Topic: Basic Identities

Sub: Mathematics **Assignment: 01** Prof. Chetan Sir

Que 1: Prove the following identities (SL. LONEY Ex. 5)

1. $\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$

2. $(\sin A + \cos A)(1 - \sin A \cos A) = \sin^3 A + \cos^3 A$

3. $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$

4. $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$

5. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

6. $\frac{\operatorname{cosec} A}{\cot A + \tan A} = \cos A$

7. $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$

8. $\frac{1}{\cot A + \tan A} = \sin A \cos A$

9. $\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$

10. $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A + 1$

11. $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

12. $(\sin A + \cos A)(\cot A + \tan A) = \sec A + \operatorname{cosec} A$

13. $\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$

14. $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$

15. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

16. $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$

17. $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

18. $\left(\frac{1}{\sec^2 \alpha - \cos^2 \alpha} + \frac{1}{\operatorname{cosec}^2 \alpha - \sin^2 \alpha}\right)\cos^2 \alpha \sin^2 \alpha = \frac{1 - \cos^2 \alpha \sin^2 \alpha}{2 + \cos^2 \alpha \sin^2 \alpha}$

19. $\sin^8 A - \cos^8 A = (\sin^2 A - \cos^2 A)(1 - 2\sin^2 A \cos^2 A)$

20. $\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$

21. $(\tan \alpha + \operatorname{cosec} \beta)^2 - (\cot \beta - \sec \alpha)^2 = 2\tan \alpha \cot \beta (\operatorname{cosec} \alpha + \sec \beta)$

22. $2\sec^2 \alpha - \sec^4 \alpha - 2\operatorname{cosec}^2 \alpha + \operatorname{cosec}^4 \alpha = \cot^4 \alpha - \tan^4 \alpha$

23. $(\sin \alpha + \operatorname{cosec} \alpha)^2 + (\cos \alpha + \sec \alpha)^2 = \tan^2 \alpha + \cot^2 \alpha + 7$

Que 2: Solve the following

1. If $x + \frac{1}{x} = 2\cos\alpha$, then $x^n + \frac{1}{x^n} =$

(A) $2^n \cos\alpha$ (B) $2^n \cos n\alpha$ (C) $2i \sin n\alpha$ (D) $2\cos n\alpha$

2. If $\operatorname{cosec} A + \cot A = \frac{11}{2}$, then $\tan A =$

(A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{43}$

3. If $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$

(A) 0 (B) 1 (C) 1/6 (D) 6

4. If $\tan\theta = \frac{1}{\sqrt{7}}$, find the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.

(A) 1/2 (B) 3/4 (C) 4/3 (D) 1/4

5. If $8\sin\theta = 4\cos\theta$, find the value of $\sin\theta$.

(A) 1/2 (B) $2/\sqrt{5}$ (C) $1/\sqrt{5}$ (D) 1

6. If $\cot\theta + \operatorname{cosec}\theta = 5$, find the value of $\cos\theta$.

(A) 5/13 (B) 12/13 (C) 13/12 (D) 5/12

7. If $\tan^2\theta + \sec\theta = 5$, find the value of $\cos\theta$.

(A) 1/2 (B) −1/3 (C) 1/3 (D) Both A and B

8. If $\tan\theta + \cot\theta = 2$, find the value of $\sin\theta$.

(A) 1/2 (B) $\sqrt{3}/2$ (C) $1/\sqrt{2}$ (D) 1

9. If $\sec^2\theta = 2 + 2\tan\theta$, find the value of $\tan\theta$.

(A) $1 + \sqrt{2}$ (B) $1 - \sqrt{2}$ (C) $1 \pm \sqrt{2}$ (D) $\sqrt{2} - 1$

10. The value of $(\operatorname{cosec} A \operatorname{cosec} B + \cot A \cot B)^2 - (\operatorname{cosec} A \cot B + \operatorname{cosec} B \cot A)^2$ is

(A) 1 (B) 2 (C) 0 (D) -1

11. If $\tan\alpha + \cot\alpha = a$, then the value of $\tan^4\alpha + \cot^4\alpha$ is equal to

(A) $a^4 + 4a^2 + 2$ (B) $a^4 - 4a^2 + 2$ (C) $a^4 - 4a^2 - 2$ (D) $-a^4 + 2a^2 + 4$

12. If $a\cos\theta + b\sin\theta = 3$ and $a\sin\theta - b\cos\theta = 4$, then $a^2 + b^2$ has the value

(A) 25 (B) 14 (C) 7 (D) 15

13. If $\frac{\tan^3 A}{1+\tan^2 A} + \frac{\cot^3 A}{1+\cot^2 A} = p\sec A \operatorname{cosec} A + q\sin A\cos A$, then

(A) $p = 2, q = 1$ (B) $p = 1, q = 2$ (C) $p = 1, q = -2$ (D) $p = 2, q = -1$

14. If $\sin A \tan A = \cos^2 A$ then $\cos^3 A + \cos^2 A$ is equal to

(A) 1 (B) 2 (C) 4 (D) None of these

15. If $\tan\theta + \sec\theta = \frac{2}{3}$ then $\sec\theta$ is

(A) −13/12 (B) 5/12 (C) 13/12 (D) −5/12

Answer Key (Que 2)

1 (D)	**2** (C)	**3** (C)	**4** (B)	**5** (C)	**6** (B)	**7** (D)	**8** (C)	**9** (C)	**10** (A)
11 (B)	**12** (A)	**13** (C)	**14** (A)	**15** (C)					