Mathematical Logic

mathbyiiserite · Class 12 Board & MHT-CET

PART 1

Statements & Logical Connectives

Statement

Statement and truth value

- A **statement** is a declarative sentence which is *either* true *or* false, but not both.
- Statements are denoted by p, q, r, ...
- Its **truth value** is T (true) or F (false).
- *e.g.* "Sun rises in the East" (T)
- *e.g.* "$5 \times 2 = 11$" (F)

These are NOT Statements

Not statements

- Question: *What is your name?*
- Exclamation: *What a beautiful place it is!*
- Command / order: *Open the window.*
- Request / suggestion: *Please, give your pen.*
- **Open sentence** — truth value depends on a variable or on the person:

  $x + 6 = 9$ "He is tall" "Maths is an interesting subject"

Simple and Compound Statements

Simple and compound

- **Simple**: cannot be split further into two or more statements.
- **Compound**: two or more simple statements joined by connectives.
- "3 is prime and 4 is even" $\rightarrow$ compound.
- "3 and 5 are twin primes" $\rightarrow$ simple.

The Five Logical Connectives

Connectives

Word	**Symbol**	**Name**	**Hint**
and	$p \wedge q$	Conjunction	T only when *both* are T
or	$p \vee q$	Disjunction	F only when *both* are F
if…then	$p \to q$	Conditional	F only when p is T and q is F
if and only if	$p \leftrightarrow q$	Biconditional	T when p, q have the *same* value
not	$\sim p$	Negation	opposite value; $\sim(\sim p) \equiv p$

Note

- In $p \to q$: p = antecedent (hypothesis), q = consequent (conclusion).

Truth Tables: Conjunction and Disjunction

Conjunction (and) $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Hint: T only when *both* are T.

Remember with: "I finished my homework *and* revised." Sir is happy only if both are done.

Disjunction (or) $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Hint: F only when *both* are F.

Remember with: "I will come by bus *or* by bike." A lie only if I come by neither.

Truth Tables: Conditional and Biconditional

Conditional (if. . . then) $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Hint: F only when p is T and q is F.

Remember with: Mom: "*If* you score 90%, *then* I'll buy you a phone." Broken *only* if you score 90% and get no phone.

Biconditional (iff) $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Hint: T when p and q have the *same* value.

Remember with: "The gate opens *if and only if* you show your ID card." Both happen together or neither.

Truth Table: Negation

Negation (not) $\sim p$

p	$\sim p$
T	F
F	T

Hint: opposite truth value.

$\sim(\sim p) \equiv p$ (negation of negation is the statement itself)

Remember with: "It is raining" $\rightarrow$ "It is not raining."

Words to Symbols

Translations

- "but", "while", "yet" = **and**
- "neither p nor q" = $\sim p \wedge \sim q$
- "It is not true that p" = $\sim p$
- $\sim(\sim p) \equiv p$

Finding Truth Values Quickly (CET)

Quick rules

- $A \to B$ is F $\Rightarrow$ A is T and B is F
- F $\to$ anything $=$ T
- anything $\to$ T $=$ T
- $A \wedge B$ is T $\Rightarrow$ both T
- $A \vee B$ is F $\Rightarrow$ both F
- $A \leftrightarrow B$ is T $\Rightarrow$ same values
- Fix every letter you can from these, then fill in the rest.

Remember

- Every real number is a complex number: $0 = 0 + 0i$, $3 + \sqrt{5}$ is complex.

Question 1

Board pattern

Which of the following are statements? Write the truth values of the statements.

i) $6 \times 4 = 25$

ii) $x + 6 = 9$

iii) Please, sit down.

iv) Every real number is a complex number.

Answer · Question 1

i) $6 \times 4 = 24 \neq 25$. It is a statement which is false, hence its truth value is F.

ii) Its truth value depends on the value of x. It is an open sentence, hence it is not a statement.

iii) It is a request, hence it is not a statement.

iv) Every real number a can be written as $a + 0i$. It is a statement which is true, hence its truth value is T.

Answer: i) Statement, F ii) Not a statement iii) Not a statement iv) Statement, T

Question 2 Ex 1.1

State which of the following are statements. Justify. In case of a statement, state its truth value.

i) Zero is a complex number.

ii) $x^2 = x$.

iii) Do you like Mathematics?

iv) $x^2 - 6x - 7 = 0$, when $x = 7$.

v) The sum of cube roots of unity is zero.

Answer · Question 2

i) $0 = 0 + 0i$, so zero is a complex number. It is a statement which is true, hence its truth value is T.

ii) Its truth value depends on x (true for $x = 0, 1$ only). It is an open sentence, hence it is not a statement.

iii) It is an interrogative sentence, hence it is not a statement.

iv) For $x = 7$: $x^2 - 6x - 7 = 49 - 42 - 7 = 0$. It is a statement which is true, hence its truth value is T.

v) The cube roots of unity are $1, \omega, \omega^2$ and $1 + \omega + \omega^2 = 0$. It is a statement which is true, hence its truth value is T.

Answer: i) Statement, T ii) Not a statement iii) Not a statement iv) Statement, T v) Statement, T

Question 3

Board pattern

Express symbolically:

"Delhi is in India but Dhaka is not in Sri Lanka."

Answer · Question 3

Let p : Delhi is in India, q : Dhaka is in Sri Lanka.

Then "Dhaka is not in Sri Lanka" is $\sim q$.

"but" means "and". $\therefore$ The symbolic form is $p \wedge \sim q$.

Answer: $p \wedge \sim q$

Question 4 Ex 1.1

Write the following compound statements symbolically.

i) Angle is neither acute nor obtuse.

ii) If $\triangle ABC$ is right angled at B, then $m\angle A + m\angle C = 90^\circ$.

iii) x is not an irrational number but is a square of an integer.

Answer · Question 4

i) Let p : Angle is acute, q : Angle is obtuse.

"neither p nor q" means "not p and not q". $\therefore$ The symbolic form is $\sim p \wedge \sim q$.

ii) Let p : $\triangle ABC$ is right angled at B, q : $m\angle A + m\angle C = 90^\circ$.

"If p then q" $\therefore$ The symbolic form is $p \to q$.

iii) Let p : x is an irrational number, q : x is a square of an integer.

"but" means "and". $\therefore$ The symbolic form is $\sim p \wedge q$.

Answer: i) $\sim p \wedge \sim q$ ii) $p \to q$ iii) $\sim p \wedge q$

Question 5

Board pattern

Write the truth value of:

"Milk is white if and only if the Sun rises in the West."

Answer · Question 5

Let p : Milk is white, q : The Sun rises in the West.

Truth values of p and q are T and F respectively.

The given statement in symbolic form is $p \leftrightarrow q$.

$\therefore\ p \leftrightarrow q \equiv \mathrm{T} \leftrightarrow \mathrm{F} \equiv \mathrm{F}$ [$\leftrightarrow$ is T only when both have the same value]

$\therefore$ Truth value of the given statement is F.

Answer: F

Question 6 Ex 1.1

Write the truth values of the following.

i) It is not true that $5 - 3i$ is a real number.

ii) If $3 \times 5 = 8$ then $3 + 5 = 15$.

iii) Milk is white if and only if sky is blue.

Answer · Question 6

i) Let $p : 5 - 3i$ is a real number. Its imaginary part is $-3 \neq 0$, so the truth value of p is F.

The given statement in symbolic form is $\sim p$.

$\therefore \sim p \equiv \sim \mathrm{F} \equiv \mathrm{T}$. $\therefore$ Truth value of the given statement is T.

ii) Let $p : 3 \times 5 = 8$, $q : 3 + 5 = 15$.

Since $3 \times 5 = 15$ and $3 + 5 = 8$, truth values of p and q are F and F respectively.

The given statement in symbolic form is $p \to q$.

