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SRI VIDYA ARADHANA ACADEMY

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PCMB — PHYSICS CHEMISTRY MATHEMATICS BIOLOGY

NEET/JEE

Topic: LCM and HCF

Sub: Mathematics

Solutions to Assignment: 05

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Solutions to Que 1

1. **Find the HCF of 36 and 48 using the prime factorization method.**
Prime factorization of 36: $36 = 2 \times 18 = 2 \times 2 \times 9 = 2^2 \times 3^2$.
Prime factorization of 48: $48 = 2 \times 24 = 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$.
HCF is the product of the lowest powers of common prime factors.

$$\begin{aligned} \text{HCF}(36, 48) &= 2^{\min(2,4)} \times 3^{\min(2,1)} \\ &= 2^2 \times 3^1 \\ &= 4 \times 3 = 12. \end{aligned}$$

Answer: (C) 12

2. **Find the LCM of 8, 12, and 18.**
Prime factorization of 8: $8 = 2^3$.
Prime factorization of 12: $12 = 2^2 \times 3^1$.
Prime factorization of 18: $18 = 2^1 \times 3^2$.
LCM is the product of the highest powers of all prime factors involved.

$$\begin{aligned} \text{LCM}(8, 12, 18) &= 2^{\max(3,2,1)} \times 3^{\max(0,1,2)} \\ &= 2^3 \times 3^2 \\ &= 8 \times 9 = 72. \end{aligned}$$

Answer: (B) 72

3. **The HCF of two numbers is 6, and their LCM is 180. If one number is 30, find the other number.**
Let the two numbers be N_1 and N_2. We are given $N_1 = 30$, HCF $= 6$, LCM $= 180$.
We know that for two numbers, HCF $\times$ LCM $= N_1 \times N_2$.

$$\begin{aligned} 6 \times 180 &= 30 \times N_2 \\ 1080 &= 30 \times N_2 \\ N_2 &= \frac{1080}{30} \\ N_2 &= 36. \end{aligned}$$

Answer: (D) 36

4. **Find the HCF of 315 and 495 using Euclid's division algorithm.**

$$\begin{aligned} 495 &= 315 \times 1 + 180 \\ 315 &= 180 \times 1 + 135 \\ 180 &= 135 \times 1 + 45 \\ 135 &= 45 \times 3 + 0 \end{aligned}$$

The last non-zero remainder is 45. So, HCF(315, 495) = 45.
Answer: (C) 45

5. **Two numbers are in the ratio 3:4, and their HCF is 5. Find the numbers.**
Let the numbers be $3x$ and $4x$.
The HCF of $3x$ and $4x$ is x (since 3 and 4 are coprime).
Given HCF $= 5$, so $x = 5$.
The numbers are $3x = 3 \times 5 = 15$ and $4x = 4 \times 5 = 20$.
Answer: (C) 15, 20

6. **Find the greatest number that divides 42, 70, and 112 without leaving a remainder. (Find HCF)**
Prime factorization of 42: $42 = 2 \times 3 \times 7$.
Prime factorization of 70: $70 = 2 \times 5 \times 7$.
Prime factorization of 112: $112 = 2^4 \times 7$.
HCF is the product of the lowest powers of common prime factors.

$$\begin{aligned} \text{HCF}(42, 70, 112) &= 2^{\min(1,1,4)} \times 7^{\min(1,1,1)} \\ &= 2^1 \times 7^1 = 14. \end{aligned}$$

Answer: (A) 14

7. **Find the smallest number that is divisible by 15, 20, and 24. (Find LCM)**
Prime factorization of 15: $15 = 3 \times 5$.
Prime factorization of 20: $20 = 2^2 \times 5$.
Prime factorization of 24: $24 = 2^3 \times 3$.
LCM is the product of the highest powers of all prime factors involved.

$$\begin{aligned} \text{LCM}(15, 20, 24) &= 2^{\max(0,2,3)} \times 3^{\max(1,0,1)} \times 5^{\max(1,1,0)} \\ &= 2^3 \times 3^1 \times 5^1 \\ &= 8 \times 3 \times 5 = 120. \end{aligned}$$

Answer: (B) 120

8. **The HCF of two numbers is 12, and their LCM is 144. If one number is 36, find the other number.**
Let the other number be N_2. Given $N_1 = 36$, HCF = 12, LCM = 144.

$$\begin{aligned} \text{HCF} \times \text{LCM} &= N_1 \times N_2 \\ 12 \times 144 &= 36 \times N_2 \\ 1728 &= 36 \times N_2 \\ N_2 &= \frac{1728}{36} \\ N_2 &= 48. \end{aligned}$$

Answer: (C) 48

9. **The product of two numbers is 2028, and their HCF is 13. Find their LCM.**
We know that Product of two numbers = HCF $\times$ LCM.

$$\begin{aligned} 2028 &= 13 \times \text{LCM} \\ \text{LCM} &= \frac{2028}{13} \\ \text{LCM} &= 156. \end{aligned}$$

Answer: (D) 156

10. **Find the HCF of $2^3 \times 3^2 \times 5$ and $2^2 \times 3^3 \times 7$.**
HCF is the product of the lowest powers of common prime factors.

$$\begin{aligned} \text{HCF} &= 2^{\min(3,2)} \times 3^{\min(2,3)} \times 5^{\min(1,0)} \times 7^{\min(0,1)} \\ &= 2^2 \times 3^2 \times 5^0 \times 7^0 \\ &= 2^2 \times 3^2 = 4 \times 9 = 36. \end{aligned}$$

