# Basic Mathematics: Section 4

# Polynomials

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# Definition of a Polynomial

An algebraic expression of the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

#### where:

- $ightharpoonup a_n, a_{n-1}, \ldots, a_0$  are constants called **coefficients**.
- ► The powers of *x* must be **whole numbers**
- ightharpoonup  $a_n \neq 0$ .
- $ightharpoonup a_n x^n$  is the **leading term**.
- ightharpoonup a<sub>n</sub> is the **leading coefficient**.
- $\triangleright$  *n* (the highest power) is the **degree** of the polynomial.

# Division of Polynomials

If P(x) (dividend) and d(x) (divisor) are two polynomials such that the degree of d(x) is less than or equal to the degree of P(x), and  $d(x) \neq 0$ , then there exist unique polynomials Q(x) (quotient) and R(x) (remainder) such that:

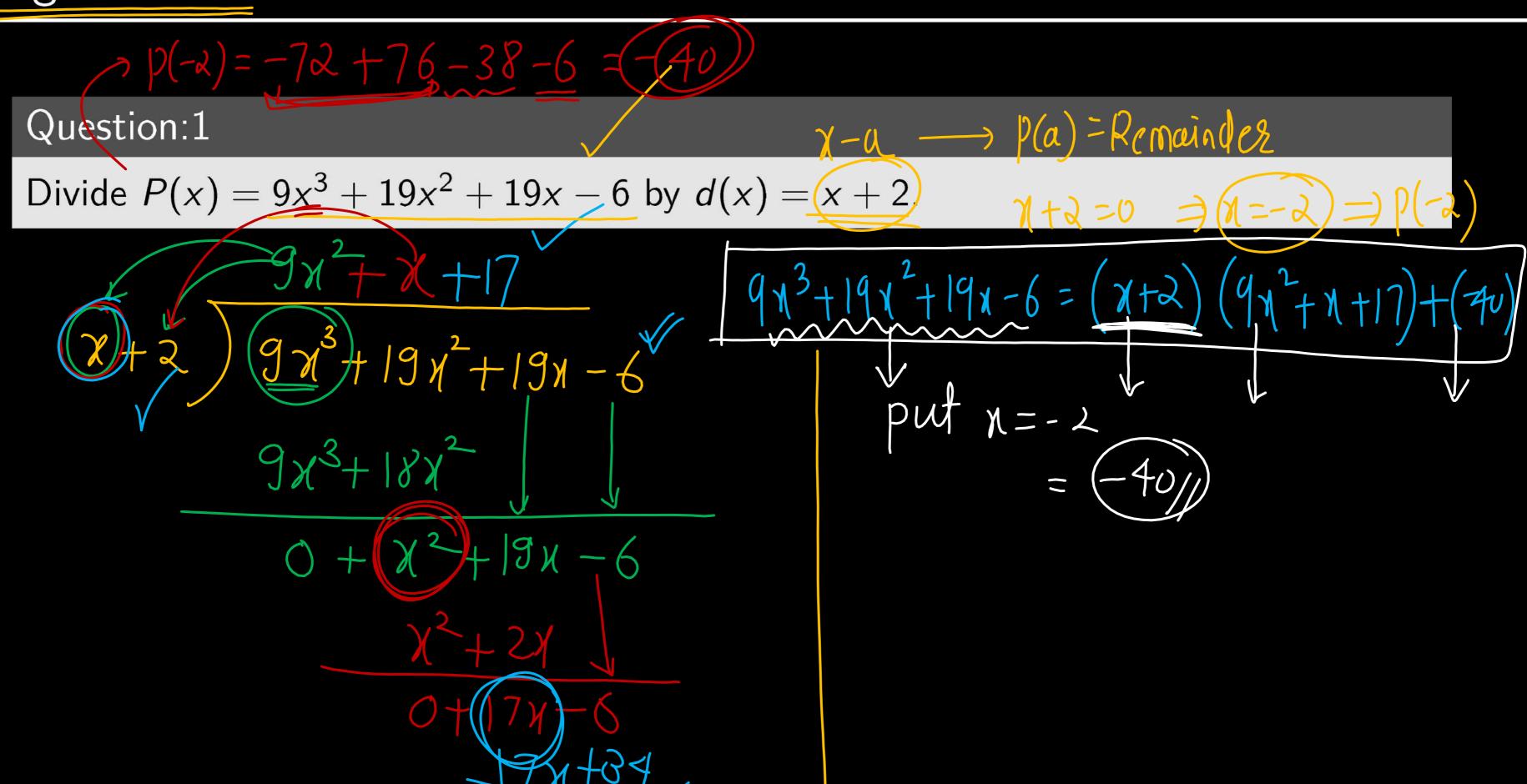
$$P(x) = d(x) \times Q(x) + R(x)$$

where the degree of the remainder R(x) is strictly less than the degree of the divisor d(x),

