Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Solutions to JEE CT-01

Date of Exam: 15 June

Syllabus: Foundation + Trigonometric Ratios & Identities (till basic identities)

Topic: Algebra & Trigonometry

Sub: Mathematics **CT-01** Prof. Chetan Sir

SECTION-A

51. If $6\sin^2\theta - 11\sin\theta + 4 = 0$, a possible value for $\sin\theta$ is:

(A) $\frac{4}{3}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{2}{3}$

Solution:

$6\sin^2\theta - 11\sin\theta + 4 = 0$

$6\sin^2\theta - 8\sin\theta - 3\sin\theta + 4 = 0$ [Splitting the middle term: $-11 = -8 - 3$]

$2\sin\theta(3\sin\theta - 4) - 1(3\sin\theta - 4) = 0$

$(2\sin\theta - 1)(3\sin\theta - 4) = 0$

This gives two possibilities:

$2\sin\theta - 1 = 0 \implies \sin\theta = \dfrac{1}{2}$

$3\sin\theta - 4 = 0 \implies \sin\theta = \dfrac{4}{3}$

Since the range of $\sin\theta$ is $[-1, 1], \sin\theta = \dfrac{4}{3}$ is not possible.

Therefore, the only possible value is $\sin\theta = \mathbf{\dfrac{1}{2}}$.

The correct option is **(B)**.

52. The value of $\sin(-1710^\circ)$ is:

(A) 0 (B) $-\frac{1}{2}$ (C) 1 (D) -1

Solution:

$\sin(-1710°) = -\sin(1710°)$ [Using the identity $\sin(-x) = -\sin(x)$]

We can write 1710° as a multiple of 360° plus a remainder.

$1710° = 4 \times 360° + 270°$ [$4 \times 360 = 1440$, so $1710 - 1440 = 270$]

Alternatively, $1710° = 5 \times 360° - 90°$ [$5 \times 360 = 1800$, so $1710 - 1800 = -90$]

$-\sin(1710°) = -\sin(4 \times 360° + 270°)$

$= -\sin(270°)$ [Since $\sin(n \cdot 360° + x) = \sin(x)$]

$= -(-1)$ [The value of $\sin(270°)$ is -1]

$= \mathbf{1}$

The correct option is **(C)**.

53. The value of $\frac{\sin(\pi-\theta)\tan(\frac{\pi}{2}+\theta)}{\cot(2\pi-\theta)\cos(\pi+\theta)}$ is:

(A) $\tan\theta$ (B) -1 (C) 1 (D) $-\tan\theta$

Solution:

We simplify each term using trigonometric identities for allied angles:

$\sin(\pi - \theta) = \sin\theta$

$\tan(\frac{\pi}{2} + \theta) = -\cot\theta$

$\cot(2\pi - \theta) = -\cot\theta$

$\cos(\pi + \theta) = -\cos\theta$

Substituting these back into the expression:

$$\frac{(\sin\theta)(-\cot\theta)}{(-\cot\theta)(-\cos\theta)}$$

$$= \frac{\sin\theta}{-\cos\theta}$$

$$= -\tan\theta$$

The correct option is **(D)**.

54. Simplify $(\frac{81}{16})^{-3/4}$.

(A) $\frac{27}{8}$ (B) $\frac{8}{27}$ (C) $\frac{9}{4}$ (D) $\frac{4}{9}$

Solution:

$$\left(\frac{81}{16}\right)^{-3/4}$$

$$= \left(\frac{16}{81}\right)^{3/4} \qquad \text{[Using the property } (a/b)^{-n} = (b/a)^n\text{]}$$

$$= \left(\left(\frac{16}{81}\right)^{1/4}\right)^3 \qquad \text{[Using the property } (x^a)^b = x^{ab}\text{]}$$

$$= \left(\frac{16^{1/4}}{81^{1/4}}\right)^3$$

$$= \left(\frac{\sqrt[4]{16}}{\sqrt[4]{81}}\right)^3 \qquad \text{[Since } 16 = 2^4 \text{ and } 81 = 3^4\text{]}$$

$$= \left(\frac{2}{3}\right)^3$$

$$= \frac{2^3}{3^3}$$

$$= \mathbf{\frac{8}{27}}$$

The correct option is **(B)**.

