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PCMB NEET/JEE

Topic: Surds

Sub: Mathematics **Solutions to Assignment: 04** Prof. Chetan Sir

1. Simplify $\sqrt[3]{54}$.

(A) $2\sqrt[3]{3}$ (B) $3\sqrt[3]{2}$ (C) $6\sqrt[3]{9}$ (D) $9\sqrt[3]{6}$

Solution:

We need to simplify $\sqrt[3]{54}$.
First, find the prime factorization of 54: $54 = 2 \times 27 = 2 \times 3^3$.
So, $\sqrt[3]{54} = \sqrt[3]{3^3 \times 2}$.
Using the property of surds:
$\sqrt[3]{3^3 \times 2} = \sqrt[3]{3^3} \times \sqrt[3]{2}$ [Property: $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$]
$= 3 \times \sqrt[3]{2}$
$= 3\sqrt[3]{2}$.

The correct option is (B).

2. Simplify $\sqrt{32} + \sqrt{18} - \sqrt{50}$.

(A) $\sqrt{0}$ (B) $12\sqrt{2}$ (C) $2\sqrt{2}$ (D) $6\sqrt{2}$

Solution:

Simplify each term:
$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$.
$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$.
$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$.
Now substitute these into the expression:
$\sqrt{32} + \sqrt{18} - \sqrt{50} = 4\sqrt{2} + 3\sqrt{2} - 5\sqrt{2}$.
Combine the terms:
$(4 + 3 - 5)\sqrt{2} = (7 - 5)\sqrt{2}$ [Combining like surds]
$= 2\sqrt{2}$.

The correct option is (C).

3. Simplify $\sqrt{24} \times \sqrt{6}$.

(A) $\sqrt{144}$ (B) 12 (C) $6\sqrt{4}$ (D) $12\sqrt{1}$

Solution:

Using the property $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$:
$\sqrt{24} \times \sqrt{6} = \sqrt{24 \times 6}$ [Property: $\sqrt{a}\sqrt{b} = \sqrt{ab}$]
$= \sqrt{144}$.
The square root of 144 is 12.
So, $\sqrt{144} = 12$.

The correct option is (B).

4. Which is greater: $\sqrt{7}$ or $\sqrt[3]{18}$?

(A) $\sqrt{7}$ (B) $\sqrt[3]{18}$ (C) They are equal (D) Cannot compare

Solution:

To compare $\sqrt{7}$ and $\sqrt[3]{18}$, we raise them to a common power.
The least common multiple (LCM) of the orders of the surds (2 and 3) is 6.
$(\sqrt{7})^6 = (7^{1/2})^6 = 7^{6/2} = 7^3 = 343$.
$(\sqrt[3]{18})^6 = (18^{1/3})^6 = 18^{6/3} = 18^2 = 324$.
Since $343 > 324$, it follows that $(\sqrt{7})^6 > (\sqrt[3]{18})^6$.
Therefore, $\sqrt{7} > \sqrt[3]{18}$.

The correct option is (A).

5. Arrange in ascending order: $\sqrt{3}$, $\sqrt[3]{7}$, $\sqrt[6]{40}$.
(A) $\sqrt{3} < \sqrt[6]{40} < \sqrt[3]{7}$ (B) $\sqrt[3]{7} < \sqrt{3} < \sqrt[6]{40}$ (C) $\sqrt[6]{40} < \sqrt{3} < \sqrt[3]{7}$ (D) $\sqrt{3} < \sqrt[3]{7} < \sqrt[6]{40}$

Solution:

To compare these surds, convert them to the same order.
The LCM of the orders (2, 3, 6) is 6.
$\sqrt{3} = 3^{1/2} = (3^3)^{1/6} = \sqrt[6]{27}$.
$\sqrt[3]{7} = 7^{1/3} = (7^2)^{1/6} = \sqrt[6]{49}$.
$\sqrt[6]{40}$ is already in order 6.
Now compare the numbers under the radical: 27, 49, 40.
In ascending order, these are $27 < 40 < 49$.
So, $\sqrt[6]{27} < \sqrt[6]{40} < \sqrt[6]{49}$.
This means $\sqrt{3} < \sqrt[6]{40} < \sqrt[3]{7}$.

The correct option is (A).