$\therefore p \to q \equiv \mathrm{F} \to \mathrm{F} \equiv \mathrm{T}$ [$\to$ is F only when T $\to$ F]

$\therefore$ Truth value of the given statement is T.

iii) Let p : Milk is white, q : Sky is blue.

Truth values of p and q are T and T respectively.

The given statement in symbolic form is $p \leftrightarrow q$.

$\therefore p \leftrightarrow q \equiv \mathrm{T} \leftrightarrow \mathrm{T} \equiv \mathrm{T}$. $\therefore$ Truth value of the given statement is T.

Answer: i) T ii) T iii) T

Question 7 MHT-CET 2025

Consider the following three statements:

(A) If $3 + 2 = 7$ then $4 + 3 = 8$.

(B) If $5 + 2 = 7$ then earth is flat.

(C) If both (A) and (B) are true then $5 + 6 = 11$.

Which of the following is correct?

(A) (A) and (C) are true while (B) is false

(B) (A) is true while (B) and (C) are false

(C) (A) is false but (B) and (C) are true

(D) (A) is false while (C) is true

Answer · Question 7

Statement (A): $3+2=7$ is F, $4+3=8$ is F. $\mathrm{F} \to \mathrm{F} \equiv \mathrm{T}$. $\therefore$ (A) is true.

Statement (B): $5+2=7$ is T, earth is flat is F. $\mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$. $\therefore$ (B) is false.

Statement (C): its consequent "$5+6=11$" is T, and anything $\to$ T is T.

(Here "(A) and (B) are true" is $\mathrm{T} \wedge \mathrm{F} \equiv \mathrm{F}$, so (C) is $\mathrm{F} \to \mathrm{T} \equiv \mathrm{T}$.) $\therefore$ (C) is true.

$\therefore$ (A), (C) are true and (B) is false.

Answer: (A)

Question 8

Board pattern

If p, q are true and r, s are false statements, find the truth value of

$$\sim(p \to q) \leftrightarrow (r \wedge s)$$

Answer · Question 8

Truth values of p, q, r, s are T, T, F, F respectively.

$$\begin{aligned}\sim(p \to q) \leftrightarrow (r \wedge s) &\equiv \sim(\mathrm{T} \to \mathrm{T}) \leftrightarrow (\mathrm{F} \wedge \mathrm{F}) \\ &\equiv \sim\mathrm{T} \leftrightarrow \mathrm{F} \\ &\equiv \mathrm{F} \leftrightarrow \mathrm{F} \\ &\equiv \mathrm{T}\end{aligned}$$

$\therefore$ Truth value of the given statement pattern is T.

Answer: T

Question 9 — Ex 1.1

If p, q are true and r, s are false, find the truth values of:

i) $(p \to q) \vee (r \to s)$

ii) $(\sim r \leftrightarrow p) \to \sim q$

iii) $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r)$

Answer · Question 9

Truth values of p, q, r, s are T, T, F, F respectively.

i) $(p \to q) \vee (r \to s) \equiv (\mathrm{T} \to \mathrm{T}) \vee (\mathrm{F} \to \mathrm{F})$

$\equiv \mathrm{T} \vee \mathrm{T} \equiv \mathrm{T}$

$\therefore$ Truth value is T.

ii) $(\sim r \leftrightarrow p) \to \sim q \equiv (\sim \mathrm{F} \leftrightarrow \mathrm{T}) \to \sim \mathrm{T}$

$\equiv (\mathrm{T} \leftrightarrow \mathrm{T}) \to \mathrm{F}$

$\equiv \mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$

$\therefore$ Truth value is F.

iii) $\sim[(\sim p \wedge r) \vee (s \to \sim q)] \leftrightarrow (p \wedge r) \equiv \sim[(\sim \mathrm{T} \wedge \mathrm{F}) \vee (\mathrm{F} \to \sim \mathrm{T})] \leftrightarrow (\mathrm{T} \wedge \mathrm{F})$

$\equiv \sim[(\mathrm{F} \wedge \mathrm{F}) \vee (\mathrm{F} \to \mathrm{F})] \leftrightarrow \mathrm{F}$

$\equiv \sim[\mathrm{F} \vee \mathrm{T}] \leftrightarrow \mathrm{F}$

$\equiv \sim \mathrm{T} \leftrightarrow \mathrm{F} \equiv \mathrm{F} \leftrightarrow \mathrm{F} \equiv \mathrm{T}$

$\therefore$ Truth value is T.

Answer: i) T ii) F iii) T

Question 10

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If p, q, r have truth values F, T, F respectively, then the truth values of $(p \wedge \sim q) \rightarrow r$ and $(p \vee q) \rightarrow r$ are respectively

(A) T, T (B) T, F (C) F, T (D) F, F

Answer · Question 10

$p \equiv \mathrm{F},\ q \equiv \mathrm{T},\ r \equiv \mathrm{F}$, so $\sim q \equiv \mathrm{F}$.

$(p \wedge \sim q) \to r \equiv (\mathrm{F} \wedge \mathrm{F}) \to \mathrm{F} \equiv \mathrm{F} \to \mathrm{F} \equiv \mathrm{T}$.

$(p \vee q) \to r \equiv (\mathrm{F} \vee \mathrm{T}) \to \mathrm{F} \equiv \mathrm{T} \to \mathrm{F} \equiv \mathrm{F}$.

$\therefore$ The truth values are T, F.

Answer: (B)

Question 11

Board pattern

Determine the truth values of p and q if

- $(p \vee q)$ is T, and
- $(p \vee q) \rightarrow q$ is F.

Answer · Question 11

$(p \vee q) \to q$ is F.

$\therefore$ $(p \vee q)$ is T and q is F [$\because$ $a \to b$ is F only when a is T, b is F]

Now $p \vee q$ is T and q is F, $\therefore$ p is T. [$\because$ $a \vee b$ is F only when both are F]

This agrees with the given condition that $p \vee q$ is T.

Hence p is T and q is F.

Answer: p is T, q is F

Question 12

Misc Ex 1

Determine the truth values of p and q if

- $(p \vee q)$ is T, and
- $(p \wedge q)$ is T.

Answer · Question 12

$p \wedge q$ is T.

$\therefore$ p is T and q is T [$\because$ $a \wedge b$ is T only when both are T]

Check: $p \vee q \equiv \mathrm{T} \vee \mathrm{T} \equiv \mathrm{T}$, which agrees with the given condition.

Hence p is T and q is T.

Answer: p is T, q is T

Question 13

MHT-CET 2024

If $p \rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively

(A) F, F, F (B) T, T, F (C) T, F, F (D) F, T, T

Answer · Question 13

A conditional $A \to B$ is false *only* when A is T and B is F.

$\therefore$ p is T and $q \vee r$ is F.

$q \vee r$ is F only when both are F $\Rightarrow$ q is F and r is F.

$\therefore$ $(p, q, r) = (\mathrm{T}, \mathrm{F}, \mathrm{F})$.

Answer: (C)

Question 14

MHT-CET 2024

If the statement $p \vee \sim(q \wedge r)$ is false, then the truth values of p, q and r are respectively

(A) F, T, F (B) T, F, F (C) F, T, T (D) F, F, T

Answer · Question 14

A disjunction $A \vee B$ is false *only* when both A and B are F.

$\therefore$ p is F and $\sim(q \wedge r)$ is F.

$\sim(q \wedge r)$ is F $\Rightarrow$ $q \wedge r$ is T $\Rightarrow$ q is T and r is T.

$\therefore$ $(p, q, r) = (\mathrm{F}, \mathrm{T}, \mathrm{T})$.

Answer: (C)

Question 15 Ex 1.1

Write the negations of the following.

i) Tirupati is in Andhra Pradesh.

ii) 3 is not a root of the equation $x^2 + 3x - 18 = 0$.

iii) $7 + 3 > 5$.