The option is $2^2 \times 3^2$.
Answer: (A) $2^2 \times 3^2$

11. **Find the LCM of $2^4 \times 3^2 \times 5$, $2^3 \times 3^3 \times 7$, and $2^2 \times 5^2 \times 7^2$.**
LCM is the product of the highest powers of all prime factors present in any of the numbers.

$$\begin{aligned} \text{LCM} &= 2^{\max(4,3,2)} \times 3^{\max(2,3,0)} \times 5^{\max(1,0,2)} \times 7^{\max(0,1,2)} \\ &= 2^4 \times 3^3 \times 5^2 \times 7^2. \end{aligned}$$

Answer: (C) $2^4 \times 3^3 \times 5^2 \times 7^2$

12. **Find the LCM of $\frac{3}{4}$ and $\frac{5}{6}$.**
LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$.
Numerators are 3 and 5. LCM(3, 5) = 15.
Denominators are 4 and 6. $4 = 2^2$, $6 = 2 \times 3$. HCF(4, 6) = $2^1 = 2$.
So, LCM$(\frac{3}{4}, \frac{5}{6}) = \frac{15}{2}$.
Answer: (B) $\frac{15}{2}$

13. Find the HCF of $\frac{3}{4}$ and $\frac{5}{6}$.
HCF of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$.
Numerators are 3 and 5. HCF(3, 5) = 1 (since they are coprime).
Denominators are 4 and 6. $4 = 2^2$, $6 = 2 \times 3$. LCM(4, 6) $= 2^2 \times 3 = 4 \times 3 = 12$.
So, HCF$(\frac{3}{4}, \frac{5}{6}) = \frac{1}{12}$.
Answer: (D) $\frac{1}{12}$

14. If the GCD (HCF) of a number n and 91 is 1, which of the following could be n?
First, find the prime factors of 91: $91 = 7 \times 13$.
For HCF$(n, 91) = 1$, n must not have 7 or 13 as a prime factor.
Let's check the options:

- (A) 30: Prime factors of 30 are 2, 3, 5. None are 7 or 13. So, HCF(30, 91) = 1.
- (B) 21: $21 = 3 \times 7$. It has 7 as a factor. HCF(21, 91) = 7.
- (C) 26: $26 = 2 \times 13$. It has 13 as a factor. HCF(26, 91) = 13.
- (D) 35: $35 = 5 \times 7$. It has 7 as a factor. HCF(35, 91) = 7.

So, n could be 30.
Answer: (A) 30

15. The LCM of a number n and 18 is 72. Which of the following can be n?
Prime factorization of 18: $18 = 2 \times 3^2 = 2^1 \times 3^2$.
Prime factorization of 72: $72 = 8 \times 9 = 2^3 \times 3^2$.
Let $n = 2^a \times 3^b \times k$, where k has no factors of 2 or 3.
$\text{LCM}(n, 18) = 2^{\max(a,1)} \times 3^{\max(b,2)} \times k = 2^3 \times 3^2$.
This implies:

- $\max(a, 1) = 3 \implies a = 3$.
- $\max(b, 2) = 2 \implies b$ can be 0, 1, or 2.
- $k = 1$.

So, n must be of the form $2^3 \times 3^b$ where $b \in \{0, 1, 2\}$. Possible values for n:

- $b = 0 : n = 2^3 \times 3^0 = 8 \times 1 = 8$. LCM(8, 18) $= \text{LCM}(2^3, 2 \times 3^2) = 2^3 \times 3^2 = 72$.
- $b = 1 : n = 2^3 \times 3^1 = 8 \times 3 = 24$. LCM(24, 18) $= \text{LCM}(2^3 \times 3, 2 \times 3^2) = 2^3 \times 3^2 = 72$.
- $b = 2 : n = 2^3 \times 3^2 = 8 \times 9 = 72$. LCM(72, 18) $= \text{LCM}(2^3 \times 3^2, 2 \times 3^2) = 2^3 \times 3^2 = 72$.

Checking the options:

- (A) 24: This is one of the possible values.
- (B) 48: $48 = 2^4 \times 3$. Here $a = 4 \neq 3$.
- (C) 16: $16 = 2^4$. Here $a = 4 \neq 3$.
- (D) 36: $36 = 2^2 \times 3^2$. Here $a = 2 \neq 3$.

Answer: (A) 24

16. The least number that is divisible by all the numbers from 1 to 10 (both inclusive).
This is asking for LCM(1, 2, 3, 4, 5, 6, 7, 8, 9, 10). We need to find the highest power of each prime factor up to 10. The primes are 2, 3, 5, 7.

- Highest power of 2: $2^3 = 8$ (from 8).
- Highest power of 3: $3^2 = 9$ (from 9).
- Highest power of 5: $5^1 = 5$ (from 5 or 10).
- Highest power of 7: $7^1 = 7$ (from 7).

