HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

Mathematical Logic • Assignment • Solutions

Topic: Statements, Truth Tables, Quantifiers, Negation, Algebra of Statements & Switching Circuits

Sub: Mathematics **12th • Board & MHT-CET** Chetan Sir

SECTION A : STATEMENTS & CONNECTIVES

1. **[Board Solved]** Which of the following are statements? Write the truth values of the statements.
i) $6 \times 4 = 25$
ii) $x + 6 = 9$
iii) Please, sit down.
iv) Every real number is a complex number.

Solution:
i) $6 \times 4 = 24 \neq 25$. It is a statement which is false, hence its truth value is F.
ii) Its truth value depends on the value of x. It is an open sentence, hence it is not a statement.
iii) It is a request, hence it is not a statement.
iv) Every real number a can be written as $a + 0i$. It is a statement which is true, hence its truth value is T.

Answer: i) Statement, F ii) Not a statement iii) Not a statement iv) Statement, T

2. **[Ex 1.1]** State which of the following are statements. Justify. In case of a statement, state its truth value.
i) Zero is a complex number.
ii) $x^2 = x$.
iii) Do you like Mathematics?
iv) $x^2 - 6x - 7 = 0$, when $x = 7$.
v) The sum of cube roots of unity is zero.

Solution:
i) $0 = 0 + 0i$, so zero is a complex number. It is a statement which is true, hence its truth value is T.
ii) Its truth value depends on x (true for $x = 0, 1$ only). It is an open sentence, hence it is not a statement.
iii) It is an interrogative sentence, hence it is not a statement.
iv) For $x = 7$: $x^2 - 6x - 7 = 49 - 42 - 7 = 0$. It is a statement which is true, hence its truth value is T.
v) The cube roots of unity are $1, \omega, \omega^2$ and $1 + \omega + \omega^2 = 0$. It is a statement which is true, hence its truth value is T.

Answer: i) Statement, T ii) Not a statement iii) Not a statement iv) Statement, T v) Statement, T

3. **[Board Solved]** Express symbolically: "Delhi is in India but Dhaka is not in Sri Lanka."

Solution:
Let p : Delhi is in India, q : Dhaka is in Sri Lanka.
Then "Dhaka is not in Sri Lanka" is $\sim q$.
"but" means "and". $\therefore$ The symbolic form is $p \wedge \sim q$.

Answer: $p \wedge \sim q$

4. **[Ex 1.1]** Write the following compound statements symbolically.
i) Angle is neither acute nor obtuse.
ii) If $\triangle ABC$ is right angled at B, then $m\angle A + m\angle C = 90°$.
iii) x is not an irrational number but is a square of an integer.

Solution:
i) Let p : Angle is acute, q : Angle is obtuse.
"neither p nor q" means "not p and not q". $\therefore$ The symbolic form is $\sim p \wedge \sim q$.
ii) Let $p : \triangle ABC$ is right angled at B, $q : m\angle A + m\angle C = 90°$.
"If p then q" $\therefore$ The symbolic form is $p \to q$.
iii) Let $p : x$ is an irrational number, $q : x$ is a square of an integer.
"but" means "and". $\therefore$ The symbolic form is $\sim p \wedge q$.

Answer: i) $\sim p \wedge \sim q$ ii) $p \to q$ iii) $\sim p \wedge q$

5. **[Board Solved]** Write the truth value of: "Milk is white if and only if the Sun rises in the West."

Solution:
Let p : Milk is white, q : The Sun rises in the West.
Truth values of p and q are T and F respectively.
The given statement in symbolic form is $p \leftrightarrow q$.
$\therefore\ p \leftrightarrow q \equiv \text{T} \leftrightarrow \text{F} \equiv \text{F}$ [$\leftrightarrow$ is T only when both have the same value]
$\therefore$ Truth value of the given statement is F.

Answer: F

6. **[Ex 1.1]** Write the truth values of the following.
i) It is not true that $5 - 3i$ is a real number.
ii) If $3 \times 5 = 8$ then $3 + 5 = 15$.
iii) Milk is white if and only if sky is blue.

Solution:
i) Let $p : 5 - 3i$ is a real number. Its imaginary part is $-3 \neq 0$, so the truth value of p is F.
The given statement in symbolic form is $\sim p$.
$\therefore\ \sim p \equiv \sim\text{F} \equiv \text{T}$. $\therefore$ Truth value of the given statement is T.
ii) Let $p : 3 \times 5 = 8$, $q : 3 + 5 = 15$.
Since $3 \times 5 = 15$ and $3 + 5 = 8$, truth values of p and q are F and F respectively.
The given statement in symbolic form is $p \to q$.
$\therefore\ p \to q \equiv \text{F} \to \text{F} \equiv \text{T}$ [$\to$ is F only when T $\to$ F]
$\therefore$ Truth value of the given statement is T.
iii) Let p : Milk is white, q : Sky is blue.
Truth values of p and q are T and T respectively.
The given statement in symbolic form is $p \leftrightarrow q$.
$\therefore\ p \leftrightarrow q \equiv \text{T} \leftrightarrow \text{T} \equiv \text{T}$. $\therefore$ Truth value of the given statement is T.

Answer: i) T ii) T iii) T

7. **[MHT-CET 2025]** Consider the following three statements:
(A) If $3 + 2 = 7$ then $4 + 3 = 8$.
(B) If $5 + 2 = 7$ then earth is flat.
(C) If both (A) and (B) are true then $5 + 6 = 11$.
Which of the following is correct?
(A) (A), (C) true; (B) false (B) (A) true; (B), (C) false
(C) (A) false; (B), (C) true (D) (A) false; (C) true

Solution:
Statement (A): $3 + 2 = 7$ is F, $4 + 3 = 8$ is F. $\text{F} \to \text{F} \equiv \text{T}$. $\therefore$ (A) is true.
Statement (B): $5 + 2 = 7$ is T, earth is flat is F. $\text{T} \to \text{F} \equiv \text{F}$. $\therefore$ (B) is false.
Statement (C): its consequent "$5 + 6 = 11$" is T, and anything $\to$ T is T.

(Here "(A) and (B) are true" is $T \wedge F \equiv F$, so (C) is $F \to T \equiv T$.) $\therefore$ (C) is true.
$\therefore$ (A), (C) are true and (B) is false.

Answer: (A)

8. **[Board Solved]** If p, q are true and r, s are false statements, find the truth value of $\sim(p \to q) \leftrightarrow (r \wedge s)$.

Solution:
Truth values of p, q, r, s are T, T, F, F respectively.
$$\begin{aligned}\sim(p \to q) \leftrightarrow (r \wedge s) &\equiv \sim(T \to T) \leftrightarrow (F \wedge F)\\ &\equiv \sim T \leftrightarrow F\\ &\equiv F \leftrightarrow F\\ &\equiv T\end{aligned}$$
$\therefore$ Truth value of the given statement pattern is T.

Answer: T

9. **[Ex 1.1]** If p, q are true and r, s are false, find the truth values of:
i) $(p \to q) \vee (r \to s)$
ii) $(\sim r \leftrightarrow p) \to \sim q$
iii) $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

Solution: Truth values of p, q, r, s are T, T, F, F respectively.
i) $$\begin{aligned}(p \to q) \vee (r \to s) &\equiv (T \to T) \vee (F \to F)\\ &\equiv T \vee T \equiv T\end{aligned}$$
$\therefore$ Truth value is T.
ii) $$\begin{aligned}(\sim r \leftrightarrow p) \to \sim q &\equiv (\sim F \leftrightarrow T) \to \sim T\\ &\equiv (T \leftrightarrow T) \to F\\ &\equiv T \to F \equiv F\end{aligned}$$
$\therefore$ Truth value is F.
iii) $$\begin{aligned}\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r) &\equiv \sim[(\sim T \wedge F) \vee (F \to \sim T)] \leftrightarrow (T \wedge F)\\ &\equiv \sim[(F \wedge F) \vee (F \to F)] \leftrightarrow F\\ &\equiv \sim[F \vee T] \leftrightarrow F\\ &\equiv \sim T \leftrightarrow F \equiv F \leftrightarrow F \equiv T\end{aligned}$$
$\therefore$ Truth value is T.

Answer: i) T ii) F iii) T

10. **[MHT-CET 2024]** If p, q, r have truth values F, T, F respectively, then the truth values of $(p \wedge \sim q) \to r$ and $(p \vee q) \to r$ are respectively
(A) T, T (B) T, F (C) F, T (D) F, F

Solution:
$p \equiv F$, $q \equiv T$, $r \equiv F$, so $\sim q \equiv F$.
$(p \wedge \sim q) \to r \equiv (F \wedge F) \to F \equiv F \to F \equiv T$.
$(p \vee q) \to r \equiv (F \vee T) \to F \equiv T \to F \equiv F$.
$\therefore$ The truth values are T, F.

Answer: (B)

11. **[Misc Ex 1]** Determine the truth values of p and q if $(p \vee q)$ is T and $(p \vee q) \to q$ is F.

Solution:
$(p \vee q) \to q$ is F.
$\therefore$ $(p \vee q)$ is T and q is F [$\because$ $a \to b$ is F only when a is T, b is F]
Now $p \vee q$ is T and q is F, $\therefore$ p is T. [$\because$ $a \vee b$ is F only when both are F]
This agrees with the given condition that $p \vee q$ is T.
Hence p is T and q is F.

Answer: p is T, q is F

12. **[Misc Ex 1]** Determine the truth values of p and q if $(p \vee q)$ is T and $(p \wedge q)$ is T.

Solution:
$p \wedge q$ is T.
$\therefore$ p is T and q is T [$\because$ $a \wedge b$ is T only when both are T]
Check: $p \vee q \equiv \text{T} \vee \text{T} \equiv \text{T}$, which agrees with the given condition.
Hence p is T and q is T.

Answer: p is T, q is T

13. **[MHT-CET 2024]** If $p \to (q \vee r)$ is false, then the truth values of p, q, r are respectively
(A) F, F, F (B) T, T, F (C) T, F, F (D) F, T, T

Solution:
A conditional $A \to B$ is false *only* when A is T and B is F.
$\therefore$ p is T and $q \vee r$ is F.
$q \vee r$ is F only when both are F $\Rightarrow$ q is F and r is F.
$\therefore$ $(p, q, r) = (\text{T}, \text{F}, \text{F})$.

Answer: (C)

14. **[MHT-CET 2024]** If the statement $p \vee \sim(q \wedge r)$ is false, then the truth values of p, q and r are respectively
(A) F, T, F (B) T, F, F (C) F, T, T (D) F, F, T

Solution:
A disjunction $A \vee B$ is false *only* when both A and B are F.
$\therefore$ p is F and $\sim(q \wedge r)$ is F.
$\sim(q \wedge r)$ is F $\Rightarrow$ $q \wedge r$ is T $\Rightarrow$ q is T and r is T.
$\therefore$ $(p, q, r) = (\text{F}, \text{T}, \text{T})$.

