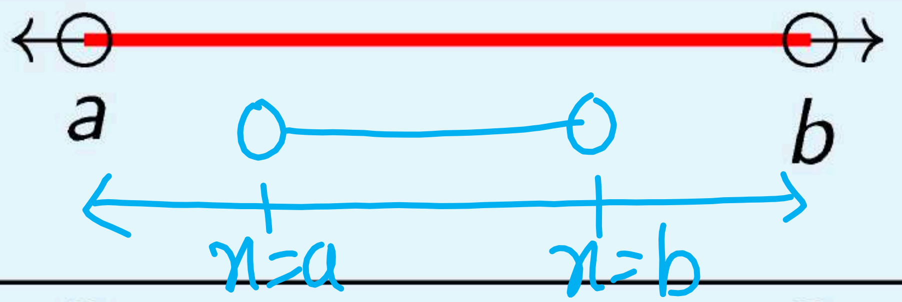
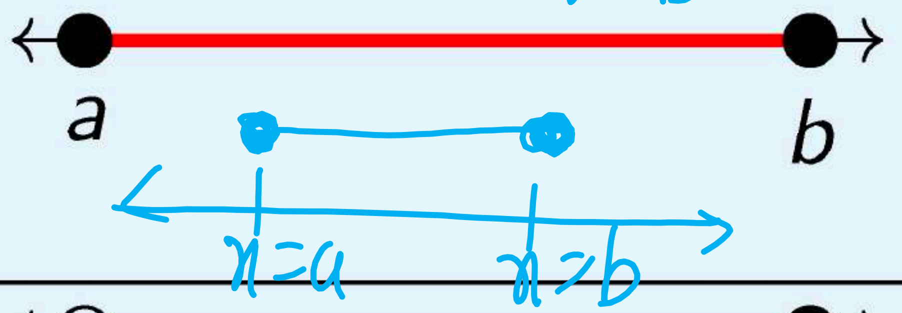
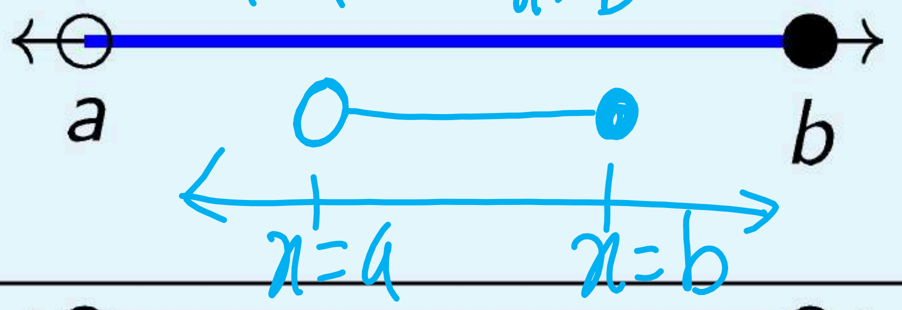
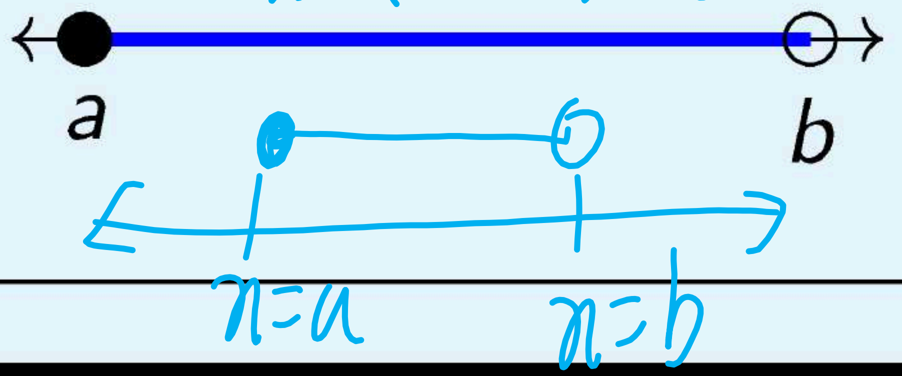
Basic Mathematics: Section 6

