A Premier Institute for Pre-Medical & Pre Engineering

SRI VIDYA ARADHANA ACADEMY

"Transforming Your DREAMS Into Reality...!"

PCMB PHYSICS CHEMISTRY MATHEMATICS BIOLOGY

NEET/JEE

Topic: Polynomial

Sub: Mathematics **Solutions to Assignment: 01** Prof. Chetan Sir

Focus: Polynomial long division, applying the Remainder Theorem and Factor Theorem.

Polynomial Basics (Definition, Degree, Coefficient, Evaluation)

1. Which of the following expressions is **not** a polynomial?
(A) $5x^3 - \sqrt{2}x + 4$ (B) $\frac{x^2}{3} + 7$ (C) 10 (D) $x^{-2} + 3x + 1$
Solution:

- A polynomial in x is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where the exponents $(n, n-1, \ldots, 1, 0)$ must be non-negative integers.
- Option (A), $5x^3 - \sqrt{2}x + 4$, has exponents 3, 1, 0. $\sqrt{2}$ is a coefficient, which is allowed. This is a polynomial.
- Option (B), $\frac{x^2}{3} + 7 = \frac{1}{3}x^2 + 7$, has exponents 2, 0. This is a polynomial.
- Option (C), $10 = 10x^0$, has exponent 0. This is a constant polynomial.
- Option (D), $x^{-2} + 3x + 1$, has an exponent -2, which is negative.

Therefore, $x^{-2} + 3x + 1$ is not a polynomial.
The correct option is (D).

2. Identify the expression that does **not** represent a polynomial.
(A) $y^5 - 3y^2 + \frac{1}{4}$ (B) $t^2 + 2\sqrt{t} - 1$ (C) $a^3 + 3a^2b - 4b^3 + 7ab$ (D) $-8x^4 + 2x^3 - 5x^2 + x$
Solution:

- A polynomial must have non-negative integer exponents for its variables.
- Option (A) has exponents 5, 2, 0 for y. This is a polynomial.
- Option (B), $t^2 + 2\sqrt{t} - 1 = t^2 + 2t^{1/2} - 1$, has an exponent 1/2, which is not an integer.
- Option (C) is a polynomial in two variables a and b. All exponents are non-negative integers.
- Option (D) has exponents 4, 3, 2, 1 for x. This is a polynomial.

Therefore, $t^2 + 2\sqrt{t} - 1$ is not a polynomial.
The correct option is (B).

3. Which of the following polynomials has a degree equal to 3?
(A) $7x^2 - 9x + 15$ (B) $4x^3 - 2x(2x^2 + 5)$
(C) $(x^2 + 1)(x^2 - 1) - x^4 + 5x^2 + 2x^3$ (D) $(x + 1)^3 - x^3 - 3x^2$
Solution: We find the degree (highest power of the variable after simplification) for each option:

- (A) $7x^2 - 9x + 15$: The highest power is 2. Degree = 2.
- (B) $4x^3 - 2x(2x^2 + 5) = 4x^3 - 4x^3 - 10x = -10x$: The highest power is 1. Degree = 1.
- (C) $(x^2 + 1)(x^2 - 1) - x^4 + 5x^2 + 2x^3 = (x^4 - 1) - x^4 + 5x^2 + 2x^3$ .... (Using $(a + b)(a - b) = a^2 - b^2$) $= x^4 - 1 - x^4 + 5x^2 + 2x^3 = 2x^3 + 5x^2 - 1$: The highest power is 3. Degree = 3.
- (D) $(x + 1)^3 - x^3 - 3x^2 = (x^3 + 3x^2 + 3x + 1) - x^3 - 3x^2$ .... (Using $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$) $= x^3 + 3x^2 + 3x + 1 - x^3 - 3x^2 = 3x + 1$: The highest power is 1. Degree = 1.

The polynomial in option (C) has a degree of 3.
The correct option is (C).

4. What is the leading coefficient of the polynomial $P(x) = 8 - 2x + 5x^4 - x^3$?
(A) 5 (B) -2 (C) 8 (D) -1
Solution:

- First, write the polynomial in standard form (descending powers of x): $P(x) = 5x^4 - x^3 - 2x + 8$.
- The term with the highest degree is $5x^4$.
- The leading coefficient is the coefficient of this term.

The leading coefficient is 5.
The correct option is (A).

5. Evaluate the polynomial $P(y) = 2y^3 - y^2 + 5y - 3$ when $y = -1$.
(A) -11 (B) -7 (C) -1 (D) 3
Solution:

- Substitute $y = -1$ into the polynomial expression: $P(-1) = 2(-1)^3 - (-1)^2 + 5(-1) - 3$
- Calculate the powers: $(-1)^3 = -1$, $(-1)^2 = 1$. $P(-1) = 2(-1) - (1) + 5(-1) - 3$
- Perform multiplication and addition/subtraction: $P(-1) = -2 - 1 - 5 - 3 = -11$.

