Quadratic Equation

Basic

1. If p and q are real number such that $p+q=3$ and $p^4+q^4=369$, then the value of $(\frac{1}{p}+\frac{1}{q})^{-2}$ is equal to _____. **[26 Jun 2022 (E)]**

2. The number of solutions of the equation $(\frac{9}{x}-\frac{9}{\sqrt{x}}+2)(2-\frac{7}{\sqrt{x}}+3)=0$ is : **[29 Jan 2025 (R)]**

(A) 3 (B) 2 (C) 1 (D) 4

3. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is: **[31 Jan 2023 (A)]**

(A) 0 (B) 1 (C) 3 (D) 2

4. The product of all the rational roots of the equation $(x^2-9x+11)^2-(x-4)(x-5)=3$, is equal to: **[24 Jan 2025 (M)]**

(A) 7 (B) 21 (C) 28 (D) 14

5. If $a+b+c=1$, $ab+bc+ca=2$ and $abc=3$ then the value of $a^4+b^4+c^4$ is equal to: _____. **[25 Jul 2021 (M)]**

6. The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots}}}}$ is equal to **[20 Jul 2021 (M)]**

(A) $1.5+\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $3+2\sqrt{3}$ (D) $4+\sqrt{3}$

7. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots}}}}$ is: **[18 Mar 2021 (M)]**

(A) $2+\frac{2}{5}\sqrt{30}$ (B) $2+\frac{4}{\sqrt{5}}\sqrt{30}$ (C) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (D) $5+\frac{2}{5}\sqrt{30}$

8. Let p and q be two positive numbers such that $p+q=2$ and $p^4+q^4=272$. Then p and q are roots of the equation: **[24 Feb 2021 (M)]**

(A) $x^2-2x+2=0$ (B) $x^2-2x+8=0$
(C) $x^2-2x+136=0$ (D) $x^2-2x+16=0$

Common Roots

9. If the value of real number $\alpha>0$ for which $x^2-5\alpha x+1=0$ and $x^2-\alpha x-5=0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____. **[30 Jan 2023 (R)]**

10. Let $\alpha,\beta\in R$ be such that the equation $ax^2-2bx+15=0$ has repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to: **[25 Jun 2022 (R)]**

(A) 37 (B) 58 (C) 68 (D) 92

11. Let $\lambda\neq 0$ be in R. If α and β are the roots of the equation, $x^2-x+2\lambda=0$ and α and γ are the roots of the equation, $3x^2-10x+27\lambda=0$, then $\frac{\beta\gamma}{\lambda}$ is equal to: **[04 Sep 2020 (R)]**

(A) 27 (B) 18 (C) 9 (D) 36

12. If for some p, q, $r \in R$, all having positive sign, one of the roots of the equation $(p^2+q^2)x^2-2q(p+r)x+q^2+r^2=0$ is also a root of the equation $x^2+2x-8=0$ then $\frac{q^2+r^2}{p^2}$ is equal to _____.
[26 Jul 2022 (A)]

13. Let α,β be the roots of the equation $x^2-4\lambda x+5=0$ and α,γ be the roots of the equation $x^2-(3\sqrt{2}+2\sqrt{3})x+7+3\lambda\sqrt{3}=0$. If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to _____. **[27 Jun 2022 (A)]**

14. Let $a,b \in R, a \neq 0$ be such that the equation, $ax^2-2bx+5=0$ has a repeated root α, which is also a root of the equation, $x^2-2bx-10=0$. If β is the other root of this equation, then $\alpha^2+\beta^2$ is equal to:
[09 Jan 2020 (M)]

(A) 25 (B) 26 (C) 28 (D) 24

15. If α,β and γ are three consecutive terms of a non-constant G.P. Such that the equations $\alpha x^2+2\beta x+\gamma=0$ and $x^2+x-1=0$ have a common root, then $\alpha(\beta+\gamma)$ is equal to: **[12 Apr 2019 (M)]**