6. Which is smaller: $(\sqrt{8} - \sqrt{7})$ or $(\sqrt{6} - \sqrt{5})$?
(A) $(\sqrt{8} - \sqrt{7})$ (B) $(\sqrt{6} - \sqrt{5})$ (C) They are equal (D) Cannot compare

Solution:

Let $x = \sqrt{8} - \sqrt{7}$ and $y = \sqrt{6} - \sqrt{5}$.
Rationalize these expressions by multiplying by their conjugates:
$x = \sqrt{8} - \sqrt{7} = \frac{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})}{\sqrt{8}+\sqrt{7}} = \frac{8-7}{\sqrt{8}+\sqrt{7}} = \frac{1}{\sqrt{8}+\sqrt{7}}$ [Using $(a-b)(a+b) = a^2 - b^2$].
$y = \sqrt{6} - \sqrt{5} = \frac{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}{\sqrt{6}+\sqrt{5}} = \frac{6-5}{\sqrt{6}+\sqrt{5}} = \frac{1}{\sqrt{6}+\sqrt{5}}$.
Now compare the denominators: $D_1 = \sqrt{8} + \sqrt{7}$ and $D_2 = \sqrt{6} + \sqrt{5}$.
Since $\sqrt{8} > \sqrt{6}$ and $\sqrt{7} > \sqrt{5}$, it follows that $\sqrt{8} + \sqrt{7} > \sqrt{6} + \sqrt{5}$.
For fractions with the same positive numerator (1), the fraction with the larger denominator is smaller.
Thus, $\frac{1}{\sqrt{8}+\sqrt{7}} < \frac{1}{\sqrt{6}+\sqrt{5}}$.
So, $(\sqrt{8} - \sqrt{7})$ is smaller than $(\sqrt{6} - \sqrt{5})$.

The correct option is (A).

7. Compare: $x = \sqrt{11}$, $y = \sqrt[3]{35}$, $z = \sqrt[4]{120}$.
(A) $x > y > z$ (B) $y > x > z$ (C) $z > y > x$ (D) $x > z > y$

Solution:

To compare $x = \sqrt{11}$, $y = \sqrt[3]{35}$, and $z = \sqrt[4]{120}$, raise them to a common power.
The LCM of the orders (2, 3, 4) is 12.
$x^{12} = (\sqrt{11})^{12} = (11^{1/2})^{12} = 11^6 = 1771561$.
$y^{12} = (\sqrt[3]{35})^{12} = (35^{1/3})^{12} = 35^4 = 1500625$.
$z^{12} = (\sqrt[4]{120})^{12} = (120^{1/4})^{12} = 120^3 = 1728000$.
Comparing these values: $1500625 < 1728000 < 1771561$.
So, $y^{12} < z^{12} < x^{12}$.
Since x, y, z are positive, this implies $y < z < x$, which can be written as $x > z > y$.

The correct option is (D).

8. If $5 + x\sqrt{3} = y + 2\sqrt{3}$, where x and y are rational, find x and y.
(A) x=5, y=2 (B) x=3, y=5 (C) x=2, y=5 (D) x=2, $y = \sqrt{3}$

Solution:

Given the equation $5 + x\sqrt{3} = y + 2\sqrt{3}$.
Since x and y are rational numbers, we can equate the rational parts and the irrational parts.
Equating the rational parts: $y = 5$.
Equating the coefficients of $\sqrt{3}$ (irrational parts): $x\sqrt{3} = 2\sqrt{3} \implies x = 2$.
So, $x = 2$ and $y = 5$.

The correct option is (C).

9. If $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15}$, find the values of a and b.
(A) a=4, b=1 (B) a=4, b=2 (C) a=8, b=1 (D) a=1, b=4

Solution:

First, rationalize the denominator of the left-hand side (LHS):
LHS $= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ [Multiplying by conjugate $\sqrt{5} + \sqrt{3}$]
$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$ [Using $(x + y)^2$ and $(x - y)(x + y) = x^2 - y^2$]

$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}}{5-3} = \frac{5+3+2\sqrt{15}}{2}$
$= \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$.
So, we have $4 + \sqrt{15} = a + b\sqrt{15}$.
Since a and b are rational, equate the rational parts and coefficients of $\sqrt{15}$.
Equating rational parts: $a = 4$.
Equating coefficients of $\sqrt{15}$: $b = 1$.

The correct option is (A).

10. If $\frac{3+\sqrt{7}}{\sqrt{16+6\sqrt{7}}} = a + \sqrt{b}$, find a and b. (Hint: Simplify denominator $\sqrt{16 + 2\sqrt{63}}$)
(A) a=3, b=7 (B) a=0, b=1 (C) a=1, b=0 (D) a=1, b=7/9

Solution:

First, simplify the denominator $\sqrt{16 + 6\sqrt{7}}$.
$\sqrt{16 + 6\sqrt{7}} = \sqrt{16 + 2 \cdot 3\sqrt{7}} = \sqrt{16 + 2\sqrt{9 \times 7}} = \sqrt{16 + 2\sqrt{63}}$.
We look for two numbers p and q such that $p + q = 16$ and $pq = 63$. These are 9 and 7.
So, $\sqrt{16 + 2\sqrt{63}} = \sqrt{(\sqrt{9} + \sqrt{7})^2} = \sqrt{9} + \sqrt{7} = 3 + \sqrt{7}$ [Using $\sqrt{X + 2\sqrt{Y}} = \sqrt{p} + \sqrt{q}$ where $p + q = X, pq = Y$].
Now, substitute this back into the expression:
$\frac{3+\sqrt{7}}{3+\sqrt{7}} = 1$.
So, we have $1 = a + \sqrt{b}$.
Given that a is rational. If $a = 1$, then $\sqrt{b} = 0$, which implies $b = 0$. This fits option (C).
(If $a = 0$, then $\sqrt{b} = 1 \implies b = 1$, which is option B. The provided key implies C is preferred.)