Answer: (C)

15. **[Ex 1.1]** Write the negations of the following.
i) Tirupati is in Andhra Pradesh.
ii) 3 is not a root of the equation $x^2 + 3x - 18 = 0$.
iii) $7 + 3 > 5$.

Solution:
i) Let p : Tirupati is in Andhra Pradesh. Its negation is $\sim p$.
$\sim p$: Tirupati is not in Andhra Pradesh.
ii) Let p : 3 is a root of the equation $x^2 + 3x - 18 = 0$. The given statement is $\sim p$.
Its negation is $\sim(\sim p) \equiv p$. [Involution law]
$\therefore$ The negation is: 3 is a root of the equation $x^2 + 3x - 18 = 0$.
iii) Let $p : 7 + 3 > 5$. Its negation is $\sim p$. [negation of $>$ is $\leq$]
$\sim p : 7 + 3 \leq 5$, i.e. $7 + 3 \not> 5$.

Answer: i) Tirupati is not in Andhra Pradesh. ii) 3 is a root of $x^2 + 3x - 18 = 0$. iii) $7 + 3 \leq 5$.

16. **[MHT-CET 2026]** Which of the following is/are *not* true?
I) If 1 is not a prime number, then 2 is not a prime number.
II) e is a vowel and $12 \times 3 = 36$.
III) It is not true that 14 is a composite number and 3 is an even number.
IV) $\sqrt{5}$ is an irrational number, but $3 + \sqrt{5}$ is a complex number.
(A) Only I and IV (B) Only I (C) Only II and III (D) Only I, II and IV

Solution: Facts used: 1 is neither prime nor composite; 2 is prime; $14 = 2 \times 7$ is composite; every real number is a complex number.
I) "1 is not a prime number" is T, "2 is not a prime number" is F. $T \to F \equiv F$. $\therefore$ I is false.
II) "e is a vowel" is T, "$12 \times 3 = 36$" is T. $T \wedge T \equiv T$. $\therefore$ II is true.
III) "14 is composite" is T, "3 is even" is F. $\sim(T \wedge F) \equiv \sim F \equiv T$. $\therefore$ III is true.
IV) "$\sqrt{5}$ is irrational" is T, "$3 + \sqrt{5}$ is complex" is T ($3 + \sqrt{5} = (3 + \sqrt{5}) + 0i$). $T \wedge T \equiv T$. $\therefore$ IV is true.
$\therefore$ Only I is not true.

Answer: (B)

17. **[MHT-CET 2025]** Which of the following statements has truth value T?
A: Cube roots of unity are in G.P. and their sum is 1.
B: $4 + 7 > 10$ iff $2 + 8 < 10$.
C: $\exists\, x \in \mathbb{N}$ such that $x^2 - 3x + 2 = 0$ and $\exists\, n \in \mathbb{N}$ such that n is odd.
D: $3 + i$ is a complex number or $\sqrt{2} + \sqrt{3} = \sqrt{5}$.
(A) only A (B) B, C, D (C) A and C (D) C and D

Solution:
A: The cube roots of unity $1, \omega, \omega^2$ are in G.P. (common ratio ω): T. Their sum $1 + \omega + \omega^2 = 0 \neq 1$: F.
$T \wedge F \equiv F$. $\therefore$ A is F.
B: $4 + 7 = 11 > 10$: T. $2 + 8 = 10$, which is not < 10: F. $T \leftrightarrow F \equiv F$. $\therefore$ B is F.
C: $x^2 - 3x + 2 = (x - 1)(x - 2) = 0$ gives $x = 1, 2 \in \mathbb{N}$: T. $n = 1 \in \mathbb{N}$ is odd: T. $T \wedge T \equiv T$. $\therefore$ C is T.
D: $3 + i$ is a complex number: T. So $T \vee$ (anything) $\equiv T$ (note $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$, F). $\therefore$ D is T.
$\therefore$ C and D have truth value T.

Answer: (D)

18. **[MHT-CET 2026]** p : If $3^4 > 4^3$, then $3^3 > 4^4$.
q : The roots of $x^2 - 2x + 2 = 0$ are real if and only if Mumbai is in Maharashtra.
r : Statement p is true or statement q is false.
Which of the following has truth value T?
(A) $(p \vee q) \wedge r$ (B) $p \vee (q \wedge r)$ (C) $p \wedge (q \vee r)$ (D) $(p \wedge q) \vee r$

Solution:
p: $3^4 = 81 > 64 = 4^3$ is T; $3^3 = 27 > 256 = 4^4$ is F. $T \to F \equiv F$. $\therefore\ p \equiv$ F.
q: For $x^2 - 2x + 2 = 0$, discriminant $b^2 - 4ac = 4 - 8 = -4 < 0$, so the roots are not real: F.
Mumbai is in Maharashtra: T. $F \leftrightarrow T \equiv F$. $\therefore\ q \equiv$ F.
r: $p \vee \sim q \equiv F \vee T \equiv T$. $\therefore\ r \equiv$ T.
(A) $(p \vee q) \wedge r \equiv (F \vee F) \wedge T \equiv F \wedge T \equiv F$
(B) $p \vee (q \wedge r) \equiv F \vee (F \wedge T) \equiv F \vee F \equiv F$
(C) $p \wedge (q \vee r) \equiv F \wedge (F \vee T) \equiv F \wedge T \equiv F$
(D) $(p \wedge q) \vee r \equiv (F \wedge F) \vee T \equiv F \vee T \equiv T$

Answer: (D)

19. **[MHT-CET 2025]** If the statement pattern $(p \wedge q) \to (r \vee \sim s)$ is false, then the truth values of p, q, r, s are respectively
(A) T, F, T, F (B) T, T, T, F (C) T, T, F, F (D) T, T, F, T

Solution:
$(p \wedge q) \to (r \vee \sim s)$ is F $\Rightarrow$ $p \wedge q$ is T and $r \vee \sim s$ is F. [$A \to B$ is F only when A is T, B is F]
$p \wedge q$ is T $\Rightarrow$ p is T and q is T.
$r \vee \sim s$ is F $\Rightarrow$ r is F and $\sim s$ is F $\Rightarrow$ r is F and s is T.
$\therefore\ (p, q, r, s) = (T, T, F, T)$.

Answer: (D)

20. **[MHT-CET 2026]** If the truth value of $[(p \vee q) \wedge (q \to r) \wedge (\sim r)] \to (p \wedge q)$ is false, then the truth values of $p \to q$ and $q \to p$ are respectively
(A) T, T (B) T, F (C) F, T (D) F, F

Solution:
The conditional is F $\Rightarrow$ $(p \vee q) \wedge (q \to r) \wedge (\sim r)$ is T and $p \wedge q$ is F.
The conjunction is T $\Rightarrow$ $p \vee q$ is T, $q \to r$ is T, $\sim r$ is T.
$\sim r$ is T $\Rightarrow$ r is F.
$q \to r$ is T with r F $\Rightarrow$ q is F. [T $\to$ F would be F]
$p \vee q$ is T with q F $\Rightarrow$ p is T.
Check: $p \wedge q \equiv$ T $\wedge$ F $\equiv$ F, as required.
$\therefore$ $p \to q \equiv$ T $\to$ F $\equiv$ F and $q \to p \equiv$ F $\to$ T $\equiv$ T.

Answer: (C)

21. **[MHT-CET 2026]** If $p \to (q \vee \sim r)$ is false, then the truth values of $(p \leftrightarrow q) \wedge r$ and $\sim p \to \sim q$ are
(A) T, T (B) T, F (C) F, T (D) F, F

Solution:
$p \to (q \vee \sim r)$ is F $\Rightarrow$ p is T and $q \vee \sim r$ is F.
$q \vee \sim r$ is F $\Rightarrow$ q is F and $\sim r$ is F $\Rightarrow$ q is F and r is T.
$\therefore$ $p \equiv$ T, $q \equiv$ F, $r \equiv$ T.
$(p \leftrightarrow q) \wedge r \equiv$ (T $\leftrightarrow$ F) $\wedge$ T $\equiv$ F $\wedge$ T $\equiv$ F.
$\sim p \to \sim q \equiv$ F $\to$ T $\equiv$ T.

Answer: (C)

22. **[MHT-CET 2026]** If $\sim p \to q$ is false and $q \leftrightarrow r$ is false, then the truth values of (p, q, r) are
(A) T, F, T (B) F, T, F (C) F, F, T (D) F, T, T

Solution:
$\sim p \to q$ is F $\Rightarrow$ $\sim p$ is T and q is F $\Rightarrow$ p is F and q is F.
$q \leftrightarrow r$ is F $\Rightarrow$ q and r have opposite values. Since q is F, r is T.
$\therefore$ $(p, q, r) =$ (F, F, T).

Answer: (C)

23. **[MHT-CET 2024]** The truth value of $p \to r$ is F and of $p \leftrightarrow q$ is F. Then the truth values of $(\sim p \vee q) \to (p \vee \sim q)$ and $(p \wedge \sim q) \to (\sim p \wedge q)$ are respectively
(A) T, F (B) F, T (C) T, T (D) F, F

Solution:
$p \to r$ is F $\Rightarrow$ p is T and r is F.
$p \leftrightarrow q$ is F $\Rightarrow$ p and q have opposite values. Since p is T, q is F.
$\therefore$ $p \equiv$ T, $q \equiv$ F, so $\sim p \equiv$ F, $\sim q \equiv$ T.
$(\sim p \vee q) \to (p \vee \sim q) \equiv$ (F $\vee$ F) $\to$ (T $\vee$ T) $\equiv$ F $\to$ T $\equiv$ T.
$(p \wedge \sim q) \to (\sim p \wedge q) \equiv$ (T $\wedge$ T) $\to$ (F $\wedge$ F) $\equiv$ T $\to$ F $\equiv$ F.

Answer: (A)

24. **[MHT-CET 2026]** r : If $p \to q$ is false then $p \vee q$ is false.
s : If $p \leftrightarrow q$ is false then $p \vee q$ is false.
The truth values of $r \to s$ and $s \to r$ are respectively
(A) T, T (B) T, F (C) F, T (D) F, F

Solution:
Statement r: $p \to q$ is false $\Rightarrow$ p is T and q is F $\Rightarrow$ $p \vee q \equiv$ T $\vee$ F $\equiv$ T.
So whenever "$p \to q$ is false" holds, "$p \vee q$ is false" fails. The if-part is T and the then-part is F.

