



A Premier Institute for Pre-Medical & Pre Engineering

**SRI VIDYA
ARADHANA
ACADEMY**
"Transforming Your DREAMS Into Reality...!"**NEET/JEE****Topic: SL LONEY Exercises**

Sub: Mathematics

Assignment: 03

Prof. Chetan Sir

Que.1: Solve the following (SL.LONEY Ex.13)

1. If $\sin \alpha = \frac{45}{53}$ and $\sin \beta = \frac{33}{65}$, find the values of $\sin(\alpha - \beta)$ and $\sin(\alpha + \beta)$.
2. If $\sin \alpha = \frac{15}{17}$ and $\cos \beta = \frac{12}{13}$, find the values of $\sin(\alpha + \beta)$, $\cos(\alpha - \beta)$ and $\tan(\alpha + \beta)$.
3. Prove that: $\cos(45^\circ - A) \cos(45^\circ - B) - \sin(45^\circ - A) \sin(45^\circ - B) = \sin(A + B)$.
4. Prove that: $\sin(45^\circ + A) \cos(45^\circ - B) + \cos(45^\circ + A) \sin(45^\circ - B) = \cos(A - B)$.
5. Prove that: $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$.
6. Prove that: $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$.
7. Prove that: $\sin 75^\circ - \sin 15^\circ = \cos 105^\circ + \cos 15^\circ$.
8. Prove that: $\cos \alpha \cos(\gamma - \alpha) - \sin \alpha \sin(\gamma - \alpha) = \cos \gamma$.
9. Prove that: $\cos(\alpha + \beta) \cos \gamma - \cos(\beta + \gamma) \cos \alpha = \sin \beta \sin(\gamma - \alpha)$.
10. Prove that: $\sin(n+1)A \sin(n-1)A + \cos(n+1)A \cos(n-1)A = \cos 2A$.
11. Prove that: $\sin(n+1)A \sin(n+2)A + \cos(n+1)A \cos(n+2)A = \cos A$.

Que.2: Prove that (SL.LONEY Ex.14)

1. $\frac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta} = \tan \theta$
2. $\frac{\cos 6\theta - \cos 4\theta}{\sin 6\theta + \sin 4\theta} = -\tan \theta$
3. $\frac{\sin 7A - \sin A}{\sin 8A - \sin 2A} = \cos 4A \sec 5A$
4. $\frac{\cos 2B + \cos 2A}{\cos 2B - \cos 2A} = \cot(A + B) \cot(A - B)$
5. $\frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \frac{\tan(A+B)}{\tan(A-B)}$
6. $\frac{\sin A + \sin 2A}{\cos A - \cos 2A} = \cot \frac{A}{2}$
7. $\frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan(A - B)$
8. $\cos(A + B) + \sin(A - B) = 2 \sin(45^\circ + A) \cos(45^\circ + B)$
9. $\frac{\sin(4A - 2B) + \sin(4B - 2A)}{\cos(4A - 2B) + \cos(4B - 2A)} = \tan(A + B)$
10. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$

11. $\frac{\cos 3\theta + 2 \cos 5\theta + \cos 7\theta}{\cos \theta + 2 \cos 3\theta + \cos 5\theta} = \cos 2\theta - \sin 2\theta \tan 3\theta$
12. $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$
13. $\frac{\sin(\theta + \phi) - 2 \sin \theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2 \cos \theta + \cos(\theta - \phi)} = \tan \theta$
14. $\frac{\sin A - \sin 5A + \sin 9A - \sin 13A}{\cos A - \cos 5A - \cos 9A + \cos 13A} = \cot 4A$
15. $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \frac{A+B}{2} \cot \frac{A-B}{2}$
16. $\frac{\sin A - \sin B}{\cos B - \cos A} = \cot \frac{A+B}{2}$
17. $\frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{\sin(A+B+C) + \sin(-A+B+C) - \sin(A-B+C) + \sin(A+B-C)} = \cot B$
18. $\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$
19. $\cos(-A + B + C) + \cos(A - B + C) + \cos(A + B - C) + \cos(A + B + C) = 4 \cos A \cos B \cos C$
20. $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$
21. $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$
22. $\sin \alpha + \sin 2\alpha + \sin 4\alpha + \sin 5\alpha = 4 \cos \frac{\alpha}{2} \cos \frac{3\alpha}{2} \sin 3\alpha$

Que 3: SL LONEY Ex.15

Express as a sum or difference the following:

1. $2 \sin 5\theta \sin 7\theta$
2. $2 \cos 7\theta \sin 5\theta$
3. $2 \cos 11\theta \cos 3\theta$
4. $2 \sin 54^\circ \sin 66^\circ$

Prove that:

5. $\sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2} = \sin 2\theta \sin 5\theta$
6. $\cos 2\theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 5\theta \sin \frac{5\theta}{2}$
7. $\sin A \sin(A + 2B) - \sin B \sin(B + 2A) = \sin(A - B) \sin(A + B)$
8. $(\sin 3A + \sin A) \sin A + (\cos 3A - \cos A) \cos A = 0$
9. $\frac{2 \sin(A-C) \cos C - \sin(A-2C)}{2 \sin(B-C) \cos C - \sin(B-2C)} = \frac{\sin A}{\sin B}$
10. $\frac{\sin A \sin 2A + \sin 3A \sin 6A + \sin 4A \sin 13A}{\sin A \cos 2A + \sin 3A \cos 6A + \sin 4A \cos 13A} = \tan 9A$
11. $\cos(36^\circ - A) \cos(36^\circ + A) + \cos(54^\circ + A) \cos(54^\circ - A) = \cos 2A$
12. $\cos A \sin(B - C) + \cos B \sin(C - A) + \cos C \sin(A - B) = 0$
13. $\sin(45^\circ + A) \sin(45^\circ - A) = \frac{1}{2} \cos 2A$
14. $\sin(\beta - \gamma) \cos(\alpha - \delta) + \sin(\gamma - \alpha) \cos(\beta - \delta) + \sin(\alpha - \beta) \cos(\gamma - \delta) = 0$
15. $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$