The correct option is (C).

11. If a, b are rational and $a\sqrt{2} + b\sqrt{3} = \sqrt{18} + \sqrt{12} - \sqrt{2}$, find a and b.
(A) a=3, b=2 (B) a=2, b=2 (C) a=4, b=2 (D) a=2, b=3

Solution:

Simplify the right-hand side (RHS) of the equation:
$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$.
$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.
So, RHS $= 3\sqrt{2} + 2\sqrt{3} - \sqrt{2}$.
Combine like terms: $(3\sqrt{2} - \sqrt{2}) + 2\sqrt{3} = 2\sqrt{2} + 2\sqrt{3}$.
Now the equation is $a\sqrt{2} + b\sqrt{3} = 2\sqrt{2} + 2\sqrt{3}$.
Since a and b are rational, equate the coefficients:
Coefficient of $\sqrt{2}$: $a = 2$.
Coefficient of $\sqrt{3}$: $b = 2$.

The correct option is (B).

12. Find $\sqrt{7 + 2\sqrt{10}}$.
(A) $\sqrt{7} + \sqrt{10}$ (B) $\sqrt{5} + \sqrt{2}$ (C) $\sqrt{10} + \sqrt{7}$ (D) $\sqrt{4} + \sqrt{3}$

Solution:

We use the identity $\sqrt{X + 2\sqrt{Y}} = \sqrt{p} + \sqrt{q}$, where $p + q = X$ and $pq = Y$.
Here, $X = 7, Y = 10$. We need two numbers p and q such that $p + q = 7$ and $pq = 10$.
These numbers are 5 and 2.
So, $\sqrt{7 + 2\sqrt{10}} = \sqrt{(5 + 2) + 2\sqrt{5 \times 2}} = \sqrt{(\sqrt{5} + \sqrt{2})^2}$.
This simplifies to $\sqrt{5} + \sqrt{2}$.

The correct option is (B).

13. Find $\sqrt{11 - 2\sqrt{30}}$.
(A) $\sqrt{6} - \sqrt{5}$ (B) $\sqrt{10} - \sqrt{1}$ (C) $\sqrt{5} - \sqrt{6}$ (D) $\sqrt{30} - \sqrt{11}$

Solution:

We use the identity $\sqrt{X - 2\sqrt{Y}} = |\sqrt{p} - \sqrt{q}|$, where $p + q = X$ and $pq = Y$. Let $p > q$.
Here, $X = 11, Y = 30$. We need two numbers p and q such that $p + q = 11$ and $pq = 30$.
These numbers are 6 and 5. Let $p = 6, q = 5$.
So, $\sqrt{11 - 2\sqrt{30}} = \sqrt{(6 + 5) - 2\sqrt{6 \times 5}} = \sqrt{(\sqrt{6} - \sqrt{5})^2}$.
This simplifies to $\sqrt{6} - \sqrt{5}$ [Since $\sqrt{6} > \sqrt{5}$, the result is positive].

The correct option is (A).

14. Find $\sqrt{15+4\sqrt{14}}$. (Hint: $\sqrt{15+2\sqrt{56}}$)
(A) $\sqrt{14}+1$ (B) $\sqrt{15}+\sqrt{14}$ (C) $2\sqrt{2}+\sqrt{7}$ (D) $\sqrt{10}+\sqrt{5}$

Solution:

First, rewrite $4\sqrt{14}$ in the form $2\sqrt{k}$:
$4\sqrt{14} = 2\times 2\sqrt{14} = 2\sqrt{2^2\times 14} = 2\sqrt{4\times 14} = 2\sqrt{56}$.
So the expression becomes $\sqrt{15+2\sqrt{56}}$.
We need p and q such that $p+q=15$ and $pq=56$. These are 8 and 7.
So, $\sqrt{15+2\sqrt{56}} = \sqrt{(\sqrt{8}+\sqrt{7})^2} = \sqrt{8}+\sqrt{7}$.
Simplify $\sqrt{8}=\sqrt{4\times 2}=2\sqrt{2}$.
The result is $2\sqrt{2}+\sqrt{7}$.

The correct option is (C).