Answer · Question 15

i) Let p : Tirupati is in Andhra Pradesh. Its negation is $\sim p$.

$\sim p$: Tirupati is not in Andhra Pradesh.

ii) Let p : 3 is a root of the equation $x^2 + 3x - 18 = 0$. The given statement is $\sim p$.

Its negation is $\sim(\sim p) \equiv p$. [Involution law]

$\therefore$ The negation is: 3 is a root of the equation $x^2 + 3x - 18 = 0$.

iii) Let p : $7 + 3 > 5$. Its negation is $\sim p$. [negation of $>$ is $\leq$]

$\sim p$: $7 + 3 \leq 5$, i.e. $7 + 3 \not> 5$.

Answer: i) Tirupati is not in Andhra Pradesh. ii) 3 is a root of $x^2 + 3x - 18 = 0$. iii) $7 + 3 \leq 5$.

PART 2

Truth Tables, Tautology & Logical Equivalence

Statement Pattern and Truth Table

Statement pattern

- A proper combination of statement letters and connectives.
- e.g. $p \to q$, $\sim p \vee q$.
- The letters are its *prime components*.

Filling the truth table

- n letters $\Rightarrow 2^n$ rows.
- 2 letters: p: TTFF, q: TFTF
- 3 letters: p: TTTTFFFF, q: TTFFTTFF, r: TFTFTFTF
- Board priority of symbols: $\sim$, $\vee$, $\wedge$, $\to$, $\leftrightarrow$ (use brackets!)

Tautology, Contradiction, Contingency

Classifying a pattern

Last column	**Name**	**Example**
all T	Tautology (t)	$p \vee \sim p$
all F	Contradiction (c)	$p \wedge \sim p$
some T, some F	Contingency	$p \wedge q$

Logical equivalence

- $A \equiv B$ if their truth tables (last columns) are identical.

Remember With & CET Shortcut

Remember with

- Tautology: "India will win *or* India will not win." — always T.
- Contradiction: "I am awake *and* I am not awake." — always F.
- Contingency: "I wake up early *and* reach class on time." — depends on the day.

CET shortcut

- To test $A \to B$ for tautology: try to make $A = \mathrm{T}$ and $B = \mathrm{F}$.
- If that is impossible $\Rightarrow$ $A \to B$ is a tautology.

Standard Tautologies to Remember

Always true (tautologies)

- $p \to p$
- $p \to (p \vee q)$
- $(p \wedge q) \to p$
- $p \to (q \to p)$
- $[p \wedge (p \to q)] \to q$
- $[(p \to q) \wedge \sim q] \to \sim p$
- $[(p \to q) \wedge (q \to r)] \to (p \to r)$
- $(p \to q) \vee (q \to p)$

Always false

- $(p \wedge q) \wedge (\sim p \vee \sim q)$
- $(p \to q) \wedge (p \wedge \sim q)$

Question 25

Board pattern

Construct the truth table of

$$p \to (q \to p)$$

Answer · Question 25

Two letters, so $2^2 = 4$ rows.

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

All entries of the last column are T, so $p \to (q \to p)$ is a tautology.

Answer: Last column T, T, T, T (tautology).

Question 26 Ex 1.2

Construct the truth table for each of the following.

i) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii) $[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$

Answer · Question 26

i) Two letters, so $2^2 = 4$ rows.

p	q	$\sim q$	$p \wedge \sim q$	$p \to q$	$(p \wedge \sim q) \leftrightarrow (p \to q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

All entries of the last column are F, so the statement pattern is a contradiction.

ii) Three letters, so $2^3 = 8$ rows.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All entries of the last column are T, so the statement pattern is a tautology.

Answer: i) Last column F, F, F, F (contradiction) ii) Last column all T (tautology)

Question 27

Board pattern

Using truth table, prove that

$$p \wedge q \equiv \sim(p \rightarrow \sim q)$$

Answer · Question 27

I	II	III	IV	V	VI
p	q	$\sim q$	$p \wedge q$	$p \to \sim q$	$\sim(p \to \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

The entries in columns IV and VI are identical.

$\therefore\ p \wedge q \equiv \sim(p \to \sim q)$.

Answer: Columns IV and VI identical, hence $p \wedge q \equiv \sim(p \to \sim q)$.

Question 28 Ex 1.2

Using truth table, prove that

$$p \leftrightarrow q \equiv \sim[(p \vee q) \wedge \sim(p \wedge q)]$$

Answer · Question 28

I	II	III	IV	V	VI	VII	VIII
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$	$\sim[(p \vee q) \wedge \sim(p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

The entries in columns III and VIII are identical.

$\therefore\ p \leftrightarrow q \equiv \sim[(p \vee q) \wedge \sim(p \wedge q)]$.

Answer: Columns III and VIII identical, hence the equivalence is proved.

Question 29

Board pattern

Using truth table, examine whether the following statement pattern is a tautology, a contradiction or a contingency.

$$(p \wedge q) \wedge (\sim p \vee \sim q)$$

Answer · Question 29

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$(p \wedge q) \wedge (\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the truth values in the last column are F.

Hence $(p \wedge q) \wedge (\sim p \vee \sim q)$ is a contradiction.

Answer: Contradiction

Question 30 Ex 1.2

Examine whether each of the following is a tautology, a contradiction or a contingency.

i) $[(p \to q) \wedge \sim q] \to \sim p$

ii) $(p \leftrightarrow q) \wedge (p \to \sim q)$

Answer · Question 30

i)

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the truth values in the last column are T. Hence it is a tautology.

ii)

p	q	$\sim q$	$p \leftrightarrow q$	$p \to \sim q$	$(p \leftrightarrow q) \wedge (p \to \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

The last column contains both T and F. Hence it is a contingency.

Answer: i) Tautology ii) Contingency

Question 31

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If p and q are statements, then _____ is a contingency.

(A) $p \wedge \sim p$ (B) $p \vee \sim p$ (C) $p \vee q$ (D) $(p \wedge (p \rightarrow q)) \rightarrow q$

Answer · Question 31

Truth table of all four options:

p	q	$\sim p$	(A) $p \wedge \sim p$	(B) $p \vee \sim p$	(C) $p \vee q$	$p \to q$	$p \wedge (p \to q)$	(D) $(p \wedge (p \to q)) \to q$
T	T	F	F	T	T	T	T	T
T	F	F	F	T	T	F	F	T
F	T	T	F	T	T	T	F	T
F	F	T	F	T	F	T	F	T

(A) all F: contradiction. (B) all T: tautology. (D) all T: tautology.

(C) $p \vee q$ has both T and F: contingency.

Answer: (C)

Question 32 MHT-CET 2026

Consider the statement patterns

A. $(q \to p) \vee (p \to q)$

B. $(\sim p \vee \sim q) \leftrightarrow \sim(p \wedge q)$

C. $[(p \vee q) \wedge \sim p] \wedge \sim q$

D. $(p \wedge q) \wedge (\sim p \vee \sim q)$

then the statement patterns

(A) A and B are contradictions, C and D are tautologies

(B) A and B are tautologies, C and D are contradictions

(C) A, B, C, D all are tautologies

(D) A, B, C, D all are contradictions

Answer · Question 32

Patterns A and B:

p	q	$\sim p$	$\sim q$	$q \to p$	$p \to q$	A	$\sim p \vee \sim q$	$\sim(p \wedge q)$	B
T	T	F	F	T	T	T	F	F	T
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T	T

Patterns C and D:

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim p$	C	$p \wedge q$	$\sim p \vee \sim q$	D
T	T	F	F	T	F	F	T	F	F
T	F	F	T	T	F	F	F	T	F
F	T	T	F	T	T	F	F	T	F
F	F	T	T	F	F	F	F	T	F

Columns A and B are all T: tautologies. Columns C and D are all F: contradictions.

