A Premier Institute for Pre-Medical & Pre Engineering
SRI VIDYA ARADHANA ACADEMY
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PCMB — PHYSICS CHEMISTRY MATHEMATICS BIOLOGY
NEET/JEE

Topic: Mathematical Logic

Mathematics **Assignment • Board & MHT-CET** Chetan Sir

"A strong finish to the last chapter is the strongest start to your revision."

How to use: each section = **Theory** → **Remember with** → **Class questions** (taught on the slides; the first question of every board type carries a **Board Answer** showing how to write it) → **Practice**. The answer key (with tables and circuits) is at the end.

SECTION A : STATEMENTS & CONNECTIVES

THEORY

Statement

A **statement** is a declarative sentence which is *either true or false, but not both*. Statements are denoted by p, q, $r, \ldots$ and the **truth value** of a statement is T (true) or F (false).

- "Sun rises in the East" (T) "$5 \times 2 = 11$" (F)

These are NOT statements

- Question: *What is your name?*
- Exclamation: *What a beautiful place it is!*
- Command / order: *Open the window.* *Sit down.*
- Request / suggestion: *Please, give your pen.* *Let us go for tea.*
- **Open sentence**: its truth value depends on a variable or on the person: $x + 6 = 9$, "He is tall", "Mathematics is an interesting subject".

Simple and compound statements

Simple: cannot be split further. **Compound**: two or more simple statements joined by connectives.
"3 is prime and 4 is even" is compound; "3 and 5 are twin primes" is simple.

The five logical connectives

Word	Symbol	Name	Hint
and	$p \wedge q$	Conjunction	T only if T $\wedge$ T
or	$p \vee q$	Disjunction	F only if F $\vee$ F
if…then	$p \to q$	Conditional	F only if T $\to$ F
if and only if	$p \leftrightarrow q$	Biconditional	T if both same
not	$\sim p$	Negation	opposite value

In $p \to q$: p = antecedent (hypothesis), q = consequent (conclusion).

THEORY (contd.)

Truth tables of the five connectives (learn by heart)

Conjunction $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Hint: T only when *both* are T.
Remember with: "I finished my homework *and* revised." Sir is happy only if both are done.

Disjunction $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Hint: F only when *both* are F.
Remember with: "I will come by bus *or* by bike." A lie only if I come by neither.

THEORY (contd.)

Conditional $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Hint: F only when p is T and q is F.
Remember with: Mom: "*If* you score 90%, *then* I'll buy you a phone." Broken *only* if you score 90% and get no phone. Score less, and whatever she does, she hasn't lied.

Biconditional $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Hint: T when p, q have the *same* value.
Remember with: "The gate opens *if and only if* you show your ID card." Both happen together or neither.

Negation $\sim p$

p	$\sim p$
T	F
F	T

Hint: opposite truth value; $\sim(\sim p) \equiv p$.
Remember with: "It is raining" → "It is not raining."

THEORY (contd.)

Words → symbols

- "but", "while", "yet" mean **and**. "neither p nor q" = $\sim p \wedge \sim q$.
- "It is not true that p" = $\sim p$. $\sim(\sim p) \equiv p$.

Finding truth values quickly (CET)

- $A \to B$ is F $\Rightarrow$ A is T and B is F. F $\to$ anything = T; anything $\to$ T = T.
- $A \wedge B$ is T $\Rightarrow$ both T. $A \vee B$ is F $\Rightarrow$ both F. $A \leftrightarrow B$ is T $\Rightarrow$ same values.
- Fix every letter you can from these, then fill in the rest.
- Every real number is a complex number ($0 = 0 + 0i$, $3 + \sqrt{5}$ is complex).

CLASS QUESTIONS

1. Which of the following are statements? Write the truth values of the statements. [Board Solved]

i) $6 \times 4 = 25$

ii) $x + 6 = 9$

iii) Please, sit down.

iv) Every real number is a complex number.

BOARD ANSWER — write it like this
i) It is a statement which is false, hence its truth value is F.
ii) It is an open sentence, hence it is not a statement.
iii) It is a request, hence it is not a statement.
iv) It is a statement which is true, hence its truth value is T.

2. State which of the following are statements. Justify. In case of a statement, state its truth value. [Ex 1.1]

i) Zero is a complex number.

ii) $x^2 = x$.

iii) Do you like Mathematics?

iv) $x^2 - 6x - 7 = 0$, when $x = 7$.

v) The sum of cube roots of unity is zero.

3. Express symbolically: "Delhi is in India but Dhaka is not in Sri Lanka." [Board Solved]

BOARD ANSWER — write it like this
Let p : Delhi is in India, q : Dhaka is in Sri Lanka.
"but" means "and". $\therefore$ The symbolic form is $p \wedge \sim q$.

4. Write the following compound statements symbolically. [Ex 1.1]

i) Angle is neither acute nor obtuse.

ii) If $\triangle ABC$ is right angled at B, then $m\angle A + m\angle C = 90^\circ$.

iii) x is not an irrational number but is a square of an integer.

5. Write the truth value of: "Milk is white if and only if the Sun rises in the West." [Board Solved]

BOARD ANSWER — write it like this
Let p : Milk is white, q : The Sun rises in the West.
Truth values of p and q are T and F respectively.
The given statement in symbolic form is $p \leftrightarrow q$.
$\therefore$ $p \leftrightarrow q \equiv \mathrm{T} \leftrightarrow \mathrm{F} \equiv \mathrm{F}$. $\therefore$ Truth value of the given statement is F.

6. Write the truth values of the following. [Ex 1.1]

i) It is not true that $5 - 3i$ is a real number.

ii) If $3 \times 5 = 8$ then $3 + 5 = 15$.

iii) Milk is white if and only if sky is blue.

7. Consider the following three statements:

(A) If $3 + 2 = 7$ then $4 + 3 = 8$.
(B) If $5 + 2 = 7$ then earth is flat.
(C) If both (A) and (B) are true then $5 + 6 = 11$.

Which of the following is correct? [MHT-CET 2025]

(A) (A), (C) true; (B) false (B) (A) true; (B), (C) false
(C) (A) false; (B), (C) true (D) (A) false; (C) true

8. If p, q are true and r, s are false statements, find the truth value of $\sim(p \to q) \leftrightarrow (r \wedge s)$. [Board Solved]

BOARD ANSWER — write it like this
$\sim(p \to q) \leftrightarrow (r \wedge s) \equiv \sim(\mathrm{T} \to \mathrm{T}) \leftrightarrow (\mathrm{F} \wedge \mathrm{F})$
$\equiv \sim\mathrm{T} \leftrightarrow \mathrm{F} \equiv \mathrm{F} \leftrightarrow \mathrm{F} \equiv \mathrm{T}$.
Hence the truth value is T.

