

JEE-M Full Syllabus Test 01

Paper- Discussion

Mathematics · Questions 51 to 75

The sum of all the real roots of the equation $(x + 2)^2 - 7|x + 2| + 10 = 0$ is :

- 1) -4
- 2) -8
- 3) 4
- 4) 8

Let $a_1, a_2, a_3, \dots$ be an A.P. and $g_1 = a_1, g_2, g_3, \dots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 2$ and $a_3 + g_3 = 8$, then $a_8 + g_4$ is equal to :

- 1) 10
- 2) -26
- 3) 18
- 4) 14

The number of solutions of the equation $\cos 2\theta + 7 \cos \theta + 4 = 0$, $\theta \in [-\pi, 3\pi]$ is

- 1) 3
- 2) 4
- 3) 5
- 4) 6

The coefficient of x^3 in the expansion of $\left(2x^2 + \frac{1}{x}\right)^9$, $x \neq 0$, is

- 1) 1008
- 2) 4032
- 3) 2016
- 4) 2520

Let

$$f(x) = \int \frac{7x^6 + 5x^8}{(1 + x^2 + 2x^7)^2} dx, \quad x > 0, \quad \lim_{x \rightarrow 0} f(x) = 0 \text{ and } f(1) = \frac{1}{4}.$$

If

$$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$$

and $B = \text{adj}(\text{adj } A)$ is such that $|B| = 625$, then α^2 is equal to :

- 1) 4
- 2) 8
- 3) 12
- 4) 6

If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, then the determinant of the matrix

$$\left(A^{2026} - 2A^{2025} + A^{2024} \right)$$

is

- 1) 9
- 2) 3
- 3) 6
- 4) 18

A rectangle is formed by the lines $x = 0$, $y = 0$, $x = 5$ and $y = 2$. Let the line L be perpendicular to $x + 2y + 5 = 0$ and divide the area of the rectangle into two equal parts. Then the distance of the point $(-1, 3)$ from the line L is equal to :

- 1) $\frac{9}{5}$
- 2) $\frac{18}{\sqrt{5}}$
- 3) $9\sqrt{5}$
- 4) $\frac{9}{\sqrt{5}}$

Let C be a circle having its centre in the first quadrant and touching the x -axis at a distance of 6 units from the origin. If C has an intercept of length 16 on the y -axis, then the length of the chord of C on the line $x - y = 6$ is :

- 1) 10
- 2) $8\sqrt{5}$
- 3) $10\sqrt{2}$
- 4) 20

Let S and S' be the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and $P(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(SP)^2 + (S'P)^2 - SP \cdot S'P = 28$, then $\alpha^2 + \beta^2$ is equal to :

1) 11

2) 9

3) 12

4) 13

Let $P(8, 3\sqrt{3})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S' . If the length of its latus rectum is $\frac{9}{2}$, then the square of the area of $\triangle PSS'$ is equal to :

- 1) 600
- 2) 675
- 3) 750
- 4) 900

Let $\vec{a} = \hat{i} - 3\hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - \hat{k}$, $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} + 2\vec{b}) \cdot \vec{d}$ is equal to :

- 1) 6
- 2) -12
- 3) 12
- 4) 18

Let the line L pass through the point $(4, 2, -3)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(5, r, -4)$ is $2\sqrt{2}$, then the sum of all possible values of r is :

- 1) 2
- 2) 6
- 3) 8
- 4) 4

If the image of the point $P(2, 1, a)$ in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{3-z}{2}$ is $Q(4, b, c)$, then $a^2 + b^2 + c^2$ is equal to

1) 35

2) 29

3) 41

4) 45

If the domain of the function

$$f(x) = \log_3 \log_5 \left(8 - \log_2 (x^2 - 10x + 104) \right) + \sin^{-1} \left| \frac{2x - 3}{9 - x} \right|$$

is $(\alpha, \beta]$, then $\alpha + 2\beta$ is equal to :

- 1) 4
- 2) 8
- 3) 6
- 4) 10

Considering the principal values of the inverse trigonometric functions, the value of $\tan\left(2\sin^{-1}\frac{1}{\sqrt{10}} - 2\cos^{-1}\frac{4}{\sqrt{17}}\right)$ is equal to :

- 1) $\frac{77}{36}$
- 2) $\frac{13}{84}$
- 3) $-\frac{13}{84}$
- 4) $\frac{36}{77}$

Let $[t]$ denote the greatest integer less than or equal to t . If the function

$$f(x) = \begin{cases} b^2 \sin\left(\frac{\pi}{2} \left[\frac{\pi}{2}(\cos x + \sin x) \cos x\right]\right), & x < 0 \\ \frac{e^x - 1 - x - \frac{x^2}{2}}{x^3}, & x > 0 \\ a, & x = 0 \end{cases}$$

is continuous at $x = 0$, then $36(a^2 + b^2)$ is equal to :

- 1) 5
- 2) 6
- 3) 8
- 4) 7

Let $f(x)$ be a polynomial of degree 5 having extrema at $x = 2$ and $x = -2$. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = -20$, then $f(3) - f(-3)$ is equal to :

- 1) 378
- 2) 270
- 3) 324
- 4) 432

The value of the integral $\int_{\pi/20}^{\pi/5} \frac{dx}{1 + (\tan 2x)^{1/5}}$ is :

- 1) $\frac{\pi}{40}$
- 2) $\frac{\pi}{20}$
- 3) $\frac{3\pi}{40}$
- 4) $\frac{3\pi}{20}$

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Area Under Curves · Area between two parabolas

The area of the region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 4$ is equal to :

1) $\frac{16}{3}$

2) $\frac{32}{3}$

3) 8

4) $\frac{64}{3}$

Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (3x^2 y + 4 \cos x) dx = 0$, $x > 0$, $y\left(\frac{\pi}{2}\right) = 0$. Then $\pi^3 y\left(\frac{\pi}{6}\right)$ is equal to :

- 1) 216
- 2) 324
- 3) 864
- 4) 432

If $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \cdots + 50(1+x)^{50}$, $x \neq 0$, and $(50)^2 S(50) = a(b)^b + b$, where $a, b \in \mathbb{N}$, then $(a+b)/10$ is equal to _____.

Let ABC be a triangle. Consider three points p_1, p_2, p_3 on the side AB , four points p_4, p_5, p_6, p_7 on the side BC and five points $p_8, p_9, p_{10}, p_{11}, p_{12}$ on the side AC . None of these points is a vertex of the triangle ABC . Then the total number of pentagons that can be formed by taking all the vertices from the points $p_1, p_2, \dots, p_{12}$ is

_____.

Let $A = \{1, 2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y) R (z, w)$ if and only if x divides z and $y < w$. Then the number of elements in R is _____.

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Parabola · Locus of centroid · Right-angled triangle inscribed in a parabola

Let A , B and C be the vertices of a variable right-angled triangle inscribed in the parabola $y^2 = 12x$. Let the vertex B containing the right angle be $(12, 12)$ and the locus of the centroid of $\triangle ABC$ be a conic C_0 . Then the sum of the abscissa of the vertex of C_0 and the length of its latus rectum is _____.

Let $[\cdot]$ denote the greatest integer function. If

$$\alpha = \int_0^{36} (\sqrt{x} - [\sqrt{x}]) dx,$$

then

$$\frac{1}{\pi} \int_0^{\alpha\pi} \frac{\sin^2 \theta + 3 \cos^2 \theta}{\sin^6 \theta + \cos^6 \theta} d\theta$$

is equal to _____.