Answer: (B)

PART 3

Quantifiers & Duality

Quantifiers

Universal quantifier $\forall$

- Read as “for all”, “for every”.
- T only if *every* object satisfies the condition.
- One counter-example makes it F.

Existential quantifier $\exists$

- Read as “there exists”, “for some”.
- T if *at least one* object satisfies the condition; F if none does.

Negation of a Quantified Statement

Rule

- Change $\forall \leftrightarrow \exists$ (all $\leftrightarrow$ some).
- Negate the condition.

Examples

Statement	**Negation**
All A are B	Some A are not B
Some A are B	No A is B
$\forall x,\ x+1>3$	$\exists\, x$ such that $x+1 \leq 3$
$\exists\, x$ such that $x-4=3$	$\forall x,\ x-4 \neq 3$

Dual

Dual of a statement pattern

- Interchange $\wedge \leftrightarrow \vee$.
- Interchange $t \leftrightarrow c$.
- $\sim$ and the letters stay as they are.
- Dual of $p \vee q$ is $p \wedge q$.
- Dual of $t \vee p$ is $c \wedge p$.
- Dual of $(p \wedge q) \vee r$ is $(p \vee q) \wedge r$.
- Dual of the dual is the original.
- If $A \equiv B$ then dual $A \equiv$ dual B.

CET trap

- Dual is defined only for $\wedge, \vee$.
- If $\rightarrow$ or $\leftrightarrow$ is present, first rewrite $(p \rightarrow q \equiv \sim p \vee q)$, then take the dual.

Remember With

Student-life examples

- $\forall$: "*Every* student of our batch has a phone." — one student without a phone makes it F.
- $\exists$: "*Some* student of our batch comes by bike." — one biker makes it T.
- Negation: "All my friends watch IPL" $\rightarrow$ "*Some* friend of mine does *not* watch IPL."

Question 41

Board pattern

If $A = \{1, 2, 3, 4, 5, 6, 7\}$, determine the truth values of:

i) $\exists\, x \in A$ such that $x - 4 = 3$

ii) $\forall\, x \in A,\ x + 1 > 3$

Answer · Question 41

i) For $x = 7$, $x - 4 = 7 - 4 = 3$.

$\therefore$ $x = 7 \in A$ satisfies $x - 4 = 3$. [One example is enough for $\exists$]

$\therefore$ The given statement is true, its truth value is T.

ii) For $x = 1$, $x + 1 = 1 + 1 = 2$, which is not greater than 3.

$\therefore$ $x = 1 \in A$ does not satisfy $x + 1 > 3$. [One counter-example is enough for $\forall$]

$\therefore$ The truth value of the given statement is F.

Answer: i) T ii) F

Question 42 Ex 1.3

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth values of:

i) $\forall x \in A,\ x^2 + x$ is an even number

ii) $\exists x \in A$ such that $x^2 < 0$

iii) $\exists x \in A$ such that $3x + 8 > 40$

iv) $\forall x \in A,\ 2x + 9 > 14$

Answer · Question 42

i) $x^2 + x = x(x+1)$ is a product of two consecutive integers, so it is always even.

Check: $3^2+3=12,\ 5^2+5=30,\ 7^2+7=56,\ 9^2+9=90,\ 11^2+11=132,\ 12^2+12=156$ — all even.

$\therefore$ Every $x \in A$ satisfies the condition. $\therefore$ The truth value is T.

ii) The square of a real number is never negative, i.e. $x^2 \geq 0$ for every $x \in A$.

$\therefore$ No $x \in A$ satisfies $x^2 < 0$. $\therefore$ The truth value is F.

iii) For $x = 11$, $3x + 8 = 33 + 8 = 41 > 40$.

$\therefore$ $x = 11 \in A$ satisfies $3x + 8 > 40$. $\therefore$ The truth value is T.

iv) $2x + 9 > 14 \iff 2x > 5 \iff x > 2.5$. The smallest element of A is 3 and $2(3) + 9 = 15 > 14$.

$\therefore$ Every $x \in A$ (each $x \geq 3$) satisfies $2x + 9 > 14$. $\therefore$ The truth value is T.

Answer: i) T ii) F iii) T iv) T

Question 43 MHT-CET 2026

If $A = \{1, 2, 3, 4, 5, 6\}$, then which of the following is *not* true?

(A) $\exists\, x \in A$ such that $x + 3 = 9$

(B) $\exists\, x \in A$ such that $x + 2 < 9$

(C) $\forall\, x \in A,\ x + 6 \geq 10$

(D) $\exists\, x \in A$ such that $x + 6 < 10$

Answer · Question 43

(A) For $x = 6$, $x + 3 = 9$. $\therefore$ T.

(B) For $x = 1$, $x + 2 = 3 < 9$. $\therefore$ T.

(C) For $x = 1$, $x + 6 = 7$, which is not ≥ 10. One counter-example makes a $\forall$ statement false. $\therefore$ F.

(D) For $x = 1$, $x + 6 = 7 < 10$. $\therefore$ T.

$\therefore$ The statement which is not true is (C).

Answer: (C)

Question 44

Board pattern

Write the duals of:

i) $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$

ii) $(p \wedge t) \vee (c \wedge \sim q)$

Answer · Question 44

To get the dual, interchange $\wedge \leftrightarrow \vee$ and $t \leftrightarrow c$; the letters and $\sim$ stay as they are.

i) Given: $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$

Dual: $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$ $[\wedge \leftrightarrow \vee]$

ii) Given: $(p \wedge t) \vee (c \wedge \sim q)$

Dual: $(p \vee c) \wedge (t \vee \sim q)$ $[\wedge \leftrightarrow \vee,\ \ t \leftrightarrow c]$

Answer: i) $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$ ii) $(p \vee c) \wedge (t \vee \sim q)$

Question 45 Ex 1.3

Write the duals of:

i) $(p \vee q) \wedge (r \vee s)$

ii) $(\sim p \vee q) \wedge (\sim r \wedge s)$

iii) $c \vee \{p \wedge (q \vee r)\}$

Answer · Question 45

Interchange $\wedge \leftrightarrow \vee$ and $t \leftrightarrow c$; keep the letters and $\sim$ unchanged.

i) Given: $(p \vee q) \wedge (r \vee s)$

Dual: $(p \wedge q) \vee (r \wedge s)$ $[\wedge \leftrightarrow \vee]$

ii) Given: $(\sim p \vee q) \wedge (\sim r \wedge s)$

Dual: $(\sim p \wedge q) \vee (\sim r \vee s)$ $[\wedge \leftrightarrow \vee]$

iii) Given: $c \vee \{p \wedge (q \vee r)\}$

Dual: $t \wedge \{p \vee (q \wedge r)\}$ $[\wedge \leftrightarrow \vee,\ \ c \to t]$

Answer: i) $(p \wedge q) \vee (r \wedge s)$ ii) $(\sim p \wedge q) \vee (\sim r \vee s)$ iii) $t \wedge \{p \vee (q \wedge r)\}$

Question 46 MHT-CET 2026

The statements p, q and r have truth values T, F, F respectively. The truth values of $\sim(p \wedge \sim q) \vee (q \vee \sim r)$ and its dual are respectively

(A) T, T (B) T, F (C) F, T (D) F, F

Answer · Question 46

$p \equiv \mathrm{T},\ q \equiv \mathrm{F},\ r \equiv \mathrm{F},$ so $\sim q \equiv \mathrm{T},\ \sim r \equiv \mathrm{T}.$

Given statement: $\sim(p \wedge \sim q) \vee (q \vee \sim r) \equiv \sim(\mathrm{T} \wedge \mathrm{T}) \vee (\mathrm{F} \vee \mathrm{T}) \equiv \sim\mathrm{T} \vee \mathrm{T} \equiv \mathrm{F} \vee \mathrm{T} \equiv \mathrm{T}.$

Its dual (interchange $\wedge \leftrightarrow \vee$; $\sim$ stays): $\sim(p \vee \sim q) \wedge (q \wedge \sim r).$

$\sim(p \vee \sim q) \wedge (q \wedge \sim r) \equiv \sim(\mathrm{T} \vee \mathrm{T}) \wedge (\mathrm{F} \wedge \mathrm{T}) \equiv \sim\mathrm{T} \wedge \mathrm{F} \equiv \mathrm{F} \wedge \mathrm{F} \equiv \mathrm{F}.$

$\therefore$ The truth values are T, F respectively.