9. If p, q are true and r, s are false, find the truth values of: [Ex 1.1]

i) $(p \to q) \vee (r \to s)$

ii) $(\sim r \leftrightarrow p) \to \sim q$

iii) $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

10. If p, q, r have truth values F, T, F respectively, then the truth values of $(p \wedge \sim q) \to r$ and $(p \vee q) \to r$ are respectively [MHT-CET 2024]

(A) T, T (B) T, F (C) F, T (D) F, F

11. Determine the truth values of p and q if $(p \vee q)$ is T and $(p \vee q) \to q$ is F. [Misc Ex 1]

BOARD ANSWER — write it like this
$(p \vee q) \to q$ is F.
$\therefore$ $(p \vee q)$ is T and q is F [$\because$ $a \to b$ is F only when a is T, b is F]
Now $p \vee q$ is T and q is F, $\therefore$ p is T.
Hence p is T and q is F.

12. Determine the truth values of p and q if $(p \vee q)$ is T and $(p \wedge q)$ is T. [Misc Ex 1]

13. If $p \to (q \vee r)$ is false, then the truth values of p, q, r are respectively [MHT-CET 2024]

(A) F, F, F (B) T, T, F (C) T, F, F (D) F, T, T

14. If the statement $p \vee \sim(q \wedge r)$ is false, then the truth values of p, q and r are respectively [MHT-CET 2024]

(A) F, T, F (B) T, F, F (C) F, T, T (D) F, F, T

15. Write the negations of the following. [Ex 1.1]

i) Tirupati is in Andhra Pradesh.

ii) 3 is not a root of the equation $x^2 + 3x - 18 = 0$.

iii) $7 + 3 > 5$.

PRACTICE QUESTIONS

16. Which of the following is/are *not* true? [MHT-CET 2026]

I) If 1 is not a prime number, then 2 is not a prime number.
II) e is a vowel and $12 \times 3 = 36$.
III) It is not true that 14 is a composite number and 3 is an even number.
IV) $\sqrt{5}$ is an irrational number, but $3 + \sqrt{5}$ is a complex number.

(A) Only I and IV (B) Only I
(C) Only II and III (D) Only I, II and IV

17. Which of the following statements has truth value T? [MHT-CET 2025]

A: Cube roots of unity are in G.P. and their sum is 1.
B: $4 + 7 > 10$ iff $2 + 8 < 10$.

C: $\exists\, x \in \mathbb{N}$ such that $x^2 - 3x + 2 = 0$ and $\exists\, n \in \mathbb{N}$ such that n is odd.
D: $3 + i$ is a complex number or $\sqrt{2} + \sqrt{3} = \sqrt{5}$.

(A) only A (B) B, C, D (C) A and C (D) C and D

18. Consider the following statements:

p : If $3^4 > 4^3$, then $3^3 > 4^4$.
q : The roots of $x^2 - 2x + 2 = 0$ are real if and only if Mumbai is in Maharashtra.
r : Statement p is true or statement q is false.

Which of the following has truth value T? [MHT-CET 2026]

(A) $(p \vee q) \wedge r$ (B) $p \vee (q \wedge r)$
(C) $p \wedge (q \vee r)$ (D) $(p \wedge q) \vee r$

19. If the statement pattern $(p \wedge q) \rightarrow (r \vee \sim s)$ is false, then the truth values of p, q, r, s are respectively [MHT-CET 2025]

(A) T, F, T, F (B) T, T, T, F
(C) T, T, F, F (D) T, T, F, T

20. If the truth value of $[(p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)] \rightarrow (p \wedge q)$ is false, then the truth values of $p \rightarrow q$ and $q \rightarrow p$ are respectively [MHT-CET 2026]

(A) T, T (B) T, F (C) F, T (D) F, F

21. If $p \rightarrow (q \vee \sim r)$ is false, then the truth values of $(p \leftrightarrow q) \wedge r$ and $\sim p \rightarrow \sim q$ are [MHT-CET 2026]

(A) T, T (B) T, F (C) F, T (D) F, F

22. If $\sim p \rightarrow q$ is false and $q \leftrightarrow r$ is false, then the truth values of (p, q, r) are [MHT-CET 2026]

(A) T, F, T (B) F, T, F (C) F, F, T (D) F, T, T

23. The truth value of $p \rightarrow r$ is F and of $p \leftrightarrow q$ is F. Then the truth values of $(\sim p \vee q) \rightarrow (p \vee \sim q)$ and $(p \wedge \sim q) \rightarrow (\sim p \wedge q)$ are respectively [MHT-CET 2024]

(A) T, F (B) F, T (C) T, T (D) F, F

24. Consider the following statements:

r : If $p \rightarrow q$ is false then $p \vee q$ is false.
s : If $p \leftrightarrow q$ is false then $p \vee q$ is false.

The truth values of $r \rightarrow s$ and $s \rightarrow r$ are respectively [MHT-CET 2026]

(A) T, T (B) T, F (C) F, T (D) F, F

SECTION B : TRUTH TABLES & TAUTOLOGY

THEORY

Statement pattern

A proper combination of statement letters and connectives, e.g. $p \rightarrow q$, $\sim p \vee q$. The letters $p, q, \ldots$ are its *prime components*.

Truth table of a pattern

Shows the truth value of the pattern for every combination of its components.

- n letters $\Rightarrow 2^n$ rows.
- 2 letters: p: TTFF, q: TFTF.
- 3 letters: p: TTTTFFFF, q: TTFFTTFF, r: TFTFTFTF.
- Board priority of symbols: $\sim$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$ — always use brackets.

Tautology, contradiction, contingency

Last column	Name	Example
all T	Tautology (t)	$p \vee \sim p$
all F	Contradiction (c)	$p \wedge \sim p$
some T, some F	Contingency	$p \wedge q$

THEORY (contd.)

Logical equivalence

$A \equiv B$ if their truth tables are *identical* (same last columns).

