Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Solutions to Assignment: 02

Trigonometric Ratios & Identities

Topic: Allied Angle

Sub: Mathematics **Assignment: 02** Prof. Chetan Sir

Que.1: Measurement of Angles

1. Find the radian measures corresponding to the following degree measures:

(i) $15°$

(ii) $240°$

(iii) $530°$

(iv) $-1215°$

Solution:

To convert degrees to radians, we use the formula: Radians = Degrees $\times \frac{\pi}{180}$.

(i)

$$15° = 15 \times \frac{\pi}{180} \text{ radians} = \frac{\pi}{\mathbf{12}} \text{ radians.}$$

(ii)

$$240° = 240 \times \frac{\pi}{180} \text{ radians} = \frac{\mathbf{4}\pi}{\mathbf{3}} \text{ radians.}$$

(iii)

$$530° = 530 \times \frac{\pi}{180} \text{ radians} = \frac{\mathbf{53}\pi}{\mathbf{18}} \text{ radians.}$$

(iv)

$$-1215° = -1215 \times \frac{\pi}{180} \text{ radians} = -\frac{\mathbf{27}\pi}{\mathbf{4}} \text{ radians.}$$

2. Find the degree measures corresponding to the following radian measures:

(i) $\frac{3\pi}{4}$

(ii) -4π

(iii) $\frac{5\pi}{3}$

(iv) $\frac{7\pi}{6}$

Solution:

To convert radians to degrees, we use the formula: Degrees = Radians $\times \frac{180}{\pi}$.

(i)

$$\frac{3\pi}{4} \text{ radians} = \frac{3\pi}{4} \times \frac{180}{\pi} = 3 \times 45 = \mathbf{135}^\circ.$$

(ii)

$$-4\pi \text{ radians} = -4\pi \times \frac{180}{\pi} = -4 \times 180 = -\mathbf{720}^\circ.$$

(iii)

$$\frac{5\pi}{3} \text{ radians} = \frac{5\pi}{3} \times \frac{180}{\pi} = 5 \times 60 = \mathbf{300}^\circ.$$

(iv)

$$\frac{7\pi}{6} \text{ radians} = \frac{7\pi}{6} \times \frac{180}{\pi} = 7 \times 30 = \mathbf{210}^\circ.$$

3. Find the value of:

Solution:

We use allied angle formulas to reduce the angles to acute angles in the first quadrant.

(i)

$$\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}.$$

(ii)

$$\sin 225^\circ = \sin(180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}.$$

(iii)

$$\tan 330^\circ = \tan(360^\circ - 30^\circ) = -\tan 30^\circ = -\frac{\mathbf{1}}{\sqrt{\mathbf{3}}}.$$

(iv)

$$\cot(-315^\circ) = -\cot(315^\circ) = -\cot(360^\circ - 45^\circ) = -(-\cot 45^\circ) = \mathbf{1}.$$

(v)

$$\sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}.$$

(vi)

$$\operatorname{cosec}(-1410^\circ) = -\operatorname{cosec}(1410^\circ) = -\operatorname{cosec}(4 \times 360^\circ - 30^\circ) = -(-\operatorname{cosec} 30^\circ) = \mathbf{2}.$$

(vii)

$$\tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{\mathbf{3}}.$$

(viii)

$$\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right) = -(-\sin\frac{\pi}{3}) = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}.$$

(ix)

$$\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = -(-\cot\frac{\pi}{4}) = \mathbf{1}.$$

(x)

$$\cos\left(\frac{53\pi}{6}\right) = \cos\left(9\pi - \frac{\pi}{6}\right) = \cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}.$$

(xi)

$$\sin\left(\frac{25\pi}{3}\right) = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}.$$

(xii)

$$\cos\left(-\frac{17\pi}{3}\right) = \cos\left(\frac{17\pi}{3}\right) = \cos\left(6\pi - \frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{\mathbf{1}}{\mathbf{2}}.$$

Que.2: Solve the following questions

1. The value of $\frac{\sin 300^\circ \tan 330^\circ \sec 420^\circ}{\tan 135^\circ \sin 210^\circ \sec 315^\circ}$ is:

Solution:

We evaluate each trigonometric function using allied angles.
Numerator: $\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$, $\tan 330^\circ = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$, $\sec 420^\circ = \sec 60^\circ = 2$.
Denominator: $\tan 135^\circ = -1$, $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$, $\sec 315^\circ = \sec 45^\circ = \sqrt{2}$.

$$\text{Expression} = \frac{(-\frac{\sqrt{3}}{2})(-\frac{1}{\sqrt{3}})(2)}{(-1)(-\frac{1}{2})(\sqrt{2})} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \mathbf{\sqrt{2}}.$$

The correct option is **(C)**.

