Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Solutions to Assignment: 01

Trigonometric Ratios & Identities

Topic: Basic Identities

Sub: Mathematics **Assignment: 01** Prof. Chetan Sir

Que 1: Prove the following identities

1. Prove that $\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \cos^4 A - \sin^4 A + 1 \\
&= (\cos^2 A)^2 - (\sin^2 A)^2 + 1 && \text{[Expressing in the form of squares]} \\
&= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A) + 1 && \text{[Using difference of squares: } a^2 - b^2 = (a-b)(a+b)\text{]} \\
&= (\cos^2 A - \sin^2 A)(1) + 1 && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \cos^2 A - \sin^2 A + 1 \\
&= \cos^2 A - (1 - \cos^2 A) + 1 && \text{[Pythagorean Identity: } \sin^2 A = 1 - \cos^2 A\text{]} \\
&= \cos^2 A - 1 + \cos^2 A + 1 \\
&= \mathbf{2\cos^2 A} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

2. Prove that $(\sin A + \cos A)(1 - \sin A\cos A) = \sin^3 A + \cos^3 A$.

Proof:

$$
\begin{aligned}
\text{R.H.S.} &= \sin^3 A + \cos^3 A \\
&= (\sin A + \cos A)(\sin^2 A - \sin A\cos A + \cos^2 A) && \text{[Sum of cubes: } a^3 + b^3 = (a+b)(a^2 - ab + b^2)\text{]} \\
&= (\sin A + \cos A)((\sin^2 A + \cos^2 A) - \sin A\cos A) && \text{[Rearranging terms]} \\
&= (\sin A + \cos A)(1 - \sin A\cos A) && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \text{L.H.S.}
\end{aligned}
$$

Hence Proved.

3. Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} \\
&= \frac{\sin^2 A + (1+\cos A)^2}{\sin A(1+\cos A)} && \text{[Taking the common denominator (LCM)]} \\
&= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{\sin A(1+\cos A)} && \text{[Expanding } (a+b)^2 = a^2+2ab+b^2\text{]} \\
&= \frac{(\sin^2 A + \cos^2 A) + 1 + 2\cos A}{\sin A(1+\cos A)} && \text{[Grouping terms]} \\
&= \frac{1+1+2\cos A}{\sin A(1+\cos A)} && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \frac{2+2\cos A}{\sin A(1+\cos A)} \\
&= \frac{2(1+\cos A)}{\sin A(1+\cos A)} && \text{[Factoring out 2 from the numerator]} \\
&= \frac{2}{\sin A} && \text{[Cancelling the common factor } (1+\cos A)\text{]} \\
&= \mathbf{2\,cosec\,A} && \text{[Reciprocal Identity: } \operatorname{cosec} A = \tfrac{1}{\sin A}\text{]} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

4. Prove that $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \sqrt{\frac{1-\sin A}{1+\sin A}} \\
&= \sqrt{\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}} && \text{[Rationalizing by multiplying Nr \& Dr by } (1-\sin A)\text{]} \\
&= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}} && \text{[Using } (a-b)(a+b) = a^2-b^2 \text{ in the denominator]} \\
&= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}} && \text{[Pythagorean Identity: } 1-\sin^2 A = \cos^2 A\text{]} \\
&= \frac{1-\sin A}{\cos A} && \text{[Taking the square root (assuming acute angle)]} \\
&= \frac{1}{\cos A} - \frac{\sin A}{\cos A} && \text{[Splitting the fraction into two parts]} \\
&= \mathbf{sec\,A - tan\,A} && \text{[Reciprocal and Quotient Identities]} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

5. Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A+1} = 2\sec^2 A$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} \\
&= \operatorname{cosec} A \left(\frac{1}{\operatorname{cosec} A - 1} + \frac{1}{\operatorname{cosec} A + 1} \right) && \text{[Factoring out cosec } A\text{]} \\
&= \operatorname{cosec} A \left(\frac{(\operatorname{cosec} A + 1) + (\operatorname{cosec} A - 1)}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)} \right) && \text{[Taking LCM]} \\
&= \operatorname{cosec} A \left(\frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A - 1} \right) && \text{[Using } (a-b)(a+b) = a^2 - b^2\text{]} \\
&= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} && \text{[Pythagorean Identity: } \operatorname{cosec}^2 A - 1 = \cot^2 A\text{]} \\
&= \frac{2(1/\sin^2 A)}{(\cos^2 A/\sin^2 A)} && \text{[Writing in terms of } \sin A \text{ and } \cos A\text{]} \\
&= \frac{2}{\cos^2 A} && \text{[Simplifying the complex fraction]} \\
&= \mathbf{2\sec^2 A} && \text{[Reciprocal Identity: } \sec A = \tfrac{1}{\cos A}\text{]} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