The value of the polynomial is -11.
The correct option is (A).

6. If $p(x) = x + 4$, then $p(x) + p(-x)$ is equal to:
(A) 0 (B) $2x$ (C) 4 (D) 8
Solution:

- Given $p(x) = x + 4$.
- Find $p(-x)$ by substituting $-x$ for x: $p(-x) = (-x) + 4 = -x + 4$.
- Calculate the sum $p(x)+p(-x)$: $p(x)+p(-x) = (x+4)+(-x+4) = x+4-x+4 = (x-x)+(4+4) = 0+8 = 8$.

The value of $p(x) + p(-x)$ is 8.
The correct option is (D).

Polynomial Long Division

7. Using long division, find the quotient when $x^3 + 5x^2 + 7x + 2$ is divided by $x + 2$.
(A) $x^2 + 7x + 21$ (B) $x^2 + 3x + 1$ (C) $x^2 - 3x - 1$ (D) $x^2 + 5x + 7$
Solution: Perform polynomial long division:

$$\begin{array}{r}
x^2 + 3x + 1 \\
x + 2 \overline{)x^3 + 5x^2 + 7x + 2} \\
-(x^3 + 2x^2) \\
\hline
3x^2 + 7x \\
-(3x^2 + 6x) \\
\hline
x + 2 \\
-(x + 2) \\
\hline
0
\end{array}$$

The quotient is $x^2 + 3x + 1$.
The correct option is (B).

8. What is the remainder when $2x^3 - 3x^2 + 4x - 5$ is divided by $x - 1$ using long division?
(A) -2 (B) 0 (C) -5 (D) 2
Solution: We can use the Remainder Theorem or long division. Method 1: Remainder Theorem

- Let $P(x) = 2x^3 - 3x^2 + 4x - 5$. The divisor is $x - 1$.
- The remainder is $P(1)$. .... (Remainder Theorem)
- $P(1) = 2(1)^3 - 3(1)^2 + 4(1) - 5 = 2 - 3 + 4 - 5 = -2$.

Method 2: Long Division

$$\begin{array}{r}
2x^2 - x + 3 \\
x - 1 \overline{)2x^3 - 3x^2 + 4x - 5} \\
-(2x^3 - 2x^2) \\
\hline
-x^2 + 4x \\
-(-x^2 + x) \\
\hline
3x - 5 \\
-(3x - 3) \\
\hline
-2
\end{array}$$

The remainder is -2.
The correct option is (A).

9. Divide $6x^4 + 5x^3 - 3x^2 + 5x - 1$ by $2x^2 - x + 1$. What is the quotient?
(A) $3x^2 - 4x - 1$ (B) $3x^2 + 4x - 1$ (C) $3x^2 - 4x + 1$ (D) $3x^2 + 4x + 1$
Solution: Perform polynomial long division:

$$\begin{array}{r}
3x^2 + 4x - 1 \\
2x^2 - x + 1 \overline{)6x^4 + 5x^3 - 3x^2 + 5x - 1} \\
-(6x^4 - 3x^3 + 3x^2) \\
\hline
8x^3 - 6x^2 + 5x \\
-(8x^3 - 4x^2 + 4x) \\
\hline
-2x^2 + x - 1 \\
-(-2x^2 + x - 1) \\
\hline
0
\end{array}$$

The quotient is $3x^2 + 4x - 1$.
The correct option is (B).

10. When dividing $x^4 - 2x^2 + 5x - 3$ by $x + 3$, what is the remainder obtained via long division?
(A) -3 (B) 51 (C) 45 (D) 57
Solution: Using the Remainder Theorem:

- Let $P(x) = x^4 - 2x^2 + 5x - 3$. The divisor is $x + 3 = x - (-3)$.
- The remainder is $P(-3)$. .... (Remainder Theorem)
- $P(-3) = (-3)^4 - 2(-3)^2 + 5(-3) - 3$
- $P(-3) = 81 - 2(9) - 15 - 3 = 81 - 18 - 15 - 3 = 45$.

The remainder is 45.
The correct option is (C).

11. Find the quotient and remainder when $4x^3 - 8x^2 + 3x + 9$ is divided by $2x + 1$.
(A) Quotient $2x^2 - 5x + 4$, Remainder 5 (B) Quotient $2x^2 + 5x - 4$, Remainder -5
(C) Quotient $2x^2 + 5x + 4$, Remainder 5 (D) Quotient $2x^2 - 5x - 4$, Remainder 13
Solution: Perform polynomial long division:

$$\begin{array}{r}
2x^2 - 5x + 4 \\
2x + 1 \overline{)4x^3 - 8x^2 + 3x + 9} \\
-(4x^3 + 2x^2) \\
\hline
-10x^2 + 3x \\
-(-10x^2 - 5x) \\
\hline
8x + 9 \\
-(8x + 4) \\
\hline
5
\end{array}$$

The quotient is $2x^2 - 5x + 4$ and the remainder is 5.
The correct option is (A).