Que 4: SL LONEY EX.17

1. Find the value of $\sin 2\alpha$ when
 (i) $\cos \alpha = \frac{3}{5}$ (ii) $\sin \alpha = \frac{12}{13}$ (iii) $\tan \alpha = \frac{16}{63}$
2. Find the value of $\cos 2\alpha$ when
 (i) $\cos \alpha = \frac{15}{17}$ (ii) $\sin \alpha = \frac{4}{5}$ (iii) $\tan \alpha = \frac{5}{12}$
3. If $\tan \theta = \frac{b}{a}$, find the value of $a \cos 2\theta + b \sin 2\theta$.
4. Prove that: $\frac{\sin 2A}{1+\cos 2A} = \tan A$
5. Prove that: $\frac{\sin 2A}{1-\cos 2A} = \cot A$
6. Prove that: $\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$
7. Prove that: $\tan A + \cot A = 2 \operatorname{cosec} 2A$
8. Prove that: $\tan A - \cot A = -2 \cot 2A$
9. Prove that: $\operatorname{cosec} 2A + \cot 2A = \cot A$
10. Prove that: $\frac{\cos A}{1+\sin A} = \tan(45^\circ - \frac{A}{2})$
11. Prove that: $\frac{\sec 8A-1}{\sec 4A-1} = \frac{\tan 8A}{\tan 2A}$
12. Prove that: $\frac{1+\tan^2(45^\circ-A)}{1-\tan^2(45^\circ-A)} = \operatorname{cosec} 2A$
13. Prove that: $\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$
14. Prove that: $\frac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A+B)$
15. Prove that: $\tan(\frac{\pi}{4} + \theta) - \tan(\frac{\pi}{4} - \theta) = 2 \tan 2\theta$
16. Prove that: $\frac{\cos A + \sin A}{\cos A - \sin A} - \frac{\cos A - \sin A}{\cos A + \sin A} = 2 \tan 2A$
17. Prove that: $\cot(A+15^\circ) - \tan(A-15^\circ) = \frac{4 \cos 2A}{1+2 \sin 2A}$
18. Prove that: $\cos^3 2\theta + 3 \cos 2\theta = 4(\cos^6 \theta - \sin^6 \theta)$
19. Prove that: $1 + \cos^2 2\theta = 2(\cos^4 \theta + \sin^4 \theta)$
20. Prove that: $\sec^2 A(1 + \sec 2A) = 2 \sec 2A$
21. Prove that: $\operatorname{cosec} A - 2 \cot 2A \cos A = 2 \sin A$
22. Prove that: $\cot A = \frac{1}{2} \left(\cot \frac{A}{2} - \tan \frac{A}{2} \right)$
23. Prove that: $\cos 4\alpha = 1 - 8 \cos^2 \alpha + 8 \cos^4 \alpha$
24. Prove that: $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$

Que 5: SL LONEY Ex 18 (Prove that)

1. $\cos^2 \alpha + \cos^2(\alpha + 120^\circ) + \cos^2(\alpha - 120^\circ) = \frac{3}{2}$
2. $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}$
3. $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$

Que 6: SL LONEY Ex 19

1. Prove that: $\sin^2 72^\circ - \sin^2 60^\circ = \frac{\sqrt{5}-1}{8}$
2. Prove that: $\cos^2 48^\circ - \sin^2 12^\circ = \frac{\sqrt{5}+1}{8}$
3. Prove that: $\cos 12^\circ + \cos 60^\circ + \cos 84^\circ = \cos 24^\circ + \cos 48^\circ$
4. Prove that: $\sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$
5. Prove that: $\sin \frac{\pi}{10} + \sin \frac{13\pi}{10} = -\frac{1}{2}$
6. Prove that: $\sin \frac{\pi}{10} \sin \frac{13\pi}{10} = -\frac{1}{4}$
7. Prove that: $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$
8. Prove that: $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{2^7}$
9. Prove that: $16 \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = 1$

Que 7: SL LONEY Ex 20

If $A + B + C = 180^\circ$, prove that:

1. $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$
2. $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$
3. $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$
4. $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$
5. $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$
6. $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
7. $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$
8. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$
9. $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$
10. $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
11. $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
12. $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$
13. $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$

Answer Key**Que.1: (SL.LONEY Ex.13)**

1. (i) $\frac{1596}{3445}, \frac{3444}{3445}$ (ii) $\frac{220}{221}, \frac{171}{221}, \frac{220}{21}$

Que.3: (SL.LONEY Ex.15)

- (1) $\cos 2\theta - \cos 12\theta$ (2) $\sin 12\theta - \sin 2\theta$ (3) $\cos 8\theta + \cos 14\theta$ (4) $\cos 12^\circ - \cos 120^\circ$

Que.4 (SL.LONEY Ex.17)

1. (i) $\pm \frac{24}{25}$ (ii) $\pm \frac{120}{169}$ (iii) $\frac{2016}{4225}$
2. (i) $\frac{161}{289}$ (ii) $-\frac{7}{25}$ (iii) $\frac{119}{169}$
3. a