$\therefore\ r \equiv \mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$.
Statement s: $p \leftrightarrow q$ is false $\Rightarrow$ p, q have opposite values, i.e. one of them is T $\Rightarrow$ $p \vee q \equiv \mathrm{T}$.
Again the if-part is T and the then-part is F. $\therefore\ s \equiv \mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$.
$\therefore\ r \to s \equiv \mathrm{F} \to \mathrm{F} \equiv \mathrm{T}$ and $s \to r \equiv \mathrm{F} \to \mathrm{F} \equiv \mathrm{T}$.

Answer: (A)

SECTION B : TRUTH TABLES & TAUTOLOGY

25. **[Board Solved]** Construct the truth table of $p \to (q \to p)$.

Solution: Two letters, so $2^2 = 4$ rows.

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

All entries of the last column are T, so $p \to (q \to p)$ is a tautology.

Answer: Last column T, T, T, T (tautology).

26. **[Ex 1.2]** Construct the truth table for each of the following.
i) $(p \wedge \sim q) \leftrightarrow (p \to q)$
ii) $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$

Solution:
i) Two letters, so $2^2 = 4$ rows.

p	q	$\sim q$	$p \wedge \sim q$	$p \to q$	$(p \wedge \sim q) \leftrightarrow (p \to q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

All entries of the last column are F, so the statement pattern is a contradiction.
ii) Three letters, so $2^3 = 8$ rows.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All entries of the last column are T, so the statement pattern is a tautology.

Answer: i) Last column F, F, F, F (contradiction) ii) Last column all T (tautology)

27. **[Board Solved]** Using truth table, prove that $p \wedge q \equiv \sim(p \to \sim q)$.

Solution:

I	II	III	IV	V	VI
p	q	$\sim q$	$p \wedge q$	$p \to \sim q$	$\sim(p \to \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

The entries in columns IV and VI are identical.
$\therefore\ p \wedge q \equiv \sim(p \to \sim q)$.

Answer: Columns IV and VI identical, hence $p \wedge q \equiv \sim(p \to \sim q)$.

28. **[Ex 1.2]** Using truth table, prove: $p \leftrightarrow q \equiv \sim[(p \vee q) \wedge \sim(p \wedge q)]$.

Solution:

I	II	III	IV	V	VI	VII	VIII
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$	$\sim[(p \vee q) \wedge \sim(p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

The entries in columns III and VIII are identical.
$\therefore\ p \leftrightarrow q \equiv \sim[(p \vee q) \wedge \sim(p \wedge q)]$.

Answer: Columns III and VIII identical, hence the equivalence is proved.

29. **[Board Solved]** Using truth table, examine whether $(p \wedge q) \wedge (\sim p \vee \sim q)$ is a tautology, contradiction or contingency.

Solution:

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$(p \wedge q) \wedge (\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the truth values in the last column are F.
Hence $(p \wedge q) \wedge (\sim p \vee \sim q)$ is a contradiction.

Answer: Contradiction

30. **[Ex 1.2]** Examine whether each is a tautology, a contradiction or a contingency.
i) $[(p \to q) \wedge \sim q] \to \sim p$
ii) $(p \leftrightarrow q) \wedge (p \to \sim q)$

Solution:
i)

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the truth values in the last column are T. Hence it is a tautology.
ii)

p	q	$\sim q$	$p \leftrightarrow q$	$p \rightarrow \sim q$	$(p \leftrightarrow q) \wedge (p \rightarrow \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

The last column contains both T and F. Hence it is a contingency.

Answer: i) Tautology ii) Contingency

31. **[MHT-CET 2024]** If p and q are statements, then _____ is a contingency.
(A) $p \wedge \sim p$ (B) $p \vee \sim p$ (C) $p \vee q$ (D) $(p \wedge (p \rightarrow q)) \rightarrow q$

Solution: Truth table of all four options:

p	q	$\sim p$	(A) $p \wedge \sim p$	(B) $p \vee \sim p$	(C) $p \vee q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	(D) $(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	F	F	T	T	T	T	T
T	F	F	F	T	T	F	F	T
F	T	T	F	T	T	T	F	T
F	F	T	F	T	F	T	F	T

(A) all F: contradiction. (B) all T: tautology. (D) all T: tautology.
(C) $p \vee q$ has both T and F: contingency.

Answer: (C)

32. **[MHT-CET 2026]** Consider the statement patterns
A. $(q \rightarrow p) \vee (p \rightarrow q)$
B. $(\sim p \vee \sim q) \leftrightarrow \sim(p \wedge q)$
C. $[(p \vee q) \wedge \sim p] \wedge \sim q$
D. $(p \wedge q) \wedge (\sim p \vee \sim q)$.
Then
(A) A, B are contradictions; C, D are tautologies
(B) A, B are tautologies; C, D are contradictions
(C) A, B, C, D all are tautologies
(D) A, B, C, D all are contradictions

Solution: Patterns A and B:

p	q	$\sim p$	$\sim q$	$q \rightarrow p$	$p \rightarrow q$	A	$\sim p \vee \sim q$	$\sim(p \wedge q)$	B
T	T	F	F	T	T	T	F	F	T
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T	T

Patterns C and D:

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim p$	C	$p \wedge q$	$\sim p \vee \sim q$	D
T	T	F	F	T	F	F	T	F	F
T	F	F	T	T	F	F	F	T	F
F	T	T	F	T	T	F	F	T	F
F	F	T	T	F	F	F	F	T	F

Columns A and B are all T: tautologies. Columns C and D are all F: contradictions.

Answer: (B)

33. **[Ex 1.2]** Using truth table, prove: $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$.

Solution: Three letters, so $2^3 = 8$ rows.

I	II	III	IV	V	VI	VII	VIII
p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns V and VIII are identical.
$\therefore\ (p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$.

Answer: Columns V and VIII identical, hence the equivalence is proved.

34. **[Ex 1.2]** Examine whether $[(p \to q) \wedge q] \to p$ is a tautology, contradiction or contingency.

Solution:

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The last column contains both T and F. Hence it is a contingency.

Answer: Contingency

35. **[MHT-CET 2025]** The last column in the truth table of $[p \to (q \wedge \sim p)] \vee [(p \vee \sim q) \wedge p]$ is
(A) TTTF (B) TFFF (C) TTTT (D) FFTT

Solution: Let $X \equiv [p \to (q \wedge \sim p)] \vee [(p \vee \sim q) \wedge p]$.

p	q	$\sim p$	$\sim q$	$q \wedge \sim p$	$p \to (q \wedge \sim p)$	$p \vee \sim q$	$(p \vee \sim q) \wedge p$	X
T	T	F	F	F	F	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	F	T	T	F	F	T
F	F	T	T	F	T	T	F	T

$\therefore$ The last column is T, T, T, T (the pattern is a tautology).

Answer: (C)

36. **[MHT-CET 2024]** The statement $(p \to q) \to ((\sim p \to q) \to q)$ is
(A) a fallacy (B) equivalent to $\sim p \to q$ (C) equivalent to $p \to \sim q$ (D) a tautology

Solution: Let $X \equiv (p \to q) \to ((\sim p \to q) \to q)$.

p	q	$\sim p$	$p \to q$	$\sim p \to q$	$(\sim p \to q) \to q$	X
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

All entries of the last column are T, so the statement is a tautology.
(Check by laws: $(\sim p \to q) \to q \equiv \sim(p \vee q) \vee q \equiv (\sim p \wedge \sim q) \vee q \equiv (\sim p \vee q) \wedge (\sim q \vee q) \equiv \sim p \vee q \equiv p \to q$, so $X \equiv (p \to q) \to (p \to q) \equiv t$.)

Answer: (D)

37. **[MHT-CET 2026]** The statement pattern $[(p \wedge q) \to (\sim p \vee r)] \vee [(\sim p \vee r) \to (p \wedge q)]$ is
(A) a contradiction (B) a tautology (C) equivalent to $(p \wedge q) \vee r$ (D) equivalent to $p \vee q$

Solution:
Let $A \equiv p \wedge q$, $B \equiv \sim p \vee r$. The given pattern is $(A \to B) \vee (B \to A)$.
$(A \to B) \vee (B \to A) \equiv (\sim A \vee B) \vee (\sim B \vee A)$ [Conditional law: $x \to y \equiv \sim x \vee y$]
$\equiv (A \vee \sim A) \vee (B \vee \sim B)$ [Commutative and associative laws]
$\equiv t \vee t \equiv t$ [Complement law, Idempotent law]
Verification by truth table:

p	q	r	$\sim p$	$A \equiv p \wedge q$	$B \equiv \sim p \vee r$	$A \to B$	$B \to A$	$(A \to B) \vee (B \to A)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	F	F	T	T	F	T
T	F	F	F	F	F	T	T	T
F	T	T	T	F	T	T	F	T
F	T	F	T	F	T	T	F	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	T	F	T

All entries of the last column are T. Hence the pattern is a tautology.

Answer: (B)

38. **[MHT-CET 2025]** The statement pattern $[(p \to q) \wedge \sim q] \to r$ is a tautology when r is equivalent to
(A) $p \wedge \sim q$ (B) $q \vee p$ (C) $p \wedge q$ (D) $\sim q$

Solution:
$(p \to q) \wedge \sim q \equiv (\sim p \vee q) \wedge \sim q$ [Conditional law]
$\equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q)$ [Distributive law]
$\equiv (\sim p \wedge \sim q) \vee c$ [Complement law]
$\equiv \sim p \wedge \sim q$ [Identity law]
So the antecedent is T only when p is F and q is F. For a tautology, r must be T when $p \equiv$ F, $q \equiv$ F.
(A) $p \wedge \sim q \equiv$ F $\wedge$ T $\equiv$ F (B) $q \vee p \equiv$ F $\vee$ F $\equiv$ F (C) $p \wedge q \equiv$ F $\wedge$ F $\equiv$ F (D) $\sim q \equiv$ T
In (A), (B), (C) the pattern becomes T $\to$ F $\equiv$ F for $p \equiv q \equiv$ F, so they are not tautologies.
With $r \equiv \sim q$: $(\sim p \wedge \sim q) \to \sim q$ is a tautology, since whenever $\sim p \wedge \sim q$ is T, $\sim q$ is also T.