Answer: (B)

Question 47

Board pattern

Write the negations of:

i) All natural numbers are rational.

ii) $\exists\, n \in \mathbb{N}$ such that $n + 8 > 11$.

Answer · Question 47

Negation rule: change "all" $\leftrightarrow$ "some" ($\forall \leftrightarrow \exists$) and negate the condition.

i) Given: All natural numbers are rational. [All A are B]

Negation: Some natural numbers are not rational. [Some A are not B]

ii) Given: $\exists\, n \in \mathbb{N}$ such that $n + 8 > 11$.

Negation: $\forall\, n \in \mathbb{N},\ n + 8 \leq 11$. [$\exists \to \forall,\ > \to \leq$]

Answer: i) Some natural numbers are not rational. ii) $\forall\, n \in \mathbb{N},\ n + 8 \leq 11$.

Question 48 — Misc Ex 1

Write the negations of:

i) $\forall\, n \in A,\ n + 7 > 6$, where $A = \{1, 2, \ldots, 9\}$.

ii) Some triangles are equilateral triangles.

Answer · Question 48

Negation rule: change $\forall \leftrightarrow \exists$ and negate the condition.

i) Given: $\forall\, n \in A,\ n + 7 > 6$.

Negation: $\exists\, n \in A$ such that $n + 7 \leq 6$. $[\forall \to \exists,\ > \to \leq]$

ii) Given: Some triangles are equilateral triangles. [Some A are B]

Negation: No triangle is an equilateral triangle. [No A is B]

Answer: i) $\exists\, n \in A$ such that $n + 7 \leq 6$. ii) No triangle is an equilateral triangle.

PART 4

Negation & Converse, Inverse, Contrapositive

Negation of Compound Statements

Rules

Statement	**Negation**	**Rule**
$p \wedge q$	$\sim p \vee \sim q$	De Morgan's law
$p \vee q$	$\sim p \wedge \sim q$	De Morgan's law
$p \rightarrow q$	$p \wedge \sim q$	never an if–then
$p \leftrightarrow q$	$(p \wedge \sim q) \vee (q \wedge \sim p)$	$\equiv p \leftrightarrow \sim q \equiv \sim p \leftrightarrow q$
$\sim p$	p	$\sim(\sim p) \equiv p$

Negating relations

- $=$ becomes $\neq$
- $>$ becomes $\leq$
- $<$ becomes $\geq$

Remember With

Student-life examples

- "I will go to the movie *and* eat popcorn."

  Negation: "I will not go to the movie *or* I will not eat popcorn."
- "If it rains, I carry an umbrella."

  Negation: "It rains *and* I do not carry an umbrella."

Converse, Inverse, Contrapositive

Four faces of $p \to q$

Name	**Form**	**Equivalent to**
Statement	$p \to q$	contrapositive
Converse	$q \to p$	inverse
Inverse	$\sim p \to \sim q$	converse
Contrapositive	$\sim q \to \sim p$	statement

Truth table

p	q	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Remember With & CET Tricks

Remember with

- "If I study daily, then I pass."
- Contrapositive: "If I did not pass, then I did not study daily." — *same meaning*.
- Converse: "If I passed, then I studied daily." — *not* the same!

CET tricks

- Negation of the contrapositive = negation of the statement = $A \wedge \sim B$.
- No arrow? Rewrite first: $\sim p \vee q \equiv p \to q$
- and $p \vee q \equiv \sim p \to q$.

Question 54

Board pattern

Write the negations of:

i) $7 > 3$ and $4 > 11$.

ii) The number is an even number if and only if it is divisible by 2.

Answer · Question 54

i) Let $p : 7 > 3, \quad q : 4 > 11.$

The given statement in symbolic form is $p \wedge q$.

Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$. [De Morgan's law]

$\therefore$ The negation is: $7 \leq 3$ or $4 \leq 11$.

ii) Let p : The number is an even number, $\quad q$: The number is divisible by 2.

The given statement in symbolic form is $p \leftrightarrow q$.

Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]

$\therefore$ The negation is: The number is an even number but not divisible by 2, or the number is divisible by 2 but not an even number.

Answer: i) $7 \leq 3$ or $4 \leq 11$. ii) The number is even but not divisible by 2, or it is divisible by 2 but not even.

Question 55 Ex 1.3

Write the negations of the following.

i) $x + 8 > 11$ or $y - 3 = 6$.

ii) Quadrilateral is a square if and only if it is a rhombus.

iii) If it is raining then we will go and play football.

iv) $\forall\, n \in \mathbb{N},\ n^2 + n + 2$ is divisible by 4.

Answer · Question 55

i) Let $p : x + 8 > 11$, $q : y - 3 = 6$.

The given statement in symbolic form is $p \vee q$.

Its negation is $\sim(p \vee q) \equiv \sim p \wedge \sim q$. [De Morgan's law]

$\therefore$ The negation is: $x + 8 \leq 11$ and $y - 3 \neq 6$.

ii) Let p : Quadrilateral is a square, q : Quadrilateral is a rhombus.

The given statement in symbolic form is $p \leftrightarrow q$.

Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]

$\therefore$ The negation is: Quadrilateral is a square but not a rhombus, or quadrilateral is a rhombus but not a square.

iii) Let p : It is raining, q : We will go, r : We will play football.

The given statement in symbolic form is $p \rightarrow (q \wedge r)$.

Its negation is $\sim[p \rightarrow (q \wedge r)] \equiv p \wedge \sim(q \wedge r)$ $[\sim(p \rightarrow q) \equiv p \wedge \sim q]$

$\equiv p \wedge (\sim q \vee \sim r)$ [De Morgan's law]

$\therefore$ The negation is: It is raining and (we will not go or we will not play football).

iv) Given: $\forall\, n \in \mathbb{N}$, $n^2 + n + 2$ is divisible by 4.

Negation: $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is not divisible by 4. [$\forall \rightarrow \exists$, negate the condition]

Answer: i) $x + 8 \leq 11$ and $y - 3 \neq 6$. ii) Quadrilateral is a square but not a rhombus, or it is a rhombus but not a square. iii) It is raining and (we will not go or we will not play football). iv) $\exists\, n \in \mathbb{N}$ such that $n^2 + n + 2$ is not divisible by 4.

Question 56 MHT-CET 2024

Negation of "The payment will be made if and only if the work is finished in time." is

(A) The work is finished in time and the payment is not made.

(B) The payment is made and the work is not finished in time.

(C) The work is finished in time and the payment is not made, or the payment is made and the work is finished in time.

(D) Either the work is finished in time and the payment is not made, or the payment is made and the work is not finished in time.

Answer · Question 56

Let p : The payment will be made, q : The work is finished in time.

The given statement in symbolic form is $p \leftrightarrow q$.

Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$. [Negation of biconditional]

$\therefore$ The negation is: The payment is made and the work is not finished in time, or the work is finished in time and the payment is not made.

This is option (D) (the two parts written in the other order). (A) and (B) give only one part each; (C) has the second part wrong.

Answer: (D)

Question 57

Board pattern

Write the converse, inverse and contrapositive of:

"If it rains then the match will be cancelled."

