

Probability

Random experiment · Sample space · Event · Classical definition · Questions based on P & C

Class notes · Part 1

Random Experiment

An experiment which satisfies the following conditions:

1. It should have **more than one outcome**
2. Outcomes are **non predictable**

Tossing a coin

Rolling a die

Drawing a card

Not random: dropping a stone from the roof. The result can be predicted every time.

Sample Space

The set of **all possible outcomes** of a random experiment.

Written as S . The number of outcomes is $n(S)$.

Experiment	Sample space S	$n(S)$
1 coin tossed	$\{H, T\}$	2
2 coins tossed	$\{HH, HT, TH, TT\}$	4
1 die thrown	$\{1, 2, 3, 4, 5, 6\}$	6

Event

An event is a **subset of the sample space**: the outcomes we are interested in (favourable outcomes).

Example: throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

$A = \text{getting an odd outcome} = \{1, 3, 5\}$, $n(A) = 3$

- Sure event: $A = S$
- Impossible event: $A = \emptyset$

Classical Definition of Probability

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = n(A) / n(S)$$

$$0 \leq P(A) \leq 1$$

Valid only when all outcomes are **equally likely** and S is **finite**.

$P(A) = 0$: impossible event · $P(A) = 1$: sure event · $P(\text{not } A) = 1 - P(A)$

When Does Favourable $\div$ Total Work?

Same sample space $S = \{o_1, o_2, o_3, o_4, o_5\}$, same event $E = \{o_1, o_3, o_4\}$. Everything depends on **how likely each outcome is**.

Equally likely

Each outcome has probability p . Total = 1, so $5p = 1$ and $p = 1/5$.

$$P(E) = 1/5 + 1/5 + 1/5 = 3/5$$

Formula $n(E)/n(S)$ works ✓

Not equally likely

$P(o_1..o_5) = 3/10, 1/10, 3/10, 1/10, 2/10$ (sum = 1)

$$P(E) = 3/10 + 3/10 + 1/10 = 7/10, \text{ not } 3/5$$

Formula $n(E)/n(S)$ fails ✗

Always: **$P(E)$ = sum of probabilities of the favourable outcomes**. The shortcut $n(E)/n(S)$ is just this sum when every outcome has the same probability.

Odds in Favour and Odds Against

Odds in favour of A

favourable : unfavourable

$$= n(A) : n(\text{not } A)$$

Odds against A

unfavourable : favourable

$$= n(\text{not } A) : n(A)$$

Odds to probability

$$\text{Odds in favour } a : b \Rightarrow P(A) = a / (a + b), \quad P(\text{not } A) = b / (a + b)$$

Example: one die, $A = \{4, 5\}$. Odds in favour = $2 : 4 = 1 : 2$, odds against = $2 : 1$, $P(A) = 1/3$.

Important Notes

1. Sum of all sample points

$$S = \{x_1, x_2, \dots, x_n\}: P(x_1) + P(x_2) + \dots + P(x_n) = 1$$

2. Limits

$P(E) = 0$: impossible event · $P(E) = 1$: sure event · always $0 \leq P(E) \leq 1$

3. Odds

Odds in favour of E = favourable : unfavourable · Odds against E = unfavourable : favourable

4. Non-occurrence

$P(\text{not } E) = P(\bar{E}) = 1 - P(E)$ (use the complement for "at least one" questions)

1 Coin vs 2 Coins vs 3 Coins ...

Coins tossed	Outcomes	$n(S)$
1 coin	H, T	$2 = 2^1$
2 coins	HH, HT, TH, TT	$4 = 2^2$
3 coins	HHH, HHT, HTH, THH, HTT, THT, TTH, TTT	$8 = 2^3$
n coins	each coin has 2 choices	2^n

$$P(\text{exactly } r \text{ heads in } n \text{ tosses}) = {}^nC_r / 2^n$$

Example: 3 tosses, exactly 2 heads: ${}^3C_2 / 8 = 3/8$

Pair of Dice: 36 Outcomes

a \ b	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$n(S) = 6 \times 6 = \mathbf{36}$$

(a, b) is an ordered pair: (2, 5) and (5, 2) are different outcomes.