Analogy with numbers:  $49 \div 6 \implies 49 = 6 \times 8 + 1$ 

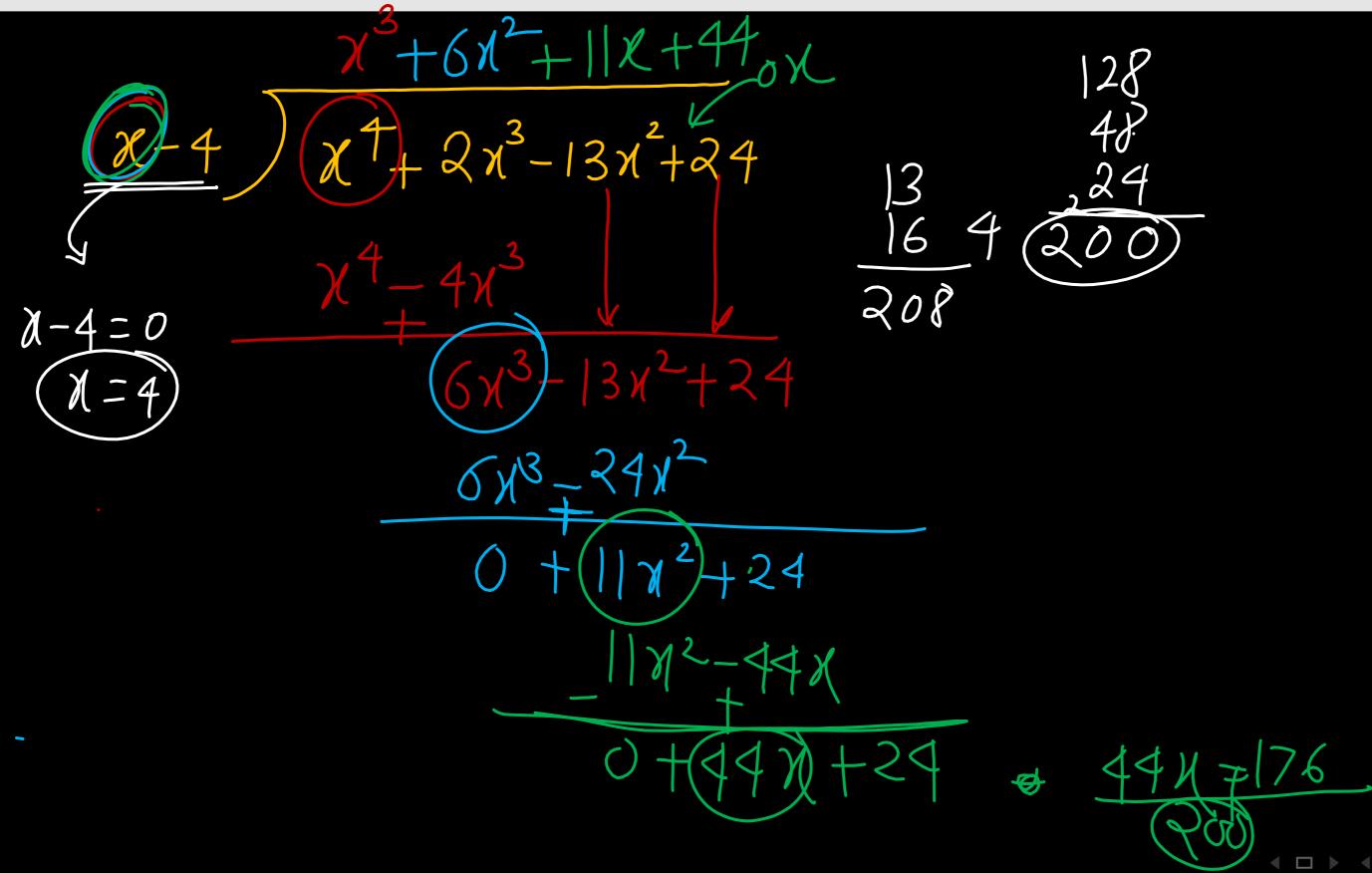
 $Dividend = Divisor \times Quotient + Remainder$ 

# Long Division Method



Question:2 
$$P(4) = 256 + 128 - 298 + 24 = 1$$

Divide the polynomial  $x^4 + 2x^3 - 13x^2 + 24$  by the linear factor (x - 4).