$\text{LCM} = 2^3 \times 3^2 \times 5^1 \times 7^1 = 8 \times 9 \times 5 \times 7$.

$$\begin{aligned} \text{LCM} &= 72 \times 35 \\ &= 72 \times (30 + 5) \\ &= 72 \times 30 + 72 \times 5 \\ &= 2160 + 360 \\ &= 2520. \end{aligned}$$

Answer: (D) 2520

Solutions to Que 2

1. Original problem: $\frac{2x+1}{y} - \frac{3x+2}{2y}$

$$\begin{aligned}\frac{2x+1}{y} - \frac{3x+2}{2y} &= \frac{2(2x+1)}{2y} - \frac{3x+2}{2y} \\ &= \frac{4x+2-(3x+2)}{2y} \\ &= \frac{4x+2-3x-2}{2y} \\ &= \frac{x}{2y}\end{aligned}$$

2. Original problem: $\frac{2a-3b}{a} + \frac{4a^2-5b^2}{ab}$

$$\begin{aligned}\frac{2a-3b}{a} + \frac{4a^2-5b^2}{ab} &= \frac{b(2a-3b)}{ab} + \frac{4a^2-5b^2}{ab} \\ &= \frac{2ab-3b^2+4a^2-5b^2}{ab} \\ &= \frac{4a^2+2ab-8b^2}{ab} \\ &= \frac{2(2a^2+ab-4b^2)}{ab}\end{aligned}$$

3. Original problem: $\frac{a+3b}{(a-b)^2} + \frac{a-3b}{a^2-b^2}$

$$\begin{aligned}\frac{a+3b}{(a-b)^2} + \frac{a-3b}{a^2-b^2} &= \frac{a+3b}{(a-b)^2} + \frac{a-3b}{(a-b)(a+b)} \\ &= \frac{(a+3b)(a+b)}{(a-b)^2(a+b)} + \frac{(a-3b)(a-b)}{(a-b)^2(a+b)} \\ &= \frac{a^2+ab+3ab+3b^2+a^2-ab-3ab+3b^2}{(a-b)^2(a+b)} \\ &= \frac{2a^2+6b^2}{(a-b)^2(a+b)} \\ &= \frac{2(a^2+3b^2)}{(a-b)^2(a+b)}\end{aligned}$$

4. Original problem: $\frac{5a^2-b^2}{ab} - \frac{3a-2b}{b}$

$$\begin{aligned}\frac{5a^2-b^2}{ab} - \frac{3a-2b}{b} &= \frac{5a^2-b^2}{ab} - \frac{a(3a-2b)}{ab} \\ &= \frac{5a^2-b^2-(3a^2-2ab)}{ab} \\ &= \frac{5a^2-b^2-3a^2+2ab}{ab} \\ &= \frac{2a^2+2ab-b^2}{ab}\end{aligned}$$

5. Original problem: $\frac{x}{ac} - \frac{x}{bc} + \frac{x}{ab}$

$$\begin{aligned}\frac{x}{ac} - \frac{x}{bc} + \frac{x}{ab} &= \frac{xb}{abc} - \frac{xa}{abc} + \frac{xc}{abc} \\ &= \frac{xb-xa+xc}{abc} \\ &= \frac{x(b-a+c)}{abc}\end{aligned}$$

6. Original problem: $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ba}$

$$\begin{aligned}\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab} &= \frac{a \cdot a}{abc} + \frac{b \cdot b}{abc} + \frac{c \cdot c}{abc} \\ &= \frac{a^2+b^2+c^2}{abc}\end{aligned}$$

7. Original problem: $\frac{1}{x^2y^3} + \frac{2}{x^3y^4}$

$$\begin{aligned}\frac{1}{x^2y^3} + \frac{2}{x^3y^4} &= \frac{1 \cdot xy}{x^2y^3 \cdot xy} + \frac{2}{x^3y^4} \\ &= \frac{xy}{x^3y^4} + \frac{2}{x^3y^4} \\ &= \frac{xy+2}{x^3y^4}\end{aligned}$$

8. Original problem: $\frac{x+1}{x^2+1} - \frac{x+2}{2x^2-2}$

$$\begin{aligned}\frac{x+1}{x^2+1} - \frac{x+2}{2x^2-2} &= \frac{x+1}{x^2+1} - \frac{x+2}{2(x^2-1)} \\ &= \frac{x+1}{x^2+1} - \frac{x+2}{2(x-1)(x+1)} \\ &= \frac{2(x-1)(x+1)(x+1)}{(x^2+1)2(x-1)(x+1)} - \frac{(x+2)(x^2+1)}{2(x-1)(x+1)(x^2+1)} \\ &= \frac{2(x-1)(x+1)^2 - (x+2)(x^2+1)}{2(x^2+1)(x-1)(x+1)} \\ &= \frac{2(x-1)(x^2+2x+1) - (x^3+x+2x^2+2)}{2(x^2+1)(x-1)(x+1)} \\ &= \frac{2(x^3+2x^2+x-x^2-2x-1) - (x^3+2x^2+x+2)}{2(x^2+1)(x-1)(x+1)} \\ &= \frac{2(x^3+x^2-x-1) - x^3-2x^2-x-2}{2(x^2+1)(x-1)(x+1)} \\ &= \frac{2x^3+2x^2-2x-2-x^3-2x^2-x-2}{2(x^2+1)(x-1)(x+1)} \\ &= \frac{x^3-3x-4}{2(x^2+1)(x-1)(x+1)}\end{aligned}$$

9. Original problem: $\frac{a+1}{a^2-a} - \frac{a+2}{2a^2-2}$

$$\begin{aligned}\frac{a+1}{a^2-a} - \frac{a+2}{2a^2-2} &= \frac{a+1}{a(a-1)} - \frac{a+2}{2(a^2-1)} \\ &= \frac{a+1}{a(a-1)} - \frac{a+2}{2(a-1)(a+1)} \\ &= \frac{2(a+1)(a+1)}{2a(a-1)(a+1)} - \frac{a(a+2)}{2a(a-1)(a+1)} \\ &= \frac{2(a^2+2a+1) - (a^2+2a)}{2a(a-1)(a+1)} \\ &= \frac{2a^2+4a+2-a^2-2a}{2a(a-1)(a+1)} \\ &= \frac{a^2+2a+2}{2a(a-1)(a+1)}\end{aligned}$$