12. Divide $x^5 + x^3 - 2x$ by $x^2 + 1$. What is the quotient?
(A) $x^3 - 2$ (B) $x^3 + 1$ (C) x^3 (D) $x^3 + 2x$
Solution: Perform polynomial long division (include 0 coefficients for missing terms):

$$\begin{array}{r}
x^3 \\
x^2 + 0x + 1 \overline{)x^5 + 0x^4 + x^3 + 0x^2 - 2x + 0} \\
-(x^5 + 0x^4 + x^3) \\
\hline
0 + 0 - 2x + 0
\end{array}$$

The quotient is x^3 and the remainder is $-2x$. The question asks for the quotient.
The correct option is (C).

13. Using long division for $(3x^3 - 7x^2 + 10x - 8) \div (x - 1)$, the remainder is:
(A) 0 (B) -2 (C) 2 (D) -8
Solution: Using the Remainder Theorem:

- Let $P(x) = 3x^3 - 7x^2 + 10x - 8$. The divisor is $x - 1$.
- The remainder is $P(1)$. .... (Remainder Theorem)
- $P(1) = 3(1)^3 - 7(1)^2 + 10(1) - 8 = 3 - 7 + 10 - 8 = -2$.

The remainder is -2.
The correct option is (B).

14. Find the quotient when $x^3 - 8$ is divided by $x - 2$.
(A) $x^2 + 2x + 4$ (B) $x^2 - 2x + 4$ (C) $x^2 + 4$ (D) $x^2 - 4$
Solution: Method 1: Using formula for difference of cubes

- $x^3 - 8 = x^3 - 2^3$.
- Using the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
- $x^3 - 2^3 = (x - 2)(x^2 + x(2) + 2^2) = (x - 2)(x^2 + 2x + 4)$.
- When $(x - 2)(x^2 + 2x + 4)$ is divided by $(x - 2)$, the quotient is $(x^2 + 2x + 4)$.

Method 2: Long Division

$$
\begin{array}{r}
x^2 + 2x + 4 \\
x - 2\overline{)x^3 + 0x^2 + 0x - 8} \\
-(x^3 - 2x^2) \\
\hline
2x^2 + 0x \\
-(2x^2 - 4x) \\
\hline
4x - 8 \\
-(4x - 8) \\
\hline
0
\end{array}
$$

The quotient is $x^2 + 2x + 4$.
The correct option is (A).

Remainder Theorem & Applications

15. The polynomial $4x^2 - kx + 7$ leaves a remainder of -2 when divided by $x - 3$. Find the value of k.
(A) $k = -15$ (B) $k = 15$ (C) $k = 43/3$ (D) $k = 45$
Solution:

- Let $P(x) = 4x^2 - kx + 7$.
- By the Remainder Theorem, the remainder when $P(x)$ is divided by $x - 3$ is $P(3)$.
- We are given that the remainder is -2. So, $P(3) = -2$.
- $P(3) = 4(3)^2 - k(3) + 7 = 4(9) - 3k + 7 = 36 - 3k + 7 = 43 - 3k$.
- Set the calculated remainder equal to the given remainder: $43 - 3k = -2$.
- Solve for k: $-3k = -2 - 43 \implies -3k = -45 \implies k = 15$.

The value of k is 15.
The correct option is (B).

16. If $P_1(x) = 2x^3 + kx^2 + 4x - 12$ and $P_2(x) = x^3 + x^2 - 2x + k$ leave the same remainder when divided by $(x - 3)$, find the value of k.
(A) $k = 3$ (B) $k = -3$ (C) $k = 27$ (D) $k = -27$
Solution:

- By the Remainder Theorem, the remainder when $P_1(x)$ is divided by $x - 3$ is $R_1 = P_1(3)$.
- The remainder when $P_2(x)$ is divided by $x - 3$ is $R_2 = P_2(3)$.
- We are given $R_1 = R_2$, so $P_1(3) = P_2(3)$.
- Calculate $P_1(3)$: $P_1(3) = 2(3)^3 + k(3)^2 + 4(3) - 12 = 2(27) + 9k + 12 - 12 = 54 + 9k$.
- Calculate $P_2(3)$: $P_2(3) = (3)^3 + (3)^2 - 2(3) + k = 27 + 9 - 6 + k = 30 + k$.
- Set $P_1(3) = P_2(3)$: $54 + 9k = 30 + k$.
- Solve for k: $9k - k = 30 - 54 \implies 8k = -24 \implies k = -3$.