Doublets: 6, the green diagonal (1,1) ... (6,6)

Sum of Two Dice: Shortcut

Sum	2	3	4	5	6	7	8	9	10	11	12
Ways	1	2	3	4	5	6	5	4	3	2	1

Ways for sum $s = 6 - |s - 7|$ (sum 7 is the peak with 6 ways)

Check: $1+2+3+4+5+6+5+4+3+2+1 = 36$

Example: sum is a prime number

Sums 2, 3, 5, 7, 11: $1 + 2 + 4 + 6 + 2 = 15$ ways, so $P = 15/36 = 5/12$

Deck of Cards = 52 Cards

Suit	Colour	Cards
Spades ♠	Black	13
Clubs ♣	Black	13
Hearts ♥	Red	13
Diamonds ♦	Red	13

Each suit: A, 2, 3, ... 10, J, Q, K

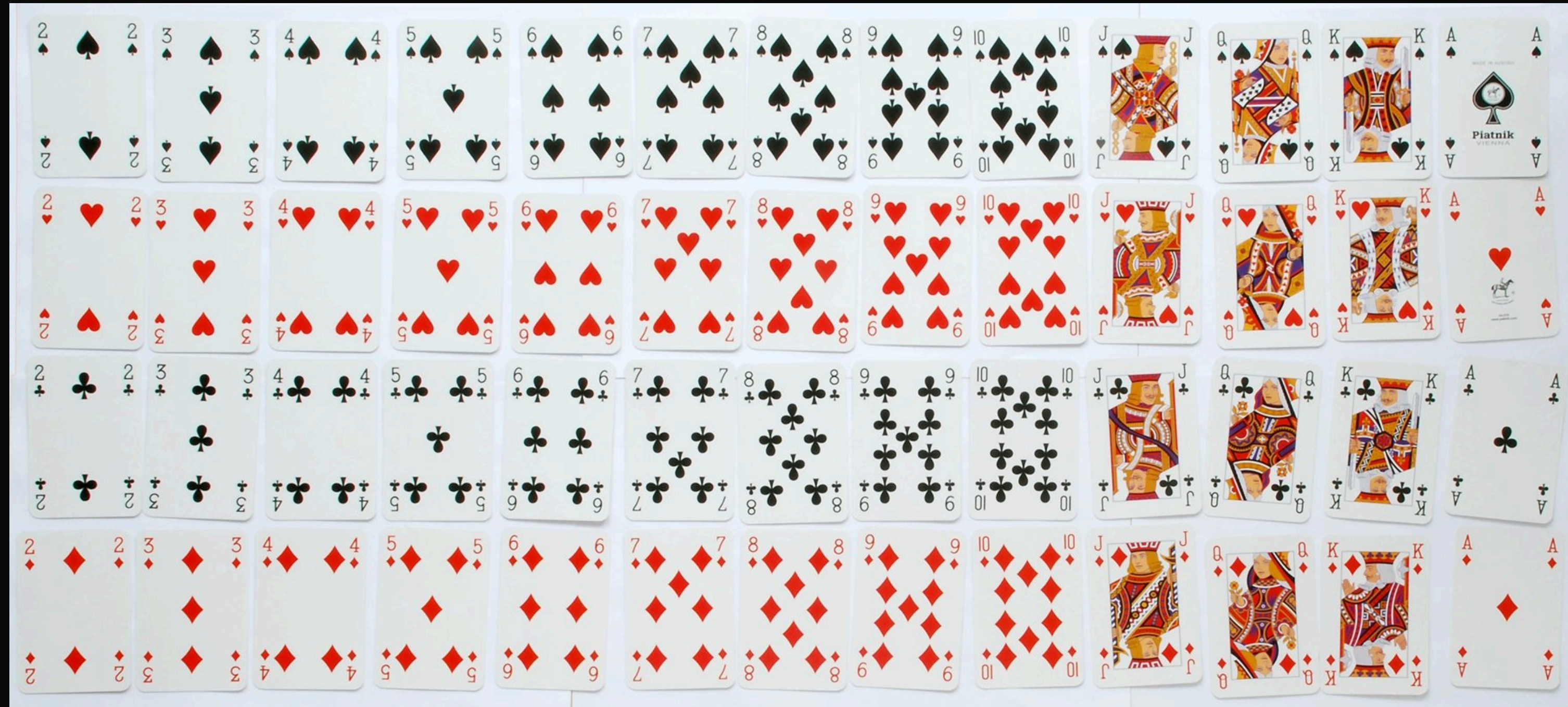
Face cards: J, Q, K = 12

Aces 4 · Kings 4 · Queens 4

Red 26 · Black 26

One card: $n(S) = 52$ · Two cards: $n(S) = {}^{52}C_2 = 1326$ · Three cards: $n(S) = {}^{52}C_3$

The 52 Cards



1

MHT CET 2020

Ans: C

The letters of the word **LOGARITHM** are arranged at random. The probability that an arrangement **starts with a vowel and ends with a consonant** is

- A.** $71/9!$ **B.** $18/9!$ **C.** $1/4$ **D.** $1/9$

2

JEE Main 2010 · MHT CET 2024

Ans: B

An urn contains nine balls of which **three are red, four are blue and two are green**. Three balls are drawn at random without replacement. The probability that the three balls have **different colours** is

- A.** $1/3$ **B.** $2/7$ **C.** $1/21$ **D.** $2/23$

3

MHT CET 2010

Ans: C

A **four-digit number** is to be formed using the digits 1, 2, 3, 4, 5, 6, 7 (no digit repeated in any number). The probability that it is **greater than 4000** is

- A.** $3/2$ **B.** $1/2$ **C.** $4/7$ **D.** $3/7$

4

MHT CET 2018

Ans: C

The letters of the word **HULULULU** are rearranged. The probability that **all three L's are together** is

- A.** $3/20$ **B.** $2/5$ **C.** $3/28$ **D.** $5/23$

5

JEE Main 2021 (20 Jul Shift 1)

Ans: B

Words with or without meaning are to be formed using all the letters of the word **EXAMINATION**. The probability that the letter **M** appears at the **fourth position** in any such word is