(A) $\beta\gamma$ (B) $\alpha\beta$ (C) $\alpha\gamma$ (D) 0

Complex Number

16. The sum of 162^{th} power of the roots of the equation $x^3-2x^2+2x-1=0$ is _____. **[17 Mar 2021 (E)]**

17. If $\alpha,\beta \in R$ are such that $1-2i$ (here $i^2=-1$) is a root of $z^2+\alpha z+\beta=0$, then $(\alpha-\beta)$ is equal to:
[26 Feb 2021 (E)]

(A) -7 (B) 7 (C) -3 (D) 3

18. Let α be a root of the equation $1+x^2+x^4=0$. Then the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ is equal to:
[29 Jun 2022 (R)]

(A) 1 (B) α (C) $1+\alpha$ (D) $1+2\alpha$

19. Let α,β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1$, $\frac{1}{\beta^2}+1$ be the roots of the equation $x^2+ax+b=0$. Then the roots of the equation $x^2-(a+b-2)x+(a+b+2)=0$ are:
[28 Jul 2022 (A)]

(A) non-real complex numbers
(B) real and both negative
(C) real and both positive
(D) real and exactly one of them is positive

Diophantine Equation

20. Let the set $C=\{(x,y)|x^2-2^y=2023, x,y \in \mathbb{N}\}$. Then $\sum_{(x,y)\in C}(x+y)$ is equal to _____.
[29 Jan 2024 (A)]

Exponential/Logarithmic Equation

21. The sum of all the solutions of the equation $(8)^{2x}-16(8)^x+48=0$ is: **[08 Apr 2024 (E)]**

(A) $1+log_8(6)$ (B) $1+log_6(8)$ (C) $log_8(6)$ (D) $log_8(4)$

22. The sum of all real roots of equation $(e^{2x}-4)(6e^{2x}-5e^x+1)=0$ is: **[24 Jun 2022 (E)]**

(A) $ln4$ (B) $-ln3$ (C) $ln3$ (D) $ln5$

23. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is: **[31 Aug 2021 (E)]**

(A) 0 (B) 2 (C) 1 (D) 3

24. Let $S = \{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the number of elements in S is: **[01 Feb 2024 (R)]**

(A) 4 (B) 0 (C) 2 (D) 1

25. The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is: **[31 Jan 2024 (R)]**

(A) 2 (B) more than 2 (C) 1 (D) 0

26. Let $S = \{x : x \in \mathbb{R}$ and $(\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$. Then $n(S)$ is equal to: **[01 Feb 2023 (R)]**

(A) 2 (B) 4 (C) 6 (D) 0

27. Let $S = \{\alpha : log_2(9^{2\alpha-4}+13) - log_2(\frac{5}{2}\cdot 3^{2\alpha-4}+1) = 2\}$. Then the maximum value of β for which the equation $x^2 - 2(\sum_{\alpha\in s}\alpha)^2 x + \sum_{\alpha\in s}(\alpha+1)^2\beta = 0$ has real roots, is _____. **[25 Jan 2023 (A)]**

28. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $log_e P$ then P is equal to _____. **[27 Jun 2022 (A)]**

Location of Roots

29. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $(1,5)$ then m lies in the interval: **[09 Jan 2019 (E)]**

(A) $(-5,-4)$ (B) $(3,4)$ (C) $(5,6)$ (D) $(4,5)$

30. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval $(1,2)$ and its other root lies in the interval $(2,3)$ is: **[01 Feb 2023 (R)]**

(A) 2 (B) 0 (C) 1 (D) 3

31. The set of all real values of λ for which the quadratic equation $(\lambda^2+1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0,1)$ is: **[03 Sep 2020 (R)]**

(A) $(-3,-1)$ (B) $(0,2)$ (C) $(1,3]$ (D) $(2,4]$

32. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval $(0,2)$ and its other root lies in the interval $(2,3)$. Then the number of elements in S is: **[10 Jan 2019 (R)]**