The value of k is -3.
The correct option is (B).

17. Find the value of k if $p(x) = (3x - 2)(x - k) - 8$ is divided by $(x - 2)$ leaving the remainder 4.
(A) $k = 1$ (B) $k = -1$ (C) $k = 2$ (D) $k = -2$
Solution:

- By the Remainder Theorem, the remainder when $p(x)$ is divided by $x - 2$ is $p(2)$.
- We are given that the remainder is 4. So, $p(2) = 4$.
- Calculate $p(2)$: $p(2) = (3(2) - 2)(2 - k) - 8 = (6 - 2)(2 - k) - 8 = 4(2 - k) - 8$.
- Set $p(2) = 4$: $4(2 - k) - 8 = 4$.

- Solve for k: $8 - 4k - 8 = 4 \implies -4k = 4 \implies k = -1$.

The value of k is -1.
The correct option is (B).

18. Let R_1 and R_2 be the remainders when $P_1(x) = x^3 + 2x^2 - 5ax - 7$ and $P_2(x) = x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, find the value of a.
(A) $a = -2$ (B) $a = 1$ (C) $a = 6/7$ (D) $a = 2$
Solution:

- Find R_1 using the Remainder Theorem with $P_1(x)$ and divisor $x + 1 = x - (-1)$: $R_1 = P_1(-1) = (-1)^3 + 2(-1)^2 - 5a(-1) - 7 = -1 + 2(1) + 5a - 7 = 1 + 5a - 7 = 5a - 6$.
- Find R_2 using the Remainder Theorem with $P_2(x)$ and divisor $x - 2$: $R_2 = P_2(2) = (2)^3 + a(2)^2 - 12(2) + 6 = 8 + 4a - 24 + 6 = 4a - 10$.
- Use the given condition $2R_1 + R_2 = 6$: $2(5a - 6) + (4a - 10) = 6$.
- Solve for a: $10a - 12 + 4a - 10 = 6 \implies 14a - 22 = 6 \implies 14a = 28 \implies a = 2$.

The value of a is 2.
The correct option is (D).

19. If the polynomials $P_1(x) = ax^3 + 4x^2 + 3x - 4$ and $P_2(x) = x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, find the value of a.
(A) $a = 1$ (B) $a = -1$ (C) $a = 2$ (D) $a = -2$
Solution:

- By the Remainder Theorem, the remainders are $P_1(3)$ and $P_2(3)$.
- We are given $P_1(3) = P_2(3)$.
- Calculate $P_1(3)$: $P_1(3) = a(3)^3 + 4(3)^2 + 3(3) - 4 = 27a + 4(9) + 9 - 4 = 27a + 36 + 9 - 4 = 27a + 41$.
- Calculate $P_2(3)$: $P_2(3) = (3)^3 - 4(3) + a = 27 - 12 + a = 15 + a$.
- Set $P_1(3) = P_2(3)$: $27a + 41 = 15 + a$.
- Solve for a: $27a - a = 15 - 41 \implies 26a = -26 \implies a = -1$.

The value of a is -1.
The correct option is (B).

20. Find the value of k if the remainder is -3 when $kx^3 + 8x^2 - 4x + 10$ is divided by $x + 1$.
(A) $k = -25$ (B) $k = 25$ (C) $k = 21$ (D) $k = -21$
Solution:

- Let $P(x) = kx^3 + 8x^2 - 4x + 10$. The divisor is $x + 1 = x - (-1)$.
- By the Remainder Theorem, the remainder is $P(-1)$.
- We are given the remainder is -3. So, $P(-1) = -3$.
- Calculate $P(-1)$: $P(-1) = k(-1)^3 + 8(-1)^2 - 4(-1) + 10 = k(-1) + 8(1) + 4 + 10 = -k + 22$.
- Set $P(-1) = -3$: $-k + 22 = -3$.
- Solve for k: $-k = -3 - 22 \implies -k = -25 \implies k = 25$.

The value of k is 25.
The correct option is (B).

21. What number should be added to $x^2 + 5$ so that the resulting polynomial leaves the remainder 3 when divided by $x + 3$?
(A) 11 (B) -11 (C) 14 (D) -14
Solution:

- Let the number added be C. The resulting polynomial is $P(x) = x^2 + 5 + C$.
- The divisor is $x + 3 = x - (-3)$.
- By the Remainder Theorem, the remainder is $P(-3)$.
- We are given the remainder is 3. So, $P(-3) = 3$.
- Calculate $P(-3)$: $P(-3) = (-3)^2 + 5 + C = 9 + 5 + C = 14 + C$.
- Set $P(-3) = 3$: $14 + C = 3$.
- Solve for C: $C = 3 - 14 = -11$.