Answer · Question 57

Let p : It rains, q : The match will be cancelled.

$\therefore$ The given statement in symbolic form is $p \to q$.

i) Converse $q \to p$: If the match is cancelled then it rains.

ii) Inverse $\sim p \to \sim q$: If it does not rain then the match will not be cancelled.

iii) Contrapositive $\sim q \to \sim p$: If the match is not cancelled then it does not rain.

Answer: Converse: If the match is cancelled then it rains. Inverse: If it does not rain then the match will not be cancelled. Contrapositive: If the match is not cancelled then it does not rain.

Question 58 Ex 1.3

Write the converse, inverse and contrapositive of:

i) If $x < y$ then $x^2 < y^2$ $(x, y \in \mathbb{R})$.

ii) A family becomes literate if the woman in it is literate.

Answer · Question 58

i) Let $p : x < y$, $q : x^2 < y^2$. The given statement in symbolic form is $p \to q$.

Converse $q \to p$: If $x^2 < y^2$ then $x < y$.

Inverse $\sim p \to \sim q$: If $x \geq y$ then $x^2 \geq y^2$. [$\sim(x < y)$ is $x \geq y$]

Contrapositive $\sim q \to \sim p$: If $x^2 \geq y^2$ then $x \geq y$.

ii) "q if p" means $p \to q$.

Let p : The woman in the family is literate, q : The family becomes literate.

The given statement in symbolic form is $p \to q$.

Converse $q \to p$: If a family becomes literate then the woman in it is literate.

Inverse $\sim p \to \sim q$: If the woman in a family is not literate then the family does not become literate.

Contrapositive $\sim q \to \sim p$: If a family does not become literate then the woman in it is not literate.

Answer: i) C: If $x^2 < y^2$ then $x < y$; I: If $x \geq y$ then $x^2 \geq y^2$; CP: If $x^2 \geq y^2$ then $x \geq y$. ii) C: If a family becomes literate then the woman in it is literate; I: If the woman is not literate then the family does not become literate; CP: If a family does not become literate then the woman in it is not literate.

Question 59 MHT-CET 2024

Contrapositive of the statement "If two numbers are not equal, then their squares are not equal" is

(A) If the squares of two numbers are not equal, then the numbers are equal.

(B) If the squares of two numbers are equal, then the numbers are not equal.

(C) If the squares of two numbers are equal, then the numbers are equal.

(D) If the squares of two numbers are not equal, then the numbers are not equal.

Answer · Question 59

Let p : Two numbers are not equal, q : Their squares are not equal.

The given statement in symbolic form is $p \to q$.

Contrapositive $\sim q \to \sim p$, where $\sim q$: The squares are equal, $\sim p$: The numbers are equal.

$\therefore$ Contrapositive: If the squares of two numbers are equal, then the numbers are equal.

Answer: (C)

Question 60

MHT-CET 2024

The negation of the contrapositive of $p \rightarrow (\sim q \wedge r)$ is

(A) $(\sim q \vee \sim r) \wedge \sim p$ (B) $(q \vee \sim r) \wedge p$

(C) $(q \wedge \sim r) \vee p$ (D) $(\sim q \wedge \sim r) \vee \sim p$

Answer · Question 60

The contrapositive is equivalent to the statement, so the negation of the contrapositive = negation of the statement.

$\sim[p \to (\sim q \wedge r)] \equiv p \wedge \sim(\sim q \wedge r)$ $[\sim(p \to q) \equiv p \wedge \sim q]$

$\equiv p \wedge (q \vee \sim r)$ [De Morgan's law, Double negation]

$\equiv (q \vee \sim r) \wedge p$ [Commutative law]

Answer: (B)

PART 5

Algebra of Statements

Two Important Results

Conditional and biconditional laws

- $p \to q \equiv \sim p \vee q$
- $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
- $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Board tips

- "Without truth table": change one step at a time and write the *law used* in brackets at every step.
- "Without if...then": use $p \to q \equiv \sim p \vee q$, i.e. "not p *or* q".

Algebra of Statements

Laws

Idempotent	$p \wedge p \equiv p, \quad p \vee p \equiv p$	De Morgan	$\sim(p \wedge q) \equiv \sim p \vee \sim q$
Commutative	$p \wedge q \equiv q \wedge p, \quad p \vee q \equiv q \vee p$		$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Identity	$p \wedge t \equiv p, \quad p \wedge c \equiv c$
	$(p \vee q) \vee r \equiv p \vee (q \vee r)$		$p \vee c \equiv p, \quad p \vee t \equiv t$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Complement	$p \wedge \sim p \equiv c, \quad p \vee \sim p \equiv t$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$		$\sim t \equiv c, \quad \sim c \equiv t$
Involution	$\sim(\sim p) \equiv p$	Absorption	$p \vee (p \wedge q) \equiv p, \quad p \wedge (p \vee q) \equiv p$

Remember With

Student-life examples

- **Absorption**: "I'll eat pizza, *or* I'll eat pizza *and* drink cola" = I eat pizza: $p \vee (p \wedge q) \equiv p$.
- **Identity**: "I passed *and* the Sun rises in the east" = "I passed": $p \wedge t \equiv p$.
- **Conditional law**: "If it is Sunday, I sleep late" = "It is not Sunday *or* I sleep late."

Question 71

Board pattern

Write the negation of the following, stating the rules used.

$$(p \vee q) \wedge (q \vee \sim r)$$

Answer · Question 71

$$\sim[(p \vee q) \wedge (q \vee \sim r)]$$

$$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r) \qquad \text{[De Morgan's law]}$$

$$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r) \qquad \text{[De Morgan's law]}$$

$$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r) \qquad \text{[Commutative law]}$$

$$\equiv \sim q \wedge (\sim p \vee r) \qquad \text{[Distributive law]}$$

Answer: $\sim[(p \vee q) \wedge (q \vee \sim r)] \equiv \sim q \wedge (\sim p \vee r)$

Question 72 Ex 1.4

Using rules of negation, write the negations of the following with justification.

i) $(p \vee \sim q) \wedge r$

ii) $(p \vee \sim q) \rightarrow (p \wedge \sim q)$

Answer · Question 72

i) $\sim[(p \vee \sim q) \wedge r]$

$\equiv \sim(p \vee \sim q) \vee \sim r$ [De Morgan's law]

$\equiv [\sim p \wedge \sim(\sim q)] \vee \sim r$ [De Morgan's law]

$\equiv (\sim p \wedge q) \vee \sim r$ [Involution law: $\sim(\sim q) \equiv q$]

ii) $\sim[(p \vee \sim q) \to (p \wedge \sim q)]$

$\equiv (p \vee \sim q) \wedge \sim(p \wedge \sim q)$ [$\sim(p \to q) \equiv p \wedge \sim q$]

$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim(\sim q)]$ [De Morgan's law]

$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ [Involution law]

Answer: i) $(\sim p \wedge q) \vee \sim r$ ii) $(p \vee \sim q) \wedge (\sim p \vee q)$

Question 73

Board pattern

Rewrite the following statement without using if. . . then.

"If it is cold, then we wear woollen clothes."

Answer · Question 73

Let p : It is cold, q : We wear woollen clothes.

The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]

$\therefore$ The given statement can be written as: "It is not cold or we wear woollen clothes."

Answer: It is not cold or we wear woollen clothes.

Question 74 Ex 1.4

Rewrite the following statements without using if. . . then.

i) If a man is a judge then he is honest.

ii) If $f(2) = 0$ then $f(x)$ is divisible by $(x - 2)$.

Answer · Question 74

i) Let p : A man is a judge, q : He is honest.

The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]

$\therefore$ The given statement can be written as: "A man is not a judge or he is honest."

ii) Let $p : f(2) = 0$, $q : f(x)$ is divisible by $(x - 2)$.

