Topic: Trigonometric Identities

Sub: Mathematics **Assignment: 02** Prof. Chetan Sir

Prove the following identities:

1. $\sin\theta\cot\theta = \cos\theta$
2. $\cos\theta\tan\theta = \sin\theta$
3. $\sec\theta\cot\theta = \operatorname{cosec}\theta$
4. $\operatorname{cosec}\theta\tan\theta = \sec\theta$
5. $(1-\sin^2\theta)\sec^2\theta = 1$
6. $(1-\cos^2\theta)\operatorname{cosec}^2\theta = 1$
7. $\tan^2\theta\cos^2\theta = \sin^2\theta$
8. $(\sec^2\theta-1)\cot^2\theta = 1$
9. $(\operatorname{cosec}^2\theta-1)\tan^2\theta = 1$
10. $(1+\cot^2 A)\sin^2 A = 1$
11. $\sin^2 x\sec^2 x = \tan^2 x$
12. $\cos^2 x\operatorname{cosec}^2 x = \cot^2 x$
13. $\cos\theta(\sec\theta-\cos\theta) = \sin^2\theta$
14. $\tan\theta(\cot\theta+\tan\theta) = \sec^2\theta$
15. $\sin^2\theta\cot^2\theta+\sin^2\theta = 1$
16. $\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$
17. $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$
18. $\frac{\sec^2\theta}{1+\cot^2\theta} = \tan^2\theta$
19. $\frac{\operatorname{cosec}^2\theta}{1+\tan^2\theta} = \cot^2\theta$
20. $\cos^2 A+\frac{1}{1+\cot^2 A} = 1$
21. $\sin^2 A+\frac{1}{1+\tan^2 A} = 1$
22. $\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A} = 1$
23. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$
24. $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$
25. $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$
26. $\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta+\cot\theta$
27. $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$
28. $\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$
29. $\frac{\sin A}{1+\cos A} = \operatorname{cosec} A-\cot A$
30. $\frac{\cos B}{1+\sin B} = \sec B-\tan B$
31. $\tan\theta+\cot\theta = \sec\theta\operatorname{cosec}\theta$
32. $\cos\theta(\tan\theta+\cot\theta) = \operatorname{cosec}\theta$
33. $\sin\theta(\cot\theta+\tan\theta) = \sec\theta$
34. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2 = 2$
35. $\cos^4 A-\sin^4 A = \cos^2 A-\sin^2 A$
36. $\frac{\tan x}{\sec x-1} = \frac{\sec x+1}{\tan x}$
37. $\frac{\cot x}{\operatorname{cosec} x-1} = \frac{\operatorname{cosec} x+1}{\cot x}$
38. $\frac{\sec\theta}{\operatorname{cosec}\theta}+\frac{\sin\theta}{\cos\theta} = 2\tan\theta$
39. $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$
40. $\frac{1}{1+\sin A}+\frac{1}{1-\sin A} = 2\sec^2 A$
41. $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$
42. $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$
43. $\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$
44. $\frac{1}{1-\sin x}-\frac{1}{1+\sin x} = 2\tan x\sec x$
45. $\frac{1}{1-\cos x}-\frac{1}{1+\cos x} = 2\cot x\operatorname{cosec} x$
46. $\frac{1}{\sec A-1}+\frac{1}{\sec A+1} = 2\operatorname{cosec} A\cot A$
47. $\frac{\sin\alpha}{1+\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha} = 2\operatorname{cosec}\alpha$
48. $\frac{\tan\beta}{1+\sec\beta}+\frac{1+\sec\beta}{\tan\beta} = 2\operatorname{cosec}\beta$
49. $\frac{\cot\theta}{1+\operatorname{cosec}\theta}-\frac{\cot\theta}{1-\operatorname{cosec}\theta} = 2\sec\theta$
50. $\frac{\tan\theta}{1+\sec\theta}-\frac{\tan\theta}{1-\sec\theta} = 2\operatorname{cosec}\theta$
51. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1} = 2\sec^2 A$

52. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = 1$
53. $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$
54. $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$
55. $\frac{1-\cos A}{1+\cos A} = (\operatorname{cosec} A - \cot A)^2$
56. $\operatorname{cosec}\theta\sqrt{1-\cos^2\theta} = 1$ (Assume $\sin\theta > 0$)
57. $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \operatorname{cosec}^2 x$
58. $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) = 1$
59. $\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = 1$
60. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$
61. $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$
62. $\tan^2 x - \cot^2 x = \sec^2 x - \operatorname{cosec}^2 x$
63. $(\operatorname{cosec}\theta + \sin\theta)(\operatorname{cosec}\theta - \sin\theta) = \cot^2\theta + \cos^2\theta$
64. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$
65. $\sec A(1-\sin A)(\sec A + \tan A) = 1$
66. $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = 0$
67. $\frac{(\sin\theta+\cos\theta)^2}{\cos\theta} - \sec\theta = 2\sin\theta$
68. $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$
69. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$
70. $\operatorname{cosec}^4\theta - \operatorname{cosec}^2\theta = \cot^4\theta + \cot^2\theta$
71. $\sec^4 A - \tan^4 A = \sec^2 A + \tan^2 A$
72. $\operatorname{cosec}^4 A - \cot^4 A = \operatorname{cosec}^2 A + \cot^2 A$
73. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
74. $\frac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta} = \sin^2\theta - \cos^2\theta$
75. $\frac{\cot A-\cos A}{\cot A+\cos A} = \frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$
76. $\frac{\sec\theta+\tan\theta}{\cot\theta+\cos\theta} = \tan\theta\sec\theta$
77. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = |\operatorname{cosec}\theta - \cot\theta|$
78. $\sqrt{\frac{1+\sin A}{1-\sin A}} = |\sec A + \tan A|$
79. $\sqrt{\frac{1-\cos A}{1+\cos A}} = |\operatorname{cosec} A - \cot A|$
80. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = |\tan\theta + \cot\theta|$
81. $\frac{\tan^3\theta-1}{\tan\theta-1} = \sec^2\theta + \tan\theta$
82. $\frac{\cot^3\theta-1}{\cot\theta-1} = \operatorname{cosec}^2\theta + \cot\theta$
83. $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x} = 1 - \sin x\cos x$
84. $\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x} = 1 + \sin x\cos x$
85. $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$
86. $\frac{\sec A-\tan A}{\sec A+\tan A} = \left(\frac{\cos A}{1+\sin A}\right)^2$
87. $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$
88. $\frac{\sec\theta+\operatorname{cosec}\theta}{\tan\theta+\cot\theta} = \sin\theta + \cos\theta$
89. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$
90. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$
91. $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$
92. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$
93. $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 = \tan^2 x + \cot^2 x + 7$
94. $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \tan\theta$
95. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$
96. $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$
97. $\operatorname{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\operatorname{cosec}^2\theta + 1$
98. $(1+\tan^2 A) + (1 + \frac{1}{\tan^2 A}) = \frac{1}{\sin^2 A - \sin^4 A}$
99. $\frac{\tan A+\tan B}{\cot A+\cot B} = \tan A\tan B$
100. $\cot^2 A\operatorname{cosec}^2 B - \cot^2 B\operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$
101. $\tan^2 A\sec^2 B - \tan^2 B\sec^2 A = \tan^2 A - \tan^2 B$
102. $\sin^2 x\cos^2 y - \cos^2 x\sin^2 y = \sin^2 x - \sin^2 y$
103. $\frac{\cos A\cot A-\sin A\tan A}{\operatorname{cosec} A-\sec A} = 1 + \sin A\cos A$
104. $(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2\tan A$
105. $(\operatorname{cosec} A+\cot A-1)(\operatorname{cosec} A-\cot A+1) = 2\cot A$
106. $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$
107. $\sin^8 x - \cos^8 x = (\sin^2 x - \cos^2 x)(1 - 2\sin^2 x\cos^2 x)$
108. $\frac{\cos A\operatorname{cosec} A-\sin A\sec A}{\cos A+\sin A} = \operatorname{cosec} A - \sec A$
109. $\frac{\tan A+\sec A-1}{\tan A-\sec A+1} = \frac{1+\sin A}{\cos A}$
110. $\frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta} = 1$
111. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, prove that $m^2 - n^2 = 4\sqrt{mn}$
112. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$
113. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
114. Prove that $(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$
115. If $\operatorname{cosec}\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, prove that $a^2b^2(a^2 + b^2) = 1$
116. Prove that $\frac{(1-\sin\theta+\cos\theta)^2}{(1+\cos\theta)(1-\sin\theta)} = 2$