10. Original problem: $\frac{2x-3y}{x^2y} - \frac{4x-5y}{xy^2}$

$$\begin{aligned}\frac{2x-3y}{x^2y} - \frac{4x-5y}{xy^2} &= \frac{y(2x-3y)}{x^2y^2} - \frac{x(4x-5y)}{x^2y^2} \\ &= \frac{2xy-3y^2-(4x^2-5xy)}{x^2y^2} \\ &= \frac{2xy-3y^2-4x^2+5xy}{x^2y^2} \\ &= \frac{-4x^2+7xy-3y^2}{x^2y^2}\end{aligned}$$

11. Original problem: $\frac{2a^2+3a-5}{a^2b} - \frac{1-4a}{ab}$

$$\begin{aligned}\frac{2a^2+3a-5}{a^2b} - \frac{1-4a}{ab} &= \frac{2a^2+3a-5}{a^2b} - \frac{a(1-4a)}{a^2b}\\ &= \frac{2a^2+3a-5-(a-4a^2)}{a^2b}\\ &= \frac{2a^2+3a-5-a+4a^2}{a^2b}\\ &= \frac{6a^2+2a-5}{a^2b}\end{aligned}$$

12. Original problem: $\frac{1}{a-b} - \frac{1}{a+b}$

$$\begin{aligned}\frac{1}{a-b} - \frac{1}{a+b} &= \frac{1(a+b)}{(a-b)(a+b)} - \frac{1(a-b)}{(a-b)(a+b)}\\ &= \frac{a+b-(a-b)}{(a-b)(a+b)}\\ &= \frac{a+b-a+b}{a^2-b^2}\\ &= \frac{2b}{a^2-b^2}\end{aligned}$$

13. Original problem: $\frac{5}{x-y} - \frac{3}{2x-2y}$

$$\begin{aligned}\frac{5}{x-y} - \frac{3}{2x-2y} &= \frac{5}{x-y} - \frac{3}{2(x-y)}\\ &= \frac{2\cdot 5}{2(x-y)} - \frac{3}{2(x-y)}\\ &= \frac{10-3}{2(x-y)}\\ &= \frac{7}{2(x-y)}\end{aligned}$$

14. Original problem: $\frac{4}{a-b} - \frac{1}{b-a}$

$$\begin{aligned}\frac{4}{a-b} - \frac{1}{b-a} &= \frac{4}{a-b} - \frac{1}{-(a-b)}\\ &= \frac{4}{a-b} + \frac{1}{a-b}\\ &= \frac{4+1}{a-b}\\ &= \frac{5}{a-b}\end{aligned}$$

15. Original problem: $\frac{4}{r+2} + \frac{3}{r-2} - \frac{7r}{r^2-4}$

$$\begin{aligned}\frac{4}{r+2} + \frac{3}{r-2} - \frac{7r}{r^2-4} &= \frac{4}{r+2} + \frac{3}{r-2} - \frac{7r}{(r-2)(r+2)}\\ &= \frac{4(r-2)}{(r+2)(r-2)} + \frac{3(r+2)}{(r-2)(r+2)} - \frac{7r}{(r-2)(r+2)}\\ &= \frac{4r-8+3r+6-7r}{(r-2)(r+2)}\\ &= \frac{7r-2-7r}{(r-2)(r+2)}\\ &= \frac{-2}{(r-2)(r+2)}\\ &= \frac{-2}{r^2-4}\end{aligned}$$

16. Original problem: $\frac{7a^2}{a^2-9} + \frac{5a}{a-3} + \frac{a}{a+3}$

$$\begin{aligned}\frac{7a^2}{a^2-9} + \frac{5a}{a-3} + \frac{a}{a+3} &= \frac{7a^2}{(a-3)(a+3)} + \frac{5a(a+3)}{(a-3)(a+3)} + \frac{a(a-3)}{(a+3)(a-3)} \\ &= \frac{7a^2+5a^2+15a+a^2-3a}{(a-3)(a+3)} \\ &= \frac{13a^2+12a}{(a-3)(a+3)} \\ &= \frac{13a^2+12a}{a^2-9}\end{aligned}$$

17. Original problem: $\frac{a-b}{2a+2b} + \frac{a^2+b^2}{a^2-a}$

$$\begin{aligned}\frac{a-b}{2a+2b} + \frac{a^2+b^2}{a^2-a} &= \frac{a-b}{2(a+b)} + \frac{a^2+b^2}{a(a-1)} \\ &= \frac{a(a-1)(a-b)}{2a(a+b)(a-1)} + \frac{2(a+b)(a^2+b^2)}{2a(a+b)(a-1)} \\ &= \frac{a(a^2-ab-a+b)+2(a^3+ab^2+a^2b+b^3)}{2a(a+b)(a-1)} \\ &= \frac{a^3-a^2b-a^2+ab+2a^3+2ab^2+2a^2b+2b^3}{2a(a+b)(a-1)} \\ &= \frac{3a^3+a^2b-a^2+ab+2ab^2+2b^3}{2a(a-1)(a+b)}\end{aligned}$$