(A) 11 (B) 12 (C) 18 (D) 10

33. If the set of all $a \in \mathbb{R} - \{1\}$, for which the roots of the equation $(1-a)x^2 + 2(a-3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to: **[02 Apr 2025 (A)]**

34. Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p+2)x + (2p+9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to: **[07 Apr 2025 (M)]**

(A) 5 (B) 0 (C) 20 (D) 9

Logarithmic Properties

35. If α, β are the roots of the equation $x^2-(5+3^{\sqrt{log_3 5}}-5^{\sqrt{log_5 3}})x+3(3^{(log_3 5)^{\frac{1}{3}}}-5^{(log_5 3)^{\frac{2}{3}}}-1)=0$, then the equation, whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$, is: **[27 Jul 2022 (M)]**

(A) $3x^2-20x-12=0$ (B) $3x^2-10x-4=0$
(C) $3x^2-10x+2=0$ (D) $3x^2-20x+16=0$

Modulus Equation

36. The number of real solutions of the equation $x(x^2+3|x|+5|x-1|+6|x-2|)=0$ is _____. **[30 Jan 2024 (E)]**

37. The number of real solutions of the equation, $x^2-|x|-12=0$ is: **[27 Jul 2021 (E)]**

(A) 2 (B) 3 (C) 1 (D) 4

38. The product of the roots of the equation $9x^2-18|x|+5=0$ is: **[05 Sep 2020 (E)]**

(A) $5/9$ (B) $25/81$ (C) $5/27$ (D) $25/9$

39. The number of real roots of the equation $x|x|-5|x+2|+6=0$ is: **[15 Apr 2023 (R)]**

(A) 5 (B) 4 (C) 6 (D) 3

40. The sum of all the roots of the equation $|x^2-8x+15|-2x+7=0$ is: **[06 Apr 2023 (R)]**

(A) $9-\sqrt{3}$ (B) $9+\sqrt{3}$ (C) $11-\sqrt{3}$ (D) $11+\sqrt{3}$

41. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is _____. **[24 Feb 2021 (R)]**

42. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is: **[10 Apr 2019 (R)]**

(A) 2 (B) 3 (C) 1 (D) 4

43. The number of real solution(s) of the equation $x^2+3x+2=\min\{|x-3|,|x+2|\}$ is : **[24 Jan 2025 (A)]**

(A) 2 (B) 3 (C) 1 (D) 0

44. The number of distinct real roots of the equation $|x+1||x+3|-4|x+2|+5=0$ is _____. **[08 Apr 2024 (A)]**

45. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____. **[05 Apr 2024 (A)]**

46. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2-2|x|+|\lambda-3|=0$. Then the largest element in the set $S=\{x+\lambda : x \text{ is an integer solution of E}\}$ is _____. **[24 Jan 2023 (A)]**

47. Let S be the set of all real roots of the equation, $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$, then S: **[08 Jan 2020 (A)]**

(A) contains exactly two elements. (B) is a singleton.
(C) is an empty set. (D) contains at least four elements.

48. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0, (x>0)$ is equal to: **[08 Apr 2019 (M)]**

(A) 10 (B) 9 (C) 12 (D) 4

Nature of Roots

49. Consider the equation $x^2+4x-n=0$, where $n \in [20,100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to: **[04 Apr 2025 (E)]**

(A) 6 (B) 5 (C) 8 (D) 7

50. The sum of all integral values of $k(k \neq 0)$ for which the equation $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$ in x has no real roots, is _____. **[26 Aug 2021 (E)]**

51. Consider the two sets:$A=\{m \in R$: both the roots of $x^2-(m+1)x+m+4=0$ are real and $B=[-3,5)$.of the following is not true? **[03 Sep 2020 (E)]**

(A) A-B=(-∞,-3)∪(5,∞) (B) A∩B={-3} (C) B-A=(-3,5) (D) A∪B=R

52. The least positive value of 'a' for which the equation, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots is _____. **[08 Jan 2020 (E)]**