The given statement is $p \to q$, but $p \to q \equiv \sim p \vee q$. [Conditional law]

$\therefore$ The given statement can be written as: "$f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$."

Answer: i) A man is not a judge or he is honest. ii) $f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$.

Question 75

Board pattern

Without using truth table, prove that

$$\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Answer · Question 75

$$\text{LHS} = \sim(p \vee q) \vee (\sim p \wedge q)$$

$$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q) \qquad \text{[De Morgan's law]}$$

$$\equiv \sim p \wedge (\sim q \vee q) \qquad \text{[Distributive law]}$$

$$\equiv \sim p \wedge t \qquad \text{[Complement law]}$$

$$\equiv \sim p \qquad \text{[Identity law]}$$

$$= \text{RHS}.$$

Answer: $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (proved).

Question 76 Ex 1.4

Without using truth table, prove that:

i) $(p \vee q) \wedge (p \vee \sim q) \equiv p$

ii) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

Answer · Question 76

i) LHS $= (p \vee q) \wedge (p \vee \sim q)$

$\equiv p \vee (q \wedge \sim q)$ [Distributive law]

$\equiv p \vee c$ [Complement law]

$\equiv p$ [Identity law]

$=$ RHS.

ii) LHS $= (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$

$\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q)$ [Distributive law]

$\equiv (t \wedge q) \vee (p \wedge \sim q)$ [Complement law]

$\equiv q \vee (p \wedge \sim q)$ [Identity law]

$\equiv (q \vee p) \wedge (q \vee \sim q)$ [Distributive law]

$\equiv (q \vee p) \wedge t$ [Complement law]

$\equiv q \vee p$ [Identity law]

$\equiv p \vee q$ [Commutative law]

$=$ RHS.

Answer: Both identities proved: i) $\equiv p$ ii) $\equiv p \vee q$.

Question 77 MHT-CET 2024

The statement pattern $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$ is equivalent to

(A) $q \vee r$ (B) $p \vee r$ (C) q (D) p

Answer · Question 77

$[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$

$\equiv [p \wedge (q \vee r)] \vee [p \wedge (\sim q \wedge \sim r)]$ [Commutative and Associative laws]

$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim (q \vee r)]$ [De Morgan's law]

$\equiv p \wedge [(q \vee r) \vee \sim (q \vee r)]$ [Distributive law]

$\equiv p \wedge t$ [Complement law]

$\equiv p$ [Identity law]

Answer: (D)

Question 78

MHT-CET 2024

The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to

(A) $p \wedge \sim q$ (B) $p \rightarrow \sim q$ (C) $p \vee q$ (D) $q \rightarrow p$

Answer · Question 78

$\sim p \vee (p \wedge \sim q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$ [Distributive law]

$\equiv t \wedge (\sim p \vee \sim q)$ [Complement law]

$\equiv \sim p \vee \sim q$ [Identity law]

$\equiv p \rightarrow \sim q$ [Conditional law: $p \rightarrow q \equiv \sim p \vee q$]

Answer: (B)

PART 6

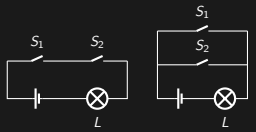
Switching Circuits

Switching Circuits

Logic $\leftrightarrow$ circuits

- Switch: two-state device (ON / OFF).
- p : switch S_1 is closed.
- S_1' is always opposite to S_1: $S_1' \rightarrow \sim p$.
- Same letter on two switches $\Rightarrow$ same state.
- **Series** $\leftrightarrow \ \wedge$
- **Parallel** $\leftrightarrow \ \vee$
- ON $= 1 \leftrightarrow$ T
- OFF $= 0 \leftrightarrow$ F

Series $p \wedge q$ Parallel $p \vee q$



Switching Table & Simplifying

Switching (input–output) table

- Truth table written with 1 and 0.
- Lamp glows in the rows where the expression is 1.
- All 1 $\Rightarrow$ lamp always glows (tautology).
- All 0 $\Rightarrow$ lamp never glows (contradiction).

Simplifying a circuit

1. Write the symbolic form of the circuit.
2. Simplify using the laws of algebra of statements.
3. Draw the circuit of the simplified form (minimum switches).

Remember With

Student-life examples

- **Series** = two doors one after another on the way to class — you get in only if *both* are open.
- **Parallel** = two gates of the school — *either* gate open lets you in.
- S and S' = two bulbs on a "one ON, other OFF" toggle.

Question 85

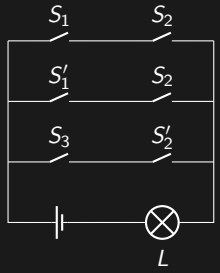
Board pattern

For the circuit:

i) write the symbolic form,

ii) construct the switching table,

iii) simplify the circuit.



Answer · Question 85

Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.

Then $\sim p$: switch S_1' is closed (or S_1 is open), $\sim q$: switch S_2' is closed.

i) The three branches are in parallel and each branch has two switches in series.

$\therefore$ Symbolic form:

$X \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$.

ii) Switching table:

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$r \wedge \sim q$	X
1	1	1	0	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	1	0	1
0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	0	1	1
0	0	0	1	1	0	0	0	0

iii) $X \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$

$\equiv [(p \vee \sim p) \wedge q] \vee (r \wedge \sim q)$ [Distributive law]

$\equiv (t \wedge q) \vee (r \wedge \sim q)$ [Complement law]

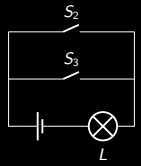
$\equiv q \vee (r \wedge \sim q)$ [Identity law]

$\equiv (q \vee r) \wedge (q \vee \sim q)$ [Distributive law]

$\equiv (q \vee r) \wedge t$ [Complement law]

$\equiv q \vee r$ [Identity law]

$\therefore$ The simplified circuit has switches S_2 and S_3 in parallel:



The lamp glows when S_2 or S_3 (or both) is closed.

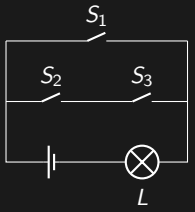
Answer: $X \equiv q \vee r$: S_2 and S_3 in parallel.

Question 86

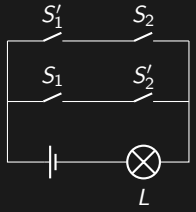
Ex 1.5

Express the following circuits in the symbolic form of logic and write the input–output table.

i)



ii)



Answer · Question 86

Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.

i) S_2 and S_3 are in series, and this combination is in parallel with S_1.

$\therefore$ Symbolic form: $X \equiv p \vee (q \wedge r)$.

Input–output table:

p	q	r	$q \wedge r$	$X \equiv p \vee (q \wedge r)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

The lamp glows when S_1 is closed, or when S_2 and S_3 are both closed.

ii) Upper branch: S_1' and S_2 in series; lower branch: S_1 and S_2' in series; the branches are in parallel.

$\therefore$ Symbolic form: $X \equiv (\sim p \wedge q) \vee (p \wedge \sim q)$.

Input–output table:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \wedge \sim q$	X
1	1	0	0	0	0	0
1	0	0	1	0	1	1
0	1	1	0	1	0	1
0	0	1	1	0	0	0

The lamp glows only when exactly one of S_1, S_2 is closed.

Answer: i) $p \vee (q \wedge r)$; X column: 1,1,1,1,1,0,0,0 ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$; X column: 0,1,1,0

Question 87

Board pattern

Construct the switching circuit of

$$(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)]$$

Answer · Question 87

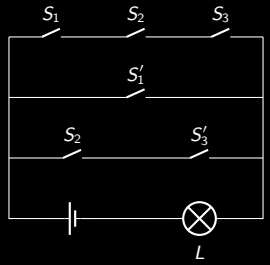
Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.