Answer: (D)

39. **[MHT-CET 2026]** The correct logical equivalence(s):
(I) $p \to (q \to r) \equiv (p \wedge q) \to r$
(II) $(p \to q) \to r \equiv p \to (q \vee r)$
(III) $(p \to q) \to r \equiv (p \to r) \wedge (\sim q \to r)$
(IV) $p \to (q \to r) \equiv q \to (p \to r)$
(A) I, II (B) III, IV (C) II, IV (D) I, IV

Solution:
(I) $p \to (q \to r) \equiv \sim p \vee (\sim q \vee r)$ [Conditional law]
$\equiv (\sim p \vee \sim q) \vee r$ [Associative law]
$\equiv \sim(p \wedge q) \vee r$ [De Morgan's law]
$\equiv (p \wedge q) \to r$ [Conditional law]
$\therefore$ (I) is correct.
(II) Take $p \equiv$ F, $q \equiv$ F, $r \equiv$ F: LHS (F $\to$ F) $\to$ F $\equiv$ T $\to$ F $\equiv$ F; RHS F $\to$ (F $\vee$ F) $\equiv$ T.
The values differ, $\therefore$ (II) is not correct.
(III) Take $p \equiv$ F, $q \equiv$ T, $r \equiv$ F: LHS (F $\to$ T) $\to$ F $\equiv$ T $\to$ F $\equiv$ F;
RHS (F $\to$ F) $\wedge$ (F $\to$ F) $\equiv$ T $\wedge$ T $\equiv$ T. The values differ, $\therefore$ (III) is not correct.

(IV) $q \to (p \to r) \equiv \sim q \vee (\sim p \vee r)$ [Conditional law]
$\equiv \sim p \vee (\sim q \vee r)$ [Commutative and associative laws]
$\equiv p \to (q \to r)$ [Conditional law]
$\therefore$ (IV) is correct.
$\therefore$ I and IV are the correct equivalences.

Answer: (D)

40. **[MHT-CET 2024]** The statement $\sim(p \leftrightarrow \sim q)$ is
(A) equivalent to $p \leftrightarrow q$ (B) a fallacy (C) a tautology (D) equivalent to $\sim p \leftrightarrow q$

Solution:

I	II	III	IV	V	VI	VII	VIII
p	q	$\sim p$	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$	$\sim p \leftrightarrow q$
T	T	F	F	F	T	T	F
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	F

Column VI has both T and F, so it is neither a fallacy nor a tautology.
Columns VI and VII are identical, so $\sim(p \leftrightarrow \sim q) \equiv p \leftrightarrow q$ (column VIII is different).

Answer: (A)

SECTION C : QUANTIFIERS AND DUALITY

41. **[Board Solved]** If $A = \{1,2,3,4,5,6,7\}$, determine the truth values of:
i) $\exists\, x \in A$ such that $x - 4 = 3$
ii) $\forall\, x \in A,\ x + 1 > 3$

Solution:
i) For $x = 7,\ x - 4 = 7 - 4 = 3$.
$\therefore\ x = 7 \in A$ satisfies $x - 4 = 3$. [One example is enough for $\exists$]
$\therefore$ The given statement is true, its truth value is T.
ii) For $x = 1,\ x + 1 = 1 + 1 = 2$, which is not greater than 3.
$\therefore\ x = 1 \in A$ does not satisfy $x + 1 > 3$. [One counter-example is enough for $\forall$]
$\therefore$ The truth value of the given statement is F.

Answer: i) T ii) F

42. **[Ex 1.3]** If $A = \{3,5,7,9,11,12\}$, determine the truth values of:
i) $\forall\, x \in A,\ x^2 + x$ is an even number
ii) $\exists\, x \in A$ such that $x^2 < 0$
iii) $\exists\, x \in A$ such that $3x + 8 > 40$
iv) $\forall\, x \in A,\ 2x + 9 > 14$

Solution:
i) $x^2 + x = x(x+1)$ is a product of two consecutive integers, so it is always even.
Check: $3^2 + 3 = 12,\ 5^2 + 5 = 30,\ 7^2 + 7 = 56,\ 9^2 + 9 = 90,\ 11^2 + 11 = 132,\ 12^2 + 12 = 156$ — all even.
$\therefore$ Every $x \in A$ satisfies the condition. $\therefore$ The truth value is T.
ii) The square of a real number is never negative, i.e. $x^2 \geq 0$ for every $x \in A$.
$\therefore$ No $x \in A$ satisfies $x^2 < 0$. $\therefore$ The truth value is F.
iii) For $x = 11,\ 3x + 8 = 33 + 8 = 41 > 40$.
$\therefore\ x = 11 \in A$ satisfies $3x + 8 > 40$. $\therefore$ The truth value is T.
iv) $2x + 9 > 14 \iff 2x > 5 \iff x > 2.5$. The smallest element of A is 3 and $2(3) + 9 = 15 > 14$.
$\therefore$ Every $x \in A$ (each $x \geq 3$) satisfies $2x + 9 > 14$. $\therefore$ The truth value is T.

Answer: i) T ii) F iii) T iv) T

43. **[MHT-CET 2026]** If $A = \{1, 2, 3, 4, 5, 6\}$, then which of the following is *not* true?
(A) $\exists\, x \in A$ such that $x + 3 = 9$ (B) $\exists\, x \in A$ such that $x + 2 < 9$
(C) $\forall\, x \in A,\ x + 6 \geq 10$ (D) $\exists\, x \in A$ such that $x + 6 < 10$

Solution:
(A) For $x = 6$, $x + 3 = 9$. $\therefore$ T.
(B) For $x = 1$, $x + 2 = 3 < 9$. $\therefore$ T.
(C) For $x = 1$, $x + 6 = 7$, which is not ≥ 10. One counter-example makes a $\forall$ statement false. $\therefore$ F.
(D) For $x = 1$, $x + 6 = 7 < 10$. $\therefore$ T.
$\therefore$ The statement which is not true is (C).

Answer: (C)

44. **[Board Solved]** Write the duals of:
i) $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$
ii) $(p \wedge t) \vee (c \wedge \sim q)$

Solution: To get the dual, interchange $\wedge \leftrightarrow \vee$ and $t \leftrightarrow c$; the letters and $\sim$ stay as they are.
i) Given: $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$
Dual: $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$ $[\wedge \leftrightarrow \vee]$
ii) Given: $(p \wedge t) \vee (c \wedge \sim q)$
Dual: $(p \vee c) \wedge (t \vee \sim q)$ $[\wedge \leftrightarrow \vee,\ t \leftrightarrow c]$

Answer: i) $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$ ii) $(p \vee c) \wedge (t \vee \sim q)$

45. **[Ex 1.3]** Write the duals of:
i) $(p \vee q) \wedge (r \vee s)$
ii) $(\sim p \vee q) \wedge (\sim r \wedge s)$
iii) $c \vee \{p \wedge (q \vee r)\}$

Solution: Interchange $\wedge \leftrightarrow \vee$ and $t \leftrightarrow c$; keep the letters and $\sim$ unchanged.
i) Given: $(p \vee q) \wedge (r \vee s)$
Dual: $(p \wedge q) \vee (r \wedge s)$ $[\wedge \leftrightarrow \vee]$
ii) Given: $(\sim p \vee q) \wedge (\sim r \wedge s)$
Dual: $(\sim p \wedge q) \vee (\sim r \vee s)$ $[\wedge \leftrightarrow \vee]$
iii) Given: $c \vee \{p \wedge (q \vee r)\}$
Dual: $t \wedge \{p \vee (q \wedge r)\}$ $[\wedge \leftrightarrow \vee,\ c \to t]$

Answer: i) $(p \wedge q) \vee (r \wedge s)$ ii) $(\sim p \wedge q) \vee (\sim r \vee s)$ iii) $t \wedge \{p \vee (q \wedge r)\}$

46. **[MHT-CET 2026]** p, q, r have truth values T, F, F respectively. The truth values of $\sim(p \wedge \sim q) \vee (q \vee \sim r)$ and its dual are respectively
(A) T, T (B) T, F (C) F, T (D) F, F

Solution: $p \equiv$ T, $q \equiv$ F, $r \equiv$ F, so $\sim q \equiv$ T, $\sim r \equiv$ T.
Given statement: $\sim(p \wedge \sim q) \vee (q \vee \sim r) \equiv \sim(\text{T} \wedge \text{T}) \vee (\text{F} \vee \text{T}) \equiv \sim\text{T} \vee \text{T} \equiv \text{F} \vee \text{T} \equiv$ T.
Its dual (interchange $\wedge \leftrightarrow \vee$; $\sim$ stays): $\sim(p \vee \sim q) \wedge (q \wedge \sim r)$.
$\sim(p \vee \sim q) \wedge (q \wedge \sim r) \equiv \sim(\text{T} \vee \text{T}) \wedge (\text{F} \wedge \text{T}) \equiv \sim\text{T} \wedge \text{F} \equiv \text{F} \wedge \text{F} \equiv$ F.
$\therefore$ The truth values are T, F respectively.

Answer: (B)

47. **[Board Solved]** Write the negations of:
i) All natural numbers are rational.
ii) $\exists\, n \in \mathbb{N}$ such that $n + 8 > 11$.

Solution: Negation rule: change "all" $\leftrightarrow$ "some" ($\forall \leftrightarrow \exists$) and negate the condition.
i) Given: All natural numbers are rational. [All A are B]
Negation: Some natural numbers are not rational. [Some A are not B]
ii) Given: $\exists\, n \in \mathbb{N}$ such that $n + 8 > 11$.
Negation: $\forall\, n \in \mathbb{N},\ n + 8 \leq 11$. [$\exists \to \forall,\ > \to \leq$]

Answer: i) Some natural numbers are not rational. ii) $\forall\, n \in \mathbb{N},\ n + 8 \leq 11$.

48. **[Misc Ex 1]** Write the negations of:
i) $\forall\, n \in A,\ n + 7 > 6$, where $A = \{1, 2, \ldots, 9\}$.
ii) Some triangles are equilateral triangles.

Solution: Negation rule: change $\forall \leftrightarrow \exists$ and negate the condition.
i) Given: $\forall\, n \in A,\ n + 7 > 6$.
Negation: $\exists\, n \in A$ such that $n + 7 \leq 6$. [$\forall \to \exists,\ > \to \leq$]
ii) Given: Some triangles are equilateral triangles. [Some A are B]
Negation: No triangle is an equilateral triangle. [No A is B]

Answer: i) $\exists\, n \in A$ such that $n + 7 \leq 6$. ii) No triangle is an equilateral triangle.

49. **[Misc Ex 1]** If $A = \{1, 2, \ldots, 9\}$, find the truth values of:
i) $\forall\, x \in A,\ x + 5 < 12$
ii) $\forall\, x \in A,\ 3x \leq 25$.

Solution:
i) For $x = 7,\ x + 5 = 7 + 5 = 12$, which is not less than 12.
$\therefore\ x = 7 \in A$ does not satisfy $x + 5 < 12$. [One counter-example is enough for $\forall$]
$\therefore$ The truth value of the given statement is F.
ii) For $x = 9,\ 3x = 27$, which is not ≤ 25.
$\therefore\ x = 9 \in A$ does not satisfy $3x \leq 25$.
$\therefore$ The truth value of the given statement is F.