53. Let p, $q \in Q$. If $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$ then: **[09 Apr 2019 (E)]**

(A) $p^2-4q+12=0$ (B) $q^2+4p+14=0$
(C) $p^2-4q-12=0$ (D) $q^2-4p-16=0$

54. The number of integral values of m for which the equation, $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root, is: **[08 Apr 2019 (E)]**

(A) 2 (B) 3 (C) Infinitely many (D) 1

55. The number of all possible positive integral values of α for which the roots of the quadratic equation $6x^2-11x+\alpha=0$ are rational numbers is: **[09 Jan 2019 (E)]**

(A) 5 (B) 3 (C) 4 (D) 2

56. If the set of all $a \in \mathbb{R}$, for which the equation $2x^2+(a-5)x+15=3a$ has no real root, is the interval (α,β), and $X=\{x \in \mathbb{Z}: \alpha<x<\beta\}$, then $\sum_{x\in X} x^2$ is equal to: **[29 Jan 2025 (A)]**

(A) 2139 (B) 2119 (C) 2109 (D) 2129

57. The set of all values of $k>-1$, for which the equation $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$ has real roots, is: **[27 Aug 2021 (A)]**

(A) $[-\frac{1}{2},1)$ (B) $(1,\frac{5}{2}]$ (C) $(\frac{1}{2},\frac{3}{2}]-\{1\}$ (D) [2,3)

58. Let the equation $x(x+2)(12-k)=2$ have equal roots. Then the distance of the point $(k,\frac{k}{2})$ from the line $3x+4y+5=0$ is: **[03 Apr 2025 (M)]**

(A) 15 (B) 12 (C) $5\sqrt{3}$ (D) $15\sqrt{5}$

59. If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where $a+c=15$ and $b=\frac{36}{5}$, then a^2+c^2 is equal to: **[23 Jan 2025 (M)]**

Newton Sums

60. Let α and β be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta=-1$. Let $p_n=(\alpha)^n+(\beta)^n$, $p_{n-1}=11$ and $p_{n+1}=29$ for some integer $n \geq 1$. Then, the value of p_n^2 is _____. **[26 Feb 2021 (E)]**

61. Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n=\alpha^n+\beta^n$, $n=1,2,3,....$, then: **[02 Sep 2020 (E)]**

(A) $6S_6+5S_5=2S_4$ (B) $5S_6+6S_5+2S_4=0$
(C) $5S_6+6S_5=2S_4$ (D) $6S_6+5S_5+2S_4=0$

62. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k$, $k\geq 1$, then which one of the following statements is not true? **[07 Jan 2020 (E)]**

(A) $p_3=p_5-p_4$ (B) $p_5=11$
(C) $(p_1+p_2+p_3+p_4+p_5)=26$ (D) $p_5=p_2\cdot p_3$

63. Let α,β be the distinct roots of the equation $x^2-(t^2-5t+6)x+1=0$, $t\in\mathbb{R}$ and $a_n=\alpha^n+\beta^n$. Then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is: **[06 Apr 2024 (R)]**

(A) $-1/4$ (B) $-1/2$ (C) $-1/4$ (D) $1/4$

64. Let α,β be roots of $x^2+\sqrt{2}x-8=0$. If $U_n=\alpha^n+\beta^n$, then $\frac{U_{10}+\sqrt{2}U_9}{2U_8}$ is equal to _____. **[06 Apr 2024 (R)]**

65. Let α,β be the roots of the equation $x^2-\sqrt{2}x+2=0$. Then $\alpha^{14}+\beta^{14}$ is equal to: **[13 Apr 2023 (R)]**

(A) -64 (B) $-64\sqrt{2}$ (C) -128 (D) $-128\sqrt{2}$

66. Let α,β be the roots of the quadratic equation $x^2+\sqrt{6}x+3=0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to: **[12 Apr 2023 (R)]**