Standard tautologies to remember

- $p \rightarrow p$, $p \rightarrow (p \vee q)$, $(p \wedge q) \rightarrow p$, $p \rightarrow (q \rightarrow p)$
- $[p \wedge (p \rightarrow q)] \rightarrow q$, $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
- $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$, $(p \rightarrow q) \vee (q \rightarrow p)$
- **Contradictions:** $(p \wedge q) \wedge (\sim p \vee \sim q)$, $(p \rightarrow q) \wedge (p \wedge \sim q)$

CET shortcut

To test $A \rightarrow B$ for a tautology, try to make A = T and B = F. If that is impossible, $A \rightarrow B$ is a tautology.

REMEMBER WITH

- **Tautology:** "India will win the match *or* India will not win the match." — always true.
- **Contradiction:** "I am awake *and* I am not awake." — always false.
- **Contingency:** "I wake up early *and* reach class on time." — true on some days, false on others.

CLASS QUESTIONS

25. Construct the truth table of $p \rightarrow (q \rightarrow p)$. [Board Solved]

BOARD ANSWER — write it like this

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

26. Construct the truth table for each of the following. [Ex 1.2]

i) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
ii) $[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$

27. Using truth table, prove that $p \wedge q \equiv \sim(p \to \sim q)$. [Board Solved]

BOARD ANSWER — write it like this

I	II	III	IV	V	VI
p	q	$\sim q$	$p \wedge q$	$p \to \sim q$	$\sim(p \to \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

Columns IV and VI are identical. $\therefore\ p \wedge q \equiv \sim(p \to \sim q)$.

28. Using truth table, prove: $p \leftrightarrow q \equiv \sim[(p \vee q) \wedge \sim(p \wedge q)]$. [Ex 1.2]

29. Using truth table, examine whether $(p \wedge q) \wedge (\sim p \vee \sim q)$ is a tautology, contradiction or contingency. [Board Solved]

BOARD ANSWER — write it like this

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$(p \wedge q) \wedge (\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the truth values in the last column are F. Hence it is a contradiction.

30. Examine whether each is a tautology, a contradiction or a contingency. [Ex 1.2]

i) $[(p \to q) \wedge \sim q] \to \sim p$
ii) $(p \leftrightarrow q) \wedge (p \to \sim q)$

31. If p and q are statements, then _____ is a contingency. [MHT-CET 2024]

(A) $p \wedge \sim p$ (B) $p \vee \sim p$
(C) $p \vee q$ (D) $(p \wedge (p \to q)) \to q$

32. Consider the statement patterns

A. $(q \to p) \vee (p \to q)$
B. $(\sim p \vee \sim q) \leftrightarrow \sim(p \wedge q)$
C. $[(p \vee q) \wedge \sim p] \wedge \sim q$
D. $(p \wedge q) \wedge (\sim p \vee \sim q)$

Then [MHT-CET 2026]

(A) A, B are contradictions; C, D are tautologies
(B) A, B are tautologies; C, D are contradictions
(C) A, B, C, D all are tautologies
(D) A, B, C, D all are contradictions

PRACTICE QUESTIONS

33. Using truth table, prove: $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$. [Ex 1.2]

34. Examine whether $[(p \to q) \wedge q] \to p$ is a tautology, contradiction or contingency. [Ex 1.2]

35. The last column in the truth table of $[p \to (q \wedge \sim p)] \vee [(p \vee \sim q) \wedge p]$ is [MHT-CET 2025]

(A) TTTF (B) TFFF (C) TTTT (D) FFTT

36. The statement $(p \to q) \to ((\sim p \to q) \to q)$ is [MHT-CET 2024]

(A) a fallacy (B) equivalent to $\sim p \to q$
(C) equivalent to $p \to \sim q$ (D) a tautology

37. The statement pattern $[(p \wedge q) \to (\sim p \vee r)] \vee [(\sim p \vee r) \to (p \wedge q)]$ is [MHT-CET 2026]

(A) a contradiction (B) a tautology
(C) equivalent to $(p \wedge q) \vee r$ (D) equivalent to $p \vee q$

38. The statement pattern $[(p \to q) \wedge \sim q] \to r$ is a tautology when r is equivalent to [MHT-CET 2025]

(A) $p \wedge \sim q$ (B) $q \vee p$ (C) $p \wedge q$ (D) $\sim q$

39. The correct logical equivalence(s) from the following is/are [MHT-CET 2026]

(I) $p \to (q \to r) \equiv (p \wedge q) \to r$
(II) $(p \to q) \to r \equiv p \to (q \vee r)$
(III) $(p \to q) \to r \equiv (p \to r) \wedge (\sim q \to r)$
(IV) $p \to (q \to r) \equiv q \to (p \to r)$

(A) I, II (B) III, IV (C) II, IV (D) I, IV

40. The statement $\sim(p \leftrightarrow \sim q)$ is [MHT-CET 2024]

(A) equivalent to $p \leftrightarrow q$ (B) a fallacy
(C) a tautology (D) equivalent to $\sim p \leftrightarrow q$

SECTION C : QUANTIFIERS AND DUALITY

THEORY

Quantifiers

Symbol	Read as	True when
$\forall$	for all, for every	*every* object satisfies
$\exists$	there exists, for some	*at least one* satisfies

One counter-example makes a $\forall$ statement F. No example at all makes an $\exists$ statement F.

Negation of a quantified statement

Change $\forall \leftrightarrow \exists$ (all $\leftrightarrow$ some) and negate the condition.

Statement	Negation
All A are B	Some A are not B
Some A are B	No A is B
$\forall\, x,\ x+1>3$	$\exists\, x$ such that $x+1 \le 3$
$\exists\, x$ such that $x-4=3$	$\forall\, x,\ x-4 \neq 3$

Dual

Interchange $\wedge \leftrightarrow \vee$ and $t \leftrightarrow c$. The letters and $\sim$ stay as they are.

- $p \vee q \to p \wedge q$; $t \vee p \to c \wedge p$; $(p \wedge q) \vee r \to (p \vee q) \wedge r$.
- Dual of the dual is the original. If $A \equiv B$ then dual $A \equiv$ dual B.
- **CET trap:** a dual is taken only with $\wedge, \vee$. If $\to$ or $\leftrightarrow$ is present, first write it with $\wedge, \vee, \sim$ ($p \to q \equiv \sim p \vee q$), then take the dual.