6. Prove that $\frac{\operatorname{cosec} A}{\cot A+\tan A} = \cos A$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \frac{\operatorname{cosec} A}{\cot A + \tan A} \\
&= \frac{1/\sin A}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}} && \text{[Expressing all terms in sin and cos]} \\
&= \frac{1/\sin A}{\frac{\cos^2 A + \sin^2 A}{\sin A \cos A}} && \text{[Taking LCM in the denominator]} \\
&= \frac{1/\sin A}{\frac{1}{\sin A \cos A}} && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \frac{1}{\sin A} \times \frac{\sin A \cos A}{1} && \text{[Multiplying by the reciprocal of the denominator]} \\
&= \cos \mathbf{A} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

7. Prove that $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= (\sec A + \cos A)(\sec A - \cos A) \\
&= \sec^2 A - \cos^2 A && \text{[Using difference of squares: } (a+b)(a-b) = a^2 - b^2\text{]} \\
&= (1 + \tan^2 A) - (1 - \sin^2 A) && \text{[Using Pythagorean Identities: } \sec^2 A = 1 + \tan^2 A \text{ and } \cos^2 A = 1 - \sin^2 A\text{]} \\
&= 1 + \tan^2 A - 1 + \sin^2 A \\
&= \tan^{\mathbf{2}} \mathbf{A} + \sin^{\mathbf{2}} \mathbf{A} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

8. Prove that $\frac{1}{\cot A+\tan A} = \sin A \cos A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \frac{1}{\cot A + \tan A} \\
&= \frac{1}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}} && \text{[Expressing all terms in sin and cos]} \\
&= \frac{1}{\frac{\cos^2 A+\sin^2 A}{\sin A \cos A}} && \text{[Taking LCM in the denominator]} \\
&= \frac{1}{\frac{1}{\sin A \cos A}} && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \sin \mathbf{A} \cos \mathbf{A} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

9. Prove that $\frac{\sec A-\tan A}{\sec A+\tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \frac{\sec A - \tan A}{\sec A + \tan A} \\
&= \frac{(\sec A - \tan A)(\sec A - \tan A)}{(\sec A + \tan A)(\sec A - \tan A)} && \text{[Multiplying numerator and denominator by the conjugate } (\sec A - \tan A)\text{]} \\
&= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A} && \text{[Using } (a-b)(a+b) = a^2 - b^2\text{]} \\
&= \frac{(\sec A - \tan A)^2}{1} && \text{[Pythagorean Identity: } \sec^2 A - \tan^2 A = 1\text{]} \\
&= \sec^2 A - 2\sec A \tan A + \tan^2 A && \text{[Expanding } (a-b)^2 = a^2 - 2ab + b^2\text{]} \\
&= (1 + \tan^2 A) - 2\sec A \tan A + \tan^2 A && \text{[Pythagorean Identity: } \sec^2 A = 1 + \tan^2 A\text{]} \\
&= \mathbf{1 - 2\sec A \tan A + 2\tan^2 A} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

10. Prove that $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \sec A \operatorname{cosec} A + 1$.

Proof:

$$\text{L.H.S.} = \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$$

$$= \frac{\sin A/\cos A}{1-\cos A/\sin A} + \frac{\cos A/\sin A}{1-\sin A/\cos A}$$ [Expressing in terms of sin and cos]

$$= \frac{\sin A/\cos A}{(\sin A-\cos A)/\sin A} + \frac{\cos A/\sin A}{(\cos A-\sin A)/\cos A}$$ [Simplifying denominators]

$$= \frac{\sin^2 A}{\cos A(\sin A-\cos A)} - \frac{\cos^2 A}{\sin A(\sin A-\cos A)}$$ [Factoring out -1 from second term's denominator]

$$= \frac{\sin^3 A-\cos^3 A}{\sin A\cos A(\sin A-\cos A)}$$ [Taking LCM]

$$= \frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{\sin A\cos A(\sin A-\cos A)}$$ [Using $a^3-b^3=(a-b)(a^2+ab+b^2)$]

$$= \frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\sin A\cos A}$$ [Cancelling $(\sin A-\cos A)$]

$$= \frac{1+\sin A\cos A}{\sin A\cos A}$$ [Pythagorean Identity: $\sin^2 A+\cos^2 A=1$]

$$= \frac{1}{\sin A\cos A} + \frac{\sin A\cos A}{\sin A\cos A}$$ [Splitting the fraction]

$$= \sec \mathbf{A} \operatorname{cosec} \mathbf{A} + \mathbf{1}$$

$$= \text{R.H.S.}$$

Hence Proved.