2. The value of $2\cos 10^\circ + \sin 100^\circ + \sin 1000^\circ + \sin 10000^\circ$ is:

Solution:

$$\begin{aligned}
&2\cos 10^\circ + \sin(90^\circ + 10^\circ) + \sin(1080^\circ - 80^\circ) + \sin(10080^\circ - 80^\circ)\\
&2\cos 10^\circ + \cos 10^\circ - \sin 80^\circ + \sin(-80^\circ) \quad \text{(since 10080 is multiple of 360)}\\
&3\cos 10^\circ - \cos 10^\circ - \cos 10^\circ = \cos \mathbf{10^\circ}
\end{aligned}$$

The correct option is **(C)**.

3. The value of $\cos^2 73^\circ + \cos^2 47^\circ - \sin^2 43^\circ + \sin^2 107^\circ$ is equal to:

Solution:

Using $\sin(90^\circ + A) = \cos A$ and $\sin(90^\circ - A) = \cos A$.
$\sin 107^\circ = \sin(90 + 17) = \cos 17^\circ$.
$\sin 43^\circ = \sin(90 - 47) = \cos 47^\circ$.

$$\begin{aligned}
\text{Expression} &= \cos^2 73^\circ + \cos^2 47^\circ - \cos^2 47^\circ + \cos^2 17^\circ\\
&= \cos^2 73^\circ + \cos^2 17^\circ\\
&= \cos^2(90 - 17)^\circ + \cos^2 17^\circ = \sin^2 17^\circ + \cos^2 17^\circ = \mathbf{1}.
\end{aligned}$$

The correct option is **(C)**.

4. If $2\cos^2\theta + 3\cos\theta = 2$, then a possible value of $\cos\theta$ is:

Solution:

This is a quadratic equation in terms of $\cos\theta$. Let $x = \cos\theta$.

$$\begin{aligned}
2x^2 + 3x - 2 &= 0\\
2x^2 + 4x - x - 2 &= 0\\
2x(x + 2) - 1(x + 2) &= 0\\
(2x - 1)(x + 2) &= 0
\end{aligned}$$

This gives $x = 1/2$ or $x = -2$.
Since the range of $\cos\theta$ is $[-1, 1]$, $x = -2$ is not possible.
Thus, a possible value is $\cos\theta = \mathbf{1/2}$.
The correct option is **(A)**.

5. If $3\tan^2\theta - 4\sqrt{3}\tan\theta + 3 = 0$, then a possible value of $\tan\theta$ is:

Solution:
Let $y = \tan\theta$. We solve the quadratic equation $3y^2 - 4\sqrt{3}y + 3 = 0$ using the quadratic formula.

$$\begin{aligned} y &= \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4(3)(3)}}{2(3)} \\ &= \frac{4\sqrt{3} \pm \sqrt{48-36}}{6} = \frac{4\sqrt{3} \pm \sqrt{12}}{6} = \frac{4\sqrt{3} \pm 2\sqrt{3}}{6} \end{aligned}$$

The two solutions are $y = \frac{6\sqrt{3}}{6} = \sqrt{3}$ and $y = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$.
A possible value is $\tan\theta = \mathbf{\sqrt{3}}$.
The correct option is **(A)**.

6. If $\tan\theta = -2$ and $\frac{\pi}{2} < \theta < \pi$, then the value of $\sin\theta$ is:

Solution:
The angle θ is in the second quadrant, where sine is positive.
We can form a right triangle with opposite side 2 and adjacent side 1. The hypotenuse is $\sqrt{2^2 + 1^2} = \sqrt{5}$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\mathbf{2}}{\mathbf{\sqrt{5}}}.$$

The correct option is **(A)**.

7. If $\cos\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$ is:

Solution:
The angle θ is in the third quadrant, where sine and cosine are negative, and tangent is positive. This is a 3-4-5 triangle.
$\sin\theta = -4/5$,
$\tan\theta = 4/3$.
$\operatorname{cosec}\theta = -5/4$,
$\sec\theta = -5/3$,
$\cot\theta = 3/4$.

$$\text{Expression} = \frac{-\frac{5}{4} + \frac{3}{4}}{-\frac{5}{3} - \frac{4}{3}} = \frac{-\frac{2}{4}}{-\frac{9}{3}} = \frac{-1/2}{-3} = \frac{\mathbf{1}}{\mathbf{6}}.$$

The correct option is **(A)**.

8. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, then θ lies in which quadrant?

Solution:
Sine is negative in the third and fourth quadrants.
Tangent is positive in the first and third quadrants. The only common quadrant is the third quadrant.
The correct option is **(C)**.

9. If $\sin\theta = \frac{12}{13}$ and $\frac{\pi}{2} < \theta < \pi$, find the value of $\sec\theta + \tan\theta$.

Solution:
The angle θ is in the second quadrant. Cosine and tangent are negative.
This is a 5-12-13 triangle.
$\cos\theta = -5/13 \implies \sec\theta = -13/5$. $\tan\theta = -12/5$.

$$\sec\theta + \tan\theta = -\frac{13}{5} - \frac{12}{5} = -\frac{25}{5} = -\mathbf{5}.$$

The correct option is **(D)**.

10. If $a\cos\theta + b\sin\theta = 3$ and $a\sin\theta - b\cos\theta = 4$, then $a^2 + b^2$ has the value:

Solution:

We square both given equations and add them.

$$\begin{aligned}(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 &= 3^2 + 4^2\\ (a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta) + (a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta) &= 9 + 16\\ a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) &= 25\\ a^2(1) + b^2(1) &= 25\\ \mathbf{a^2 + b^2} &= \mathbf{25}.\end{aligned}$$

The correct option is **(A)**.

11. The expression $\frac{\tan\left(x-\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right)\tan\left(\frac{3\pi}{2}+x\right)}$ when simplified reduces to:

Solution:

We simplify each term using allied angle identities.

$\tan(x - \frac{\pi}{2}) = -\tan(\frac{\pi}{2} - x) = -\cot x$.

$\cos(\frac{3\pi}{2} + x) = \sin x$.

$\sin(\frac{7\pi}{2} - x) = \sin(4\pi - \frac{\pi}{2} - x) = \sin(-\frac{\pi}{2} - x) = -\cos x$.

$\cos(x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - x) = \sin x$.

$\tan(\frac{3\pi}{2} + x) = -\cot x$.

$$\begin{aligned}\text{Expression} &= \frac{(-\cot x)(\sin x) - (-\cos x)^3}{(\sin x)(-\cot x)}\\ &= \frac{-\cos x + \cos^3 x}{-\cos x} = \frac{-\cos x(1 - \cos^2 x)}{-\cos x}\\ &= 1 - \cos^2 x = \sin^{\mathbf{2}}\mathbf{x}.\end{aligned}$$

The correct option is **(D)**.

12. The value of $\sin^2\frac{\pi}{9} + \sin^2\frac{2\pi}{9} + \cdots + \sin^2\frac{17\pi}{9}$ is:

Solution:

This question is wrong.

13. The value of $\sin^2 6^\circ + \sin^2 12^\circ + \cdots + \sin^2 84^\circ + \sin^2 90^\circ$ is:

Solution:

The angles are in an AP: 6, 12, ..., 84.

Number of terms is $\frac{84-6}{6} + 1 = 14$.

We pair terms using $\sin^2 A + \sin^2(90 - A) = \sin^2 A + \cos^2 A = 1$.

Pairs: $(\sin^2 6 + \sin^2 84)$, $(\sin^2 12 + \sin^2 78)$, ..., $(\sin^2 42 + \sin^2 48)$. There are $14/2 = 7$ such pairs.

The sum of these pairs is $7 \times 1 = 7$.

The final term is $\sin^2 90^\circ = 1^2 = 1$.

Total sum $= 7 + 1 = \mathbf{8}$.

The correct option is **(B)**.

14. The value of $\cos 0 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cdots + \cos\frac{6\pi}{7}$ is:

Solution:

We use the identity $\cos A + \cos(\pi - A) = \cos A - \cos A = 0$.

We pair the terms: $(\cos\frac{\pi}{7} + \cos\frac{6\pi}{7}) + (\cos\frac{2\pi}{7} + \cos\frac{5\pi}{7}) + (\cos\frac{3\pi}{7} + \cos\frac{4\pi}{7})$.

Each pair sums to 0.

The only term left is $\cos 0 = 1$.

The total sum is **1**.

The correct option is **(D)**.

15. The value of $\cos^2\frac{\pi}{12} + \cos^2\frac{3\pi}{12} + \cos^2\frac{5\pi}{12}$ is:

Solution:

We use $\cos A = \sin(90 - A) = \sin(\pi/2 - A)$.