18. Original problem: $\frac{7x-1}{2x^2-6x} - \frac{3x-5}{x^2-9}$

$$\begin{aligned}\frac{7x-1}{2x^2-6x} - \frac{3x-5}{x^2-9} &= \frac{7x-1}{2x(x-3)} - \frac{3x-5}{(x-3)(x+3)} \\ &= \frac{(7x-1)(x+3)}{2x(x-3)(x+3)} - \frac{2x(3x-5)}{2x(x-3)(x+3)} \\ &= \frac{7x^2+21x-x-3-(6x^2-10x)}{2x(x-3)(x+3)} \\ &= \frac{7x^2+20x-3-6x^2+10x}{2x(x-3)(x+3)} \\ &= \frac{x^2+30x-3}{2x(x-3)(x+3)}\end{aligned}$$

19. Original problem: $\frac{3a-b}{3a^2b} + \frac{a^2+b^2}{2a^2b^2} - \frac{a+b}{2ab^2}$

$$\begin{aligned}&\frac{3a-b}{3a^2b} + \frac{a^2+b^2}{2a^2b^2} - \frac{a+b}{2ab^2} \\ &= \frac{2b(3a-b)}{6a^2b^2} + \frac{3(a^2+b^2)}{6a^2b^2} - \frac{3a(a+b)}{6a^2b^2} \\ &= \frac{6ab-2b^2+3a^2+3b^2-(3a^2+3ab)}{6a^2b^2} \\ &= \frac{6ab-2b^2+3a^2+3b^2-3a^2-3ab}{6a^2b^2} \\ &= \frac{3ab+b^2}{6a^2b^2} = \frac{b(3a+b)}{6a^2b^2} = \frac{3a+b}{6a^2b}\end{aligned}$$

20. Original problem: $\frac{3x}{4a^2b} - \frac{7}{6ab^5} - \frac{5x}{2ab^2}$

$$\begin{aligned}&\frac{3x}{4a^2b} - \frac{7}{6ab^5} - \frac{5x}{2ab^2} \quad \text{LCM is } 12a^2b^5 \\ &= \frac{3x\cdot 3b^4}{12a^2b^5} - \frac{7\cdot 2a}{12a^2b^5} - \frac{5x\cdot 6ab^3}{12a^2b^5} \\ &= \frac{9xb^4-14a-30axb^3}{12a^2b^5}\end{aligned}$$

21. Original problem: $\frac{5}{t-3} - \frac{t-2}{t^2-9} + \frac{t-1}{2t+6}$

$$\begin{aligned}
&\frac{5}{t-3} - \frac{t-2}{(t-3)(t+3)} + \frac{t-1}{2(t+3)} \quad \text{LCM is } 2(t-3)(t+3) \\
&= \frac{5 \cdot 2(t+3)}{2(t-3)(t+3)} - \frac{(t-2)\cdot 2}{2(t-3)(t+3)} + \frac{(t-1)(t-3)}{2(t+3)(t-3)} \\
&= \frac{10(t+3) - 2(t-2) + (t-1)(t-3)}{2(t-3)(t+3)} \\
&= \frac{10t + 30 - 2t + 4 + t^2 - 3t - t + 3}{2(t^2-9)} \\
&= \frac{t^2 + (10-2-3-1)t + (30+4+3)}{2(t^2-9)} \\
&= \frac{t^2 + 4t + 37}{2(t^2-9)}
\end{aligned}$$

22. Original problem: $\frac{5}{a+2} + \frac{2a}{a^2+4a+4} - \frac{4}{a-2}$

$$\begin{aligned}
&\frac{5}{a+2} + \frac{2a}{(a+2)^2} - \frac{4}{a-2} \quad \text{LCM is } (a+2)^2(a-2) \\
&= \frac{5(a+2)(a-2)}{(a+2)^2(a-2)} + \frac{2a(a-2)}{(a+2)^2(a-2)} - \frac{4(a+2)^2}{(a-2)(a+2)^2} \\
&= \frac{5(a^2-4) + 2a^2 - 4a - 4(a^2+4a+4)}{(a+2)^2(a-2)} \\
&= \frac{5a^2 - 20 + 2a^2 - 4a - 4a^2 - 16a - 16}{(a+2)^2(a-2)} \\
&= \frac{(5+2-4)a^2 + (-4-16)a + (-20-16)}{(a+2)^2(a-2)} \\
&= \frac{3a^2 - 20a - 36}{(a+2)^2(a-2)}
\end{aligned}$$

23. Original problem: $\frac{(a-1)a}{a^2-25} + \frac{a-2}{5-a} - \frac{a-3}{a+5}$

$$\begin{aligned}
&\frac{a(a-1)}{(a-5)(a+5)} + \frac{a-2}{-(a-5)} - \frac{a-3}{a+5} \\
&= \frac{a^2-a}{(a-5)(a+5)} - \frac{a-2}{a-5} - \frac{a-3}{a+5} \quad \text{LCM is } (a-5)(a+5) \\
&= \frac{a^2-a}{(a-5)(a+5)} - \frac{(a-2)(a+5)}{(a-5)(a+5)} - \frac{(a-3)(a-5)}{(a+5)(a-5)} \\
&= \frac{a^2 - a - (a^2 + 5a - 2a - 10) - (a^2 - 5a - 3a + 15)}{(a-5)(a+5)} \\
&= \frac{a^2 - a - (a^2 + 3a - 10) - (a^2 - 8a + 15)}{(a-5)(a+5)} \\
&= \frac{a^2 - a - a^2 - 3a + 10 - a^2 + 8a - 15}{(a-5)(a+5)} \\
&= \frac{-a^2 + 4a - 5}{(a-5)(a+5)} = \frac{-(a^2 - 4a + 5)}{(a-5)(a+5)}
\end{aligned}$$