11. Prove that $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$.

Proof:

$$\text{L.H.S.} = \frac{\cos A}{1-\sin A/\cos A} + \frac{\sin A}{1-\cos A/\sin A}$$ [Expressing in terms of sin and cos]

$$= \frac{\cos A}{(\cos A-\sin A)/\cos A} + \frac{\sin A}{(\sin A-\cos A)/\sin A}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{\sin A-\cos A}$$

$$= -\frac{\cos^2 A}{\sin A-\cos A} + \frac{\sin^2 A}{\sin A-\cos A}$$ [Making denominators same]

$$= \frac{\sin^2 A-\cos^2 A}{\sin A-\cos A}$$

$$= \frac{(\sin A-\cos A)(\sin A+\cos A)}{\sin A-\cos A}$$ [Using $a^2-b^2=(a-b)(a+b)$]

$$= \sin \mathbf{A} + \cos \mathbf{A}$$ [Cancelling common factor]

$$= \text{R.H.S.}$$

Hence Proved.

12. Prove that $(\sin A + \cos A)(\cot A + \tan A) = \sec A + \operatorname{cosec} A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= (\sin A + \cos A)(\cot A + \tan A) \\
&= (\sin A + \cos A)\left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right) && \text{[Expressing in terms of sin and cos]} \\
&= (\sin A + \cos A)\left(\frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right) && \text{[Taking LCM inside bracket]} \\
&= (\sin A + \cos A)\left(\frac{1}{\sin A \cos A}\right) && \text{[Pythagorean Identity: } \sin^2 A + \cos^2 A = 1\text{]} \\
&= \frac{\sin A + \cos A}{\sin A \cos A} \\
&= \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A} && \text{[Splitting the fraction]} \\
&= \frac{1}{\cos A} + \frac{1}{\sin A} \\
&= \sec \mathbf{A} + \operatorname{cosec} \mathbf{A} && \text{[Reciprocal Identities]} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

13. Prove that $\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$.

Proof:

$$\begin{aligned}
\text{R.H.S.} &= \tan^2 A + \cot^2 A + 2 \\
&= (\tan^2 A + 2\tan A \cot A + \cot^2 A) && \text{[Since } 2 = 2 \times 1 = 2\tan A \cot A\text{]} \\
&= (\tan A + \cot A)^2 && \text{[Using } (a+b)^2 = a^2 + 2ab + b^2\text{]} \\
&= \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)^2 && \text{[Expressing in terms of sin and cos]} \\
&= \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)^2 && \text{[Taking LCM]} \\
&= \left(\frac{1}{\sin A \cos A}\right)^2 && \text{[Pythagorean Identity]} \\
&= \frac{1}{\sin^2 A \cos^2 A} \\
&= \left(\frac{1}{\cos^2 A}\right)\left(\frac{1}{\sin^2 A}\right) \\
&= \sec^{\mathbf{2}} \mathbf{A} \operatorname{cosec}^{\mathbf{2}} \mathbf{A} && \text{[Reciprocal Identities]} \\
&= \text{L.H.S.}
\end{aligned}$$

Hence Proved.

14. Prove that $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \tan^2 A - \sin^2 A \\
&= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A && \text{[Quotient Identity]} \\
&= \sin^2 A \left(\frac{1}{\cos^2 A} - 1\right) && \text{[Factoring out } \sin^2 A\text{]} \\
&= \sin^2 A(\sec^2 A - 1) && \text{[Reciprocal Identity]} \\
&= \sin^2 A(\tan^2 A) && \text{[Pythagorean Identity: } \sec^2 A - 1 = \tan^2 A\text{]} \\
&= \sin^2 A \left(\frac{\sin^2 A}{\cos^2 A}\right) \\
&= \frac{\sin^4 A}{\cos^2 A} \\
&= \mathbf{\sin^4 A \sec^2 A} && \text{[Reciprocal Identity]} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

15. Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\
&= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) && \text{[Expressing in terms of sin and cos]} \\
&= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right) && \text{[Taking LCM in each bracket]} \\
&= \frac{[(\sin A + \cos A) - 1][(\sin A + \cos A) + 1]}{\sin A \cos A} \\
&= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A} && [(a-b)(a+b) = a^2 - b^2 \text{ with } a = \sin A + \cos A] \\
&= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A} && \text{[Expanding the square]} \\
&= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A} && \text{[Pythagorean Identity]} \\
&= \frac{2\sin A \cos A}{\sin A \cos A} \\
&= \mathbf{2} \\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