55. If $x = 3 + \sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$.

(A) 34 (B) 30 (C) 36 (D) $6 + \sqrt{8}$

Solution:

Given $x = 3 + \sqrt{8}$.

First, find $\frac{1}{x}$:

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}}$$

$$= \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} \qquad \text{[Rationalizing the denominator]}$$

$$= \frac{3-\sqrt{8}}{3^2-(\sqrt{8})^2} = \frac{3-\sqrt{8}}{9-8} = 3-\sqrt{8}$$

Now, find $x + \frac{1}{x}$:

$$x + \frac{1}{x} = (3+\sqrt{8}) + (3-\sqrt{8}) = 6$$

We need to find $x^2 + \frac{1}{x^2}$.

We know that $\left(x + \frac{1}{x}\right)^2 = x^2 + 2(x)\left(\frac{1}{x}\right) + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 2$

$$(6)^2 = x^2 + \frac{1}{x^2} + 2$$

$$36 = x^2 + \frac{1}{x^2} + 2$$

$$x^2 + \frac{1}{x^2} = 36 - 2 = \mathbf{34}$$

The correct option is **(A)**.

56. The LCM of a number n and 18 is 72. Which of the following can be n?

(A) 24 (B) 48 (C) 16 (D) All of these

Solution:

We are given LCM(n, 18) = 72.
Let's write the prime factorization of the numbers:
$18 = 2 \times 9 = 2^1 \times 3^2$
$72 = 8 \times 9 = 2^3 \times 3^2$
For LCM, we take the highest power of each prime factor present in the numbers.
Let the prime factorization of n be $n = 2^a \times 3^b \times ...$
$\text{LCM}(n, 18) = \text{LCM}(2^a \times 3^b, 2^1 \times 3^2) = 2^{\max(a,1)} \times 3^{\max(b,2)}$
Comparing with the given LCM (72):
$2^{\max(a,1)} = 2^3 \implies \max(a, 1) = 3 \implies a$ must be 3.
$3^{\max(b,2)} = 3^2 \implies \max(b, 2) = 2 \implies b$ can be $0, 1,$ or 2.
So, n must have 2^3 as a factor. Let's check the options:
(A) $n = 24 = 8 \times 3 = 2^3 \times 3^1$. Here $a = 3, b = 1$. This is a valid possibility.
Check: LCM(24, 18) = $\text{LCM}(2^3 \times 3^1, 2^1 \times 3^2) = 2^3 \times 3^2 = 8 \times 9 = 72$. Correct.
(B) $n = 48 = 16 \times 3 = 2^4 \times 3^1$. Here $a = 4$. This is not possible as $\max(4, 1)$ would be 4, not 3.
(C) $n = 16 = 2^4$. Here $a = 4$. This is also not possible.
Therefore, the only possible value for n among the options is 24.

The correct option is **(A)**.

57. If $\tan\theta = 2/3$ then the value of $\frac{\sin^3\theta - \cos^3\theta}{\sin^3\theta + \cos^3\theta}$ is:

(A) $-\frac{19}{35}$ (B) $\frac{1}{2}$ (C) $-\frac{10}{35}$ (D) 0

Solution:

Given the expression $\dfrac{\sin^3\theta - \cos^3\theta}{\sin^3\theta + \cos^3\theta}$
To use the value of $\tan\theta$, we divide the numerator and the denominator by $\cos^3\theta$.

$$= \frac{\frac{\sin^3\theta}{\cos^3\theta} - \frac{\cos^3\theta}{\cos^3\theta}}{\frac{\sin^3\theta}{\cos^3\theta} + \frac{\cos^3\theta}{\cos^3\theta}}$$

$$= \frac{\tan^3\theta - 1}{\tan^3\theta + 1}$$

We are given $\tan\theta = 2/3$.

$$\tan^3\theta = (2/3)^3 = \frac{8}{27}$$

$$= \frac{\frac{8}{27} - 1}{\frac{8}{27} + 1}$$

$$= \frac{\frac{8-27}{27}}{\frac{8+27}{27}}$$

$$= \frac{-19/27}{35/27}$$

$$= -\frac{\mathbf{19}}{\mathbf{35}}$$

The correct option is **(A)**.