Answer: i) F ii) F

50. **[MHT-CET 2026]** Which of the following is/are False?
S_1: $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is divisible by 4.
S_2: $\exists\, x \in \mathbb{N}$ such that $x - 17 < 20$.
S_3: $\forall\, x \in \mathbb{N},\ x^2 + 3x - 10 = 0$.
S_4: $\forall\, n \in \mathbb{N},\ n^2 \geq 1$.
(A) S_1, S_2 (B) S_1, S_3 (C) Only S_3 (D) S_2, S_4

Solution:
S_1: For $n = 1$, $n^2 + n + 2 = 1 + 1 + 2 = 4$, which is divisible by 4. $\therefore$ S_1 is T.
S_2: For $x = 1$, $x - 17 = -16 < 20$. $\therefore$ S_2 is T.
S_3: For $x = 1$, $x^2 + 3x - 10 = 1 + 3 - 10 = -6 \neq 0$. One counter-example. $\therefore$ S_3 is F.
S_4: Every natural number $n \geq 1$, so $n^2 \geq 1$. $\therefore$ S_4 is T.
$\therefore$ Only S_3 is false.

Answer: (C)

51. **[MHT-CET 2025]** The negation of "For all $M > 0$, there exists $x \in S$ such that $x \geq M$" is
(A) $\exists\, M > 0$ such that $x \geq M$ for all $x \in S$
(B) $\exists\, M > 0,\ \exists\, x \in S$ such that $x \geq M$
(C) $\exists\, M > 0$ such that $x < M$ for all $x \in S$
(D) $\exists\, M > 0$, there exists $x \in S$ such that $x < M$

Solution:
Given: $\forall M > 0$, $\exists x \in S$ such that $x \geq M$.
Negation: change *every* quantifier ($\forall \to \exists$, $\exists \to \forall$) and negate the condition ($\geq \to <$).
$\therefore$ Negation: $\exists M > 0$ such that $\forall x \in S$, $x < M$,
i.e. $\exists M > 0$ such that $x < M$ for all $x \in S$.

Answer: (C)

52. **[MHT-CET 2026]** The dual of $(p \wedge \sim q) \to (q \wedge \sim p)$ is equivalent to
(A) $\sim(p \to q) \wedge (q \to p)$ (B) $(p \to q) \wedge \sim(q \to p)$
(C) $(\sim p \to q) \wedge (q \to p)$ (D) $(q \to p) \vee (\sim p \to \sim q)$

Solution: A dual is taken only with $\wedge, \vee$, so first remove the $\to$.
$(p \wedge \sim q) \to (q \wedge \sim p) \equiv \sim(p \wedge \sim q) \vee (q \wedge \sim p)$ $[p \to q \equiv \sim p \vee q]$
Dual: $\sim(p \vee \sim q) \wedge (q \vee \sim p)$ $[\wedge \leftrightarrow \vee]$
Now $p \vee \sim q \equiv \sim q \vee p \equiv q \to p$, so $\sim(p \vee \sim q) \equiv \sim(q \to p)$.
And $q \vee \sim p \equiv \sim p \vee q \equiv p \to q$.
$\therefore$ Dual $\equiv \sim(q \to p) \wedge (p \to q) \equiv (p \to q) \wedge \sim(q \to p)$ [Commutative law]

Answer: (B)

53. **[MHT-CET 2026]** The dual of the converse of the inverse of $p \to (q \to r)$ is equivalent to
(A) $\sim[p \vee (r \to q)]$ (B) $p \vee (r \to q)$
(C) $\sim[p \vee (q \to r)]$ (D) $p \vee (q \to r)$

Solution: Statement: $p \to (q \to r)$, of the form $A \to B$ with $A = p$, $B = q \to r$.
Inverse ($\sim A \to \sim B$): $\sim p \to \sim(q \to r)$.
Converse of the inverse: $\sim(q \to r) \to \sim p$ (this is the contrapositive).
Remove the arrows before taking the dual:
$\sim(q \to r) \to \sim p \equiv \sim[\sim(q \to r)] \vee \sim p$ $[p \to q \equiv \sim p \vee q]$
$\equiv (q \to r) \vee \sim p$ [Double negation]
$\equiv (\sim q \vee r) \vee \sim p \equiv \sim p \vee \sim q \vee r$ $[p \to q \equiv \sim p \vee q$, Commutative law]
Dual: $\sim p \wedge \sim q \wedge r$ $[\wedge \leftrightarrow \vee]$
Check (A): $\sim[p \vee (r \to q)] \equiv \sim p \wedge \sim(r \to q)$ [De Morgan's law]
$\equiv \sim p \wedge (r \wedge \sim q) \equiv \sim p \wedge \sim q \wedge r$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\therefore$ The dual is equivalent to $\sim[p \vee (r \to q)]$.

Answer: (A)

SECTION D : NEGATION & CONVERSE, INVERSE, CONTRAPOSITIVE

54. **[Board Solved]** Write the negations of:
i) $7 > 3$ and $4 > 11$.
ii) The number is an even number if and only if it is divisible by 2.

Solution:
i) Let $p : 7 > 3$, $q : 4 > 11$.
The given statement in symbolic form is $p \wedge q$.
Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$. [De Morgan's law]
$\therefore$ The negation is: $7 \leq 3$ or $4 \leq 11$.
ii) Let p : The number is an even number, q : The number is divisible by 2.
The given statement in symbolic form is $p \leftrightarrow q$.
Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]
$\therefore$ The negation is: The number is an even number but not divisible by 2, or the number is divisible by 2 but not an even number.

Answer: i) $7 \leq 3$ or $4 \leq 11$. ii) The number is even but not divisible by 2, or it is divisible by 2 but not even.

55. **[Ex 1.3]** Write the negations of the following.
i) $x + 8 > 11$ or $y - 3 = 6$.
ii) Quadrilateral is a square if and only if it is a rhombus.
iii) If it is raining then we will go and play football.
iv) $\forall\, n \in \mathbb{N},\ n^2 + n + 2$ is divisible by 4.

Solution:
i) Let $p : x + 8 > 11, \quad q : y - 3 = 6$.
The given statement in symbolic form is $p \vee q$.
Its negation is $\sim(p \vee q) \equiv \sim p \wedge \sim q$. [De Morgan's law]
$\therefore$ The negation is: $x + 8 \leq 11$ and $y - 3 \neq 6$.
ii) Let p : Quadrilateral is a square, $\quad q$: Quadrilateral is a rhombus.
The given statement in symbolic form is $p \leftrightarrow q$.
Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]
$\therefore$ The negation is: Quadrilateral is a square but not a rhombus, or quadrilateral is a rhombus but not a square.
iii) Let p : It is raining, $\quad q$: We will go, $\quad r$: We will play football.
The given statement in symbolic form is $p \to (q \wedge r)$.
Its negation is $\sim[p \to (q \wedge r)] \equiv p \wedge \sim(q \wedge r)$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\equiv p \wedge (\sim q \vee \sim r)$ [De Morgan's law]
$\therefore$ The negation is: It is raining and (we will not go or we will not play football).
iv) Given: $\forall\, n \in \mathbb{N},\ n^2 + n + 2$ is divisible by 4.
Negation: $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is not divisible by 4. [$\forall \to \exists$, negate the condition]

Answer: i) $x + 8 \leq 11$ and $y - 3 \neq 6$. ii) Quadrilateral is a square but not a rhombus, or it is a rhombus but not a square. iii) It is raining and (we will not go or we will not play football). iv) $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is not divisible by 4.

56. **[MHT-CET 2024]** Negation of "The payment will be made if and only if the work is finished in time." is
(A) The work is finished in time and the payment is not made.
(B) The payment is made and the work is not finished in time.
(C) The work is finished in time and the payment is not made, or the payment is made and the work is finished in time.
(D) Either the work is finished in time and the payment is not made, or the payment is made and the work is not finished in time.

Solution:
Let p : The payment will be made, $\quad q$: The work is finished in time.
The given statement in symbolic form is $p \leftrightarrow q$.
Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]
$\therefore$ The negation is: The payment is made and the work is not finished in time, or the work is finished in time and the payment is not made.
This is option (D) (the two parts written in the other order). (A) and (B) give only one part each; (C) has the second part wrong.

Answer: (D)

57. **[Board Solved]** Write the converse, inverse and contrapositive of: "If it rains then the match will be cancelled."

Solution:
Let p : It rains, $\quad q$: The match will be cancelled.
$\therefore$ The given statement in symbolic form is $p \to q$.
i) Converse $q \to p$: If the match is cancelled then it rains.
ii) Inverse $\sim p \to \sim q$: If it does not rain then the match will not be cancelled.
iii) Contrapositive $\sim q \to \sim p$: If the match is not cancelled then it does not rain.

Answer: Converse: If the match is cancelled then it rains. Inverse: If it does not rain then the match will not be cancelled. Contrapositive: If the match is not cancelled then it does not rain.

58. **[Ex 1.3]** Write the converse, inverse and contrapositive of:
i) If $x < y$ then $x^2 < y^2$ $(x, y \in \mathbb{R})$.
ii) A family becomes literate if the woman in it is literate.

Solution:
i) Let $p : x < y$, $q : x^2 < y^2$. The given statement in symbolic form is $p \to q$.
Converse $q \to p$: If $x^2 < y^2$ then $x < y$.
Inverse $\sim p \to \sim q$: If $x \geq y$ then $x^2 \geq y^2$. [$\sim(x < y)$ is $x \geq y$]
Contrapositive $\sim q \to \sim p$: If $x^2 \geq y^2$ then $x \geq y$.
ii) "q if p" means $p \to q$.
Let p : The woman in the family is literate, q : The family becomes literate.
The given statement in symbolic form is $p \to q$.
Converse $q \to p$: If a family becomes literate then the woman in it is literate.
Inverse $\sim p \to \sim q$: If the woman in a family is not literate then the family does not become literate.
Contrapositive $\sim q \to \sim p$: If a family does not become literate then the woman in it is not literate.

Answer: i) C: If $x^2 < y^2$ then $x < y$; I: If $x \geq y$ then $x^2 \geq y^2$; CP: If $x^2 \geq y^2$ then $x \geq y$. ii) C: If a family becomes literate then the woman in it is literate; I: If the woman is not literate then the family does not become literate; CP: If a family does not become literate then the woman in it is not literate.

59. **[MHT-CET 2024]** Contrapositive of "If two numbers are not equal, then their squares are not equal" is
(A) If the squares of two numbers are not equal, then the numbers are equal.
(B) If the squares of two numbers are equal, then the numbers are not equal.
(C) If the squares of two numbers are equal, then the numbers are equal.
(D) If the squares of two numbers are not equal, then the numbers are not equal.

Solution:
Let p : Two numbers are not equal, q : Their squares are not equal.
The given statement in symbolic form is $p \to q$.
Contrapositive $\sim q \to \sim p$, where $\sim q$: The squares are equal, $\sim p$: The numbers are equal.
$\therefore$ Contrapositive: If the squares of two numbers are equal, then the numbers are equal.