24. Original problem: $\frac{2x-1}{2x} - \frac{2x}{2x-1} - \frac{1}{2x-4x^2}$

$$\begin{aligned}
&\frac{2x-1}{2x} - \frac{2x}{2x-1} - \frac{1}{2x(1-2x)} \\
&= \frac{2x-1}{2x} - \frac{2x}{2x-1} + \frac{1}{2x(2x-1)} \quad \text{LCM is } 2x(2x-1) \\
&= \frac{(2x-1)(2x-1)}{2x(2x-1)} - \frac{2x(2x)}{2x(2x-1)} + \frac{1}{2x(2x-1)} \\
&= \frac{(2x-1)^2 - 4x^2 + 1}{2x(2x-1)} \\
&= \frac{4x^2 - 4x + 1 - 4x^2 + 1}{2x(2x-1)} \\
&= \frac{-4x+2}{2x(2x-1)} = \frac{2(-2x+1)}{2x(2x-1)} = \frac{-(2x-1)}{x(2x-1)} = -\frac{1}{x}
\end{aligned}$$

25. Original problem: $\frac{r+1}{r^2-2r} + \frac{r+1}{r^2+2r} - \frac{2r}{r^2-4}$

$$\begin{aligned}
&\frac{r+1}{r(r-2)} + \frac{r+1}{r(r+2)} - \frac{2r}{(r-2)(r+2)} \quad \text{LCM is } r(r-2)(r+2) \\
&= \frac{(r+1)(r+2)}{r(r-2)(r+2)} + \frac{(r+1)(r-2)}{r(r+2)(r-2)} - \frac{2r \cdot r}{r(r-2)(r+2)} \\
&= \frac{r^2 + 3r + 2 + r^2 - r - 2 - 2r^2}{r(r-2)(r+2)} \\
&= \frac{(1+1-2)r^2 + (3-1)r + (2-2)}{r(r-2)(r+2)} \\
&= \frac{2r}{r(r-2)(r+2)} = \frac{2}{(r-2)(r+2)} = \frac{2}{r^2-4}
\end{aligned}$$

26. Original problem: $\frac{a+b}{2(a-b)} - \frac{a-b}{2(a+b)} - \frac{2b^2}{b^2-a^2}$

$$\begin{aligned}
&\frac{a+b}{2(a-b)} - \frac{a-b}{2(a+b)} - \frac{2b^2}{-(a^2-b^2)} \\
&= \frac{a+b}{2(a-b)} - \frac{a-b}{2(a+b)} + \frac{2b^2}{(a-b)(a+b)} \quad \text{LCM is } 2(a-b)(a+b) \\
&= \frac{(a+b)(a+b)}{2(a-b)(a+b)} - \frac{(a-b)(a-b)}{2(a+b)(a-b)} + \frac{2b^2 \cdot 2}{2(a-b)(a+b)} \\
&= \frac{(a+b)^2 - (a-b)^2 + 4b^2}{2(a-b)(a+b)} \\
&= \frac{(a^2 + 2ab + b^2) - (a^2 - 2ab + b^2) + 4b^2}{2(a-b)(a+b)} \\
&= \frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2 + 4b^2}{2(a-b)(a+b)} \\
&= \frac{4ab + 4b^2}{2(a-b)(a+b)} = \frac{4b(a+b)}{2(a-b)(a+b)} = \frac{2b}{a-b}
\end{aligned}$$

27. Original problem: $\frac{2x-y}{10x} - \frac{y}{2x} + \frac{2y-x}{15x}$

$$\begin{aligned}
&\frac{2x-y}{10x} - \frac{y}{2x} + \frac{2y-x}{15x} \quad \text{LCM is } 30x \\
&= \frac{3(2x-y)}{30x} - \frac{15y}{30x} + \frac{2(2y-x)}{30x} \\
&= \frac{6x - 3y - 15y + 4y - 2x}{30x} \\
&= \frac{(6-2)x + (-3-15+4)y}{30x} \\
&= \frac{4x - 14y}{30x} = \frac{2(2x-7y)}{30x} = \frac{2x-7y}{15x}
\end{aligned}$$

28. Original problem: $\frac{3}{x+1} - \frac{2}{x-1} + \frac{5}{x^2-1}$

$$\begin{aligned}
&\frac{3}{x+1} - \frac{2}{x-1} + \frac{5}{(x-1)(x+1)} \quad \text{LCM is } (x-1)(x+1) \\
&= \frac{3(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x-1)(x+1)} + \frac{5}{(x-1)(x+1)} \\
&= \frac{3x-3-(2x+2)+5}{(x-1)(x+1)} \\
&= \frac{3x-3-2x-2+5}{(x-1)(x+1)} \\
&= \frac{x}{x^2-1}
\end{aligned}$$

29. Original problem: $\frac{4a}{a^2-b^2} + \frac{2b}{b^2-a^2}$

$$\begin{aligned}
\frac{4a}{a^2-b^2} + \frac{2b}{-(a^2-b^2)} &= \frac{4a}{a^2-b^2} - \frac{2b}{a^2-b^2} \\
&= \frac{4a-2b}{a^2-b^2} \\
&= \frac{2(2a-b)}{a^2-b^2}
\end{aligned}$$