16. Prove that $\frac{1}{\text{cosec } A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec } A + \cot A}$.

Proof:

The equation is equivalent to proving: $\dfrac{1}{\text{cosec } A - \cot A} + \dfrac{1}{\text{cosec } A + \cot A} = \dfrac{2}{\sin A}$ [Rearranging terms]

$$
\begin{aligned}
\text{L.H.S.} &= \frac{1}{\text{cosec } A - \cot A} + \frac{1}{\text{cosec } A + \cot A} \\
&= \frac{(\text{cosec } A + \cot A) + (\text{cosec } A - \cot A)}{(\text{cosec } A - \cot A)(\text{cosec } A + \cot A)} && \text{[Taking LCM]} \\
&= \frac{2 \text{ cosec } A}{\text{cosec}^2 A - \cot^2 A} && \text{[Simplifying numerator]} \\
&= \frac{2 \text{ cosec } A}{1} && \text{[Pythagorean Identity: } \text{cosec}^2 A - \cot^2 A = 1] \\
&= 2 \text{ cosec } A \\
&= \frac{\mathbf{2}}{\sin \mathbf{A}} && \text{[Reciprocal Identity]} \\
&= \text{R.H.S. of rearranged eq.}
\end{aligned}
$$

Hence Proved.

17. Prove that $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \frac{\cot A + \tan B}{\cot B + \tan A} \\
&= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} && \text{[Expressing in terms of sin and cos]} \\
&= \frac{\frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\frac{\cos B \cos A + \sin B \sin A}{\sin B \cos A}} && \text{[Taking LCM in numerator and denominator]} \\
&= \frac{\cos(A - B)/(\sin A \cos B)}{\cos(A - B)/(\sin B \cos A)} && \text{[Using compound angle formula } \cos(X - Y)] \\
&= \frac{\cos(A - B)}{\sin A \cos B} \times \frac{\sin B \cos A}{\cos(A - B)} \\
&= \frac{\sin B \cos A}{\sin A \cos B} && \text{[Cancelling } \cos(A - B)] \\
&= \left(\frac{\cos A}{\sin A}\right)\left(\frac{\sin B}{\cos B}\right) \\
&= \cot \mathbf{A} \tan \mathbf{B} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

18. Prove $\left(\frac{1}{\sec^2\alpha-\cos^2\alpha}+\frac{1}{\text{cosec}^2\alpha-\sin^2\alpha}\right)\cos^2\alpha\sin^2\alpha=\frac{1-\cos^2\alpha\sin^2\alpha}{2+\cos^2\alpha\sin^2\alpha}$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \left(\frac{1}{\frac{1}{\cos^2\alpha}-\cos^2\alpha}+\frac{1}{\frac{1}{\sin^2\alpha}-\sin^2\alpha}\right)\cos^2\alpha\sin^2\alpha \\
&= \left(\frac{\cos^2\alpha}{1-\cos^4\alpha}+\frac{\sin^2\alpha}{1-\sin^4\alpha}\right)\cos^2\alpha\sin^2\alpha \\
&= \left(\frac{\cos^2\alpha}{(1-\cos^2\alpha)(1+\cos^2\alpha)}+\frac{\sin^2\alpha}{(1-\sin^2\alpha)(1+\sin^2\alpha)}\right)\cos^2\alpha\sin^2\alpha \\
&= \left(\frac{\cos^2\alpha}{\sin^2\alpha(1+\cos^2\alpha)}+\frac{\sin^2\alpha}{\cos^2\alpha(1+\sin^2\alpha)}\right)\cos^2\alpha\sin^2\alpha \\
&= \frac{\cos^4\alpha}{1+\cos^2\alpha}+\frac{\sin^4\alpha}{1+\sin^2\alpha} && [\text{Distributing } \cos^2\alpha\sin^2\alpha] \\
&= \frac{\cos^4\alpha(1+\sin^2\alpha)+\sin^4\alpha(1+\cos^2\alpha)}{(1+\cos^2\alpha)(1+\sin^2\alpha)} && [\text{Taking LCM}] \\
&= \frac{\cos^4\alpha+\cos^4\alpha\sin^2\alpha+\sin^4\alpha+\sin^4\alpha\cos^2\alpha}{1+\sin^2\alpha+\cos^2\alpha+\sin^2\alpha\cos^2\alpha} \\
&= \frac{(\sin^4\alpha+\cos^4\alpha)+\sin^2\alpha\cos^2\alpha(\cos^2\alpha+\sin^2\alpha)}{1+(\sin^2\alpha+\cos^2\alpha)+\sin^2\alpha\cos^2\alpha} \\
&= \frac{(1-2\sin^2\alpha\cos^2\alpha)+\sin^2\alpha\cos^2\alpha(1)}{1+1+\sin^2\alpha\cos^2\alpha} && [\text{Using } \sin^4 x+\cos^4 x = \text{1-2}\sin^2 x\cos^2 x] \\
&= \frac{\mathbf{1}-\sin^{\mathbf{2}}\alpha\cos^{\mathbf{2}}\alpha}{\mathbf{2}+\sin^{\mathbf{2}}\alpha\cos^{\mathbf{2}}\alpha} \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