Answer: (C)

60. **[MHT-CET 2024]** The negation of the contrapositive of $p \to (\sim q \wedge r)$ is
(A) $(\sim q \vee \sim r) \wedge \sim p$ (B) $(q \vee \sim r) \wedge p$
(C) $(q \wedge \sim r) \vee p$ (D) $(\sim q \wedge \sim r) \vee \sim p$

Solution:
The contrapositive is equivalent to the statement, so the negation of the contrapositive = negation of the statement.
$\sim[p \to (\sim q \wedge r)] \equiv p \wedge \sim(\sim q \wedge r)$ [$\sim(p \to q) \equiv p \wedge \sim q$]
$\equiv p \wedge (q \vee \sim r)$ [De Morgan's law, Double negation]
$\equiv (q \vee \sim r) \wedge p$ [Commutative law]

Answer: (B)

61. **[Ex 1.3]** Write the converse, inverse and contrapositive of: "If voltage increases then current decreases."

Solution:
Let p : Voltage increases, q : Current decreases.
$\therefore$ The given statement in symbolic form is $p \to q$.
i) Converse $q \to p$: If current decreases then voltage increases.
ii) Inverse $\sim p \to \sim q$: If voltage does not increase then current does not decrease.
iii) Contrapositive $\sim q \to \sim p$: If current does not decrease then voltage does not increase.

Answer: Converse: If current decreases then voltage increases. Inverse: If voltage does not increase then current does not decrease. Contrapositive: If current does not decrease then voltage does not increase.

62. **[MHT-CET 2026]** Negation of "If an integer is greater than 4 and less than 5, then it is a multiple of 3" is
(A) An integer is not greater than 4 but less than 5 and it is a multiple of 3.
(B) If an integer is not greater than 4 and less than 5 then it is not a multiple of 3.
(C) An integer is greater than 4 and less than 5 but it is not a multiple of 3.
(D) An integer is not greater than 4 and not less than 5 but it is not a multiple of 3.

Solution:
Let p : An integer is greater than 4, q : It is less than 5, r : It is a multiple of 3.
The given statement in symbolic form is $(p \wedge q) \to r$.
Its negation is $\sim[(p \wedge q) \to r] \equiv (p \wedge q) \wedge \sim r$. $[\sim(p \to q) \equiv p \wedge \sim q]$
$\therefore$ The negation is: An integer is greater than 4 and less than 5 but it is not a multiple of 3.
(The negation of an if–then statement is never an if–then statement, so (B) is ruled out.)

Answer: (C)

63. **[MHT-CET 2026]** The negation of $(p \wedge q) \to ((p \vee r) \to \sim q)$ is equivalent to
(A) $p \wedge q$ (B) $p \wedge \sim r$ (C) $q \wedge (p \vee r)$ (D) $\sim p \wedge \sim q$

Solution:
$\sim[(p \wedge q) \to ((p \vee r) \to \sim q)] \equiv (p \wedge q) \wedge \sim[(p \vee r) \to \sim q]$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\equiv (p \wedge q) \wedge [(p \vee r) \wedge \sim(\sim q)]$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\equiv (p \wedge q) \wedge [(p \vee r) \wedge q]$ [Double negation]
$\equiv [(p \wedge q) \wedge q] \wedge (p \vee r)$ [Commutative, Associative laws]
$\equiv (p \wedge q) \wedge (p \vee r)$ [Idempotent law]
$\equiv q \wedge [p \wedge (p \vee r)]$ [Commutative, Associative laws]
$\equiv q \wedge p \equiv p \wedge q$ [Absorption law, Commutative law]

Answer: (A)

64. **[MHT-CET 2025]** The negation of $(p \wedge \sim q) \to (p \vee \sim q)$ is
(A) a tautology (B) a contingency (C) a contradiction (D) equivalent to $p \wedge q$

Solution:
$\sim[(p \wedge \sim q) \to (p \vee \sim q)] \equiv (p \wedge \sim q) \wedge \sim(p \vee \sim q)$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\equiv (p \wedge \sim q) \wedge (\sim p \wedge q)$ [De Morgan's law, Double negation]
$\equiv (p \wedge \sim p) \wedge (\sim q \wedge q)$ [Commutative, Associative laws]
$\equiv c \wedge c \equiv c$ [Complement law, Idempotent law]
$\therefore$ The negation is always F, i.e. it is a contradiction.
(Also: $p \wedge \sim q$ T forces p T, so $p \vee \sim q$ is T; the conditional is a tautology and its negation is a contradiction.)

Answer: (C)

65. **[MHT-CET 2026]** p: If voltage increases then current decreases.
q: If voltage does not increase then current does not decrease.
r: If current decreases then voltage increases.
s: If current does not decrease then voltage does not increase.
Which pairs have the same meaning?
(A) $p, q; r, s$ (B) $p, r; q, s$ (C) $p, s; q, r$ (D) no pair

Solution:
Let a : Voltage increases, b : Current decreases.
$p \equiv a \to b$ (statement), $q \equiv \sim a \to \sim b$ (inverse),
$r \equiv b \to a$ (converse), $s \equiv \sim b \to \sim a$ (contrapositive).
A statement $\equiv$ its contrapositive, and converse $\equiv$ inverse.
$\therefore$ $p \equiv s$ and $q \equiv r$.

Answer: (C)

66. **[MHT-CET 2025]** The equivalent statement of "If three vertices of a triangle are represented by cube roots of unity, then the triangle is equilateral" is
(A) Three vertices are cube roots of unity and the triangle is not equilateral.
(B) If a triangle is equilateral then its three vertices are cube roots of unity.
(C) If three vertices are not cube roots of unity then the triangle is not equilateral.
(D) If a triangle is not equilateral then its three vertices cannot be cube roots of unity.

Solution:
Let p : Three vertices of a triangle are cube roots of unity, q : The triangle is equilateral.
The given statement in symbolic form is $p \to q$, and $p \to q \equiv \sim q \to \sim p$ (contrapositive).
$\sim q \to \sim p$: If a triangle is not equilateral then its three vertices cannot be cube roots of unity.
(A) is $p \wedge \sim q$ (negation), (B) is $q \to p$ (converse), (C) is $\sim p \to \sim q$ (inverse) — none is equivalent.

Answer: (D)

67. **[MHT-CET 2025]** p : If 7 is an odd number then 7 is divisible by 2.
q : If 7 is a prime number then 7 is an odd number.
If V_1, V_2 are the truth values of the contrapositives of p and q, then $(V_1, V_2) \equiv$
(A) (T, T) (B) (T, F) (C) (F, T) (D) (F, F)

Solution: A statement and its contrapositive have the same truth value.
Contrapositive of p: If 7 is not divisible by 2 then 7 is not an odd number.
"7 is not divisible by 2" is T, "7 is not an odd number" is F. $\therefore$ T $\to$ F $\equiv$ F. $\therefore$ $V_1 = \text{F}$.
Contrapositive of q: If 7 is not an odd number then 7 is not a prime number.
"7 is not an odd number" is F, "7 is not a prime number" is F. $\therefore$ F $\to$ F $\equiv$ T. $\therefore$ $V_2 = \text{T}$.
$\therefore$ $(V_1, V_2) \equiv (\text{F}, \text{T})$.

Answer: (C)

68. **[MHT-CET 2026]** The contrapositive of $\sim q \to p$ is equivalent to
(A) $p \to q$ (B) $p \wedge q$ (C) $p \vee q$ (D) $\sim p \to \sim q$

Solution:
Contrapositive of $\sim q \to p$ is $\sim p \to \sim(\sim q) \equiv \sim p \to q$. [Double negation]
$\sim p \to q \equiv \sim(\sim p) \vee q$ [$p \to q \equiv \sim p \vee q$]
$\equiv p \vee q$ [Double negation]

Answer: (C)

69. **[MHT-CET 2024]** Statement I: If the work is not finished on time, the contractor is in trouble.
Statement II: Either the work is finished on time or the contractor is in trouble.
Then
(A) II is negation of I (B) II is converse of I
(C) I and II are equivalent (D) II is inverse of I

Solution:
Let p : The work is finished on time, q : The contractor is in trouble.
Statement I in symbolic form: $\sim p \to q$. Statement II in symbolic form: $p \vee q$.
$\sim p \to q \equiv \sim(\sim p) \vee q \equiv p \vee q$ [$p \to q \equiv \sim p \vee q$, Double negation]
$\therefore$ Statement I $\equiv$ Statement II, i.e. I and II are equivalent.

Answer: (C)

70. **[MHT-CET 2024]** p : X is an equilateral triangle,
q : X is an isosceles triangle,
$r : q \vee \sim p$.
The equivalent statement of r is
(A) If X is not equilateral, then X is not isosceles.
(B) X is neither isosceles nor equilateral.
(C) X is isosceles but not equilateral.
(D) If X is not isosceles, then X is not equilateral.

Solution:
$r \equiv q \vee \sim p \equiv \sim p \vee q$ [Commutative law]
$\equiv p \to q$ $[p \to q \equiv \sim p \vee q]$
$\equiv \sim q \to \sim p$ [Contrapositive]
$\sim q \to \sim p$: If X is not isosceles, then X is not equilateral.
((A) is $\sim p \to \sim q$, the inverse, which is not equivalent to $p \to q$.)

Answer: (D)

SECTION E : ALGEBRA OF STATEMENTS

71. **[Board Solved]** Write the negation of $(p \vee q) \wedge (q \vee \sim r)$, stating the rules used.

Solution:
$\sim[(p \vee q) \wedge (q \vee \sim r)]$
$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r)$ [De Morgan's law]
$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$ [De Morgan's law]
$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r)$ [Commutative law]
$\equiv \sim q \wedge (\sim p \vee r)$ [Distributive law]

Answer: $\sim[(p \vee q) \wedge (q \vee \sim r)] \equiv \sim q \wedge (\sim p \vee r)$

72. **[Ex 1.4]** Using rules of negation, write the negations with justification:
i) $(p \vee \sim q) \wedge r$
ii) $(p \vee \sim q) \to (p \wedge \sim q)$

Solution:
i) $\sim[(p \vee \sim q) \wedge r]$
$\equiv \sim(p \vee \sim q) \vee \sim r$ [De Morgan's law]
$\equiv [\sim p \wedge \sim(\sim q)] \vee \sim r$ [De Morgan's law]
$\equiv (\sim p \wedge q) \vee \sim r$ [Involution law: $\sim(\sim q) \equiv q$]
ii) $\sim[(p \vee \sim q) \to (p \wedge \sim q)]$
$\equiv (p \vee \sim q) \wedge \sim(p \wedge \sim q)$ $[\sim(p \to q) \equiv p \wedge \sim q]$
$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim(\sim q)]$ [De Morgan's law]
$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ [Involution law]

Answer: i) $(\sim p \wedge q) \vee \sim r$ ii) $(p \vee \sim q) \wedge (\sim p \vee q)$

73. **[Board Solved]** Rewrite without using if…then: "If it is cold, then we wear woollen clothes."