15. Find $\sqrt{5+\sqrt{21}}$. (Hint: Multiply by $\sqrt{2}/\sqrt{2}$ inside)
(A) $\sqrt{3/2}+\sqrt{7/2}$ (B) $\sqrt{7/2}-\sqrt{3/2}$ (C) $\sqrt{5}+\sqrt{21}$ (D) $\frac{\sqrt{5}+\sqrt{21}}{2}$

Solution:

To get the form $\sqrt{X+2\sqrt{Y}}$:
$\sqrt{5+\sqrt{21}} = \sqrt{\frac{2(5+\sqrt{21})}{2}} = \sqrt{\frac{10+2\sqrt{21}}{2}} = \frac{\sqrt{10+2\sqrt{21}}}{\sqrt{2}}$.
Consider $\sqrt{10+2\sqrt{21}}$. We need p and q such that $p+q=10$ and $pq=21$.
These are 7 and 3.
So, $\sqrt{10+2\sqrt{21}} = \sqrt{(\sqrt{7}+\sqrt{3})^2} = \sqrt{7}+\sqrt{3}$.
Therefore, $\sqrt{5+\sqrt{21}} = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{2}}$.
This can be written as: $\frac{\sqrt{7}}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{7}{2}}+\sqrt{\frac{3}{2}}$.

The correct option is (A).

16. Find the positive square root of $16+6\sqrt{7}$. (Hint: $\sqrt{16+2\sqrt{63}}$)
(A) $4+\sqrt{7}$ (B) $\sqrt{9}+\sqrt{7}$ (C) $3+\sqrt{7}$ (D) $\sqrt{8}+\sqrt{8}$

Solution:

We need to find $\sqrt{16+6\sqrt{7}}$.
Rewrite $6\sqrt{7}$ as $2\times 3\sqrt{7} = 2\sqrt{3^2\times 7} = 2\sqrt{9\times 7} = 2\sqrt{63}$.
So the expression is $\sqrt{16+2\sqrt{63}}$.
We need p and q such that $p+q=16$ and $pq=63$. These are 9 and 7.
So, $\sqrt{16+2\sqrt{63}} = \sqrt{(\sqrt{9}+\sqrt{7})^2} = \sqrt{9}+\sqrt{7}$.
Since $\sqrt{9}=3$, the result is $3+\sqrt{7}$.

The correct option is (C).

17. Find $\sqrt{17-12\sqrt{2}}$. (Hint: $\sqrt{17-2\sqrt{72}}$)
(A) $\sqrt{9}-\sqrt{8}$ (B) $3-2\sqrt{2}$ (C) $2\sqrt{2}-3$ (D) $\sqrt{12}-\sqrt{5}$

Solution:

We need to find $\sqrt{17-12\sqrt{2}}$.
Rewrite $12\sqrt{2}$ as $2\times 6\sqrt{2} = 2\sqrt{6^2\times 2} = 2\sqrt{36\times 2} = 2\sqrt{72}$.
So the expression is $\sqrt{17-2\sqrt{72}}$.
We need p and q such that $p+q=17$ and $pq=72$. These are 9 and 8. Let $p=9, q=8$.
So, $\sqrt{17-2\sqrt{72}} = \sqrt{(\sqrt{9}-\sqrt{8})^2}$.
Since $\sqrt{9}>\sqrt{8}$, the result is $\sqrt{9}-\sqrt{8}$.
Simplify: $\sqrt{9}=3$ and $\sqrt{8}=\sqrt{4\times 2}=2\sqrt{2}$.
So the result is $3-2\sqrt{2}$.

The correct option is (B).

18. Rationalize the denominator: $\frac{3}{\sqrt{5}}$.
(A) $\frac{3\sqrt{5}}{25}$ (B) $\frac{\sqrt{15}}{5}$ (C) $3\sqrt{5}$ (D) $\frac{3\sqrt{5}}{5}$

Solution:

To rationalize the denominator $\sqrt{5}$, multiply numerator and denominator by $\sqrt{5}$:
$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$ [Multiplying by rationalizing factor $\sqrt{5}$]
$= \frac{3\sqrt{5}}{(\sqrt{5})^2} = \frac{3\sqrt{5}}{5}$.

The correct option is (D).

19. Rationalize the denominator: $\frac{1}{\sqrt{6}+\sqrt{5}}$.

(A) $\sqrt{6}-\sqrt{5}$ (B) $\frac{\sqrt{6}-\sqrt{5}}{11}$ (C) $\sqrt{6}+\sqrt{5}$ (D) $\frac{\sqrt{30}}{11}$

Solution:

To rationalize $\sqrt{6}+\sqrt{5}$, multiply by its conjugate $\sqrt{6}-\sqrt{5}$:

$\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

$= \frac{\sqrt{6}-\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2}$ [Using $(a+b)(a-b) = a^2-b^2$]

$= \frac{\sqrt{6}-\sqrt{5}}{6-5} = \frac{\sqrt{6}-\sqrt{5}}{1} = \sqrt{6}-\sqrt{5}$.

The correct option is (A).