58. If $\sin\theta < 0$ and $\cot\theta < 0$, then θ lies in which quadrant?

(A) First quadrant (B) Second quadrant (C) Third quadrant (D) Fourth quadrant

Solution:

We analyze the sign of trigonometric functions in each quadrant (using ASTC rule).
First condition: $\sin\theta < 0$ (sine is negative).
This occurs in the Third (T) and Fourth (C) quadrants.
Second condition: $\cot\theta < 0$ (cotangent is negative).
This occurs in the Second (S) and Fourth (C) quadrants.
For both conditions to be true, we need to find the common quadrant.
The only quadrant that satisfies both $\sin\theta < 0$ and $\cot\theta < 0$ is the **Fourth quadrant**.

The correct option is **(D)**.

59. The value of $\cos(-930°)$ is:

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Solution:

$\cos(-930°) = \cos(930°)$ [Using the identity $\cos(-x) = \cos(x)$]
We find the coterminal angle by subtracting multiples of 360°.
$930° = 2 \times 360° + 210°$ [$2 \times 360 = 720$, so $930 - 720 = 210$]
$\cos(930°) = \cos(2 \times 360° + 210°)$
$= \cos(210°)$ [Since $\cos(n \cdot 360° + x) = \cos(x)$]
$= \cos(180° + 30°)$ [210° is in the third quadrant]
$= -\cos(30°)$ [In Q3, cosine is negative]
$= -\frac{\sqrt{3}}{2}$

The correct option is **(B)**.

60. The expression $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 - (\tan^2 x + \cot^2 x)$ simplifies to :

(A) 5 (B) 7 (C) 9 (D) 1

Solution:

$$(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 - (\tan^2 x + \cot^2 x)$$
$$= (\sin^2 x + 2\sin x \operatorname{cosec} x + \operatorname{cosec}^2 x) + (\cos^2 x + 2\cos x \sec x + \sec^2 x) - (\tan^2 x + \cot^2 x)$$
$$= (\sin^2 x + 2(1) + \operatorname{cosec}^2 x) + (\cos^2 x + 2(1) + \sec^2 x) - \tan^2 x - \cot^2 x$$

[Since $\sin x \operatorname{cosec} x = 1$ and $\cos x \sec x = 1$]

$$= \sin^2 x + 2 + \operatorname{cosec}^2 x + \cos^2 x + 2 + \sec^2 x - \tan^2 x - \cot^2 x$$
$$= (\sin^2 x + \cos^2 x) + (\operatorname{cosec}^2 x - \cot^2 x) + (\sec^2 x - \tan^2 x) + 2 + 2$$
$$= (1) + (1) + (1) + 4$$
$$= \mathbf{7}$$

The correct option is **(B)**.

61. Solve for x: $9^x = 3^{x+3}$.

(A) 1 (B) 2 (C) 3 (D) -3

Solution:

$9^x = 3^{x+3}$

$(3^2)^x = 3^{x+3}$ [Expressing the base 9 as a power of 3]

$3^{2x} = 3^{x+3}$ [Using the property $(a^m)^n = a^{mn}$]

Since the bases are equal, we can equate the exponents.

$2x = x + 3$

$2x - x = 3$

$x = \mathbf{3}$

The correct option is **(C)**.

62. The value of $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$ is equal to:

(A) $\sin A + \cos A$ (B) $\sin A - \cos A$ (C) 0 (D) 1

Solution:

$$\text{L.H.S.} = \frac{\cos A}{1 - \sin A/\cos A} + \frac{\sin A}{1 - \cos A/\sin A}$$ [Expressing in terms of sin and cos]

$$= \frac{\cos A}{(\cos A - \sin A)/\cos A} + \frac{\sin A}{(\sin A - \cos A)/\sin A}$$
$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$
$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$ [Making denominators same by taking -1 common from second term's denominator]

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$
$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A}$$ [Using $a^2 - b^2 = (a-b)(a+b)$]

$$= \cos \mathbf{A} + \sin \mathbf{A}$$ [Cancelling common factor]

The correct option is **(A)**.