REMEMBER WITH

- $\forall$: "*Every* student of our batch has a phone." One student without a phone makes it false.
- $\exists$: "*Some* student of our batch comes by bike." One biker is enough to make it true.
- Negation: "All my friends watch IPL" $\to$ "*Some* friend of mine does *not* watch IPL."

CLASS QUESTIONS

41. If $A = \{1,2,3,4,5,6,7\}$, determine the truth values of: i) $\exists\, x \in A$ such that $x - 4 = 3$ ii) $\forall\, x \in A,\ x + 1 > 3$ [Board Solved]

> **BOARD ANSWER — write it like this**
> i) For $x = 7$, $x - 4 = 7 - 4 = 3$. $\therefore\ x = 7$ satisfies $x - 4 = 3$.
> $\therefore$ The given statement is true, its truth value is T.
> ii) For $x = 1$, $x + 1 = 2$, which is not greater than 3.
> $\therefore$ The truth value of the given statement is F.

42. If $A = \{3,5,7,9,11,12\}$, determine the truth values of: [Ex 1.3]

i) $\forall\, x \in A,\ x^2 + x$ is an even number
ii) $\exists\, x \in A$ such that $x^2 < 0$
iii) $\exists\, x \in A$ such that $3x + 8 > 40$
iv) $\forall\, x \in A,\ 2x + 9 > 14$

43. If $A = \{1,2,3,4,5,6\}$, then which of the following is *not* true? [MHT-CET 2026]

(A) $\exists\, x \in A$ such that $x + 3 = 9$ (B) $\exists\, x \in A$ such that $x + 2 < 9$
(C) $\forall\, x \in A,\ x + 6 \geq 10$ (D) $\exists\, x \in A$ such that $x + 6 < 10$

44. Write the duals of: i) $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$ ii) $(p \wedge t) \vee (c \wedge \sim q)$ [Board Solved]

> **BOARD ANSWER — write it like this**
> i) $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$ ii) $(p \vee c) \wedge (t \vee \sim q)$

45. Write the duals of: [Ex 1.3]

i) $(p \vee q) \wedge (r \vee s)$
ii) $(\sim p \vee q) \wedge (\sim r \wedge s)$
iii) $c \vee \{p \wedge (q \vee r)\}$

46. p, q, r have truth values T, F, F respectively. The truth values of $\sim(p \wedge \sim q) \vee (q \vee \sim r)$ and its dual are respectively [MHT-CET 2026]

(A) T, T (B) T, F (C) F, T (D) F, F

47. Write the negations of: i) All natural numbers are rational. ii) $\exists\, n \in \mathbb{N}$ such that $n + 8 > 11$. [Board Solved]

> **BOARD ANSWER — write it like this**
> i) Some natural numbers are not rational.
> ii) $\forall\, n \in \mathbb{N},\ n + 8 \leq 11$.

48. Write the negations of: i) $\forall\, n \in A,\ n + 7 > 6$, where $A = \{1,2,\ldots,9\}$. ii) Some triangles are equilateral triangles. [Misc Ex 1]

PRACTICE QUESTIONS

49. If $A = \{1,2,\ldots,9\}$, find the truth values of: i) $\forall\, x \in A,\ x + 5 < 12$ ii) $\forall\, x \in A,\ 3x \leq 25$. [Misc Ex 1]

50. Which of the following is/are False? [MHT-CET 2026]

S_1: $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is divisible by 4.
S_2: $\exists\, x \in \mathbb{N}$ such that $x - 17 < 20$.
S_3: $\forall\, x \in \mathbb{N},\ x^2 + 3x - 10 = 0$.
S_4: $\forall\, n \in \mathbb{N},\ n^2 \geq 1$.

(A) S_1, S_2 (B) S_1, S_3 (C) Only S_3 (D) S_2, S_4

51. The negation of "For all $M > 0$, there exists $x \in S$ such that $x \geq M$" is [MHT-CET 2025]

(A) $\exists\, M > 0$ such that $x \geq M$ for all $x \in S$
(B) $\exists\, M > 0$, $\exists\, x \in S$ such that $x \geq M$
(C) $\exists\, M > 0$ such that $x < M$ for all $x \in S$
(D) $\exists\, M > 0$, there exists $x \in S$ such that $x < M$

52. The dual of $(p \wedge \sim q) \rightarrow (q \wedge \sim p)$ is equivalent to [MHT-CET 2026]

(A) $\sim(p \rightarrow q) \wedge (q \rightarrow p)$ (B) $(p \rightarrow q) \wedge \sim(q \rightarrow p)$
(C) $(\sim p \rightarrow q) \wedge (q \rightarrow p)$ (D) $(q \rightarrow p) \vee (\sim p \rightarrow \sim q)$

53. The dual of the converse of the inverse of $p \rightarrow (q \rightarrow r)$ is equivalent to [MHT-CET 2026]

(A) $\sim[p \vee (r \rightarrow q)]$ (B) $p \vee (r \rightarrow q)$
(C) $\sim[p \vee (q \rightarrow r)]$ (D) $p \vee (q \rightarrow r)$

SECTION D : NEGATION & CONVERSE, INVERSE, CONTRAPOSITIVE

THEORY

Negation of compound statements

Statement	Negation	Rule
$p \wedge q$	$\sim p \vee \sim q$	De Morgan
$p \vee q$	$\sim p \wedge \sim q$	De Morgan
$p \rightarrow q$	$p \wedge \sim q$	never an if–then
$p \leftrightarrow q$	$(p \wedge \sim q) \vee (q \wedge \sim p)$	$\equiv p \leftrightarrow \sim q \equiv \sim p \leftrightarrow q$
$\sim p$	p	$\sim(\sim p) \equiv p$

Relations: $= \rightarrow \neq$, $> \rightarrow \leq$, $< \rightarrow \geq$.

THEORY (contd.)

Four faces of $p \rightarrow q$

Name	Form	Equivalent to
Statement	$p \rightarrow q$	contrapositive
Converse	$q \rightarrow p$	inverse
Inverse	$\sim p \rightarrow \sim q$	converse
Contrapositive	$\sim q \rightarrow \sim p$	statement

p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

CET tricks

- Negation of the contrapositive = negation of the statement = $A \wedge \sim B$.
- No arrow in the statement? Rewrite first: $\sim p \vee q \equiv p \rightarrow q$, $p \vee q \equiv \sim p \rightarrow q$.