19. Prove that $\sin^8 A-\cos^8 A=(\sin^2 A-\cos^2 A)(1-2\sin^2 A\cos^2 A)$.

Proof:

$$
\begin{aligned}
\text{L.H.S.} &= \sin^8 A-\cos^8 A \\
&= (\sin^4 A)^2-(\cos^4 A)^2 \\
&= (\sin^4 A-\cos^4 A)(\sin^4 A+\cos^4 A) && [\text{Using } a^2-b^2=(a-b)(a+b)] \\
&= [(\sin^2 A)^2-(\cos^2 A)^2]\cdot[(\sin^2 A+\cos^2 A)^2-2\sin^2 A\cos^2 A] && [\text{Factor again and use } a^2+b^2=(a+b)^2-2ab] \\
&= [(\sin^2 A-\cos^2 A)(\sin^2 A+\cos^2 A)]\cdot[(1)^2-2\sin^2 A\cos^2 A] \\
&= [(\sin^2 A-\cos^2 A)(1)]\cdot[1-2\sin^2 A\cos^2 A] \\
&= (\sin^{\mathbf{2}}\mathbf{A}-\cos^{\mathbf{2}}\mathbf{A})(\mathbf{1}-\mathbf{2}\sin^{\mathbf{2}}\mathbf{A}\cos^{\mathbf{2}}\mathbf{A}) \\
&= \text{R.H.S.}
\end{aligned}
$$

Hence Proved.

20. Prove that $\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= \frac{\cos A(1/\sin A) - \sin A(1/\cos A)}{\cos A + \sin A} && \text{[Expressing in terms of sin and cos]}\\
&= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A} && \text{[Taking LCM in the numerator]}\\
&= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A(\cos A + \sin A)}\\
&= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\sin A \cos A(\cos A + \sin A)} && [\text{Using } a^2 - b^2 = (a-b)(a+b)]\\
&= \frac{\cos A - \sin A}{\sin A \cos A} && [\text{Cancelling } (\cos A + \sin A)]\\
&= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A}\\
&= \frac{1}{\sin A} - \frac{1}{\cos A}\\
&= \operatorname{cosec} \mathbf{A} - \sec \mathbf{A} && \text{[Reciprocal Identities]}\\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

21. Prove that $(\tan\alpha + \operatorname{cosec}\beta)^2 - (\cot\beta - \sec\alpha)^2 = 2\tan\alpha\cot\beta(\operatorname{cosec}\alpha + \sec\beta)$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= (\tan^2\alpha + 2\tan\alpha\operatorname{cosec}\beta + \operatorname{cosec}^2\beta) - (\cot^2\beta - 2\cot\beta\sec\alpha + \sec^2\alpha)\\
&= (\tan^2\alpha - \sec^2\alpha) + (\operatorname{cosec}^2\beta - \cot^2\beta) + 2\tan\alpha\operatorname{cosec}\beta + 2\cot\beta\sec\alpha\\
&= (-1) + (1) + 2(\tan\alpha\operatorname{cosec}\beta + \cot\beta\sec\alpha) && \text{[Pythagorean Identities]}\\
&= 2\left(\frac{\sin\alpha}{\cos\alpha}\frac{1}{\sin\beta} + \frac{\cos\beta}{\sin\beta}\frac{1}{\cos\alpha}\right) = \frac{2}{\cos\alpha\sin\beta}(\sin\alpha + \cos\beta)\\
\text{R.H.S.} &= 2\tan\alpha\cot\beta(\operatorname{cosec}\alpha + \sec\beta)\\
&= 2\frac{\sin\alpha}{\cos\alpha}\frac{\cos\beta}{\sin\beta}\left(\frac{1}{\sin\alpha} + \frac{1}{\cos\beta}\right)\\
&= 2\frac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta}\left(\frac{\cos\beta + \sin\alpha}{\sin\alpha\cos\beta}\right) = \frac{2(\sin\alpha + \cos\beta)}{\cos\alpha\sin\beta}
\end{aligned}$$