(A) 81 (B) 9 (C) 72 (D) 729

67. If α and β are the roots of the equation $x^2-7x-1=0$ then the value of $\frac{\alpha^{21}+\beta^{21}+\alpha^{17}+\beta^{17}}{\alpha^{19}+\beta^{19}}$ is equal to _____. **[11 Apr 2023 (R)]**

68. Let α,β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to: **[27 Jul 2021 (R)]**

(A) 10 (B) 100 (C) 50 (D) 160

69. If α and β are the distinct roots of the equation $x^2+(3)^{\frac{1}{4}}x+3^{\frac{1}{2}}=0$ then the value of $\alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)$ is equal to: **[25 Jul 2021 (R)]**

(A) 56×3^{25} (B) 56×3^{24} (C) 52×3^{24} (D) 28×3^{25}

70. Let α and β be the roots of $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$. then the value of $\frac{a_{10}-2a_8}{3a_9}$ is: **[25 Feb 2021 (R)]**

(A) 1 (B) 3 (C) 2 (D) 4

71. Let α and β be the roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to: **[09 Jan 2019 (R)]**

(A) -512 (B) 128 (C) 512 (D) -256

72. Let α and β be the roots of $x^2+\sqrt{3}x-16=0$, and γ and δ be the roots of $x^2+3x-1=0$. If $P_n=\alpha^n+\beta^n$ and $Q_n=\gamma^n+\delta^n$, then $\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}+\frac{Q_{25}-Q_{23}}{Q_{24}}$ is equal to: **[03 Apr 2025 (A)]**

(A) 4 (B) 3 (C) 5 (D) 7

73. Let $\alpha,\beta;\alpha>\beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n=\alpha^n-\beta^n$, $n\in N$. Then $(11\sqrt{3}-10\sqrt{2})P_{10}+(11\sqrt{2}+10)P_{11}-11P_{12}$ is equal to: **[09 Apr 2024 (A)]**

(A) $10\sqrt{3}P_9$ (B) $11\sqrt{3}P_9$ (C) $10\sqrt{2}P_9$ (D) $11\sqrt{2}P_9$

74. If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$ then **[27 Jan 2024 (A)]**

(A) $S_{12} = S_{11} + S_{10}$ (B) $2S_{12} = S_{11} + S_{10}$
(C) $2S_{11} = S_{12} + S_{10}$ (D) $S_{11} = S_{10} + S_{12}$

75. Let , $\beta(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _____. **[29 Jul 2022 (A)]**

76. For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is _____. **[25 Jun 2022 (A)]**

77. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2})$ is equal to _____. **[25 Jul 2021 (A)]**

78. Let $P_n = \alpha^n + \beta^n$, $n \in N$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is : **[02 Apr 2025 (M)]**

(A) $x^2 + x - 1 = 0$ (B) $x^2 - x + 1 = 0$
(C) $x^2 + x + 1 = 0$ (D) $x^2 - x - 1 = 0$

Number of Real Roots

79. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in \mathbb{R}$ cuts x-axis, is equal to_____. **[11 Apr 2023 (R)]**

80. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in R$ has: **[31 Jan 2023 (R)]**

(A) four solutions two of which are negative
(B) two solutions and both are negative
(C) no solution
(D) two solutions and only one of them is negative

81. The number of real solutions of the equation $3(x^2 + \frac{1}{x^2}) - 2(x + \frac{1}{x}) + 5 = 0$, is: **[24 Jan 2023 (R)]**

(A) 4 (B) 0 (C) 3 (D) 2

82. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is _____. **[28 Jun 2022 (R)]**

83. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to _____. **[27 Jul 2021 (R)]**

84. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is: **[09 Jan 2020 (R)]**

(A) 1 (B) 3 (C) 2 (D) 4

Properties of Polynomials

85. Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to: **[28 Jun 2022 (E)]**

(A) $11/3$ (B) $7/3$ (C) $13/3$ (D) $14/3$

86. Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in: **[02 Sep 2020 (M)]**