REMEMBER WITH

- "*If* I study daily, *then* I pass." Contrapositive: "If I did not pass, then I did not study daily" — *same meaning*. Converse: "If I passed, then I studied daily" — *not* the same (maybe the paper was easy!).
- Negation of "I will go to the movie *and* eat popcorn": "I will not go to the movie *or* I will not eat popcorn."
- Negation of "If it rains, I carry an umbrella": "It rains *and* I do not carry an umbrella."

CLASS QUESTIONS

54. Write the negations of: i) $7 > 3$ and $4 > 11$. ii) The number is an even number if and only if it is divisible by 2. [Board Solved]

BOARD ANSWER — write it like this
i) Let $p : 7 > 3$, $q : 4 > 11$. The given statement is $p \wedge q$.
Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$.
$\therefore$ The negation is: $7 \leq 3$ or $4 \leq 11$.
ii) Let p : The number is an even number, q : The number is divisible by 2.
The given statement is $p \leftrightarrow q$. Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$.
$\therefore$ The negation is: "The number is even but not divisible by 2, or the number is divisible by 2 but not even."

55. Write the negations of the following. [Ex 1.3]

i) $x + 8 > 11$ or $y - 3 = 6$.
ii) Quadrilateral is a square if and only if it is a rhombus.
iii) If it is raining then we will go and play football.
iv) $\forall\, n \in \mathbb{N}$, $n^2 + n + 2$ is divisible by 4.

56. Negation of "The payment will be made if and only if the work is finished in time." is [MHT-CET 2024]

(A) The work is finished in time and the payment is not made.

(B) The payment is made and the work is not finished in time.

(C) The work is finished in time and the payment is not made, or the payment is made and the work is finished in time.

(D) Either the work is finished in time and the payment is not made, or the payment is made and the work is not finished in time.

57. Write the converse, inverse and contrapositive of: "If it rains then the match will be cancelled." [Board Solved]

BOARD ANSWER — write it like this
Let p : It rains, q : The match gets cancelled. $\therefore$ Given statement is $p \to q$.
i) Converse $q \to p$: If the match gets cancelled then it rains.
ii) Inverse $\sim p \to \sim q$: If it does not rain then the match will not be cancelled.
iii) Contrapositive $\sim q \to \sim p$: If the match is not cancelled then it does not rain.

58. Write the converse, inverse and contrapositive of: [Ex 1.3]

i) If $x < y$ then $x^2 < y^2$ $(x, y \in \mathbb{R})$.
ii) A family becomes literate if the woman in it is literate.

59. Contrapositive of "If two numbers are not equal, then their squares are not equal" is [MHT-CET 2024]

(A) If the squares of two numbers are not equal, then the numbers are equal.

(B) If the squares of two numbers are equal, then the numbers are not equal.

(C) If the squares of two numbers are equal, then the numbers are equal.

(D) If the squares of two numbers are not equal, then the numbers are not equal.

60. The negation of the contrapositive of $p \to (\sim q \wedge r)$ is [MHT-CET 2024]

(A) $(\sim q \vee \sim r) \wedge \sim p$ (B) $(q \vee \sim r) \wedge p$
(C) $(q \wedge \sim r) \vee p$ (D) $(\sim q \wedge \sim r) \vee \sim p$

PRACTICE QUESTIONS

61. Write the converse, inverse and contrapositive of: "If voltage increases then current decreases." [Ex 1.3]

62. Negation of "If an integer is greater than 4 and less than 5, then it is a multiple of 3" is [MHT-CET 2026]

(A) An integer is not greater than 4 but less than 5 and it is a multiple of 3.

(B) If an integer is not greater than 4 and less than 5 then it is not a multiple of 3.

(C) An integer is greater than 4 and less than 5 but it is not a multiple of 3.

(D) An integer is not greater than 4 and not less than 5 but it is not a multiple of 3.

63. The negation of $(p \wedge q) \to ((p \vee r) \to \sim q)$ is equivalent to [MHT-CET 2026]

(A) $p \wedge q$ (B) $p \wedge \sim r$
(C) $q \wedge (p \vee r)$ (D) $\sim p \wedge \sim q$

64. The negation of $(p \wedge \sim q) \to (p \vee \sim q)$ is [MHT-CET 2025]

(A) a tautology (B) a contingency
(C) a contradiction (D) equivalent to $p \wedge q$

65. Consider the statements:

p: If voltage increases then current decreases.
q: If voltage does not increase then current does not decrease.
r: If current decreases then voltage increases.
s: If current does not decrease then voltage does not increase.

Which pairs have the same meaning? [MHT-CET 2026]

(A) $p, q; r, s$ (B) $p, r; q, s$ (C) $p, s; q, r$ (D) no pair

66. The equivalent statement of "If three vertices of a triangle are represented by cube roots of unity, then the triangle is equilateral" is [MHT-CET 2025]

(A) Three vertices are cube roots of unity and the triangle is not equilateral.

(B) If a triangle is equilateral then its three vertices are cube roots of unity.

(C) If three vertices are not cube roots of unity then the triangle is not equilateral.

(D) If a triangle is not equilateral then its three vertices cannot be cube roots of unity.

67. Consider the statements:

p : If 7 is an odd number then 7 is divisible by 2.
q : If 7 is a prime number then 7 is an odd number.

If V_1, V_2 are the truth values of the contrapositives of p and q, then $(V_1, V_2) \equiv$ [MHT-CET 2025]

(A) (T, T) (B) (T, F) (C) (F, T) (D) (F, F)

68. The contrapositive of $\sim q \rightarrow p$ is equivalent to [MHT-CET 2026]

(A) $p \rightarrow q$ (B) $p \wedge q$
(C) $p \vee q$ (D) $\sim p \rightarrow \sim q$

69. Consider the statements:

I: If the work is not finished on time, the contractor is in trouble.
II: Either the work is finished on time or the contractor is in trouble.

Then [MHT-CET 2024]

(A) II is negation of I (B) II is converse of I
(C) I and II are equivalent (D) II is inverse of I

70. Consider the statements:

p : X is an equilateral triangle,
q : X is an isosceles triangle,
r : $q \vee \sim p$.

The equivalent statement of r is [MHT-CET 2024]

(A) If X is not equilateral, then X is not isosceles.
(B) X is neither isosceles nor equilateral.
(C) X is isosceles but not equilateral.
(D) If X is not isosceles, then X is not equilateral.