30. Original problem: $\frac{1}{x^2-4} - \frac{1}{x^2+4x+4}$

$$\begin{aligned}
&\frac{1}{(x-2)(x+2)} - \frac{1}{(x+2)^2} \quad \text{LCM is } (x-2)(x+2)^2 \\
&= \frac{1(x+2)}{(x-2)(x+2)^2} - \frac{1(x-2)}{(x+2)^2(x-2)} \\
&= \frac{x+2-(x-2)}{(x-2)(x+2)^2} \\
&= \frac{x+2-x+2}{(x-2)(x+2)^2} \\
&= \frac{4}{(x-2)(x+2)^2}
\end{aligned}$$

31. Original problem: $\frac{2}{x^2-9} + \frac{3}{x^2+3x}$

$$\begin{aligned}
&\frac{2}{(x-3)(x+3)} + \frac{3}{x(x+3)} \quad \text{LCM is } x(x-3)(x+3) \\
&= \frac{2x}{x(x-3)(x+3)} + \frac{3(x-3)}{x(x+3)(x-3)} \\
&= \frac{2x+3x-9}{x(x-3)(x+3)} \\
&= \frac{5x-9}{x(x^2-9)}
\end{aligned}$$

32. Original problem: $\frac{5}{x^2-x-2} - \frac{3}{x^2+x-6}$

$$\begin{aligned}
&\frac{5}{(x-2)(x+1)} - \frac{3}{(x+3)(x-2)} \quad \text{LCM is } (x-2)(x+1)(x+3) \\
&= \frac{5(x+3)}{(x-2)(x+1)(x+3)} - \frac{3(x+1)}{(x+3)(x-2)(x+1)} \\
&= \frac{5x+15-(3x+3)}{(x-2)(x+1)(x+3)} \\
&= \frac{5x+15-3x-3}{(x-2)(x+1)(x+3)} \\
&= \frac{2x+12}{(x-2)(x+1)(x+3)} = \frac{2(x+6)}{(x-2)(x+1)(x+3)}
\end{aligned}$$

33. Original problem: $\frac{a}{a^2-4}+\frac{b}{b^2-4}$

$$\frac{a}{(a-2)(a+2)}+\frac{b}{(b-2)(b+2)} \quad \text{LCM is } (a-2)(a+2)(b-2)(b+2)$$
$$=\frac{a(b-2)(b+2)}{(a-2)(a+2)(b-2)(b+2)}+\frac{b(a-2)(a+2)}{(b-2)(b+2)(a-2)(a+2)}$$
$$=\frac{a(b^2-4)+b(a^2-4)}{(a-2)(a+2)(b-2)(b+2)}$$
$$=\frac{ab^2-4a+a^2b-4b}{(a-2)(a+2)(b-2)(b+2)}$$

34. Original problem: $\frac{2x}{x^2-1}-\frac{3}{x+1}+\frac{1}{x-1}$

$$\frac{2x}{(x-1)(x+1)}-\frac{3}{x+1}+\frac{1}{x-1} \quad \text{LCM is } (x-1)(x+1)$$
$$=\frac{2x}{(x-1)(x+1)}-\frac{3(x-1)}{(x+1)(x-1)}+\frac{1(x+1)}{(x-1)(x+1)}$$
$$=\frac{2x-(3x-3)+(x+1)}{(x-1)(x+1)}$$
$$=\frac{2x-3x+3+x+1}{(x-1)(x+1)}$$
$$=\frac{4}{(x-1)(x+1)}=\frac{4}{x^2-1}$$

35. Original problem: $\frac{4}{x^2-2x}+\frac{3}{x^2+2x}-\frac{2}{x^2-4}$

$$\frac{4}{x(x-2)}+\frac{3}{x(x+2)}-\frac{2}{(x-2)(x+2)} \quad \text{LCM is } x(x-2)(x+2)$$
$$=\frac{4(x+2)}{x(x-2)(x+2)}+\frac{3(x-2)}{x(x+2)(x-2)}-\frac{2x}{x(x-2)(x+2)}$$
$$=\frac{4x+8+3x-6-2x}{x(x-2)(x+2)}$$
$$=\frac{5x+2}{x(x-2)(x+2)}=\frac{5x+2}{x(x^2-4)}$$

36. Original problem: $\frac{1}{x^2+x}-\frac{1}{x^2-x}+\frac{2}{x^2-1}$

$$\frac{1}{x(x+1)}-\frac{1}{x(x-1)}+\frac{2}{(x-1)(x+1)} \quad \text{LCM is } x(x-1)(x+1)$$
$$=\frac{1(x-1)}{x(x+1)(x-1)}-\frac{1(x+1)}{x(x-1)(x+1)}+\frac{2x}{x(x-1)(x+1)}$$
$$=\frac{x-1-(x+1)+2x}{x(x-1)(x+1)}$$
$$=\frac{x-1-x-1+2x}{x(x-1)(x+1)}$$
$$=\frac{2x-2}{x(x-1)(x+1)}=\frac{2(x-1)}{x(x-1)(x+1)}=\frac{2}{x(x+1)}$$