$\cos\frac{5\pi}{12} = \cos(75°) = \sin(15°) = \sin\frac{\pi}{12}$.

The expression becomes $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \sin^2\frac{\pi}{12}$.

$$\begin{aligned}\text{Expression} &= (\cos^2\frac{\pi}{12} + \sin^2\frac{\pi}{12}) + \cos^2\frac{\pi}{4} \\ &= 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 + \frac{1}{2} = \mathbf{\frac{3}{2}}.\end{aligned}$$

The correct option is **(A)**.

16. The value of $\tan(\frac{\pi}{20})\tan(\frac{3\pi}{20})\tan(\frac{5\pi}{20})\tan(\frac{7\pi}{20})\tan(\frac{9\pi}{20})$ is:

Solution:

We use $\tan A \tan(90 - A) = \tan A \cot A = 1$.

Let's convert to degrees: $9°, 27°, 45°, 63°, 81°$.

$$\begin{aligned}\text{Expression} &= \tan 9° \tan 27° \tan 45° \tan 63° \tan 81° \\ &= (\tan 9° \tan 81°)(\tan 27° \tan 63°) \tan 45° \\ &= (\tan 9° \cot 9°)(\tan 27° \cot 27°)(1) = (1)(1)(1) = \mathbf{1}.\end{aligned}$$

The correct option is **(B)**.

17. The value of $\sin(\frac{\pi}{5}) + \sin(\frac{2\pi}{5}) + \sin(\frac{3\pi}{5}) + \cdots + \sin(\frac{9\pi}{5})$ is:

Solution:

We use the identity $\sin A + \sin(2\pi - A) = \sin A - \sin A = 0$.

$\sin(\frac{9\pi}{5}) = \sin(2\pi - \frac{\pi}{5}) = -\sin(\frac{\pi}{5})$.

Pairs: $(\sin\frac{\pi}{5} + \sin\frac{9\pi}{5}), (\sin\frac{2\pi}{5} + \sin\frac{8\pi}{5}), (\sin\frac{3\pi}{5} + \sin\frac{7\pi}{5}), (\sin\frac{4\pi}{5} + \sin\frac{6\pi}{5})$.

All these pairs sum to 0.

The remaining term is $\sin(\frac{5\pi}{5}) = \sin(\pi) = 0$.

The total sum is **0**.

The correct option is **(C)**.

Que.3: Proofs and Simplifications

1. Prove that: $2\sin^2\frac{\pi}{6} + \operatorname{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 0.$

Solution:

We evaluate the Left Hand Side (LHS) by substituting the known values of the trigonometric functions.

$$\begin{aligned}\text{LHS} &= 2\sin^2(30^\circ) + \operatorname{cosec}(210^\circ)\cos^2(60^\circ)\\ &= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}(180^\circ + 30^\circ)\left(\frac{1}{2}\right)^2\\ &= 2\left(\frac{1}{4}\right) + (-\operatorname{cosec}30^\circ)\left(\frac{1}{4}\right)\\ &= \frac{1}{2} + (-2)\left(\frac{1}{4}\right) = \frac{1}{2} - \frac{1}{2} = \mathbf{0}.\end{aligned}$$

Since LHS $= 0 =$ RHS, the identity is proven.

2. Prove that: $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6.$

Solution:

We evaluate the LHS by substituting the known values.

$$\begin{aligned}\text{LHS} &= \cot^2(30^\circ) + \operatorname{cosec}(150^\circ) + 3\tan^2(30^\circ)\\ &= (\sqrt{3})^2 + \operatorname{cosec}(180^\circ - 30^\circ) + 3\left(\frac{1}{\sqrt{3}}\right)^2\\ &= 3 + \operatorname{cosec}(30^\circ) + 3\left(\frac{1}{3}\right)\\ &= 3 + 2 + 1 = \mathbf{6}.\end{aligned}$$

Since LHS $= 6 =$ RHS, the identity is proven.

3. Prove that: $\sec(\frac{3\pi}{2} - A)\sec(\frac{\pi}{2} - A) - \tan(\frac{3\pi}{2} - A)\tan(\frac{\pi}{2} + A) + 1 = 0.$

Solution:

We simplify the terms on the LHS using allied angle identities.

$$\begin{aligned}\text{LHS} &= (-\operatorname{cosec}A)(\operatorname{cosec}A) - (\cot A)(-\cot A) + 1\\ &= -\operatorname{cosec}^2 A + \cot^2 A + 1\\ &= -(\operatorname{cosec}^2 A - \cot^2 A) + 1\\ &= -(1) + 1 = \mathbf{0}.\end{aligned}$$

Since LHS $= 0 =$ RHS, the identity is proven.