63. If $x\cos\theta + y\sin\theta = a$ and $x\sin\theta - y\cos\theta = b$, then:

(A) $x^2 - y^2 = a^2 + b^2$ (B) $x^2 + y^2 = a^2 + b^2$ (C) $x^2 - y^2 = a^2 - b^2$ (D) $x^2 + y^2 = a^2 - b^2$

Solution:

Let $x\cos\theta + y\sin\theta = a \quad \cdots(1)$
And $x\sin\theta - y\cos\theta = b \quad \cdots(2)$
We square both equations and add them.
$(x\cos\theta + y\sin\theta)^2 + (x\sin\theta - y\cos\theta)^2 = a^2 + b^2$
$(x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta) + (x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta) = a^2 + b^2$
$x^2\cos^2\theta + y^2\sin^2\theta + x^2\sin^2\theta + y^2\cos^2\theta = a^2 + b^2$
$x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2$ [Grouping terms]
$x^2(1) + y^2(1) = a^2 + b^2$
$\mathbf{x^2 + y^2 = a^2 + b^2}$

The correct option is **(B)**.

64. Find the sum of all real solutions for x in the equation $9^{x+1} - 28 \cdot 3^x + 3 = 0$.

(A) 1 (B) 2 (C) -1 (D) -2

Solution:

$9^{x+1} - 28 \cdot 3^x + 3 = 0$
$9 \cdot 9^x - 28 \cdot 3^x + 3 = 0$
$9 \cdot (3^2)^x - 28 \cdot 3^x + 3 = 0$
$9 \cdot (3^x)^2 - 28 \cdot 3^x + 3 = 0$
Let $y = 3^x$. The equation becomes a quadratic in y:
$9y^2 - 28y + 3 = 0$
$9y^2 - 27y - y + 3 = 0$
$9y(y-3) - 1(y-3) = 0$
$(9y-1)(y-3) = 0$
Case 1: $y - 3 = 0 \implies y = 3$. Since $y = 3^x, 3^x = 3^1 \implies x_1 = 1$.
Case 2: $9y - 1 = 0 \implies y = 1/9$. Since $y = 3^x, 3^x = 1/9 = 3^{-2} \implies x_2 = -2$.
The real solutions are $x_1 = 1$ and $x_2 = -2$.
Sum of solutions $= 1 + (-2) = \mathbf{-1}$.

The correct option is **(C)**.

65. If $10\sin^4\alpha + 15\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$ then the value of $16\sec^6\alpha + 81\,\text{cosec}^6\alpha$ is equal to:

(A) 500 (B) 625 (C) 450 (D) 750

Solution:

$10\sin^4\alpha + 15\cos^4\alpha = 6$

We can write 6 as $6 \times 1 = 6(\sin^2\alpha + \cos^2\alpha)$.

$10\sin^4\alpha + 15\cos^4\alpha = 6\sin^2\alpha + 6\cos^2\alpha$

$10\sin^4\alpha - 6\sin^2\alpha + 15\cos^4\alpha - 6\cos^2\alpha = 0$

$2\sin^2\alpha(5\sin^2\alpha - 3) + 3\cos^2\alpha(5\cos^2\alpha - 2) = 0$

A different approach: divide by $\cos^4\alpha$.

$10\tan^4\alpha + 15 = 6\sec^4\alpha = 6(1+\tan^2\alpha)^2 = 6(1 + 2\tan^2\alpha + \tan^4\alpha)$

$10\tan^4\alpha + 15 = 6 + 12\tan^2\alpha + 6\tan^4\alpha$

$4\tan^4\alpha - 12\tan^2\alpha + 9 = 0$

$(2\tan^2\alpha - 3)^2 = 0 \implies 2\tan^2\alpha = 3 \implies \tan^2\alpha = 3/2$.