37. Original problem: $\frac{3a}{a^2-a-2}+\frac{2}{a^2+a-2}$

$$\frac{3a}{(a-2)(a+1)}+\frac{2}{(a+2)(a-1)} \quad \text{LCM is } (a-2)(a+1)(a+2)(a-1)$$
$$=\frac{3a(a+2)(a-1)}{(a-2)(a+1)(a+2)(a-1)}+\frac{2(a-2)(a+1)}{(a+2)(a-1)(a-2)(a+1)}$$
$$=\frac{3a(a^2+a-2)+2(a^2-a-2)}{(a^2-4)(a^2-1)}$$
$$=\frac{3a^3+3a^2-6a+2a^2-2a-4}{(a^2-1)(a^2-4)}$$
$$=\frac{3a^3+5a^2-8a-4}{(a^2-1)(a^2-4)}$$

38. Original problem: $\frac{5}{x^2-5x+6} - \frac{2}{x^2-4x+3}$

$$\begin{aligned}
&\frac{5}{(x-2)(x-3)} - \frac{2}{(x-1)(x-3)} \quad \text{LCM is } (x-1)(x-2)(x-3)\\
&= \frac{5(x-1)}{(x-2)(x-3)(x-1)} - \frac{2(x-2)}{(x-1)(x-3)(x-2)}\\
&= \frac{5x-5-(2x-4)}{(x-1)(x-2)(x-3)}\\
&= \frac{5x-5-2x+4}{(x-1)(x-2)(x-3)}\\
&= \frac{3x-1}{(x-1)(x-2)(x-3)}
\end{aligned}$$

39. Original problem: $\frac{2x}{x^2-9} + \frac{3}{x-3} - \frac{1}{x+3}$

$$\begin{aligned}
&\frac{2x}{(x-3)(x+3)} + \frac{3}{x-3} - \frac{1}{x+3} \quad \text{LCM is } (x-3)(x+3)\\
&= \frac{2x}{(x-3)(x+3)} + \frac{3(x+3)}{(x-3)(x+3)} - \frac{1(x-3)}{(x+3)(x-3)}\\
&= \frac{2x+3x+9-(x-3)}{(x-3)(x+3)}\\
&= \frac{5x+9-x+3}{(x-3)(x+3)}\\
&= \frac{4x+12}{(x-3)(x+3)} = \frac{4(x+3)}{(x-3)(x+3)} = \frac{4}{x-3}
\end{aligned}$$

40. Original problem: $\frac{4}{x^2-1} - \frac{2}{x^2+2x+1} + \frac{3}{x^2-x-2}$

$$\begin{aligned}
&\frac{4}{(x-1)(x+1)} - \frac{2}{(x+1)^2} + \frac{3}{(x-2)(x+1)} \quad \text{LCM is } (x-1)(x+1)^2(x-2)\\
&= \frac{4(x+1)(x-2)}{(x-1)(x+1)^2(x-2)} - \frac{2(x-1)(x-2)}{(x+1)^2(x-1)(x-2)} + \frac{3(x-1)(x+1)}{(x-2)(x+1)(x-1)(x+1)}\\
\text{Numerator} &= 4(x^2-x-2) - 2(x^2-3x+2) + 3(x^2-1)\\
&= 4x^2-4x-8-(2x^2-6x+4)+3x^2-3\\
&= 4x^2-4x-8-2x^2+6x-4+3x^2-3\\
&= (4-2+3)x^2+(-4+6)x+(-8-4-3)\\
&= 5x^2+2x-15
\end{aligned}$$

Therefore, the expression equals $\frac{5x^2+2x-15}{(x-1)(x+1)^2(x-2)}$

41. Original problem: $\frac{3}{x^2-x} + \frac{2}{x^2+x} - \frac{1}{x^2-1}$

$$\begin{aligned}
&\frac{3}{x(x-1)} + \frac{2}{x(x+1)} - \frac{1}{(x-1)(x+1)} \quad \text{LCM is } x(x-1)(x+1)\\
&= \frac{3(x+1)}{x(x-1)(x+1)} + \frac{2(x-1)}{x(x+1)(x-1)} - \frac{1x}{x(x-1)(x+1)}\\
&= \frac{3x+3+2x-2-x}{x(x-1)(x+1)}\\
&= \frac{4x+1}{x(x-1)(x+1)} = \frac{4x+1}{x(x^2-1)}
\end{aligned}$$

42. Original problem: $\frac{4}{x^2-4x+3} - \frac{2}{x^2-x-6} + \frac{1}{x^2+2x-3}$

$$\frac{4}{(x-1)(x-3)} - \frac{2}{(x-3)(x+2)} + \frac{1}{(x+3)(x-1)}$$

LCM is $(x-1)(x-3)(x+2)(x+3)$

$$= \frac{4(x+2)(x+3)}{(x-1)(x-3)(x+2)(x+3)} - \frac{2(x-1)(x+3)}{(x-3)(x+2)(x-1)(x+3)}$$
$$+ \frac{1(x-3)(x+2)}{(x+3)(x-1)(x-3)(x+2)}$$

$$\text{Numerator} = 4(x^2+5x+6) - 2(x^2+2x-3) + (x^2-x-6)$$
$$= 4x^2+20x+24-(2x^2+4x-6)+x^2-x-6$$
$$= 4x^2+20x+24-2x^2-4x+6+x^2-x-6$$
$$= (4-2+1)x^2+(20-4-1)x+(24+6-6)$$
$$= 3x^2+15x+24 = 3(x^2+5x+8)$$

Therefore, the expression equals $\frac{3(x^2+5x+8)}{(x-1)(x-3)(x+2)(x+3)}$