4. Prove that: $\cot A + \tan(\pi + A) + \tan(\frac{\pi}{2} + A) + \tan(2\pi - A) = 0.$

Solution:

We simplify the terms on the LHS.

$$\begin{aligned}\text{LHS} &= \cot A + \tan A - \cot A - \tan A\\ &= (\cot A - \cot A) + (\tan A - \tan A) = \mathbf{0}.\end{aligned}$$

Since LHS $= 0 =$ RHS, the identity is proven.

5. Prove that: $\frac{\cos(\pi-A)\sin(\frac{\pi}{2}+A)\cot(A)}{\tan(\frac{3\pi}{2}-A)\tan(\frac{\pi}{2}-A)\sin(2\pi-A)} = \cos A.$

Solution:
The question have a typo and should prove it equals $\cos A$. (not $\sin A$)

$$\begin{aligned} \text{LHS} &= \frac{(-\cos A)(\cos A)(\frac{\cos A}{\sin A})}{(\cot A)(\cot A)(-\sin A)} \\ &= \frac{-\frac{\cos^3 A}{\sin A}}{-\cot^2 A \sin A} \\ &= \frac{\frac{\cos^3 A}{\sin A}}{\frac{\cos^2 A}{\sin^2 A}\sin A} \\ &= \frac{\frac{\cos^3 A}{\sin A}}{\frac{\cos^2 A}{\sin A}} \\ &= \cos \mathbf{A}. \end{aligned}$$

6. Prove that: $\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos(\frac{\pi}{2}+\theta)} = \cot^2\theta$**.**

Solution:
We simplify the LHS.

$$\text{LHS} = \frac{(-\cos\theta)(\cos\theta)}{(\sin\theta)(-\sin\theta)} = \frac{-\cos^2\theta}{-\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^{\mathbf{2}}\theta.$$

Since LHS = RHS, the identity is proven.

7. Prove that: $\cos\theta + \sin(270° + \theta) - \sin(270° - \theta) + \cos(180° + \theta) = 0$**.**

Solution:
We simplify the LHS.

$$\begin{aligned} \text{LHS} &= \cos\theta + (-\cos\theta) - (-\cos\theta) + (-\cos\theta) \\ &= \cos\theta - \cos\theta + \cos\theta - \cos\theta = \mathbf{0}. \end{aligned}$$

Since LHS = 0 = RHS, the identity is proven.

8. Prove that: $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1$**.**

Solution:
We simplify the LHS.

$$\begin{aligned} \text{LHS} &= (\sin\theta)(\cos\theta)[\tan\theta + \cot\theta] \\ &= \sin\theta\cos\theta\left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right] \\ &= \sin\theta\cos\theta\left[\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right] \\ &= \sin\theta\cos\theta\left[\frac{1}{\sin\theta\cos\theta}\right] = \mathbf{1}. \end{aligned}$$

Since LHS = 1 = RHS, the identity is proven.

9. If $\tan\theta = -5/12$ **and** θ **is not in the second quadrant, then show that** $\frac{\sin(360°-\theta)+\tan(90°+\theta)}{-\sec(270°+\theta)+\text{cosec}(-\theta)} = \frac{181}{338}$**.**

Solution:

Given $\tan\theta < 0$, θ is in Q2 or Q4.

Since it's not in Q2, it must be in Q4.

In Q4: $\sin\theta < 0$, $\cos\theta > 0$. This is a 5-12-13 triangle.

$\sin\theta = -5/13$,

$\cos\theta = 12/13$.

$\sec\theta = 13/12$,

$\operatorname{cosec}\theta = -13/5$,

$\cot\theta = -12/5$.

Now we simplify the LHS expression:

$$\begin{aligned}
\text{LHS} &= \frac{-\sin\theta - \cot\theta}{-\operatorname{cosec}\theta - \operatorname{cosec}\theta} = \frac{-\sin\theta - \cot\theta}{-2\operatorname{cosec}\theta} \\
&= \frac{-(-5/13) - (-12/5)}{-2(-13/5)} = \frac{5/13 + 12/5}{26/5} \\
&= \frac{\frac{25+156}{65}}{\frac{26}{5}} = \frac{181/65}{26/5} = \frac{181}{65} \times \frac{5}{26} \\
&= \frac{181}{13 \times 26} = \frac{\mathbf{181}}{\mathbf{338}}.
\end{aligned}$$

Since LHS = RHS, the identity is shown.