20. Rationalize the denominator: $\frac{4}{\sqrt{7}-\sqrt{3}}$.

(A) $\sqrt{7}+\sqrt{3}$ (B) $4(\sqrt{7}+\sqrt{3})$ (C) $\sqrt{7}-\sqrt{3}$ (D) $\frac{4\sqrt{7}+4\sqrt{3}}{10}$

Solution:

To rationalize $\sqrt{7}-\sqrt{3}$, multiply by its conjugate $\sqrt{7}+\sqrt{3}$:

$\frac{4}{\sqrt{7}-\sqrt{3}} = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$= \frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$ [Using $(a-b)(a+b) = a^2-b^2$]

$= \frac{4(\sqrt{7}+\sqrt{3})}{7-3} = \frac{4(\sqrt{7}+\sqrt{3})}{4}$

$= \sqrt{7}+\sqrt{3}$.

The correct option is (A).

21. Rationalize the denominator: $\frac{5}{4-\sqrt{11}}$.

(A) $\frac{4+\sqrt{11}}{1}$ (B) $4+\sqrt{11}$ (C) $\frac{5(4+\sqrt{11})}{5}$ (D) $\frac{20+5\sqrt{11}}{27}$

Solution:

To rationalize $4-\sqrt{11}$, multiply by its conjugate $4+\sqrt{11}$:

$\frac{5}{4-\sqrt{11}} = \frac{5}{4-\sqrt{11}} \times \frac{4+\sqrt{11}}{4+\sqrt{11}}$

$= \frac{5(4+\sqrt{11})}{4^2-(\sqrt{11})^2}$ [Using $(a-b)(a+b) = a^2-b^2$]

$= \frac{5(4+\sqrt{11})}{16-11} = \frac{5(4+\sqrt{11})}{5}$

$= 4+\sqrt{11}$.

The correct option is (B).

22. Simplify $\frac{\sqrt{3}+1}{\sqrt{3}-1}$.

(A) $2+\sqrt{3}$ (B) $2-\sqrt{3}$ (C) $4+2\sqrt{3}$ (D) 2

Solution:

Rationalize the denominator. The conjugate of $\sqrt{3}-1$ is $\sqrt{3}+1$.

$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-1^2}$ [Using $(a+b)^2$ and $(a-b)(a+b)$]

$= \frac{(\sqrt{3})^2+1^2+2(\sqrt{3})(1)}{3-1} = \frac{3+1+2\sqrt{3}}{2}$

$= \frac{4+2\sqrt{3}}{2} = \frac{4}{2} + \frac{2\sqrt{3}}{2} = 2+\sqrt{3}$.

The correct option is (A).

23. Rationalize the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}-\sqrt{3}}$.

(A) $\frac{\sqrt{10}+2-\sqrt{6}}{4}$ (B) $\frac{\sqrt{10}+2+\sqrt{6}}{4}$ (C) $\frac{3\sqrt{2}-2\sqrt{3}+\sqrt{30}}{12}$ (D) $\frac{2\sqrt{5}+\sqrt{10}+\sqrt{30}}{12}$

Solution:

Let the expression be $E = \frac{1}{(\sqrt{5}+\sqrt{2})-\sqrt{3}}$.

Multiply by the conjugate of the denominator, treating $(\sqrt{5}+\sqrt{2})$ as one term:

$E = \frac{1}{(\sqrt{5}+\sqrt{2})-\sqrt{3}} \times \frac{(\sqrt{5}+\sqrt{2})+\sqrt{3}}{(\sqrt{5}+\sqrt{2})+\sqrt{3}} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{3}}{(\sqrt{5}+\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{5}+\sqrt{2}+\sqrt{3}}{(5+2+2\sqrt{10})-3} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{3}}{4+2\sqrt{10}}$.

Now, rationalize the new denominator $4+2\sqrt{10} = 2(2+\sqrt{10})$. Multiply by $(2-\sqrt{10})$:

$E = \frac{\sqrt{5}+\sqrt{2}+\sqrt{3}}{2(2+\sqrt{10})} \times \frac{2-\sqrt{10}}{2-\sqrt{10}} = \frac{(\sqrt{5}+\sqrt{2}+\sqrt{3})(2-\sqrt{10})}{2((2)^2-(\sqrt{10})^2)}$

$= \frac{2\sqrt{5}-\sqrt{50}+2\sqrt{2}-\sqrt{20}+2\sqrt{3}-\sqrt{30}}{2(4-10)}$
(where $\sqrt{50} = 5\sqrt{2}$, $\sqrt{20} = 2\sqrt{5}$)
$= \frac{2\sqrt{5}-5\sqrt{2}+2\sqrt{2}-2\sqrt{5}+2\sqrt{3}-\sqrt{30}}{-12}$
$= \frac{-3\sqrt{2}+2\sqrt{3}-\sqrt{30}}{-12}$.
Multiply numerator and denominator by -1:
$= \frac{3\sqrt{2}-2\sqrt{3}+\sqrt{30}}{12}$.

The correct option is (C).