(A) $(-1, 0)$ (B) $(1, 3)$ (C) $(-3, -1)$ (D) $(0, 1)$

Properties of Roots

87. Let $\alpha, \beta \in \mathbb{N}$ be roots of equation $x^2 - 70x + \lambda = 0$ where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbb{N}$. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to: _____. **[30 Jan 2024 (A)]**

88. The number of pairs (a,b) of real numbers, such that whenever α is a root of the equation $x^2+ax+b=0$, α^2-2 is also a root of this equation, is: **[01 Sep 2021 (A)]**

(A) 6 (B) 8 (C) 4 (D) 2

89. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is : **[12 Jan 2019 (M)]**

(A) $2-\sqrt{3}$ (B) $-2+\sqrt{2}$ (C) $4-2\sqrt{3}$ (D) $4-3\sqrt{2}$

90. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is : **[11 Jan 2019 (M)]**

(A) -81 (B) 100 (C) 144 (D) -300

Range/Max/Min Value

91. The minimum value of the sum of the squares of the roots of $x^2 + (3-a)x = 2a - 1$ is: **[26 Jul 2022 (E)]**

(A) 6 (B) 4 (C) 5 (D) 8

92. If $x^2 + 9y^2 - 4x + 3 = 0$, x, $y \in R$, then x and y respectively lie in the intervals: **[27 Aug 2021 (E)]**

(A) $[-\frac{1}{3}, \frac{1}{3}]$ and $[-\frac{1}{3}, \frac{1}{3}]$ (B) $[1,3]$ and $[-\frac{1}{3}, \frac{1}{3}]$
(C) $[-\frac{1}{3}, \frac{1}{3}]$ and $[1,3]$ (D) $[1,3]$ and $[1,3]$

93. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3-\lambda)x + 2 = \lambda$ has the least value is: **[10 Jan 2019 (E)]**

(A) 2 (B) $4/9$ (C) $15/8$ (D) 1

Relation between Roots and Coefficients

94. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$ is: **[09 Apr 2024 (E)]**

(A) $x^2 - 190x + 9466 = 0$ (B) $x^2 - 180x + 9506 = 0$
(C) $x^2 - 195x + 9506 = 0$ (D) $x^2 - 195x + 9466 = 0$

95. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to: **[24 Jun 2022 (E)]**

(A) 46 (B) 36 (C) 24 (D) 18

96. If α and β are the roots of the equation $2x(2x+1) = 1$, then β is equal to: **[06 Sep 2020 (E)]**

(A) $2\alpha(\alpha+1)$ (B) $-2\alpha(\alpha+1)$ (C) $2a(\alpha-1)$ (D) $2\alpha^2$

97. Let α and β be the roots of the equation $px^2 + qx - r = 0$ where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is: **[01 Feb 2024 (R)]**

(A) $80/9$ (B) 9 (C) $20/3$ (D) 8

98. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$ then the product of all possible values of a is _____. **[25 Jan 2023 (R)]**

99. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____. **[26 Aug 2021 (R)]**

100. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is: **[04 Sep 2020 (R)]**

(A) 3:1 (B) 9:7 (C) 5:3 (D) 33:31

101. Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals : **[22 Jan 2025 (A)]**

(A) 27 (B) 17 (C) 25 (D) 24

102. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation : **[29 Jan 2023 (A)]**

(A) $7x^2 + 245x - 250 = 0$ (B) $7x^2 - 245x + 250 = 0$
(C) $49x^2 - 245x + 250 = 0$ (D) $49x^2 + 245x + 250 = 0$

103. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$ such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____. **[25 Jul 2022 (A)]**

104. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $(\frac{\alpha^3}{\beta^5})^{\frac{1}{8}} + (\frac{\beta^3}{\alpha^5})^{\frac{1}{8}}$ is: **[06 Sep 2020 (M)]**

(A) 2 (B) 3 (C) 1 (D) 4

105. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to: **[05 Sep 2020 (M)]**