- A.** $1/66$ **B.** $1/11$ **C.** $1/9$ **D.** $2/11$

6

MHT CET 2025 (27 Apr Shift 2)

Ans: D

If **6 boys and 3 girls** are to be seated on chairs for a photograph, the probability that the **end seats are occupied by girls** and **no two girls are side by side** is

- A.** $5/12$ **B.** $1/28$ **C.** $1/84$ **D.** $5/84$

7

JEE Main 2012

Ans: B

If **six students**, including two particular students **A and B**, stand in a row, the probability that A and B are separated with **exactly one student in between** them is

- A.** $8/15$ **B.** $4/15$ **C.** $2/15$ **D.** $1/15$

8

JEE Main 2025 (4 Apr Shift 1)

Ans: A

The probability of forming a **12-person committee** from 4 engineers, 2 doctors and 10 professors, containing **at least 3 engineers and at least 1 doctor**, is

- A.** $129/182$ **B.** $103/182$ **C.** $17/26$ **D.** $19/26$

9

MHT CET 2023

Ans: C

Three of the six vertices of a **regular hexagon** are chosen at random. The probability that the triangle formed is **equilateral** is

- A.** $1/2$ **B.** $1/5$ **C.** $1/10$ **D.** $1/20$

10

JEE Main 2024 (6 Apr Shift 2)

Ans: A

If **three letters** can be posted to any one of **5 different addresses**, the probability that the three letters are posted to **exactly two addresses** is

- A.** $12/25$ **B.** $18/25$ **C.** $4/25$ **D.** $6/25$

11

JEE Main 2021 (16 Mar Shift 2)

Ans: B

A **6-digit integer** is formed from the digits 0, 1, 2, 3, 4, 5, 6 **without repetition**. The probability that it is **divisible by 3** is

- A.** $9/56$ **B.** $4/9$ **C.** $3/7$ **D.** $11/27$

12

JEE Main 2022 (25 Jul Shift 1)

Ans: A

If the numbers appearing on **two throws of a fair six-faced die** are α and β , the probability that $x^2 + \alpha x + \beta > 0$ for all $x \in \mathbb{R}$ is