24. The rationalising factor of $\sqrt[3]{9} + \sqrt[3]{3} + 1$ is? (Hint: Relates to $a^3 - b^3$)
(A) $\sqrt[3]{3} + 1$ (B) 2 (C) $\sqrt[3]{9} - 1$ (D) $\sqrt[3]{3} - 1$

Solution:

Let $x = \sqrt[3]{3}$. Then $x^2 = (\sqrt[3]{3})^2 = \sqrt[3]{9}$.
The given expression is $\sqrt[3]{9} + \sqrt[3]{3} + 1 = x^2 + x \cdot 1 + 1^2$.
This is in the form $a^2 + ab + b^2$ where $a = x = \sqrt[3]{3}$ and $b = 1$.
We know the identity $(a - b)(a^2 + ab + b^2) = a^3 - b^3$.
To rationalize the expression (i.e., make it an integer), we multiply by $(a - b)$.
So, the rationalising factor is $a - b = \sqrt[3]{3} - 1$.
Multiplying the expression by this factor gives:
$(\sqrt[3]{3} - 1)(\sqrt[3]{9} + \sqrt[3]{3} + 1) = (\sqrt[3]{3})^3 - 1^3 = 3 - 1 = 2$, which is rational.

The correct option is (D).

25. Simplify $\frac{\sqrt{72}-\sqrt{18}}{\sqrt{12}}$.
(A) $\sqrt{6}/2$ (B) $\sqrt{3}/2$ (C) $3\sqrt{6}/2$ (D) $3\sqrt{2}/2$

Solution:

Simplify the surds:
$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$.
$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$.
$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.
Substitute these into the expression:
$\frac{6\sqrt{2}-3\sqrt{2}}{2\sqrt{3}} = \frac{(6-3)\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{2\sqrt{3}}$.
Rationalize the denominator:
$\frac{3\sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{2\times 3}}{2(\sqrt{3})^2} = \frac{3\sqrt{6}}{2\times 3} = \frac{3\sqrt{6}}{6}$.
Simplify the fraction: $\frac{\sqrt{6}}{2}$.

The correct option is (A).

26. Simplify $\sqrt{4 + \sqrt{15}} + \sqrt{4 - \sqrt{15}}$. (Hint: Square the expression or find square roots individually)
(A) $\sqrt{10}$ (B) $\sqrt{6}$ (C) $2\sqrt{5/2}$ (D) 8

Solution:

Let $X = \sqrt{4 + \sqrt{15}} + \sqrt{4 - \sqrt{15}}$.
Since $4 > \sqrt{15}$ (as $16 > 15$), both terms are real and positive, so $X > 0$.
Square the expression:
$X^2 = (\sqrt{4 + \sqrt{15}} + \sqrt{4 - \sqrt{15}})^2$
$= (\sqrt{4 + \sqrt{15}})^2 + (\sqrt{4 - \sqrt{15}})^2 + 2(\sqrt{4 + \sqrt{15}})(\sqrt{4 - \sqrt{15}})$
$= (4 + \sqrt{15}) + (4 - \sqrt{15}) + 2\sqrt{(4 + \sqrt{15})(4 - \sqrt{15})}$
$= 8 + 2\sqrt{4^2 - (\sqrt{15})^2}$ [Using $(a - b)(a + b) = a^2 - b^2$]
$= 8 + 2\sqrt{16 - 15} = 8 + 2\sqrt{1} = 8 + 2(1) = 10$.
So, $X^2 = 10$. Since $X > 0$, $X = \sqrt{10}$.

The correct option is (A).

27. If $x = 3 + \sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$.
(A) 34 (B) 30 (C) 36 (D) $6 + \sqrt{8}$

Solution:

Given $x = 3 + \sqrt{8}$. Simplify $\sqrt{8} = 2\sqrt{2}$. So $x = 3 + 2\sqrt{2}$.
Find $\frac{1}{x}$:
$\frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{3^2-(2\sqrt{2})^2}$

$= \frac{3-2\sqrt{2}}{9-8} = \frac{3-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$.
Find $x + \frac{1}{x}$:
$x + \frac{1}{x} = (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) = 6$.
We need $x^2 + \frac{1}{x^2}$. We use the identity $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$.
$x^2 + \frac{1}{x^2} = (6)^2 - 2 = 36 - 2 = 34$.

The correct option is (A).

28. If $x = 2 + \sqrt{3}$, find the value of $\sqrt{x}$. (Hint: Find $\sqrt{2+\sqrt{3}}$)
(A) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (B) $\sqrt{2}+1$ (C) $\frac{\sqrt{6}+\sqrt{2}}{2}$ (D) $\sqrt{3}+1$

Solution:

We need to find $\sqrt{x} = \sqrt{2+\sqrt{3}}$.
To get the form $\sqrt{A + 2\sqrt{B}}$, multiply and divide by 2 inside the square root:
$\sqrt{2+\sqrt{3}} = \sqrt{\frac{2(2+\sqrt{3})}{2}} = \sqrt{\frac{4+2\sqrt{3}}{2}} = \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}}$.
For the numerator $\sqrt{4+2\sqrt{3}}$:
We need p and q such that $p + q = 4$ and $pq = 3$. These are 3 and 1.
So, $\sqrt{4+2\sqrt{3}} = \sqrt{(\sqrt{3}+\sqrt{1})^2} = \sqrt{3} + \sqrt{1} = \sqrt{3} + 1$.
Therefore, $\sqrt{x} = \frac{\sqrt{3}+1}{\sqrt{2}}$.
Rationalize this: $\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$.