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If 
$$x = 2 - \sqrt{3}$$
, then find the value of  $x^3 - x^2 - 11x + 4$ 

$$\chi - 2 = -\sqrt{3}$$

Square 
$$\chi^2 - 4\chi + 4 = 3$$

$$\sqrt{2-4\pi+1}=0$$

$$(2-\sqrt{3})^3 - (2-\sqrt{3})^2 - 11(2-\sqrt{3}) + 4$$

$$(\chi^3 - \chi^2 - 11\chi + 4) = (\chi^2 - 4\chi + 1)(\chi + 3) +$$

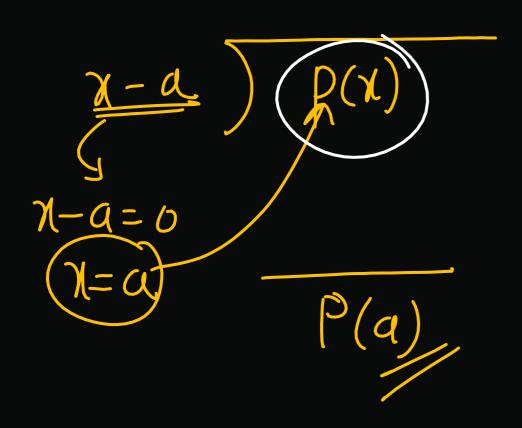
Find the quotient and remainder when the polynomial  $2x^3 + 9x^2 + 10x + 3$  is divided by  $x^2 + 1$ .

$$\begin{array}{c}
(2x+9) \\
(3x+9) \\
(4x+10x+3) \\
(4$$

# Remainder Theorem

# The Remainder Theorem

If P(x) is divided by (x - a), then remainder is P(a).



Find the remainder when  $P(x) = x^3 - 7x^2 + 6x + 9$  is divided by (x - 6).

$$\pi - 6 = 0$$

$$\pi = 6$$

$$\frac{36}{7} = 216 - 252 + 36 + 9$$

$$= 252 + 9 - 282$$

$$= 9$$

Find the remainder when  $P(x) = x^3 - 10x^2 + 6x + 10$  is divided by  $(x + \frac{1}{2})$ .

Remainder = 
$$P\left(-\frac{1}{2}\right)$$

$$P(\chi) \longrightarrow (\chi - \alpha) \longrightarrow P(\alpha)$$

$$\frac{1}{2} + \frac{1}{2} = 0$$

$$\frac{1}{2} = 0$$

# Factor Theorem

# The Factor Theorem

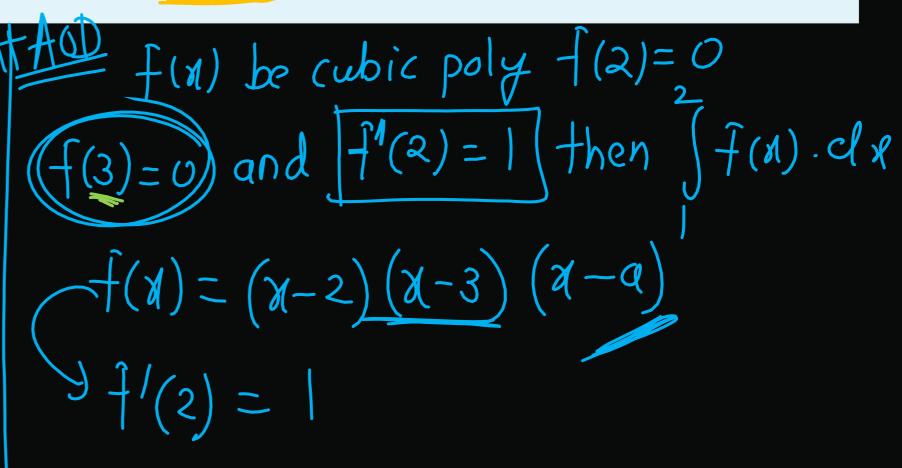
$$f(x) = (x + a)(x - a)(x - b) + 3$$

Let P(x) be a polynomial and let a be any real number.