The number to be added is -11.
The correct option is (B).

22. What is the remainder when $x^{2018} + 2018$ is divided by $(x - 1)$?
(A) 2017 (B) 2018 (C) 1 (D) 2019
Solution:

- Let $P(x) = x^{2018} + 2018$. The divisor is $x - 1$.
- By the Remainder Theorem, the remainder is $P(1)$.
- Calculate $P(1)$: $P(1) = (1)^{2018} + 2018 = 1 + 2018 = 2019$.

The remainder is 2019.
The correct option is (D).

23. A polynomial $p(x)$ when divided by $(x - 1)$, $(x + 1)$ and $(x + 2)$ gives remainder 5, 7 and 2 respectively. If $p(x)$ is divided by $(x^2 - 1)$, the remainder is $R(x)$. Find $R(50)$.
(A) 34 (B) -44 (C) 44 (D) 104
Solution:

- From the given information and the Remainder Theorem: $p(1) = 5$, $p(-1) = 7$, $p(-2) = 2$.
- When $p(x)$ is divided by $x^2 - 1 = (x - 1)(x + 1)$ (a quadratic), the remainder $R(x)$ must be linear, i.e., $R(x) = ax + b$.
- We can write $p(x) = Q(x)(x^2 - 1) + (ax + b)$ for some quotient polynomial $Q(x)$.
- Use $p(1) = 5$: $p(1) = Q(1)(1^2 - 1) + a(1) + b = Q(1)(0) + a + b = a + b$. So, $a + b = 5$.
- Use $p(-1) = 7$: $p(-1) = Q(-1)((-1)^2 - 1) + a(-1) + b = Q(-1)(0) - a + b = -a + b$. So, $-a + b = 7$.
- Solve the system:
$$a + b = 5$$
$$-a + b = 7$$
Adding the two equations: $(a + b) + (-a + b) = 5 + 7 \implies 2b = 12 \implies b = 6$. Substitute $b = 6$ into $a + b = 5$: $a + 6 = 5 \implies a = -1$.
- The remainder is $R(x) = -1x + 6 = -x + 6$.
- Find $R(50)$: $R(50) = -(50) + 6 = -44$.

The value of R(50) is -44.
The correct option is (B).

24. Using the same $p(x)$ as in the previous question, if $p(x)$ is divided by $(x - 1)(x + 2)$, the remainder is $r(x)$. Find $r(100)$.
(A) 34 (B) 44 (C) 54 (D) 104
Solution:

- We have $p(1) = 5$ and $p(-2) = 2$.
- When $p(x)$ is divided by $(x - 1)(x + 2)$ (a quadratic), the remainder $r(x)$ must be linear, i.e., $r(x) = cx + d$.
- We can write $p(x) = S(x)(x - 1)(x + 2) + (cx + d)$ for some quotient polynomial $S(x)$.
- Use $p(1) = 5$: $p(1) = S(1)(1 - 1)(1 + 2) + c(1) + d = S(1)(0)(3) + c + d = c + d$. So, $c + d = 5$.
- Use $p(-2) = 2$: $p(-2) = S(-2)(-2-1)(-2+2)+c(-2)+d = S(-2)(-3)(0)-2c+d = -2c+d$. So, $-2c+d = 2$.
- Solve the system:
$$c + d = 5$$
$$-2c + d = 2$$
Subtracting the second equation from the first: $(c + d) - (-2c + d) = 5 - 2 \implies c + d + 2c - d = 3 \implies 3c = 3 \implies c = 1$. Substitute $c = 1$ into $c + d = 5$: $1 + d = 5 \implies d = 4$.
- The remainder is $r(x) = 1x + 4 = x + 4$.
- Find $r(100)$: $r(100) = 100 + 4 = 104$.

The value of r(100) is 104.
The correct option is (D).

25. When a polynomial f(x) is divided by $(x - 1)$ the remainder is 5 and when it is divided by $(x - 2)$, the remainder is 7. Find the remainder when it is divided by $(x - 1)(x - 2)$.
(A) $2x - 3$ (B) $2x + 3$ (C) $3x + 2$ (D) $3x - 2$
Solution:

- Given $f(1) = 5$ and $f(2) = 7$. .... (Remainder Theorem)
- When $f(x)$ is divided by $(x-1)(x-2)$ (a quadratic), the remainder is linear, say $ax + b$.
- $f(x) = Q(x)(x-1)(x-2) + ax + b$.
- Use $f(1) = 5$: $f(1) = Q(1)(0)(-1) + a(1) + b = a + b$. So, $a + b = 5$.
- Use $f(2) = 7$: $f(2) = Q(2)(1)(0) + a(2) + b = 2a + b$. So, $2a + b = 7$.
- Solve the system:

$$a + b = 5$$
$$2a + b = 7$$

  Subtracting the first equation from the second: $(2a + b) - (a + b) = 7 - 5 \implies a = 2$. Substitute $a = 2$ into $a + b = 5$: $2 + b = 5 \implies b = 3$.
- The remainder is $ax + b = 2x + 3$.