SECTION E : ALGEBRA OF STATEMENTS

THEORY

Two important results

- $p \rightarrow q \equiv \sim p \vee q$ (conditional law)
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (biconditional law)
  $\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Algebra of statements

Law	Statement (and its dual)
Idempotent	$p \wedge p \equiv p,\ p \vee p \equiv p$
Commutative	$p \wedge q \equiv q \wedge p,\ p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan	$\sim(p \wedge q) \equiv \sim p \vee \sim q$
	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Identity	$p \wedge t \equiv p,\ p \wedge c \equiv c$
	$p \vee c \equiv p,\ p \vee t \equiv t$
Complement	$p \wedge \sim p \equiv c,\ p \vee \sim p \equiv t$
	$\sim t \equiv c,\ \sim c \equiv t$
Absorption	$p \vee (p \wedge q) \equiv p,\ p \wedge (p \vee q) \equiv p$
Involution	$\sim(\sim p) \equiv p$

Proving "without truth table"

Start from one side, change *one step at a time*, and write the *law used* in brackets at every step — the board gives marks for the law names.

Writing "without if…then"

Use $p \rightarrow q \equiv \sim p \vee q$, i.e. "not p *or* q".

REMEMBER WITH

- **Absorption:** "I'll eat pizza, *or* I'll eat pizza *and* drink cola" — either way, I eat pizza: $p \vee (p \wedge q) \equiv p$.
- **Identity:** "I passed *and* the Sun rises in the east" says nothing more than "I passed": $p \wedge t \equiv p$.
- **Conditional law:** "If it is Sunday, I sleep late" = "It is not Sunday *or* I sleep late."

CLASS QUESTIONS

71. Write the negation of $(p \vee q) \wedge (q \vee \sim r)$, stating the rules used. [Board Solved]

BOARD ANSWER — write it like this

$\sim[(p \vee q) \wedge (q \vee \sim r)]$
$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r)$ [De Morgan's law]
$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r)$ [De Morgan's law]
$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r)$ [Commutative law]
$\equiv \sim q \wedge (\sim p \vee r)$ [Distributive law]

72. Using rules of negation, write the negations with justification: i) $(p \vee \sim q) \wedge r$ ii) $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ [Ex 1.4]

73. Rewrite without using if…then: "If it is cold, then we wear woollen clothes." [Board Solved]

BOARD ANSWER — write it like this

Let p : It is cold, q : We wear woollen clothes.
The given statement is $p \rightarrow q$, but $p \rightarrow q \equiv \sim p \vee q$.
$\therefore$ The given statement can be written as: "It is not cold or we wear woollen clothes."

74. Rewrite without using if…then: i) If a man is a judge then he is honest. ii) If $f(2) = 0$ then $f(x)$ is divisible by $(x - 2)$. [Ex 1.4]

75. Without using truth table, prove that $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$. [Board Solved]

BOARD ANSWER — write it like this

LHS $= \sim(p \vee q) \vee (\sim p \wedge q)$
$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ [De Morgan's law]
$\equiv \sim p \wedge (\sim q \vee q)$ [Distributive law]
$\equiv \sim p \wedge t$ [Complement law]
$\equiv \sim p$ [Identity law] = RHS.

76. Without using truth table, prove that: [Ex 1.4]

i) $(p \vee q) \wedge (p \vee \sim q) \equiv p$
ii) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

77. The statement pattern $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$ is equivalent to [MHT-CET 2024]

(A) $q \vee r$ (B) $p \vee r$ (C) q (D) p

78. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to [MHT-CET 2024]

(A) $p \wedge \sim q$ (B) $p \rightarrow \sim q$ (C) $p \vee q$ (D) $q \rightarrow p$

PRACTICE QUESTIONS

79. Without using truth table, prove that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$. [Ex 1.4]

80. $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to [MHT-CET 2024]

(A) $p \wedge q$ (B) $p \vee \sim q$
(C) $p \wedge \sim q$ (D) $\sim p \wedge \sim q$

81. The logically equivalent statement of $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$ is [MHT-CET 2025]

(A) $\sim p \wedge q$ (B) $\sim p \vee \sim q$
(C) $\sim p \wedge \sim q$ (D) $p \vee q$

82. $(p \vee q) \wedge [(\sim p \wedge q) \vee (p \wedge \sim q)] \wedge \sim q$ is logically equivalent to [MHT-CET 2026]

(A) $p \wedge \sim q$ (B) $\sim p \wedge q$
(C) $p \wedge q$ (D) $\sim p \vee \sim q$

83. The statement pattern $(p \vee q) \rightarrow \sim r$ is logically equivalent to [MHT-CET 2026]

(A) $(\sim p \vee \sim q) \vee \sim r$ (B) $(\sim p \wedge \sim q) \wedge \sim r$
(C) $(\sim p \wedge \sim q) \vee \sim r$ (D) $(\sim p \vee \sim q) \wedge \sim r$

84. The negation of the contrapositive of $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is [MHT-CET 2026]

(A) $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$ (B) $(\sim p \wedge q) \vee (p \wedge \sim q)$
(C) $(\sim p \vee \sim q) \wedge (p \vee q)$ (D) $(\sim p \vee q) \wedge (p \vee \sim q)$

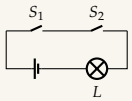
SECTION F : SWITCHING CIRCUITS

THEORY

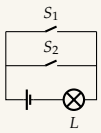
Logic ↔ circuits

A **switch** is a two-state device: closed/ON (current flows) or open/OFF.

In the circuit	In logic
Switch S_1 is closed	p
S_1' (always opposite to S_1)	$\sim p$
Same letter on two switches	same state (equivalent)
Switches in **series**	$\wedge$
Switches in **parallel**	$\vee$
ON / OFF	1 (T) / 0 (F)
Lamp always glows (all 1)	tautology
Lamp never glows (all 0)	contradiction



Series: $p \wedge q$



Parallel: $p \vee q$

THEORY (contd.)

Switching (input–output) table

A truth table written with 1 and 0. The lamp glows in the rows where the expression is 1.

Simplifying a circuit

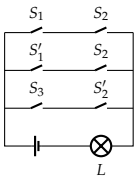
1. Write the symbolic form of the circuit.
2. Simplify it with the laws of Section E.
3. Draw the circuit of the simplified form (minimum switches).

REMEMBER WITH

- **Series** = two doors one after another on the way to class: you get in only if *both* are open ($\wedge$).
- **Parallel** = two gates of the school: *either* gate open lets you in ($\vee$).
- S and S' = two bulbs on a "one ON, other OFF" toggle: whenever one is ON the other is OFF.