Let 
$$P(x)$$
 be a polynomial and let  $a$  be any real number.  $f(-\lambda) = (3)$ 

Conversely, if (x - a) is a factor of P(x), then P(a) = 0.

eg 
$$P(n) = \chi^2 - 5\chi + 6 = (\chi - 2)(\chi - 3)$$
  
 $P(1) = 1^2 - 5 + 6 = 2$   
 $P(3) = 4 - 10 + 6 = 0$   
 $P(3) = 8 - 15 + 6 = 0$ 



Check whether (x-1) is a factor of  $F(x) = 2\sqrt{2}x^3 + 5\sqrt{2}x^2 - 7\sqrt{2}$ .

$$7\bar{\lambda} x^{3} + 5\bar{\lambda} x^{2} - 7\bar{\lambda} z = (x - 1) (quad)$$

$$F(1) = 2\sqrt{2} + 5\sqrt{2} - 7\sqrt{2} z = 0$$

Show that (x - 3) is a factor of  $F(x) = x^3 - 3x^2 + 4x - 12$ .

$$F(3) = 0$$
  
 $F(3) = 27 - 27 + 12 - 12$   
 $= 0//$ 

Prove that 
$$x^{2} - 3x + 2$$
 is a factor of  $F(x) = 2x^{4} - 6x^{3} + 3x^{2} + 3x - 2$ .

| Quad | Bi-quad |

 $Q(x^{4} - 6x^{3} + 3x^{2} + 3x - 2) = (x - 1)(x - 2) (Quad)$ 
 $F(x) = 0$ 
 $F(x) = 0$ 

# Expression vs Function vs Equation

# Distinguishing the terms:

- **Polynomial Expression:**  $3x^2 5x + 6$ .
- ▶ Polynomial Function:  $f(x) = 3x^2 5x + 6$ .
- ▶ Polynomial Equation:  $3x^2 5x + 6 = 0$ .

# Zeroes vs Roots

#### Note:

- ▶ **Zeroes:** Values of x for which the value of a *polynomial function* P(x) becomes zero. That is, P(x) = 0.
- ▶ **Roots:** Values of *x* which satisfy a *polynomial equation*.
- For the function  $f(x) = x^2 5x + 6$ : (x = 2) and (x = 3) are the **zeroes** of the function f(x).
- For the equation  $x^2 5x + 6 = 0$ : x = 2 and x = 3 are the **roots** of the equation  $x^2 - 5x + 6 = 0$ .

# Graphical Meaning of Zeroes/Roots

#### Note:

Graphically, the real zeroes or roots correspond to the **x-coordinates** of the points where the graph of y = P(x) intersects or touches the **x-axis**.

**Example:** For  $f(x) = x^2 - 7x + 6 = (x - 1)(x - 6)$ 

Given that x = 1 is a zero of the polynomial  $f(x) = x^3 - 2x^2 - 5x + 6$ , find the other two zeros.

(A) 2, 3

(B) 
$$-2, 3$$

(C) 
$$2, -3$$

(D) 
$$-2, -3$$

# P(-1)=0 P(2)=0

If x = -1 and x = 2 are zeros of  $P(x) = x^4 - x^3 - 7x^2 + x + 6$ , find the remaining zeros.

(A) 
$$1, -3$$

(B) 
$$-1, 3$$

$$(x=-1) \text{ is zero} \Rightarrow (x+1) \text{ is factor}$$

$$x=2 \text{ is zero} \Rightarrow (x-2) \text{ is factor}$$

$$P(x)=x^4-x^3-7x^2+x+6=(x+1)(x-2) \text{ (quad)}$$

$$=(x^2-x-2) \text{ (quad)}$$

$$\begin{array}{c} \chi^{2} - 5 \\ \chi^{2} - \chi^{2} - \chi^{3} - 7\chi^{2} + \chi + 6 \\ \chi^{4} - \chi^{3} - 2\chi^{2} \\ - 5\chi^{2} + \chi + 6 \\ - 5\chi^{2} + 5\chi + 10 \\ - 4\chi - 4 \end{array}$$

Question:3 (y) f(n) is quad having 1 - 2 roots. f(a) = k(n-1)(1+a)

The polynomial  $F(x) = 3x^3 + ax^2 + bx + c$  has the roots 1, -2, and 3. Find the coefficients a, b, and c.

Rooks are 
$$1, -2,$$
 and  $3$ 

$$F(x) = 3x^3 + ax^2 + bx + c = 3 \left( x - 1 \right) \left( x + x \right) \left( x - 3 \right)$$

$$3x^3 + ax^2 + bx + c = 3 \left( x + x - 2 \right) \left( x - 3 \right)$$

$$3a^3 + ax^2 + bx + c = 3x^3 - 6x^2 - 15x + 18$$

$$9 = -6$$

$$6 = -15$$

$$6 = -15$$