Since L.H.S. = R.H.S., the identity is proved. **Hence Proved.**

22. Prove that $2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = \cot^4\alpha - \tan^4\alpha$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= (\operatorname{cosec}^4\alpha - 2\operatorname{cosec}^2\alpha) - (\sec^4\alpha - 2\sec^2\alpha)\\
&= (\operatorname{cosec}^4\alpha - 2\operatorname{cosec}^2\alpha + 1 - 1) - (\sec^4\alpha - 2\sec^2\alpha + 1 - 1) && \text{[Completing the square]}\\
&= [(\operatorname{cosec}^2\alpha - 1)^2 - 1] - [(\sec^2\alpha - 1)^2 - 1]\\
&= (\cot^4\alpha - 1) - (\tan^4\alpha - 1) && \text{[Pythagorean Identities]}\\
&= \cot^4\alpha - 1 - \tan^4\alpha + 1\\
&= \cot^{\mathbf{4}}\alpha - \tan^{\mathbf{4}}\alpha\\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

23. Prove that $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = \tan^2\alpha + \cot^2\alpha + 7$.

Proof:

$$\begin{aligned}
\text{L.H.S.} &= (\sin^2\alpha + 2\sin\alpha\operatorname{cosec}\alpha + \operatorname{cosec}^2\alpha) + (\cos^2\alpha + 2\cos\alpha\sec\alpha + \sec^2\alpha) && \text{[Expanding squares]}\\
&= \sin^2\alpha + 2(1) + \operatorname{cosec}^2\alpha + \cos^2\alpha + 2(1) + \sec^2\alpha\\
&= (\sin^2\alpha + \cos^2\alpha) + \operatorname{cosec}^2\alpha + \sec^2\alpha + 4\\
&= 1 + (1 + \cot^2\alpha) + (1 + \tan^2\alpha) + 4 && \text{[Pythagorean Identities]}\\
&= \tan^{\mathbf{2}}\alpha + \cot^{\mathbf{2}}\alpha + \mathbf{7}\\
&= \text{R.H.S.}
\end{aligned}$$

Hence Proved.

Que 2: Solve the following

1. If $x + \frac{1}{x} = 2\cos\alpha$, then $x^n + \frac{1}{x^n} =$
(A) $2^n \cos\alpha$ (B) $2^n \cos n\alpha$ (C) $2i \sin n\alpha$ (D) $2\cos n\alpha$

Solution:

$$x + \frac{1}{x} = 2\cos\alpha \implies x^2 - 2x\cos\alpha + 1 = 0$$
$$x = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4}}{2} = \cos\alpha \pm i\sin\alpha$$
Let $x = \cos\alpha + i\sin\alpha$. By De Moivre's Theorem:
$$x^n = \cos(n\alpha) + i\sin(n\alpha)$$
$$\frac{1}{x^n} = \cos(n\alpha) - i\sin(n\alpha)$$
$$x^n + \frac{1}{x^n} = 2\cos(n\alpha)$$

The correct option is **(D)** $2\cos n\alpha$.

2. If $\operatorname{cosec} A + \cot A = \frac{11}{2}$, then $\tan A =$
(A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{43}$

Solution:

$$\text{We know } \operatorname{cosec}^2 A - \cot^2 A = 1 \implies (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = 1$$
$$\text{Given } \operatorname{cosec} A + \cot A = \frac{11}{2} \quad (1) \implies \operatorname{cosec} A - \cot A = \frac{2}{11} \quad (2)$$
$$\text{Subtracting (2) from (1): } 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22} \implies \cot A = \frac{117}{44}$$
$$\tan A = \frac{1}{\cot A} = \frac{44}{117}$$

The correct option is **(C)** $\frac{44}{117}$.

3. If $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$
(A) 0 (B) 1 (C) 1/6 (D) 6

Solution:

$$\text{Given } \tan\theta = \frac{4}{5}. \text{ Divide Nr and Dr by } \cos\theta:$$
$$\frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{5(4/5) - 3}{5(4/5) + 2} = \frac{4-3}{4+2} = \frac{1}{6}$$

The correct option is **(C)** 1/6.