From this, we find $\sec^2\alpha$ and $\operatorname{cosec}^2\alpha$.

$\sec^2\alpha = 1 + \tan^2\alpha = 1 + 3/2 = 5/2$.

$\cot^2\alpha = 1/\tan^2\alpha = 2/3. \implies \operatorname{cosec}^2\alpha = 1 + \cot^2\alpha = 1 + 2/3 = 5/3$.

Now find the value of the expression $16\sec^6\alpha + 81\operatorname{cosec}^6\alpha$.

$= 16(\sec^2\alpha)^3 + 81(\operatorname{cosec}^2\alpha)^3$

$= 16(5/2)^3 + 81(5/3)^3$

$= 250 + 375$

$= \mathbf{625}$.

The correct option is **(B)**.

66. The value of $\cos(\frac{3\pi}{2}+\theta)\cdot\cos(2\pi+\theta)\cdot[\cot(\frac{3\pi}{2}-\theta)+\cot(2\pi+\theta)]$ is:

(A) 0 (B) 1 (C) -1 (D) 2

Solution:

Simplify each term:

$\cos(\frac{3\pi}{2}+\theta) = \sin\theta$ [Angle in Q4, cos is positive, function changes]

$\cos(2\pi+\theta) = \cos\theta$ [Periodicity]

$\cot(\frac{3\pi}{2}-\theta) = \tan\theta$ [Angle in Q3, cot is positive, function changes]

$\cot(2\pi+\theta) = \cot\theta$ [Periodicity]

Substitute back into the expression:

$(\sin\theta)\cdot(\cos\theta)\cdot[\tan\theta + \cot\theta]$

$= \sin\theta\cos\theta\left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]$

$= \sin\theta\cos\theta\left[\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right]$

$= \sin\theta\cos\theta\left[\frac{1}{\sin\theta\cos\theta}\right]$

$= \mathbf{1}$

The correct option is **(B)**.

67. If $3\tan\theta - \sec\theta = 1$, the possible values of $\tan\theta$ are:

(A) $0, \frac{3}{4}$ (B) $0, -\frac{3}{4}$ (C) $1, \frac{3}{4}$ (D) $1, -\frac{3}{4}$

Solution:

$3\tan\theta - 1 = \sec\theta$
Square both sides:
$(3\tan\theta - 1)^2 = \sec^2\theta$
$9\tan^2\theta - 6\tan\theta + 1 = 1 + \tan^2\theta$ [Using $\sec^2\theta = 1 + \tan^2\theta$]
$8\tan^2\theta - 6\tan\theta = 0$
$2\tan\theta(4\tan\theta - 3) = 0$
This gives two potential solutions:

$$\tan\theta = 0 \quad \text{or} \quad \tan\theta = \frac{3}{4}$$

The correct option is **(A)**.

68. The value of the expression $\frac{\operatorname{cosec}(90° - A)\sin(180° - A)\cot(360° - A)}{\sec(180° + A)\tan(90° + A)\sin(-A)}$ is:

(A) 1 (B) 2 (C) 0 (D) -1

Solution:

$$\text{Numerator: } \operatorname{cosec}(90 - A) = \sec A, \quad \sin(180 - A) = \sin A, \quad \cot(360 - A) = -\cot A.$$
$$\text{Denominator: } \sec(180 + A) = -\sec A, \quad \tan(90 + A) = -\cot A, \quad \sin(-A) = -\sin A.$$
$$\text{Expression} = \frac{(\sec A)(\sin A)(-\cot A)}{(-\sec A)(-\cot A)(-\sin A)}$$
$$= \frac{-\sec A \sin A \cot A}{-\sec A \sin A \cot A}$$
$$= \mathbf{1}$$

The correct option is **(A)**.