CLASS QUESTIONS

85. For the circuit below: i) write the symbolic form, ii) construct the switching table, iii) simplify the circuit. [Board Solved]



BOARD ANSWER — write it like this

Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.

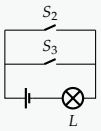
i) Symbolic form: $(p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$.

ii) Switching table:

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$r \wedge \sim q$	X
1	1	1	0	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	1	0	1
0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	0	1	1
0	0	0	1	1	0	0	0	0

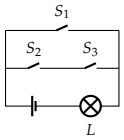
iii) $X \equiv [(p \vee \sim p) \wedge q] \vee (r \wedge \sim q)$ [Distributive law]
$\equiv (t \wedge q) \vee (r \wedge \sim q) \equiv q \vee (r \wedge \sim q)$ [Complement, Identity]
$\equiv (q \vee r) \wedge (q \vee \sim q)$ [Distributive law]
$\equiv (q \vee r) \wedge t \equiv q \vee r$ [Complement, Identity]

Simplified circuit: S_2 and S_3 in parallel.

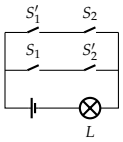


86. Express the following circuits in symbolic form and write the input–output table. [Ex 1.5]

i)



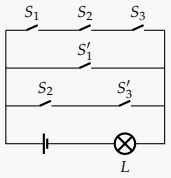
ii)



87. Construct the switching circuit of $(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)]$. [Board Solved]

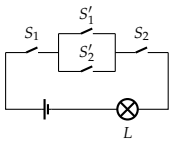
BOARD ANSWER — write it like this

Let p : switch S_1, q : switch S_2, r : switch S_3 (closed). The expression is the parallel combination of three branches: S_1, S_2, S_3 in series; S_1'; S_2, S_3' in series.

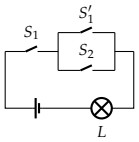


88. Construct the switching circuit of $(\sim p \wedge q) \vee (p \wedge \sim r)$. [Ex 1.5]

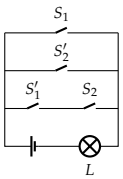
89. Simplify the given circuit by writing its logical expression. Also write your conclusion. [Textbook Ex]



90. Give an alternative equivalent simple circuit for: [Ex 1.5]



91. Write the symbolic form of the circuit, construct its switching table and interpret it. [Ex 1.5]



92. In a switching circuit, if the combination $(S_1 \wedge S_2)$ is connected in parallel to the combination $(S_1' \wedge S_2')$, then the room is lit only when [MHT-CET 2026]

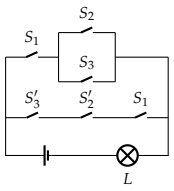
(A) S_1 is ON and S_2 is OFF

(B) S_1 is OFF and S_2 is ON

(C) S_1 and S_2 both ON or S_1 and S_2 both OFF

(D) The room is always lit

93. If p, q, r : switches S_1, S_2, S_3 are closed, then the symbolic form of the following circuit is equivalent to [MHT-CET 2025]

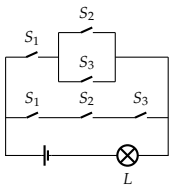


(A) p (B) q (C) $p \wedge q$ (D) $p \vee q$

PRACTICE QUESTIONS

94. Obtain the simple logical expression and draw the corresponding circuit: i) $p \vee (q \wedge \sim q)$ ii) $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$ [Ex 1.5]

95. If p, q, r : switches S_1, S_2, S_3 are closed, then the symbolic form of the following circuit is equivalent to [MHT-CET 2025]



(A) $p \wedge (q \vee r)$ (B) $q \vee r$
(C) p (D) $\sim q \wedge \sim r$

ANSWER KEY

A. Statements & connectives

1, 3, 5, 8, 11. Board Answer given

2. i) statement, T $(0 = 0 + 0i)$
ii) open sentence, not a statement
iii) interrogative, not a statement
iv) statement, T $(49 - 42 - 7 = 0)$
v) statement, T $(1 + \omega + \omega^2 = 0)$

4. i) p: Angle is acute, q: Angle is obtuse; $\sim p \wedge \sim q$
ii) p: $\triangle ABC$ is right angled at B, q: $m\angle A + m\angle C = 90°$; $p \to q$
iii) p: x is an irrational number, q: x is a square of an integer; $\sim p \wedge q$

6. i) p: $5 - 3i$ is a real number (F); $\sim p \equiv$ T
ii) p: $3 \times 5 = 8$ (F), q: $3 + 5 = 15$ (F); $p \to q \equiv$ F$\to$F $\equiv$ T
iii) p: Milk is white (T), q: Sky is blue (T); $p \leftrightarrow q \equiv$ T$\leftrightarrow$T $\equiv$ T

7. (A) **10.** (B)

9. i) $(\mathrm{T} \to \mathrm{T}) \vee (\mathrm{F} \to \mathrm{F}) \equiv \mathrm{T} \vee \mathrm{T} \equiv \mathrm{T}$
ii) $(\mathrm{T} \leftrightarrow \mathrm{T}) \to \mathrm{F} \equiv \mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$
iii) $\sim[(\mathrm{F} \wedge \mathrm{F}) \vee (\mathrm{F} \to \mathrm{F})] \leftrightarrow (\mathrm{T} \wedge \mathrm{F}) \equiv \sim[\mathrm{F} \vee \mathrm{T}] \leftrightarrow \mathrm{F} \equiv \mathrm{F} \leftrightarrow \mathrm{F} \equiv \mathrm{T}$

12. $p \wedge q$ is T $\Rightarrow$ p is T and q is T **13.** (C) **14.** (C)

15. i) Tirupati is not in Andhra Pradesh.
ii) 3 is a root of the equation $x^2 + 3x - 18 = 0$.
iii) $7 + 3 \leq 5$, i.e. $7 + 3 \not> 5$.