4. If $\tan\theta = \frac{1}{\sqrt{7}}$, find the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.
(A) 1/2 (B) 3/4 (C) 4/3 (D) 1/4

Solution:

$$\tan^2\theta = 1/7 \implies \sec^2\theta = 1 + 1/7 = 8/7$$
$$\cot^2\theta = 7 \implies \operatorname{cosec}^2\theta = 1 + 7 = 8$$
$$\text{Expression } = \frac{8 - 8/7}{8 + 8/7} = \frac{8(1 - 1/7)}{8(1 + 1/7)} = \frac{6/7}{8/7} = \frac{3}{4}$$

The correct option is **(B)** 3/4.

5. If $8\sin\theta = 4\cos\theta$, find the value of $\sin\theta$.
(A) $1/2$ (B) $2/\sqrt{5}$ (C) $1/\sqrt{5}$ (D) 1

Solution:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{8} = \frac{1}{2}$$
$$\text{From } \tan\theta = 1/2, \text{ opposite=1, adjacent=2, hypotenuse=}\sqrt{1^2+2^2} = \sqrt{5}.$$
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{5}}$$

The correct option is **(C)** $1/\sqrt{5}$.

6. If $\cot\theta + \operatorname{cosec}\theta = 5$, find the value of $\cos\theta$.
(A) 5/13 (B) 12/13 (C) 13/12 (D) 5/12

Solution:

$$\operatorname{cosec}\theta + \cot\theta = 5 \quad (1) \implies \operatorname{cosec}\theta - \cot\theta = 1/5 \quad (2)$$
$$\text{Add (1) and (2): } 2\operatorname{cosec}\theta = 5 + 1/5 = 26/5 \implies \operatorname{cosec}\theta = 13/5, \sin\theta = 5/13.$$
$$\text{Subtract (2) from (1): } 2\cot\theta = 5 - 1/5 = 24/5 \implies \cot\theta = 12/5.$$
$$\cos\theta = \cot\theta \times \sin\theta = (12/5) \times (5/13) = 12/13$$

The correct option is **(B)** 12/13.

7. If $\tan^2\theta + \sec\theta = 5$, find the value of $\cos\theta$.
(A) 1/2 (B) $-1/3$ (C) 1/3 (D) Both A and B

Solution:

$$(\sec^2\theta - 1) + \sec\theta = 5 \implies \sec^2\theta + \sec\theta - 6 = 0$$
$$(\sec\theta + 3)(\sec\theta - 2) = 0 \implies \sec\theta = -3 \text{ or } \sec\theta = 2$$
$$\cos\theta = -1/3 \text{ or } \cos\theta = 1/2$$

The possible values are $\mathbf{1/2}$ and $\mathbf{-1/3}$. These correspond to options (A) and (B).
The correct option is **(D)** Both A and B

8. If $\tan\theta + \cot\theta = 2$, find the value of $\sin\theta$.
(A) 1/2 (B) $\sqrt{3}/2$ (C) $1/\sqrt{2}$ (D) 1

Solution:

$$\tan\theta + 1/\tan\theta = 2 \implies \tan^2\theta - 2\tan\theta + 1 = 0$$
$$(\tan\theta - 1)^2 = 0 \implies \tan\theta = 1 \implies \theta = 45^\circ$$
$$\sin(45^\circ) = 1/\sqrt{2}$$

The correct option is **(C)** $1/\sqrt{2}$.

9. If $\sec^2\theta = 2 + 2\tan\theta$, find the value of $\tan\theta$.
(A) $1+\sqrt{2}$ (B) $1-\sqrt{2}$ (C) $1\pm\sqrt{2}$ (D) $\sqrt{2}-1$

Solution:

$$1+\tan^2\theta = 2+2\tan\theta \implies \tan^2\theta - 2\tan\theta - 1 = 0$$
$$\tan\theta = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)} = \frac{2\pm\sqrt{8}}{2} = 1\pm\sqrt{2}$$

The correct option is **(C)** $1\pm\sqrt{2}$.

10. The value of $(\operatorname{cosec} A \operatorname{cosec} B + \cot A \cot B)^2 - (\operatorname{cosec} A \cot B + \operatorname{cosec} B \cot A)^2$ is
(A) 1 (B) 2 (C) 0 (D) -1

Solution:

$$\begin{aligned}\text{Expression} &= (\operatorname{cosec}^2 A \operatorname{cosec}^2 B + \cot^2 A \cot^2 B) - (\operatorname{cosec}^2 A \cot^2 B + \operatorname{cosec}^2 B \cot^2 A)\\ &= \operatorname{cosec}^2 A(\operatorname{cosec}^2 B - \cot^2 B) - \cot^2 A(\operatorname{cosec}^2 B - \cot^2 B)\\ &= \operatorname{cosec}^2 A(1) - \cot^2 A(1) = \operatorname{cosec}^2 A - \cot^2 A = 1\end{aligned}$$

The correct option is **(A) 1**.