69. If $\frac{\sin x + \cos x}{\sin^3 x} = a\cot^3 x + b\cot^2 x + c\cot x + d$, then the value of $a + b + c + d$ is:

(A) 1 (B) 2 (C) 0 (D) 4

Solution:

$$\frac{\sin x + \cos x}{\sin^3 x} = \frac{\sin x}{\sin^3 x} + \frac{\cos x}{\sin^3 x}$$
$$= \frac{1}{\sin^2 x} + \frac{\cos x}{\sin x} \cdot \frac{1}{\sin^2 x}$$
$$= \operatorname{cosec}^2 x + \cot x \cdot \operatorname{cosec}^2 x$$
$$= (1 + \cot^2 x) + \cot x(1 + \cot^2 x) \qquad \text{[Using identity } \operatorname{cosec}^2 x = 1 + \cot^2 x\text{]}$$
$$= 1 + \cot^2 x + \cot x + \cot^3 x$$
$$= \cot^3 x + \cot^2 x + \cot x + 1$$

Comparing with $a\cot^3 x + b\cot^2 x + c\cot x + d$, we get:
$a = 1, \quad b = 1, \quad c = 1, \quad d = 1.$
The value of $a + b + c + d = 1 + 1 + 1 + 1 = \mathbf{4}$.

The correct option is **(D)**.

70. Let $S_n(x) = \sin^n x + \cos^n x$ for $n \in \mathbb{N}$. Then for all $x \in \mathbb{R}$, the value of $2S_6(x) - 3S_4(x) + S_2(x)$ is equal to:

(A) 1 (B) 0 (C) -1 (D) 1/2

Solution:

$$\begin{aligned}
&S_6(x) = \sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) \\
&= 1 \cdot [(\sin^2 x + \cos^2 x)^2 - 3 \sin^2 x \cos^2 x] = 1 - 3 \sin^2 x \cos^2 x \\
&S_4(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x = 1 - 2 \sin^2 x \cos^2 x \\
&S_2(x) = \sin^2 x + \cos^2 x = 1 \\
&\text{Now substitute into the expression } 2S_6(x) - 3S_4(x) + S_2(x) : \\
&= 2(1 - 3 \sin^2 x \cos^2 x) - 3(1 - 2 \sin^2 x \cos^2 x) + 1 \\
&= (2 - 6 \sin^2 x \cos^2 x) - (3 - 6 \sin^2 x \cos^2 x) + 1 \\
&= 2 - 6 \sin^2 x \cos^2 x - 3 + 6 \sin^2 x \cos^2 x + 1 \\
&= (2 - 3 + 1) + (-6 \sin^2 x \cos^2 x + 6 \sin^2 x \cos^2 x) \\
&= 0 + 0 = \mathbf{0}
\end{aligned}$$

The correct option is **(B)**.

SECTION-B

71. If $2x^3 + kx^2 + 4x - 12$ and $x^3 + x^2 - 2x + k$ leave the same remainder when divided by $(x - 3)$, find the value of $-2k$.

Solution:

$$\begin{aligned}
&\text{Let } P(x) = 2x^3 + kx^2 + 4x - 12. \\
&\text{Let } Q(x) = x^3 + x^2 - 2x + k. \\
&\text{By the Remainder Theorem, the remainder when } P(x) \text{ is divided by } (x - 3) \text{ is } P(3). \\
&R_1 = P(3) = 2(3)^3 + k(3)^2 + 4(3) - 12 = 2(27) + 9k + 12 - 12 = 54 + 9k. \\
&\text{The remainder when } Q(x) \text{ is divided by } (x - 3) \text{ is } Q(3). \\
&R_2 = Q(3) = (3)^3 + (3)^2 - 2(3) + k = 27 + 9 - 6 + k = 30 + k. \\
&\text{Given that the remainders are the same, } R_1 = R_2. \\
&54 + 9k = 30 + k \\
&8k = 30 - 54 = -24 \\
&k = -3 \\
&\text{We need to find the value of } -2k. \\
&-2k = -2(-3) = \mathbf{6}
\end{aligned}$$

The answer is **6**.

72. If $(\frac{a}{b})^{x-1} = (\frac{b}{a})^{x-3}$, find the value of x.

Solution:

$$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$$

$$\left(\frac{a}{b}\right)^{x-1} = \left(\left(\frac{a}{b}\right)^{-1}\right)^{x-3}$$ [Using the property that $(b/a) = (a/b)^{-1}$]

$$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-(x-3)}$$

$$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-x+3}$$

Since the bases are the same, we can equate the exponents.