Solution:
Let p : It is cold, q : We wear woollen clothes.
The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]
$\therefore$ The given statement can be written as: "It is not cold or we wear woollen clothes."

Answer: It is not cold or we wear woollen clothes.

74. **[Ex 1.4]** Rewrite without using if…then:
i) If a man is a judge then he is honest.
ii) If $f(2) = 0$ then $f(x)$ is divisible by $(x-2)$.

Solution:
i) Let p : A man is a judge, q : He is honest.
The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]
$\therefore$ The given statement can be written as: "A man is not a judge or he is honest."
ii) Let $p : f(2) = 0$, $q : f(x)$ is divisible by $(x-2)$.
The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]
$\therefore$ The given statement can be written as: "$f(2) \neq 0$ or $f(x)$ is divisible by $(x-2)$."

Answer: i) A man is not a judge or he is honest. ii) $f(2) \neq 0$ or $f(x)$ is divisible by $(x-2)$.

75. **[Board Solved]** Without using truth table, prove that $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$.

Solution:
LHS $= \sim(p \vee q) \vee (\sim p \wedge q)$
$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ [De Morgan's law]
$\equiv \sim p \wedge (\sim q \vee q)$ [Distributive law]
$\equiv \sim p \wedge t$ [Complement law]
$\equiv \sim p$ [Identity law]
$=$ RHS.

Answer: $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (proved).

76. **[Ex 1.4]** Without using truth table, prove that:
i) $(p \vee q) \wedge (p \vee \sim q) \equiv p$
ii) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

Solution:
i) LHS $= (p \vee q) \wedge (p \vee \sim q)$
$\equiv p \vee (q \wedge \sim q)$ [Distributive law]
$\equiv p \vee c$ [Complement law]
$\equiv p$ [Identity law]
$=$ RHS.
ii) LHS $= (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$
$\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q)$ [Distributive law]
$\equiv (t \wedge q) \vee (p \wedge \sim q)$ [Complement law]
$\equiv q \vee (p \wedge \sim q)$ [Identity law]
$\equiv (q \vee p) \wedge (q \vee \sim q)$ [Distributive law]
$\equiv (q \vee p) \wedge t$ [Complement law]
$\equiv q \vee p$ [Identity law]
$\equiv p \vee q$ [Commutative law]
$=$ RHS.

Answer: Both identities proved: i) $\equiv p$ ii) $\equiv p \vee q$.

77. **[MHT-CET 2024]** The statement pattern $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$ is equivalent to
(A) $q \vee r$ (B) $p \vee r$ (C) q (D) p

Solution:
$[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$
$\equiv [p \wedge (q \vee r)] \vee [p \wedge (\sim q \wedge \sim r)]$ [Commutative and Associative laws]
$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim(q \vee r)]$ [De Morgan's law]
$\equiv p \wedge [(q \vee r) \vee \sim(q \vee r)]$ [Distributive law]
$\equiv p \wedge t$ [Complement law]
$\equiv p$ [Identity law]

Answer: (D)

78. **[MHT-CET 2024]** The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to
(A) $p \wedge \sim q$ (B) $p \to \sim q$ (C) $p \vee q$ (D) $q \to p$

Solution:

$\sim p \vee (p \wedge \sim q)$
$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$ [Distributive law]
$\equiv t \wedge (\sim p \vee \sim q)$ [Complement law]
$\equiv \sim p \vee \sim q$ [Identity law]
$\equiv p \to \sim q$ [Conditional law: $p \to q \equiv \sim p \vee q$]

Answer: (B)

79. **[Ex 1.4]** Without using truth table, prove that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.

Solution:

LHS $= p \leftrightarrow q$
$\equiv (p \to q) \wedge (q \to p)$ [Biconditional law]
$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ [Conditional law]
$\equiv [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$ [Distributive law]
$\equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q) \vee (\sim p \wedge p) \vee (q \wedge p)$ [Distributive law]
$\equiv (\sim p \wedge \sim q) \vee c \vee c \vee (q \wedge p)$ [Complement law]
$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$ [Identity law]
$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ [Commutative law]
$=$ RHS.

Answer: $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ (proved).

80. **[MHT-CET 2024]** $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to
(A) $p \wedge q$ (B) $p \vee \sim q$ (C) $p \wedge \sim q$ (D) $\sim p \wedge \sim q$

Solution:

First, $(p \wedge q) \vee (p \vee \sim q)$
$\equiv [(p \wedge q) \vee p] \vee \sim q$ [Associative law]
$\equiv [p \vee (p \wedge q)] \vee \sim q$ [Commutative law]
$\equiv p \vee \sim q$ [Absorption law]
$\therefore$ Given pattern $\equiv (p \vee \sim q) \wedge (\sim p \wedge \sim q)$
$\equiv (p \vee \sim q) \wedge (\sim q \wedge \sim p)$ [Commutative law]
$\equiv [(p \vee \sim q) \wedge \sim q] \wedge \sim p$ [Associative law]
$\equiv [\sim q \wedge (\sim q \vee p)] \wedge \sim p$ [Commutative law]
$\equiv \sim q \wedge \sim p$ [Absorption law]
$\equiv \sim p \wedge \sim q$ [Commutative law]

Answer: (D)

81. **[MHT-CET 2025]** The logically equivalent statement of $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$ is
(A) $\sim p \wedge q$ (B) $\sim p \vee \sim q$ (C) $\sim p \wedge \sim q$ (D) $p \vee q$

Solution:

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$
$\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q)$ [Distributive law]
$\equiv (\sim p \wedge t) \vee (p \wedge \sim q)$ [Complement law]
$\equiv \sim p \vee (p \wedge \sim q)$ [Identity law]
$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$ [Distributive law]
$\equiv t \wedge (\sim p \vee \sim q)$ [Complement law]

$\equiv \sim p \vee \sim q$ [Identity law]

Answer: (B)

82. **[MHT-CET 2026]** $(p \vee q) \wedge [(\sim p \wedge q) \vee (p \wedge \sim q)] \wedge \sim q$ is logically equivalent to
(A) $p \wedge \sim q$ (B) $\sim p \wedge q$ (C) $p \wedge q$ (D) $\sim p \vee \sim q$

Solution:
First, $[(\sim p \wedge q) \vee (p \wedge \sim q)] \wedge \sim q$
$\equiv [(\sim p \wedge q) \wedge \sim q] \vee [(p \wedge \sim q) \wedge \sim q]$ [Distributive law]
$\equiv [\sim p \wedge (q \wedge \sim q)] \vee [p \wedge (\sim q \wedge \sim q)]$ [Associative law]
$\equiv (\sim p \wedge c) \vee [p \wedge (\sim q \wedge \sim q)]$ [Complement law]
$\equiv (\sim p \wedge c) \vee (p \wedge \sim q)$ [Idempotent law]
$\equiv c \vee (p \wedge \sim q)$ [Identity law]
$\equiv p \wedge \sim q$ [Identity law]
$\therefore$ Given pattern $\equiv (p \vee q) \wedge (p \wedge \sim q)$
$\equiv [(p \vee q) \wedge p] \wedge \sim q$ [Associative law]
$\equiv [p \wedge (p \vee q)] \wedge \sim q$ [Commutative law]
$\equiv p \wedge \sim q$ [Absorption law]

Answer: (A)

83. **[MHT-CET 2026]** The statement pattern $(p \vee q) \to \sim r$ is logically equivalent to
(A) $(\sim p \vee \sim q) \vee \sim r$ (B) $(\sim p \wedge \sim q) \wedge \sim r$
(C) $(\sim p \wedge \sim q) \vee \sim r$ (D) $(\sim p \vee \sim q) \wedge \sim r$

Solution:
$(p \vee q) \to \sim r$
$\equiv \sim(p \vee q) \vee \sim r$ [Conditional law: $p \to q \equiv \sim p \vee q$]
$\equiv (\sim p \wedge \sim q) \vee \sim r$ [De Morgan's law]

Answer: (C)

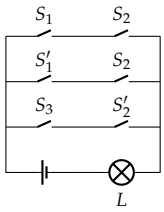
84. **[MHT-CET 2026]** The negation of the contrapositive of $(p \vee \sim q) \to (p \wedge \sim q)$ is
(A) $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$ (B) $(\sim p \wedge q) \vee (p \wedge \sim q)$
(C) $(\sim p \vee \sim q) \wedge (p \vee q)$ (D) $(\sim p \vee q) \wedge (p \vee \sim q)$

Solution:
The contrapositive of $A \to B$ is $\sim B \to \sim A$.
$\therefore$ Contrapositive of the given statement: $\sim(p \wedge \sim q) \to \sim(p \vee \sim q)$.
Its negation:
$\sim[\sim(p \wedge \sim q) \to \sim(p \vee \sim q)]$
$\equiv \sim(p \wedge \sim q) \wedge \sim[\sim(p \vee \sim q)]$ [$\sim(p \to q) \equiv p \wedge \sim q$]
$\equiv \sim(p \wedge \sim q) \wedge (p \vee \sim q)$ [Involution law]
$\equiv [\sim p \vee \sim(\sim q)] \wedge (p \vee \sim q)$ [De Morgan's law]
$\equiv (\sim p \vee q) \wedge (p \vee \sim q)$ [Involution law]

Answer: (D)

SECTION F : SWITCHING CIRCUITS

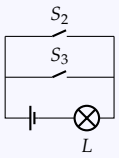
85. **[Board Solved]** For the circuit below: i) write the symbolic form, ii) construct the switching table, iii) simplify the circuit.



Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.
Then $\sim p$: switch S_1' is closed (or S_1 is open), $\sim q$: switch S_2' is closed.
i) The three branches are in parallel and each branch has two switches in series.
$\therefore$ Symbolic form: $X \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$.
ii) Switching table:

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$r \wedge \sim q$	X
1	1	1	0	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	1	0	1
0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	0	1	1
0	0	0	1	1	0	0	0	0

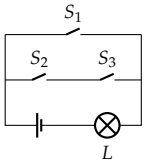
iii) $X \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$
$\equiv [(p \vee \sim p) \wedge q] \vee (r \wedge \sim q)$ [Distributive law]
$\equiv (t \wedge q) \vee (r \wedge \sim q)$ [Complement law]
$\equiv q \vee (r \wedge \sim q)$ [Identity law]
$\equiv (q \vee r) \wedge (q \vee \sim q)$ [Distributive law]
$\equiv (q \vee r) \wedge t$ [Complement law]
$\equiv q \vee r$ [Identity law]
$\therefore$ The simplified circuit has switches S_2 and S_3 in parallel:

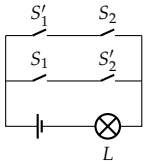


The lamp glows when S_2 or S_3 (or both) is closed.