Then $\sim p$: switch S_1' is closed, $\sim r$: switch S_3' is closed.

$(p \wedge q \wedge r) \vee [\sim p \vee (q \wedge \sim r)] \equiv (p \wedge q \wedge r) \vee \sim p \vee (q \wedge \sim r)$ [Associative law]

So the circuit has three branches in parallel:

(1) S_1, S_2, S_3 in series; (2) S_1' alone; (3) S_2 and S_3' in series.



Answer: Three parallel branches: $S_1S_2S_3$ in series $\parallel$ S_1' $\parallel$ S_2S_3' in series.

Question 88

Ex 1.5

Construct the switching circuit of

$$(\sim p \wedge q) \vee (p \wedge \sim r)$$

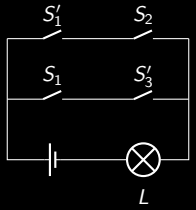
Answer · Question 88

Let p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 is closed.

Then $\sim p$: switch S_1' is closed, $\sim r$: switch S_3' is closed.

The given statement is the parallel combination ($\vee$) of two branches:

(1) $\sim p \wedge q$: S_1' and S_2 in series; (2) $p \wedge \sim r$: S_1 and S_3' in series.



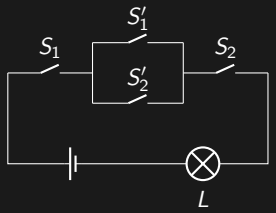
Answer: Two parallel branches: (S_1', S_2 in series) and (S_1, S_3' in series).

Question 89

Textbook Ex

Simplify the given circuit by writing its logical expression.

Also write your conclusion.



Answer · Question 89

Let p : switch S_1 is closed, q : switch S_2 is closed.

Then $\sim p$: switch S_1' is closed, $\sim q$: switch S_2' is closed.

S_1' and S_2' are in parallel, and this combination is in series with S_1 and S_2.

$\therefore$ Symbolic form: $X \equiv p \wedge (\sim p \vee \sim q) \wedge q$.

$X \equiv [p \wedge (\sim p \vee \sim q)] \wedge q$ [Associative law]

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \wedge q$ [Distributive law]

$\equiv [c \vee (p \wedge \sim q)] \wedge q$ [Complement law]

$\equiv (p \wedge \sim q) \wedge q$ [Identity law]

$\equiv p \wedge (\sim q \wedge q)$ [Associative law]

$\equiv p \wedge c$ [Complement law]

$\equiv c$ [Identity law]

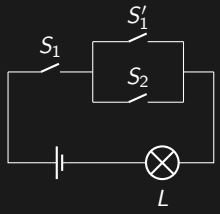
Conclusion: X is a contradiction, so the lamp will never glow, whatever the positions of the switches S_1 and S_2.

Answer: $X \equiv c$: the lamp never glows.

Question 90

Ex 1.5

Give an alternative equivalent simple circuit for the following circuit.



Answer · Question 90

Let p : switch S_1 is closed, q : switch S_2 is closed.

Then $\sim p$: switch S_1' is closed.

S_1' and S_2 are in parallel, and this combination is in series with S_1.

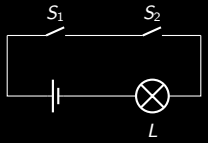
The symbolic form of the given circuit is $p \wedge (\sim p \vee q)$.

$p \wedge (\sim p \vee q) \equiv (p \wedge \sim p) \vee (p \wedge q)$ [Distributive law]

$\equiv c \vee (p \wedge q)$ [Complement law]

$\equiv p \wedge q$ [Identity law]

$\therefore$ The alternative circuit has S_1 and S_2 in series:



Answer: $p \wedge q$: S_1 and S_2 in series.

Question 91

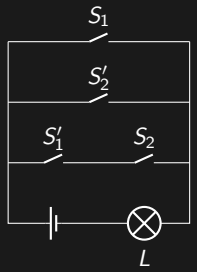
Ex 1.5

For the circuit:

i) write the symbolic form,

ii) construct its switching table,

iii) interpret it.



Answer · Question 91

Let p : switch S_1 is closed, q : switch S_2 is closed.

Then $\sim p$: switch S_1' is closed, $\sim q$: switch S_2' is closed.

The three branches S_1; S_2'; (S_1' and S_2 in series) are in parallel.

$\therefore$ Symbolic form: $X \equiv p \vee \sim q \vee (\sim p \wedge q)$.

Switching table:

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$X \equiv p \vee \sim q \vee (\sim p \wedge q)$
1	1	0	0	0	1
1	0	0	1	0	1
0	1	1	0	1	1
0	0	1	1	0	1

Interpretation: All the entries in the last column are 1. Hence the lamp will always glow, irrespective of the positions of the switches (X is a tautology).

Verification by laws:

$X \equiv p \vee [\sim q \vee (\sim p \wedge q)]$ [Associative law]

$\equiv p \vee [(\sim q \vee \sim p) \wedge (\sim q \vee q)]$ [Distributive law]

$\equiv p \vee [(\sim q \vee \sim p) \wedge t]$ [Complement law]

$\equiv p \vee (\sim q \vee \sim p)$ [Identity law]

$\equiv p \vee (\sim p \vee \sim q)$ [Commutative law]

$\equiv (p \vee \sim p) \vee \sim q$ [Associative law]

$\equiv t \vee \sim q$ [Complement law]

$\equiv t$ [Identity law]

Answer: $X \equiv p \vee \sim q \vee (\sim p \wedge q)$; last column all 1 : the lamp always glows.

Question 92 MHT-CET 2026

In a switching circuit, if the combination $(S_1 \wedge S_2)$ is connected in parallel to the combination $(S_1' \wedge S_2')$, then the room is lit only when

(A) S_1 is ON and S_2 is OFF

(B) S_1 is OFF and S_2 is ON

(C) S_1 and S_2 both ON or S_1 and S_2 both OFF

(D) The room is always lit

Answer · Question 92

Let p : switch S_1 is closed (ON), q : switch S_2 is closed (ON).

Symbolic form: $X \equiv (p \wedge q) \vee (\sim p \wedge \sim q) \equiv p \leftrightarrow q$.

p	q	$p \wedge q$	$\sim p \wedge \sim q$	X
1	1	1	0	1
1	0	0	0	0
0	1	0	0	0
0	0	0	1	1

$X = 1$ only in rows 1 and 4, i.e. when S_1, S_2 are both ON or both OFF.

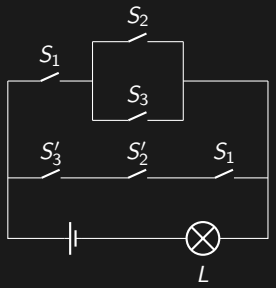
Answer: (C)

Question 93

MHT-CET 2025

If p, q, r : switches S_1, S_2, S_3 are closed, then the symbolic form of the circuit is equivalent to

(A) p (B) q (C) $p \wedge q$ (D) $p \vee q$



Answer · Question 93

Upper branch: S_1 in series with (S_2 parallel S_3) $\Rightarrow p \wedge (q \vee r)$.

Lower branch: S_3', S_2', S_1 in series $\Rightarrow \sim r \wedge \sim q \wedge p$.

$\therefore X \equiv [p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$

$\equiv [p \wedge (q \vee r)] \vee [p \wedge (\sim q \wedge \sim r)]$ [Commutative and Associative laws]

$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim (q \vee r)]$ [De Morgan's law]

$\equiv p \wedge [(q \vee r) \vee \sim (q \vee r)]$ [Distributive law]

$\equiv p \wedge t$ [Complement law]

$\equiv p$ [Identity law]

Answer: (A)

Homework

Practice Questions of every section in the Assignment

(Q. 16–24, 33–40, 49–53, 61–70, 79–84, 94–95)