The correct option is (C).

29. If $x = 9 - 4\sqrt{5}$, find the value of $x + \frac{1}{x}$.
(A) 18 (B) $-8\sqrt{5}$ (C) 81 (D) 10

Solution:

Given $x = 9 - 4\sqrt{5}$.
Find $\frac{1}{x}$:
$\frac{1}{x} = \frac{1}{9-4\sqrt{5}} = \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$
$= \frac{9+4\sqrt{5}}{9^2-(4\sqrt{5})^2} = \frac{9+4\sqrt{5}}{81-(16\times5)} = \frac{9+4\sqrt{5}}{81-80} = 9 + 4\sqrt{5}$.
Now, find $x + \frac{1}{x}$:
$x + \frac{1}{x} = (9 - 4\sqrt{5}) + (9 + 4\sqrt{5}) = 18$.

The correct option is (A).

30. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, find $x + y$.
(A) 8 (B) 16 (C) $2\sqrt{15}$ (D) 2

Solution:

Simplify x:
$x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$
$= \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$.
Notice that $y = \frac{1}{x}$. So, $y = \frac{1}{4+\sqrt{15}}$.
$y = \frac{1}{4+\sqrt{15}} \times \frac{4-\sqrt{15}}{4-\sqrt{15}} = \frac{4-\sqrt{15}}{16-15} = 4 - \sqrt{15}$.
Find $x + y$:
$x + y = (4 + \sqrt{15}) + (4 - \sqrt{15}) = 8$.

The correct option is (A).

31. Simplify the expression: $\frac{1}{2-\sqrt{3}} - \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{4}{3-\sqrt{5}}$.
(A) $3 + \sqrt{2} + \sqrt{7}$ (B) $5 + \sqrt{2} + \sqrt{5}$ (C) $\sqrt{2} + \sqrt{5}$ (D) $1 - \sqrt{5}$

Solution:

Rationalize each term separately:
Term 1: $\frac{1}{2-\sqrt{3}} = \frac{1(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$.
Term 2: $\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2}$.
Term 3: $\frac{4}{3-\sqrt{5}} = \frac{4(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{4(3+\sqrt{5})}{9-5} = \frac{4(3+\sqrt{5})}{4} = 3 + \sqrt{5}$.
Substitute back:
$(2 + \sqrt{3}) - (\sqrt{3} - \sqrt{2}) + (3 + \sqrt{5})$

$= 2 + \sqrt{3} - \sqrt{3} + \sqrt{2} + 3 + \sqrt{5}$
$= (2+3) + \sqrt{2} + \sqrt{5} + (\sqrt{3} - \sqrt{3}) = 5 + \sqrt{2} + \sqrt{5}$.

The correct option is (B).

32. The expression $\left(\frac{2}{\sqrt{6}+2} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}-\sqrt{7}} + 2 - 2\sqrt{2}\right)$ is equal to:

(A) 0 (B) $2\sqrt{2}$ (C) $\sqrt{6}$ (D) $2\sqrt{7}$

Solution:

Rationalize each fraction:
Term 1: $\frac{2}{\sqrt{6}+2} = \frac{2(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} = \frac{2(\sqrt{6}-2)}{6-4} = \sqrt{6} - 2$.
Term 2: $\frac{1}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}-\sqrt{6}}{7-6} = \sqrt{7} - \sqrt{6}$.
Term 3: $\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8} + \sqrt{7} = 2\sqrt{2} + \sqrt{7}$.
Substitute into the expression:
$(\sqrt{6} - 2) + (\sqrt{7} - \sqrt{6}) + (2\sqrt{2} + \sqrt{7}) + 2 - 2\sqrt{2}$
$= \sqrt{6} - 2 + \sqrt{7} - \sqrt{6} + 2\sqrt{2} + \sqrt{7} + 2 - 2\sqrt{2}$.
Combine like terms:
$(\sqrt{6} - \sqrt{6}) + (-2 + 2) + (\sqrt{7} + \sqrt{7}) + (2\sqrt{2} - 2\sqrt{2})$
$= 0 + 0 + 2\sqrt{7} + 0 = 2\sqrt{7}$.

The correct option is (D).