Answer: $X \equiv q \vee r$: S_2 and S_3 in parallel.

86. **[Ex 1.5]** Express the following circuits in symbolic form and write the input–output table.

i)


ii)


Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.
i) S_2 and S_3 are in series, and this combination is in parallel with S_1.
$\therefore$ Symbolic form: $X \equiv p \vee (q \wedge r)$.
Input–output table:

p	q	r	$q \wedge r$	$X \equiv p \vee (q \wedge r)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

The lamp glows when S_1 is closed, or when S_2 and S_3 are both closed.

ii) Upper branch: S_1' and S_2 in series; lower branch: S_1 and S_2' in series; the branches are in parallel.
$\therefore$ Symbolic form: $X \equiv (\sim p \wedge q) \vee (p \wedge \sim q)$.
Input–output table:

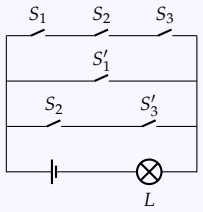
p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \wedge \sim q$	X
1	1	0	0	0	0	0
1	0	0	1	0	1	1
0	1	1	0	1	0	1
0	0	1	1	0	0	0

The lamp glows only when exactly one of S_1, S_2 is closed.

Answer: i) $p \vee (q \wedge r)$; X column: 1,1,1,1,1,0,0,0 ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$; X column: 0,1,1,0

87. **[Board Solved]** Construct the switching circuit of $(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)]$.

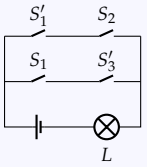
Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.
Then $\sim p$: switch S_1' is closed, $\sim r$: switch S_3' is closed.
$(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)] \equiv (p \wedge q \wedge r) \vee \sim p \vee (q \wedge \sim r)$ [Associative law]
So the circuit has three branches in parallel:
(1) S_1, S_2, S_3 in series; (2) S_1' alone; (3) S_2 and S_3' in series.



Answer: Three parallel branches: $S_1S_2S_3$ in series $\parallel$ S_1' $\parallel$ S_2S_3' in series.

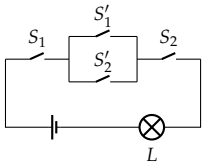
88. **[Ex 1.5]** Construct the switching circuit of $(\sim p \wedge q) \vee (p \wedge \sim r)$.

Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.
Then $\sim p$: switch S_1' is closed, $\sim r$: switch S_3' is closed.
The given statement is the parallel combination ($\vee$) of two branches:
(1) $\sim p \wedge q$: S_1' and S_2 in series; (2) $p \wedge \sim r$: S_1 and S_3' in series.



Answer: Two parallel branches: (S_1', S_2 in series) and (S_1, S_3' in series).

89. **[Textbook Ex]** Simplify the given circuit by writing its logical expression. Also write your conclusion.



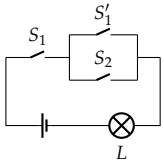
Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed.
Then $\sim p$: switch S_1' is closed, $\sim q$: switch S_2' is closed.
S_1' and S_2' are in parallel, and this combination is in series with S_1 and S_2.
$\therefore$ Symbolic form: $X \equiv p \wedge (\sim p \vee \sim q) \wedge q$.

$X \equiv [p \wedge (\sim p \vee \sim q)] \wedge q$ [Associative law]
$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \wedge q$ [Distributive law]
$\equiv [c \vee (p \wedge \sim q)] \wedge q$ [Complement law]
$\equiv (p \wedge \sim q) \wedge q$ [Identity law]
$\equiv p \wedge (\sim q \wedge q)$ [Associative law]
$\equiv p \wedge c$ [Complement law]
$\equiv c$ [Identity law]

Conclusion: X is a contradiction, so the lamp will never glow, whatever the positions of the switches S_1 and S_2.

Answer: $X \equiv c$: the lamp never glows.

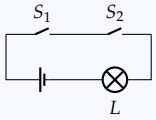
90. **[Ex 1.5]** Give an alternative equivalent simple circuit for:



Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed.
Then $\sim p$: switch S_1' is closed.
S_1' and S_2 are in parallel, and this combination is in series with S_1.
The symbolic form of the given circuit is $p \wedge (\sim p \vee q)$.

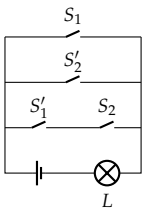
$p \wedge (\sim p \vee q) \equiv (p \wedge \sim p) \vee (p \wedge q)$ [Distributive law]
$\equiv c \vee (p \wedge q)$ [Complement law]
$\equiv p \wedge q$ [Identity law]

$\therefore$ The alternative circuit has S_1 and S_2 in series:



Answer: $p \wedge q$: S_1 and S_2 in series.

91. **[Ex 1.5]** Write the symbolic form of the circuit, construct its switching table and interpret it.



Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed.
Then $\sim p$: switch S_1' is closed, $\sim q$: switch S_2' is closed.
The three branches S_1; S_2'; (S_1' and S_2 in series) are in parallel.
$\therefore$ Symbolic form: $X \equiv p \vee \sim q \vee (\sim p \wedge q)$.
Switching table:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$X \equiv p \vee \sim q \vee (\sim p \wedge q)$
1	1	0	0	0	1
1	0	0	1	0	1
0	1	1	0	1	1
0	0	1	1	0	1

Interpretation: All the entries in the last column are 1. Hence the lamp will always glow, irrespective of the positions of the switches (X is a tautology).
Verification by laws:

$X \equiv p \vee [\sim q \vee (\sim p \wedge q)]$ [Associative law]
$\equiv p \vee [(\sim q \vee \sim p) \wedge (\sim q \vee q)]$ [Distributive law]
$\equiv p \vee [(\sim q \vee \sim p) \wedge t]$ [Complement law]
$\equiv p \vee (\sim q \vee \sim p)$ [Identity law]
$\equiv p \vee (\sim p \vee \sim q)$ [Commutative law]
$\equiv (p \vee \sim p) \vee \sim q$ [Associative law]
$\equiv t \vee \sim q$ [Complement law]
$\equiv t$ [Identity law]

Answer: $X \equiv p \vee \sim q \vee (\sim p \wedge q)$; last column all 1 : the lamp always glows.

92. **[MHT-CET 2026]** In a switching circuit, if the combination $(S_1 \wedge S_2)$ is connected in parallel to the combination $(S_1' \wedge S_2')$, then the room is lit only when
(A) S_1 is ON and S_2 is OFF
(B) S_1 is OFF and S_2 is ON
(C) S_1 and S_2 both ON or S_1 and S_2 both OFF
(D) The room is always lit

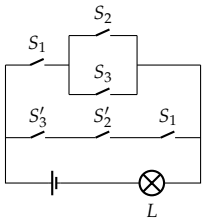
Solution:
Let p : switch S_1 is closed (ON), q : switch S_2 is closed (ON).
Symbolic form: $X \equiv (p \wedge q) \vee (\sim p \wedge \sim q) \equiv p \leftrightarrow q$.

p	q	$p \wedge q$	$\sim p \wedge \sim q$	X
1	1	1	0	1
1	0	0	0	0
0	1	0	0	0
0	0	0	1	1

$X = 1$ only in rows 1 and 4, i.e. when S_1, S_2 are both ON or both OFF.

Answer: (C)

93. **[MHT-CET 2025]** If p, q, r : switches S_1, S_2, S_3 are closed, then the symbolic form of the following circuit is equivalent to



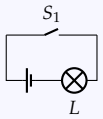
(A) p (B) q (C) $p \wedge q$ (D) $p \vee q$

Solution:
Upper branch: S_1 in series with (S_2 parallel S_3) $\Rightarrow p \wedge (q \vee r)$.
Lower branch: S_3', S_2', S_1 in series $\Rightarrow \sim r \wedge \sim q \wedge p$.
$\therefore X \equiv [p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$
$\equiv [p \wedge (q \vee r)] \vee [p \wedge (\sim q \wedge \sim r)]$ [Commutative and Associative laws]
$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim(q \vee r)]$ [De Morgan's law]
$\equiv p \wedge [(q \vee r) \vee \sim(q \vee r)]$ [Distributive law]
$\equiv p \wedge t$ [Complement law]
$\equiv p$ [Identity law]

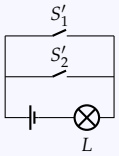
Answer: (A)

94. **[Ex 1.5]** Obtain the simple logical expression and draw the corresponding circuit:
i) $p \vee (q \wedge \sim q)$
ii) $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$

Solution:
Let p : switch S_1 is closed, q : switch S_2 is closed.
Then $\sim p$: switch S_1' is closed, $\sim q$: switch S_2' is closed.
i) $p \vee (q \wedge \sim q)$
$\equiv p \vee c$ [Complement law]
$\equiv p$ [Identity law]
$\therefore$ The circuit has the single switch S_1:

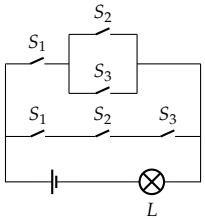


ii) $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$
$\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q)$ [Distributive law]
$\equiv (\sim p \wedge t) \vee (p \wedge \sim q)$ [Complement law]
$\equiv \sim p \vee (p \wedge \sim q)$ [Identity law]
$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$ [Distributive law]
$\equiv t \wedge (\sim p \vee \sim q)$ [Complement law]
$\equiv \sim p \vee \sim q$ [Identity law]
$\therefore$ The circuit has switches S_1' and S_2' in parallel:



Answer: i) p : single switch S_1 ii) $\sim p \vee \sim q$: S_1' and S_2' in parallel.

95. **[MHT-CET 2025]** If p, q, r : switches S_1, S_2, S_3 are closed, then the symbolic form of the following circuit is equivalent to



(A) $p \wedge (q \vee r)$ (B) $q \vee r$ (C) p (D) $\sim q \wedge \sim r$

Solution:
Upper branch: S_1 in series with (S_2 parallel S_3) $\Rightarrow p \wedge (q \vee r)$.
Lower branch: S_1, S_2, S_3 in series $\Rightarrow p \wedge q \wedge r$.

$\therefore X \equiv [p \wedge (q \vee r)] \vee [p \wedge (q \wedge r)]$

$\equiv p \wedge [(q \vee r) \vee (q \wedge r)]$ [Distributive law]

$\equiv p \wedge [(r \vee q) \vee (q \wedge r)]$ [Commutative law]

$\equiv p \wedge [r \vee \{q \vee (q \wedge r)\}]$ [Associative law]

$\equiv p \wedge (r \vee q)$ [Absorption law]

$\equiv p \wedge (q \vee r)$ [Commutative law]

Answer: (A)