(A) $27/32$ (B) $1/24$ (C) $3/8$ (D) $27/16$

106. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $(\alpha - \frac{1}{\alpha})(\beta - \frac{1}{\beta})(\alpha + \frac{1}{\beta})(\beta + \frac{1}{\alpha})$ is equal to : **[03 Sep 2020 (M)]**

(A) $\frac{9}{4}(9 + q^2)$ (B) $\frac{9}{4}(9 - q^2)$ (C) $\frac{9}{4}(9 + p^2)$ (D) $\frac{9}{4}(9 - p^2)$

107. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$ then a value of λ is: **[07 Jan 2020 (M)]**

(A) $10\sqrt{2}$ (B) 10 (C) 5 (D) $5\sqrt{2}$

108. If α and β are the roots of the quadratic equation $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in (0, \frac{\pi}{2})$ then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12}) \cdot (\alpha - \beta)^{24}}$ is equal to : **[10 Apr 2019 (M)]**

(A) $\frac{2^6}{(\sin\theta + 8)^{12}}$ (B) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (C) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$ (D) $\frac{2^{12}}{(\sin\theta - 8)^6}$

109. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: **[09 Apr 2019 (M)]**

(A) $4\sqrt{3}$ (B) $10\sqrt{5}$ (C) $8\sqrt{3}$ (D) $8\sqrt{5}$

110. Let α and β be the roots of the quadratic equation $x^2\sin\theta - x(\sin\theta\cos\theta+1)+\cos\theta = 0 (0<\theta<45^\circ)$, and $\alpha<\beta$. Then $\sum_{n=0}^{\infty}(\alpha^n + \frac{(-1)^n}{\beta^n})$ is equal to: **[11 Jan 2019 (M)]**

(A) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (B) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$
(C) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (D) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Sign of Quadratic Expression

111. The integer k, for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R is: **[25 Feb 2021 (E)]**

(A) 4 (B) 2 (C) 3 (D) 0

112. The number of integral values of m for which the quadratic expression $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is: **[12 Jan 2019 (E)]**

(A) 7 (B) 3 (C) 6 (D) 8

113. Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0$, $\forall x \in \mathbb{R}$. Then, the number of elements in S is: **[31 Jan 2024 (R)]**

(A) 1 (B) 0 (C) ∞ (D) 3

114. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is: **[20 Jul 2021 (R)]**

(A) $7/36$ (B) $2/9$ (C) $1/6$ (D) $1/4$

Theory of Equations

115. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is _____. **[26 Jun 2022 (R)]**

116. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2) = \frac{125}{16}m$. Then the value of m is _____. **[06 Apr 2024 (A)]**

117. Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$ then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to: **[08 Apr 2023 (A)]**

(A) $155/8$ (B) 21 (C) $169/8$ (D) 19

118. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq ... \geq |\alpha_7|$. Then, $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _____. **[29 Jan 2023 (A)]**

Transformation of Roots

119. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$ then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$ is: **[04 Apr 2024 (M)]**

(A) $2x^2 + 11x + 12 = 0$ (B) $x^2 + 8x + 12 = 0$
(C) $4x^2 + 14x + 12 = 0$ (D) $x^2 + 10x + 16 = 0$

Miscellaneous

120. If three distinct numbers a, b, c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct? **[08 Apr 2019 (E)]**

(A) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.
(B) d, e, f are in A.P.
(C) d, e, f are in G.P.
(D) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

121. For $0<c<b<a$, let $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements
(I) If $\alpha \in (-1,0)$ then b cannot be the geometric mean of a and c.
(II) If $\alpha \in (0,1)$ then b may be the geometric mean of a and c. **[31 Jan 2024 (A)]**

(A) Both (I) and (II) are true
(B) Neither (I) nor (II) is true
(C) Only (II) is true
(D) Only (I) is true

122. Let a, b, c be the length of three sides of a triangle satisfying the condition $(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$. If the set of all possible values of x is in the interval (α,β), then $12(\alpha^2+\beta^2)$ is equal to _____. **[31 Jan 2024 (A)]**