The remainder is $2x + 3$.
The correct option is (B).

26. Find remainder when $f(x) = x^5 - x^3 + 3x^2 + 3x + 1$ is divided by $(x^2 - 1)$.
(A) $3x - 4$ (B) $3x + 4$ (C) $4x + 3$ (D) $4x - 3$
Solution:

- The divisor is $x^2 - 1 = (x-1)(x+1)$. The remainder is linear, say $ax + b$.
- $f(x) = Q(x)(x^2 - 1) + ax + b$.
- Find $f(1)$ and $f(-1)$: $f(1) = (1)^5 - (1)^3 + 3(1)^2 + 3(1) + 1 = 1 - 1 + 3 + 3 + 1 = 7$. $f(-1) = (-1)^5 - (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 - (-1) + 3(1) - 3 + 1 = -1 + 1 + 3 - 3 + 1 = 1$.
- Using the Remainder Theorem property for factors: $f(1) = a(1)+b = a+b = 7$. $f(-1) = a(-1)+b = -a+b = 1$.
- Solve the system:

$$a + b = 7$$
$$-a + b = 1$$

  Adding the two equations: $2b = 8 \implies b = 4$. Substitute $b = 4$ into $a + b = 7$: $a + 4 = 7 \implies a = 3$.
- The remainder is $ax + b = 3x + 4$.

The remainder is $3x + 4$.
The correct option is (B).

27. A polynomial $f(x)$ when divided by $x^2 - 3x + 2$ leaves the remainder $ax + b$. If $f(1) = 4$ and $f(2) = 7$, determine a and b.
(A) $a = 3, b = -1$ (B) $a = -3, b = 1$ (C) $a = -3, b = -1$ (D) $a = 3, b = 1$
Solution:

- The divisor is $x^2 - 3x + 2 = (x-1)(x-2)$.
- We are given that $f(x) = Q(x)(x^2 - 3x + 2) + (ax + b)$.
- Substituting $x = 1$: $f(1) = Q(1)(1^2 - 3(1) + 2) + a(1) + b = Q(1)(0) + a + b = a + b$.
- We are given $f(1) = 4$, so $a + b = 4$.
- Substituting $x = 2$: $f(2) = Q(2)(2^2 - 3(2) + 2) + a(2) + b = Q(2)(0) + 2a + b = 2a + b$.
- We are given $f(2) = 7$, so $2a + b = 7$.
- Solve the system:

$$a + b = 4$$
$$2a + b = 7$$

  Subtracting the first equation from the second: $(2a + b) - (a + b) = 7 - 4 \implies a = 3$. Substitute $a = 3$ into $a + b = 4$: $3 + b = 4 \implies b = 1$.
- Thus, $a = 3$ and $b = 1$.

The values are $a = 3, b = 1$.
The correct option is (D).

Factor Theorem & Finding Zeros

28. Find the value of m, if $x = 1/2$ is one of the zeroes of the polynomial $p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$.
(A) $m = -11$ (B) $m = 5$ (C) $m = 11$ (D) $m = -5$
Solution:

- If $x = 1/2$ is a zero, then $p(1/2) = 0$. .... (Factor Theorem/Definition of Zero)
- Substitute $x = 1/2$ into $p(x)$: $p(1/2) = 4(1/2)^4 - 4(1/2)^3 - m(1/2)^2 + 12(1/2) - 3 = 0$.
- Calculate the powers and simplify: $4(1/16) - 4(1/8) - m(1/4) + 6 - 3 = 0$ $1/4 - 1/2 - m/4 + 3 = 0$.
- Multiply the entire equation by 4 to clear fractions: $4(1/4) - 4(1/2) - 4(m/4) + 4(3) = 4(0)$ $1 - 2 - m + 12 = 0$.
- Solve for m: $11 - m = 0 \implies m = 11$.

The value of m is 11.
The correct option is (C).

