, y and the minimum cost, assuming it exists for the unbounded region.



Q.12 · Solution

(i) Lines Read the intercepts: (0, 8)–(8/3, 0) gives $3x + y \geq 8$; (0, 6)–(6, 0) gives $x + y \geq 6$; (0, 5)–(10, 0) gives $x + 2y \geq 10$. Add $x, y \geq 0$.

(ii) Corners (0, 8), (1, 5), (2, 4), (10, 0). Unbounded.

Half-plane Smallest value m = 112. $16x + 20y < 112$, i.e. $4x + 5y < 28$, has no point in common with the region, so 112 is the minimum.

Corner point	Z = 16x + 20y
(0, 8)	160
(1, 5)	116
(2, 4)	112
(10, 0)	160

(i) $x+2y \geq 10$, $x+y \geq 6$, $3x+y \geq 8$, $x, y \geq 0$ (ii) $x = 2$, $y = 4$, min cost = Rs 112

13

Class Question · Manufacturing

Ans: 4 of M, 4 of N; Rs 4000

A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hrs required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Q.13 · Solution

Formulate	x units of M, y of N. Maximise Z = 600x + 400y.
Constraints	x + 2y ≤ 12, 2x + y ≤ 12, x + 1.25y ≥ 5 (i.e. 4x + 5y ≥ 20), x, y ≥ 0.
Corners	Bounded region with corners as in the table.
Compare	Greatest Z is 4000 at (4, 4).

Corner point	Z = 600x + 400y
(5, 0)	3000
(6, 0)	3600
(4, 4)	4000
(0, 6)	2400
(0, 4)	1600

4 units of M, 4 units of N; maximum profit = Rs 4000