123. Let m and n be the numbers of real roots of the quadratic equations $x^2-12x+[x]+31=0$ and $x^2-5|x+2|-4=0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then m^2+mn+n^2 is equal to _____. **[08 Apr 2023 (A)]**

124. The sum of all real values of x for which $\frac{3x^2-9x+17}{x^2+3x+10}=\frac{5x^2-7x+19}{3x^2+5x+12}$ is equal to _____. **[28 Jul 2022 (A)]**

125. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0)=p$, $p\neq 0$, and $f(1)=\frac{1}{3}$. If the equations $f(x)=0$ and $fofofof(x)=0$ have a common real root, then $f(-3)$ is equal to _____. **[25 Jul 2022 (A)]**

126. Let $\alpha = max_{x\in R}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$ and $\beta = min_{x\in R}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$. If $8x^2+bx+c=0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$; then the value of $c-b$ is equal to : **[27 Jul 2021 (A)]**

(A) 42 (B) 47 (C) 43 (D) 50

127. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x, $[x]^2+2[x+2]-7=0$ has: **[04 Sep 2020 (M)]**

(A) exactly two solutions
(B) exactly four integral solutions
(C) no integral solution
(D) infinitely many solutions

128. If α and β are the roots of the equation $375x^2-25x-2=0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to: **[12 Apr 2019 (M)]**

(A) $1/12$ (B) $21/346$ (C) $7/116$ (D) $29/358$

Answer Key - Quadratic Equation

1	(4)	**2**	(4)	**3**	(B)	**4**	(14)	**5**	(13)	**6**	(A)	**7**	(A)	**8**	(D)
9	(13)	**10**	(B)	**11**	(B)	**12**	(272)	**13**	(98)	**14**	(A)	**15**	(A)	**16**	(3)
17	(A)	**18**	(A)	**19**	(B)	**20**	(46)	**21**	(A)	**22**	(B)	**23**	(C)	**24**	(C)
25	(D)	**26**	(B)	**27**	(25)	**28**	(45)	**29**	(D)	**30**	(C)	**31**	(C)	**32**	(A)
33	(7)	**34**	(5)	**35**	(B)	**36**	(1)	**37**	(A)	**38**	(B)	**39**	(D)	**40**	(B)
41	(2)	**42**	(C)	**43**	(2)	**44**	(2)	**45**	(3)	**46**	(5)	**47**	(B)	**48**	(A)
49	(6)	**50**	(66)	**51**	(A)	**52**	(8)	**53**	(C)	**54**	(C)	**55**	(B)	**56**	(2139)
57	(B)	**58**	(15)	**59**	(117)	**60**	(324)	**61**	(C)	**62**	(D)	**63**	(B)	**64**	(4)
65	(C)	**66**	(A)	**67**	(51)	**68**	(C)	**69**	(C)	**70**	(C)	**71**	(D)	**72**	(5)
73	(A)	**74**	(B)	**75**	(16)	**76**	(4)	**77**	(1)	**78**	(A)	**79**	(2)	**80**	(B)
81	(B)	**82**	(2)	**83**	(2)	**84**	(A)	**85**	(A)	**86**	(A)	**87**	(60)	**88**	(A)
89	(D)	**90**	(D)	**91**	(A)	**92**	(B)	**93**	(A)	**94**	(C)	**95**	(C)	**96**	(B)
97	(A)	**98**	(45)	**99**	(18)	**100**	(B)	**101**	(25)	**102**	(C)	**103**	(38)	**104**	(A)
105	(D)	**106**	(D)	**107**	(B)	**108**	(C)	**109**	(D)	**110**	(C)	**111**	(C)	**112**	(A)
113	(B)	**114**	(B)	**115**	(36)	**116**	(221)	**117**	(D)	**118**	(9)	**119**	(B)	**120**	(A)
121	(A)	**122**	(36)	**123**	(9)	**124**	(6)	**125**	(25)	**126**	(A)	**127**	(D)	**128**	(A)