16. (B) **17.** (D) **18.** (D) **19.** (D) **20.** (C)
21. (C) **22.** (C) **23.** (A) **24.** (A)

B. Truth tables

25, 27, 29. Board Answer given

26. i) contradiction

p	q	$p \wedge \sim q$	$p \to q$	$\leftrightarrow$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	F
F	F	F	T	F

ii) tautology

p	q	r	$q \to r$	$p \to (q \to r)$	$(p \wedge q) \to r$	$\leftrightarrow$
T	T	T	T	T	T	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

28. $A = (p \vee q) \wedge \sim(p \wedge q)$; $\sim A$ and $p \leftrightarrow q$ identical

p	q	$p \vee q$	$\sim(p \wedge q)$	A	$\sim A$	$p \leftrightarrow q$
T	T	T	F	F	T	T
T	F	T	T	T	F	F
F	T	T	T	T	F	F
F	F	F	T	F	T	T

30. i) tautology

p	q	$p \to q$	$(p \to q) \wedge \sim q$	$\sim p$	X
T	T	T	F	F	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	T	T	T

ii) contingency

p	q	$p \leftrightarrow q$	$p \to \sim q$	X
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	T	T	T

31. (C) **32.** (B)

33. $B = (p \to r) \wedge (q \to r)$; $(p \vee q) \to r$ and B identical

p	q	r	$(p \vee q) \to r$	$p \to r$	$q \to r$	B
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

34. contingency

p	q	$p \to q$	$(p \to q) \wedge q$	X
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

35. (C) **36.** (D) **37.** (B) **38.** (D) **39.** (D) **40.** (A)

C. Quantifiers & duals

41, 44, 47. Board Answer given

42. i) T $(x^2 + x = x(x + 1))$
ii) F
iii) T $(x = 11)$
iv) T

43. (C) **46.** (B)

45. i) $(p \wedge q) \vee (r \wedge s)$

ii) $(\sim p \wedge q) \vee (\sim r \vee s)$
iii) $t \wedge \{p \vee (q \wedge r)\}$

48. i) $\exists\, n \in A$ such that $n + 7 \leq 6$
ii) No triangle is an equilateral triangle.

49. i) F $(x = 7)$
ii) F $(x = 9)$

50. (C) **51.** (C) **52.** (B) **53.** (A)

D. Negation; C/I/C

54, 57. Board Answer given

55. i) (p: $x + 8 > 11$, q: $y - 3 = 6$; $\sim(p \vee q) \equiv \sim p \wedge \sim q$) $x + 8 \leq 11$ and $y - 3 \neq 6$
ii) $(\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p))$ A quadrilateral is a square but not a rhombus, or it is a rhombus but not a square.
iii) $(\sim(p \rightarrow q) \equiv p \wedge \sim q)$ It is raining and (we will not go or we will not play football).
iv) $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is not divisible by 4.

56. (D) **59.** (C) **60.** (B)

58. i) (p: $x < y$, q: $x^2 < y^2$; $p \rightarrow q$)
C: If $x^2 < y^2$ then $x < y$.
I: If $x \geq y$ then $x^2 \geq y^2$.
CP: If $x^2 \geq y^2$ then $x \geq y$.
ii) (p: the woman is literate, q: the family becomes literate; $p \rightarrow q$)
C: If a family becomes literate then the woman in it is literate.
I: If the woman is not literate then the family does not become literate.
CP: If a family does not become literate then the woman in it is not literate.

61. (p: Voltage increases, q: Current decreases; $p \rightarrow q$)
C: If current decreases then voltage increases.
I: If voltage does not increase then current does not decrease.
CP: If current does not decrease then voltage does not increase.

62. (C) **63.** (A) **64.** (C) **65.** (C) **66.** (D)
67. (C) **68.** (C) **69.** (C) **70.** (D)

E. Algebra of statements

71, 73, 75. Board Answer given

72. i) $\sim[(p \vee \sim q) \wedge r] \equiv \sim(p \vee \sim q) \vee \sim r$ [De Morgan] $\equiv (\sim p \wedge q) \vee \sim r$ [De Morgan]
ii) $\sim[(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge \sim(p \wedge \sim q)$ $[\sim(p \rightarrow q) \equiv p \wedge \sim q]$ $\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ [De Morgan]

74. i) p: A man is a judge, q: He is honest; $p \rightarrow q \equiv \sim p \vee q$: "A man is not a judge or he is honest."
ii) p: $f(2) = 0$, q: $f(x)$ is divisible by $(x - 2)$; "$f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$."

76. i) $\equiv p \vee (q \wedge \sim q)$ [Dist.] $\equiv p \vee c$ [Compl.] $\equiv p$ [Iden.]
ii) $\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q)$ [Dist.] $\equiv q \vee (p \wedge \sim q)$ [Compl., Iden.] $\equiv (q \vee p) \wedge (q \vee \sim q)$ [Dist.] $\equiv p \vee q$ [Compl., Iden., Comm.]

77. (D) **78.** (B)

79. $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ [Bicond.] $\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge p) \vee (q \wedge \sim q) \vee (q \wedge p)$ [Dist.] $\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$ [Compl., Iden.]

80. (D) **81.** (B) **82.** (A) **83.** (C) **84.** (D)

F. Switching circuits

p, q, r : switches S_1, S_2, S_3 are closed.

85, 87. Board Answer given

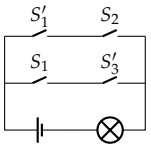
86. i) $p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$

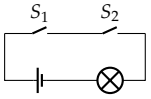
p	q	$\sim p \wedge q$	$p \wedge \sim q$	X
1	1	0	0	0
1	0	0	1	1
0	1	1	0	1
0	0	0	0	0

88. $(\sim p \wedge q) \vee (p \wedge \sim r)$:



89. $p \wedge (\sim p \vee \sim q) \wedge q \equiv (p \wedge \sim q) \wedge q \equiv p \wedge c \equiv c$. The lamp will never glow, whatever the switches.

90. $p \wedge (\sim p \vee q) \equiv (p \wedge \sim p) \vee (p \wedge q) \equiv p \wedge q$:



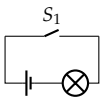
91. $X = p \vee \sim q \vee (\sim p \wedge q) \equiv p \vee [(\sim q \vee \sim p) \wedge (\sim q \vee q)] \equiv p \vee \sim q \vee \sim p \equiv t$

p	q	$\sim q$	$\sim p \wedge q$	X
1	1	0	0	1
1	0	1	0	1
0	1	0	1	1
0	0	1	0	1

All 1: the lamp always glows.

92. (C) **93.** (A) **95.** (A)

94. i) $p \vee c \equiv p$



ii) $\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q) \equiv \sim p \vee (p \wedge \sim q) \equiv \sim p \vee \sim q$