29. Obtain all zeros of $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1. The other two zeros are:
(A) $3, 1/2$ (B) $-3, 1/2$ (C) $3, -1/2$ (D) $-3, -1/2$
Solution:

- Since -2 and -1 are zeros, $(x - (-2)) = (x + 2)$ and $(x - (-1)) = (x + 1)$ are factors. .... (Factor Theorem)
- Their product $(x + 2)(x + 1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$ must also be a factor of $f(x)$.
- Divide $f(x)$ by $(x^2 + 3x + 2)$ using polynomial long division to find the other factor:

$$
\begin{array}{r}
2x^2 - 5x - 3 \\
x^2 + 3x + 2 \overline{) 2x^4 + x^3 - 14x^2 - 19x - 6} \\
-(2x^4 + 6x^3 + 4x^2) \\
\hline
-5x^3 - 18x^2 - 19x \\
-(-5x^3 - 15x^2 - 10x) \\
\hline
-3x^2 - 9x - 6 \\
-(-3x^2 - 9x - 6) \\
\hline
0
\end{array}
$$

- The other factor is the quotient $2x^2 - 5x - 3$.
- Find the zeros of this quadratic factor by setting it to zero: $2x^2 - 5x - 3 = 0$.
- Factor the quadratic: $(2x + 1)(x - 3) = 0$.
- The zeros are $2x + 1 = 0 \implies x = -1/2$ and $x - 3 = 0 \implies x = 3$.

The other two zeros are 3 and $-1/2$.
The correct option is (C).

30. Obtain all zeros of $f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2. The other two zeros are:
(A) $1, 10$ (B) $-1, 10$ (C) $1, -10$ (D) $-1, -10$
Solution:

- Since -2 is a zero, $(x + 2)$ is a factor. .... (Factor Theorem)
- Divide $f(x)$ by $(x + 2)$ using synthetic division or long division. Synthetic Division:

$$
\begin{array}{c|cccc}
-2 & 1 & 13 & 32 & 20 \\
 & & -2 & -22 & -20 \\
\hline
 & 1 & 11 & 10 & 0
\end{array}
$$

- The quotient is $x^2 + 11x + 10$.
- Find the zeros of the quotient by setting it to zero: $x^2 + 11x + 10 = 0$.
- Factor the quadratic: $(x + 1)(x + 10) = 0$.
- The zeros are $x + 1 = 0 \implies x = -1$ and $x + 10 = 0 \implies x = -10$.

The other two zeros are -1 and -10.
The correct option is (D).

31. Obtain all zeros of $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$ if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$. The other two zeros are:
(A) $-1, -2$ (B) $1, 2$ (C) $-1, 2$ (D) $1, -2$
Solution:

- Since $-\sqrt{3}$ and $\sqrt{3}$ are zeros, $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors. .... (Factor Theorem)
- Their product $(x + \sqrt{3})(x - \sqrt{3}) = x^2 - (\sqrt{3})^2 = x^2 - 3$ must also be a factor.

- Divide $f(x)$ by $(x^2 - 3)$ to find the other factor:

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x^2 + 0x - 3 \overline{) x^4 - 3x^3 - x^2 + 9x - 6} \\
-(x^4 + 0x^3 - 3x^2) \\
\hline
-3x^3 + 2x^2 + 9x \\
-(-3x^3 + 0x^2 + 9x) \\
\hline
2x^2 + 0x - 6 \\
-(2x^2 + 0x - 6) \\
\hline
0
\end{array}
$$

- The other factor is the quotient $x^2 - 3x + 2$.
- Find the zeros of this quadratic factor: $x^2 - 3x + 2 = 0$.
- Factor the quadratic: $(x - 1)(x - 2) = 0$.
- The zeros are $x - 1 = 0 \implies x = 1$ and $x - 2 = 0 \implies x = 2$.

The other two zeros are 1 and 2.
The correct option is (B).

32. Find all the zeroes of the polynomial $f(x) = x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeroes are 2 and -2. The other two zeros are:
(A) $6, 5$ (B) $-6, 5$ (C) $6, -5$ (D) $-6, -5$
Solution:

- Since 2 and -2 are zeros, $(x - 2)$ and $(x + 2)$ are factors. .... (Factor Theorem)
- Their product $(x - 2)(x + 2) = x^2 - 4$ must also be a factor.
- Divide $f(x)$ by $(x^2 - 4)$ to find the other factor:

$$
\begin{array}{r}
x^2 + x - 30 \\
x^2 + 0x - 4 \overline{) x^4 + x^3 - 34x^2 - 4x + 120} \\
-(x^4 + 0x^3 - 4x^2) \\
\hline
x^3 - 30x^2 - 4x \\
-(x^3 + 0x^2 - 4x) \\
\hline
-30x^2 + 0x + 120 \\
-(-30x^2 + 0x + 120) \\
\hline
0
\end{array}
$$

- The other factor is the quotient $x^2 + x - 30$.
- Find the zeros of this quadratic factor: $x^2 + x - 30 = 0$.
- Factor the quadratic: $(x + 6)(x - 5) = 0$.
- The zeros are $x + 6 = 0 \implies x = -6$ and $x - 5 = 0 \implies x = 5$.

The other two zeros are -6 and 5.
The correct option is (B).