- A.** $17/36$ **B.** $4/9$ **C.** $1/2$ **D.** $19/36$

13

JEE Main 2025 (24 Jan Shift 2)

Ans: D

Let $A = [a_{ij}]$ be a **square matrix of order 2** with entries either **0 or 1**. Let E be the event that **A is invertible**. Then $P(E)$ is

- A.** $5/8$ **B.** $3/16$ **C.** $1/8$ **D.** $3/8$

14

JEE Main 2020 (6 Sep Shift 1)

Ans: C

Out of **11 consecutive natural numbers**, three numbers are selected at random (without repetition). The probability that they are in **A.P. with positive common difference** is

- A.** $15/101$ **B.** $5/101$ **C.** $5/33$ **D.** $10/99$

15

JEE Main 2024 (27 Jan Shift 2)

Ans: C

An urn contains **6 white and 9 black balls**. Two successive draws of 4 balls are made **without replacement**. The probability that the first draw gives **all white** balls and the second draw gives **all black** balls is

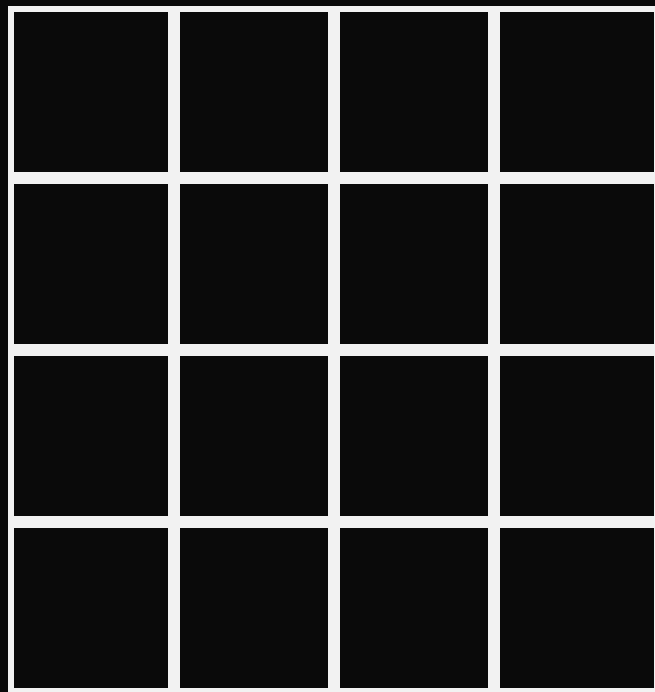
- A.** $5/256$ **B.** $5/715$ **C.** $3/715$ **D.** $3/256$

16

JEE Main 2025 (23 Jan Shift 2)

Ans: A

A board has **16 squares** (a 4×4 grid). Two squares are chosen at random. The probability that they have **no side in common** is



- A.** $4/5$ **B.** $7/10$ **C.** $3/5$ **D.** $23/30$

17

JEE Main 2025 (28 Jan Shift 2)

Ans: D

Let S be the set of all words formed by arranging all the letters of **GARDEN**. One word is selected at random. The probability that it will **NOT** have the vowels in alphabetical order is

- A.** $1/4$ **B.** $2/3$ **C.** $1/3$ **D.** $1/2$

18

JEE Main 2025 (28 Jan Shift 1)

Ans: C

Two numbers k_1 and k_2 are randomly chosen from the set of **natural numbers**. The probability that $i^{k_1} + i^{k_2}$ ($i = \sqrt{-1}$) is **non-zero** equals

- A.** $1/2$ **B.** $1/4$ **C.** $3/4$ **D.** $2/3$

19

JEE Main 2025 (3 Apr Shift 1) · Numerical

Ans: 183

All five-letter words are made using all the letters **A, B, C, D, E** and arranged as in an English dictionary with serial numbers. Let the word at serial number n be W_n . Let $P(W_n) = 2 \cdot P(W_{n-1})$, $n > 1$. If $P(\text{CDBEA}) = 2^\alpha / (2^\beta - 1)$, with $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta = \underline{\hspace{2cm}}$