$x - 1 = -x + 3$

$2x = 4$

$x = \mathbf{2}$

The answer is **2**.

73. Find the value of $\frac{7^{x+3}-7^{x+1}}{48\cdot 7^x}$.

Solution:

$$\frac{7^{x+3} - 7^{x+1}}{48 \cdot 7^x}$$

$$= \frac{7^x \cdot 7^3 - 7^x \cdot 7^1}{48 \cdot 7^x}$$ [Using the property $a^{m+n} = a^m \cdot a^n$]

$$= \frac{7^x(7^3 - 7^1)}{48 \cdot 7^x}$$ [Factoring out 7^x from the numerator]

$$= \frac{7^3 - 7}{48}$$ [Cancelling the common factor 7^x]

$$= \frac{343 - 7}{48}$$

$$= \frac{336}{48}$$

$$= \mathbf{7}$$

The answer is **7**.

74. If $\sin\theta + \cos\theta = a$ and $\tan\theta + \cot\theta = b$, find the value of $b(a^2 - 1)$.

Solution:

Consider the first equation: $\sin\theta + \cos\theta = a$
Squaring both sides:$(\sin\theta + \cos\theta)^2 = a^2$
$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$
$1 + 2\sin\theta\cos\theta = a^2 \implies 2\sin\theta\cos\theta = a^2 - 1. \quad \cdots (1)$
Consider the second equation: $\tan\theta + \cot\theta = b$
$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = b$
$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = b$
$\frac{1}{\sin\theta\cos\theta} = b \implies \sin\theta\cos\theta = \frac{1}{b}. \quad \cdots (2)$
Substitute the expression for $\sin\theta\cos\theta$ from (2) into (1).
$2\left(\frac{1}{b}\right) = a^2 - 1$
$\frac{2}{b} = a^2 - 1$
$2 = b(a^2 - 1)$

The answer is **2**.

75. If $\cos x + \cos^2 x = 1$, find the value of $\sin^8 x + 2\sin^6 x + \sin^4 x$.

Solution:

From the given equation: $\cos x = 1 - \cos^2 x$
$\cos x = \sin^2 x$ [Using the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$]

Now, consider the expression to be evaluated:
$\sin^8 x + 2\sin^6 x + \sin^4 x$
$= \sin^4 x(\sin^4 x + 2\sin^2 x + 1)$ [Factoring out $\sin^4 x$]
$= \sin^4 x(\sin^2 x + 1)^2$ [Recognizing a perfect square trinomial]
Now, substitute $\sin^2 x = \cos x$.
$= (\sin^2 x)^2(\sin^2 x + 1)^2$
$= (\cos x)^2(\cos x + 1)^2$
$= \cos^2 x(\cos x + 1)^2$
From the given equation, we have $\cos x + 1 = \sec x$? No.
From $\cos x + \cos^2 x = 1$, we have $\cos x + 1 = \frac{1}{\cos x} = \sec x$.
$= \cos^2 x(\sec x)^2$
$= \cos^2 x \cdot \frac{1}{\cos^2 x}$
$= \mathbf{1}$

Alternative method:

$$\begin{aligned}
&\sin^8 x + 2\sin^6 x + \sin^4 x \\
&= (\sin^2 x)^4 + 2(\sin^2 x)^3 + (\sin^2 x)^2 \\
&\text{Substitute } \sin^2 x = \cos x : \\
&= (\cos x)^4 + 2(\cos x)^3 + (\cos x)^2 \\
&= \cos^4 x + 2\cos^3 x + \cos^2 x \\
&= \cos^2 x(\cos^2 x + 2\cos x + 1) \\
&= \cos^2 x(\cos x + 1)^2 \\
&= (\cos x(\cos x + 1))^2 \\
&= (\cos^2 x + \cos x)^2 \\
&\text{Since we are given } \cos x + \cos^2 x = 1, \\
&= (1)^2 = \mathbf{1}
\end{aligned}$$

The answer is **1**.