11. If $\tan\alpha + \cot\alpha = a$, then the value of $\tan^4\alpha + \cot^4\alpha$ is equal to
(A) a^4+4a^2+2 (B) a^4-4a^2+2 (C) a^4-4a^2-2 (D) $-a^4+2a^2+4$

Solution:

$$\tan\alpha+\cot\alpha = a \implies \tan^2\alpha+\cot^2\alpha+2 = a^2 \implies \tan^2\alpha+\cot^2\alpha = a^2-2$$
$$\text{Squaring again: } (\tan^2\alpha+\cot^2\alpha)^2 = (a^2-2)^2$$
$$\tan^4\alpha+\cot^4\alpha+2 = a^4-4a^2+4 \implies \tan^4\alpha+\cot^4\alpha = a^4-4a^2+2$$

The correct option is **(B)** a^4-4a^2+2.

12. If $a\cos\theta + b\sin\theta = 3$ and $a\sin\theta - b\cos\theta = 4$, then a^2+b^2 has the value
(A) 25 (B) 14 (C) 7 (D) 15

Solution:

$$\text{Square and add both equations:}$$
$$(a\cos\theta+b\sin\theta)^2 + (a\sin\theta-b\cos\theta)^2 = 3^2+4^2$$
$$(a^2\cos^2\theta+b^2\sin^2\theta) + (a^2\sin^2\theta+b^2\cos^2\theta) = 25$$
$$a^2(\cos^2\theta+\sin^2\theta)+b^2(\sin^2\theta+\cos^2\theta) = 25 \implies a^2+b^2 = 25$$

The correct option is **(A) 25**.

13. If $\frac{\tan^3 A}{1+\tan^2 A} + \frac{\cot^3 A}{1+\cot^2 A} = p\sec A \operatorname{cosec} A + q\sin A\cos A$, then

(A) $p = 2, q = 1$ (B) $p = 1, q = 2$ (C) $p = 1, q = -2$ (D) $p = 2, q = -1$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \frac{\tan^3 A}{\sec^2 A} + \frac{\cot^3 A}{\operatorname{cosec}^2 A} = \frac{\sin^3 A/\cos^3 A}{1/\cos^2 A} + \frac{\cos^3 A/\sin^3 A}{1/\sin^2 A} \\ &= \frac{\sin^3 A}{\cos A} + \frac{\cos^3 A}{\sin A} = \frac{\sin^4 A + \cos^4 A}{\sin A\cos A} \\ &= \frac{1 - 2\sin^2 A\cos^2 A}{\sin A\cos A} = \frac{1}{\sin A\cos A} - 2\sin A\cos A \\ &= \sec A\operatorname{cosec} A - 2\sin A\cos A \\ \implies p = 1, q = -2.\end{aligned}$$

The correct option is **(C)** $p = 1, q = -2$.

14. If $\sin A\tan A = \cos^2 A$ then $\cos^3 A + \cos^2 A$ is equal to

(A) 1 (B) 2 (C) 4 (D) None of these

Solution:

$$\begin{aligned}\sin A\frac{\sin A}{\cos A} = \cos^2 A &\implies \sin^2 A = \cos^3 A \\ \text{We need } \cos^3 A + \cos^2 A. \text{ Substitute } \cos^3 A = \sin^2 A: \\ \sin^2 A + \cos^2 A = 1\end{aligned}$$

The correct option is **(A) 1**.

15. If $\tan\theta + \sec\theta = \frac{2}{3}$ then $\sec\theta$ is

(A) $-13/12$ (B) $5/12$ (C) $13/12$ (D) $-5/12$

Solution:

$$\begin{aligned}\sec\theta + \tan\theta = 2/3 \quad (1) \\ \text{Since } \sec^2\theta - \tan^2\theta = 1, \text{ we have } \sec\theta - \tan\theta = \frac{1}{2/3} = 3/2 \quad (2) \\ \text{Add (1) and (2):} \quad 2\sec\theta = \frac{2}{3} + \frac{3}{2} = \frac{4+9}{6} = \frac{13}{6} \\ \sec\theta = \frac{13}{12}\end{aligned}$$

The correct option is **(C)** $13/12$.