33. Find the values of l and m if $P(x) = 8x^3 + lx^2 - 27x + m$ is divisible by $D(x) = 2x^2 - x - 6$.
(A) $l = 1, m = 18$ (B) $l = 1, m = -18$ (C) $l = -1, m = 18$ (D) $l = 2, m = -18$
Solution:

- If $P(x)$ is divisible by $D(x)$, then the zeros of $D(x)$ are also zeros of $P(x)$. .... (Factor Theorem extension)
- Find the zeros of the divisor $D(x) = 2x^2 - x - 6 = 0$. Factor the quadratic: $2x^2 - 4x + 3x - 6 = 2x(x - 2) + 3(x - 2) = (2x + 3)(x - 2) = 0$. The zeros are $x = 2$ and $x = -3/2$.
- Since these are zeros of $P(x)$, we must have $P(2) = 0$ and $P(-3/2) = 0$.
- $P(2) = 8(2)^3 + l(2)^2 - 27(2) + m = 8(8) + l(4) - 54 + m = 64 + 4l - 54 + m = 4l + m + 10 = 0$. (Eq 1)
- $P(-3/2) = 8(-3/2)^3 + l(-3/2)^2 - 27(-3/2) + m = 8(-27/8) + l(9/4) + 81/2 + m = -27 + 9l/4 + 81/2 + m = 0$. Multiply by 4: $4(-27) + 4(9l/4) + 4(81/2) + 4m = 0 \implies -108 + 9l + 162 + 4m = 9l + 4m + 54 = 0$. (Eq 2)
- Solve the system of equations: 1) $4l + m = -10 \implies m = -10 - 4l$ 2) $9l + 4m = -54$ Substitute m from (1) into (2): $9l + 4(-10 - 4l) = -54$. $9l - 40 - 16l = -54 \implies -7l = -54 + 40 \implies -7l = -14 \implies l = 2$.
- Substitute $l = 2$ back into $m = -10 - 4l$: $m = -10 - 4(2) = -10 - 8 = -18$.

The values are $l = 2$ and $m = -18$.
The correct option is (D).

34. Find l and m if $P(x) = 2x^3 - (2l + 1)x^2 + (l + m)x + m$ may be exactly divisible by $D(x) = 2x^2 - x - 3$.
(A) $l = 1, m = 3$ (B) $l = 1, m = -3$ (C) $l = -1, m = 3$ (D) $l = -1, m = -3$
Solution:

- Find the zeros of the divisor $D(x) = 2x^2 - x - 3 = 0$. Factor the quadratic: $2x^2 - 3x + 2x - 3 = x(2x - 3) + 1(2x - 3) = (x + 1)(2x - 3) = 0$. The zeros are $x = -1$ and $x = 3/2$.
- Since $P(x)$ is divisible by $D(x)$, we must have $P(-1) = 0$ and $P(3/2) = 0$. .... (Factor Theorem extension)
- $P(-1) = 2(-1)^3-(2l+1)(-1)^2+(l+m)(-1)+m = 0$. $2(-1)-(2l+1)(1)-(l+m)+m = 0$. $-2-2l-1-l-m+m = 0 \implies -3l - 3 = 0 \implies -3l = 3 \implies l = -1$.
- $P(3/2) = 2(3/2)^3 - (2l+1)(3/2)^2 + (l+m)(3/2)+m = 0$. Substitute $l = -1$: $2(27/8) - (2(-1)+1)(9/4) + (-1+m)(3/2) + m = 0$. $54/8 - (-2+1)(9/4) + (-3/2+3m/2) + m = 0$. $27/4 - (-1)(9/4) - 3/2 + 3m/2 + 2m/2 = 0$. $27/4 + 9/4 - 3/2 + 5m/2 = 0$. $36/4 - 3/2 + 5m/2 = 0 \implies 9 - 3/2 + 5m/2 = 0$. Multiply by 2: $18 - 3 + 5m = 0 \implies 15 + 5m = 0 \implies 5m = -15 \implies m = -3$.

The values are $l = -1$ and $m = -3$.
The correct option is (D).

35. Obtain all zeros of $f(x) = x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and -2. The other two zeros are:
(A) $6, 5$ (B) $-6, 5$ (C) $6, -5$ (D) $-6, -5$
Solution: This question is identical to Question 32.

- Since 2 and -2 are zeros, $(x - 2)$ and $(x + 2)$ are factors. Their product $(x^2 - 4)$ is a factor.
- Divide $f(x)$ by $(x^2 - 4)$. From Q32, the quotient is $x^2 + x - 30$.
- Find the zeros of the quotient: $x^2 + x - 30 = 0$.
- Factor: $(x + 6)(x - 5) = 0$.
- The zeros are $x = -6$ and $x = 5$.

The other two zeros are -6 and 5.
The correct option is (B).