Basic Mathematics: Section 1

Number System

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The Building Blocks: N, W, Z

Basic Number Sets

- **Natural Numbers (N):** Counting numbers. $N = \{1, 2, 3, \dots\}$
- **Whole Numbers (W):** Natural numbers including zero. $W = \{0, 1, 2, 3, \dots\}$
- **Integers (Z or I):** $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
  - Positive Integers $Z^+ = \{1, 2, 3, \dots\}$
  - Negative Integers $Z^- = \{\dots, -3, -2, -1\}$
  - Non-negative Integers: $\{0, 1, 2, 3, \dots\}$
  - Non-positive Integers: $\{\dots, -3, -2, -1, 0\}$
  - Even Integers: $\{\dots, -4, -2, 0, 2, 4, \dots\}$
  - Odd Integers: $\{\dots, -3, -1, 1, 3, \dots\}$

A Note on Zero

Zero (0) is **neither positive nor negative**. It is considered an **even** number.

Rational vs. Irrational Numbers

Rational Numbers (Q)

Def 1: Numbers that can be written in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

Def 2: Decimals that are **terminating** or **non-terminating but repeating**.

Examples

- $\frac{7}{4}$
- -5
- 0.125
- $1.333\ldots$

Irrational Numbers (Q')

Def 1: Numbers that **cannot** be written in the form $\frac{p}{q}$.

Def 2: Decimals that are **non-terminating** and **non-repeating**.

Examples

- $\sqrt{2}$
- π
- e
- $1.010010001\ldots$

Important Values to Remember

Key Constants

Approximate values for common irrational numbers:

- $\pi \approx$
- $e \approx$
- $\sqrt{2} \approx$
- $\sqrt{3} \approx$

Table of Square Roots

Number	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{8}$	$\sqrt{9}$
Square Root	1.414	1.732	2	2.236	2.449	2.646	2.828	3

Rational & Rational (R, R)

Let R_1 and R_2 be rational numbers.

- $R_1 + R_2 =$
- $R_1 - R_2 =$
- $R_1 \times R_2 =$
- $R_1 \div R_2 =$

Irrational Irrational (Ir, Ir)

Let Ir_1 and Ir_2 be irrational numbers.

- $Ir_1 + Ir_2 =$
- $Ir_1 - Ir_2 =$
- $Ir_1 \times Ir_2 =$
- $Ir_1 \div Ir_2 =$

Rational & Irrational (R, Ir)

Let R be rational and Ir be irrational.

- $R + Ir =$
- $R - Ir =$
- $R \times Ir =$
- $R \div Ir =$
- $Ir \div R =$

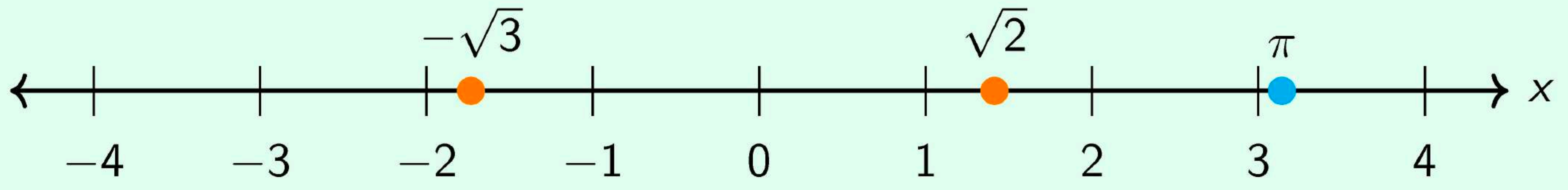
Real Numbers (R)

Definition

The set of **Real Numbers (R)** is the union of all rational and irrational numbers.

$$R = Q \cup Q'$$

These are all the numbers that can be placed on a number line.



Beyond the Real Line: Complex Numbers ($\mathbb{C}$)

Definition

A **Complex Number** is a number of the form:

$$z = a + bi$$

where:

- a and b are real numbers.
- i is the imaginary unit, defined as $i = \sqrt{-1}$.
- a is the **Real Part**.
- b is the **Imaginary Part**.

This allows us to find solutions for equations like $x^2 + 1 = 0$.

The Complete Number System

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

Check Your Understanding

True or False?

1. Every integer is a rational number. (   )
2. Every rational number is an integer. (   )
3. The product of two distinct irrational numbers is always irrational. (   )
4. Every real number is a complex number. (   )
5. The sum of a rational number and an irrational number is always irrational. (   )

Even and Odd Numbers

Definitions

- **Even Numbers:** $\{0, 2, 4, 6, \ldots\}$
  Integers divisible by 2. General form: $2n$.
- **Odd Numbers:** $\{1, 3, 5, 7, \ldots\}$
  Integers not divisible by 2. General form: $2n + 1$ or $2n - 1$.

Operations

- Even $+$ Even $=$
- Odd $+$ Odd $=$
- Even $+$ Odd $=$
- Even $\times$ Even $=$
- Odd $\times$ Odd $=$
- Even $\times$ Odd $=$

Prime and Composite Numbers

Definitions

Applies to natural numbers greater than 1.

- **Prime Number:** Has exactly two factors: 1 and itself.
  Examples: 2, 3, 5, 7, 11, ...

- **Composite Number:** Has more than two factors.
  Examples: 4, 6, 8, 9, 10, ...

Note:

- The number **1** is neither prime nor composite.
- The number **2** is the only even prime number.
- The number **4** is the smallest composite number.
- All prime numbers (except 2 and 3) can be expressed in the form $6n \pm 1$.

Special Prime Relationships

Co-prime Numbers / Relatively Prime

Two natural numbers are co-prime (or relatively prime) if their HCF 1.

Examples

(3, 4), (9, 28), (5, 7), (1, 3)

Twin Prime Numbers

A pair of prime numbers that have a difference of 2.

Examples

(3, 5), (5, 7), (11, 13), (17, 19)

Note:

- Two distinct prime numbers are always co-prime.
- Two consecutive natural numbers are always co-prime.

Point to Remember: 1

Rule 1: Cancelling in Division

When cancelling a common factor in DIVISION, you must add a condition that the factor is not zero.

Example

Find the number of real values of x satisfying:

$$x^2 - \frac{1}{x+3} = 9 - \frac{x-3}{x^2-9}$$

(A) 0 (B) 1 (C) 2 (D) 3

Rule 2: Cancelling in Equations (Equality)

When cancelling a common factor from both sides of an equation, setting that factor to zero provides a potential solution.

$$A \cdot B = A \cdot C \implies A = 0 \text{ or } B = C$$

Example

Solve for x in the equation $x^2 = 5x$

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Squares

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

Cubes

$2^3 = 8$	$9^3 = 729$
$3^3 = 27$	$10^3 = 1000$
$4^3 = 64$	$11^3 = 1331$
$5^3 = 125$	$12^3 = 1728$
$6^3 = 216$	$13^3 = 2197$
$7^3 = 343$	$14^3 = 2744$
$8^3 = 512$	$15^3 = 3375$

Divisibility Rules

Divisible by...	**Rule**
2	The last digit is an even number (0, 2, 4, 6, 8).
3	The sum of its digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is divisible by both 2 and 3.
8	The number formed by the last three digits is divisible by 8.
9	The sum of its digits is divisible by 9.

Divisibility Rules

10	The last digit is 0.
11	The difference between the sum of digits at odd places and the sum of digits at even places is 0 or is divisible by 11.
12	The number is divisible by 3 & 4
14	The number is divisible by 2 & 7
15	The number is divisible by 3 & 5