33. The value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{100}+\sqrt{99}}$ is:

(A) 1 (B) 9 (C) $\sqrt{99}$ (D) $\sqrt{99} - 1$

Solution:

Consider a general term: $\frac{1}{\sqrt{k+1}+\sqrt{k}}$.
Rationalize: $\frac{1}{\sqrt{k+1}+\sqrt{k}} = \frac{\sqrt{k+1}-\sqrt{k}}{(k+1)-k} = \sqrt{k+1} - \sqrt{k}$ [Rationalizing denominator].
The sum is:
$(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \cdots + (\sqrt{100} - \sqrt{99})$.
This is a telescoping sum:
$= -\sqrt{1} + (\cancel{\sqrt{2}} - \cancel{\sqrt{2}}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{3}}) + \cdots + (\cancel{\sqrt{99}} - \cancel{\sqrt{99}}) + \sqrt{100}$
$= -\sqrt{1} + \sqrt{100} = -1 + 10 = 9$.

The correct option is (B).

34. Simplify $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} + \cdots + \frac{1}{\sqrt{2}-\sqrt{1}}$.

(A) 0 (B) 9 (C) 10 (D) 11

Solution:

Consider a general term of the form $\frac{1}{\sqrt{k}-\sqrt{k-1}}$.
Rationalize: $\frac{1}{\sqrt{k}-\sqrt{k-1}} = \frac{\sqrt{k}+\sqrt{k-1}}{k-(k-1)} = \sqrt{k} + \sqrt{k-1}$ [Rationalizing denominator].
The series is:
$(\sqrt{100} + \sqrt{99}) - (\sqrt{99} + \sqrt{98}) + (\sqrt{98} + \sqrt{97}) - \ldots$
The last term is for $k = 2$. There are $100 - 2 + 1 = 99$ terms. The 99th term is positive.
So, $\cdots - (\sqrt{3} + \sqrt{2}) + (\sqrt{2} + \sqrt{1})$.
Expanding this:
$\sqrt{100} + \cancel{\sqrt{99}} - \cancel{\sqrt{99}} - \cancel{\sqrt{98}} + \cancel{\sqrt{98}} + \cancel{\sqrt{97}} - \cdots - \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} + \sqrt{1}$.
This is a telescoping sum, leaving:
$\sqrt{100} + \sqrt{1} = 10 + 1 = 11$.

The correct option is (D).

35. The value $\sqrt{3\sqrt{3\sqrt{3\ldots}}}$ is equal to:

(A) $\sqrt{3}$ (B) 3 (C) $2\sqrt{3}$ (D) $3\sqrt{3}$

Solution:

Let $x = \sqrt{3\sqrt{3\sqrt{3\ldots}}}$. Assume $x > 0$.

Square both sides: $x^2 = 3\sqrt{3\sqrt{3\sqrt{3\ldots}}}$.
The expression under the first radical on the RHS is x. So, $x^2 = 3x$.
Rearrange: $x^2 - 3x = 0 \implies x(x-3) = 0$.
Possible solutions are $x = 0$ or $x = 3$. Since $x > 0$, we have $x = 3$.
Alternatively, using fractional exponents:

$x = 3^{1/2} \cdot 3^{1/4} \cdot 3^{1/8} \cdots = 3^{(1/2+1/4+1/8+\dots)}$.
The exponent is an infinite geometric series $S = \frac{a}{1-r}$ with $a = 1/2, r = 1/2$.
$S = \frac{1/2}{1-1/2} = \frac{1/2}{1/2} = 1$ [Sum of infinite GP: $S = a/(1-r)$ for $|r| < 1$].
So, $x = 3^1 = 3$.

The correct option is (B).

36. Find the value of $\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$.
(A) 5 (B) $3\sqrt{10}$ (C) 6 (D) 7

Solution:

Let $x = \sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$. Assume $x > 0$.
Square both sides: $x^2 = 30 + \sqrt{30 + \sqrt{30 + \dots}}$.
The expression under the first radical on the RHS is x. So, $x^2 = 30 + x$.
Rearrange into a quadratic equation: $x^2 - x - 30 = 0$.
Factor the quadratic: $(x - 6)(x + 5) = 0$.
Possible solutions are $x = 6$ or $x = -5$. Since $x > 0$, we have $x = 6$.
Alternatively, if $k = n(n + 1)$, then $\sqrt{k + \sqrt{k + \dots}} = n + 1$.
Here $30 = 5 \times 6$, so $n = 5$. The value is $n + 1 = 5 + 1 = 6$.

The correct option is (C).

Answer Key

1	(B)	**2**	(C)	**3**	(B)	**4**	(A)	**5**	(A)	**6**	(A)	**7**	(D)	**8**	(C)	**9**	(A)	**10**	(C)
11	(B)	**12**	(B)	**13**	(A)	**14**	(C)	**15**	(A)	**16**	(C)	**17**	(B)	**18**	(D)	**19**	(A)	**20**	(A)
21	(B)	**22**	(A)	**23**	(C)	**24**	(D)	**25**	(A)	**26**	(A)	**27**	(A)	**28**	(C)	**29**	(A)	**30**	(A)
31	(B)	**32**	(D)	**33**	(B)	**34**	(D)	**35**	(B)	**36**	(C)								