Inequality and Wavy Curve Method

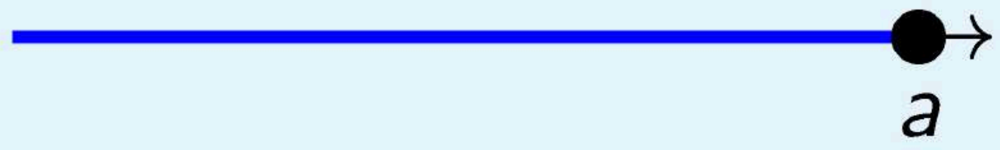
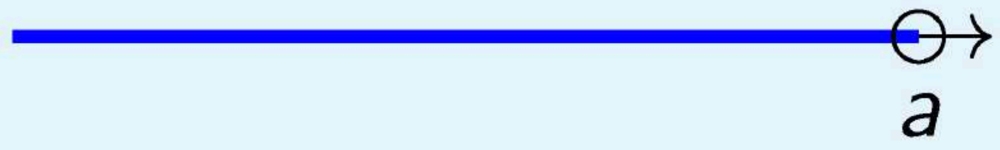
mathbyiiserite

Types of Intervals

An interval is a subset of the real numbers $\mathbb{R}$. If $a, b \in \mathbb{R}$ and $a < b$

Name	**Representation**	**Inequality**	**On Number Line**
Open Interval	$x \in (a, b)$	$a < x < b$	
Closed Interval	$x \in [a, b]$	$a \leq x \leq b$	
Open-Closed Interval	$x \in (a, b]$	$a < x \leq b$	
Closed-Open Interval	$x \in [a, b)$	$a \leq x < b$	

Infinite Intervals

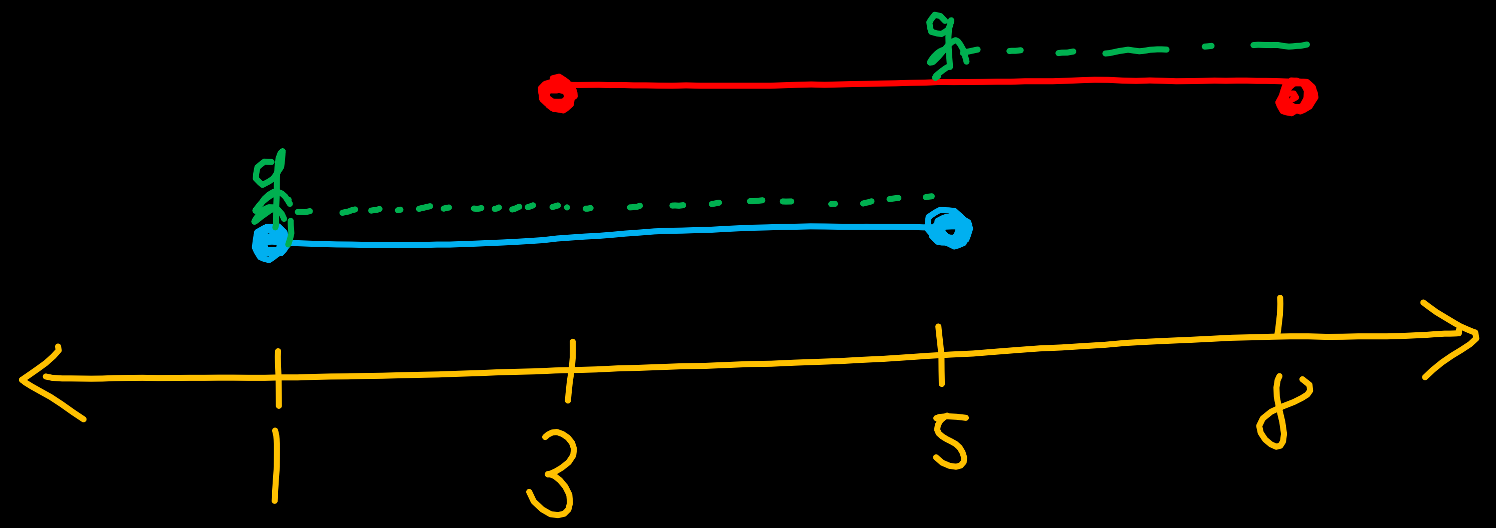
Set Notation	Interval Notation	On Number Line
$\{x \in \mathbb{R} \mid x \geq a\}$ s.t	$[a, \infty)$	
$\{x \in \mathbb{R} \mid x \leq a\}$	$(-\infty, a]$	
$\{x \in \mathbb{R} \mid x > a\}$	(a, ∞)	
$\{x \in \mathbb{R} \mid x < a\}$	$(-\infty, a)$	

We always use an open bracket,for infinity (∞) and negative infinity $(-\infty)$.

Question 1

Find the result of:

$$[1,5] \cup [3,8]$$



Ans: $[1,8]$

Question 2

Find the result of:

$$(-\infty, 0) \cup (2, \infty)$$

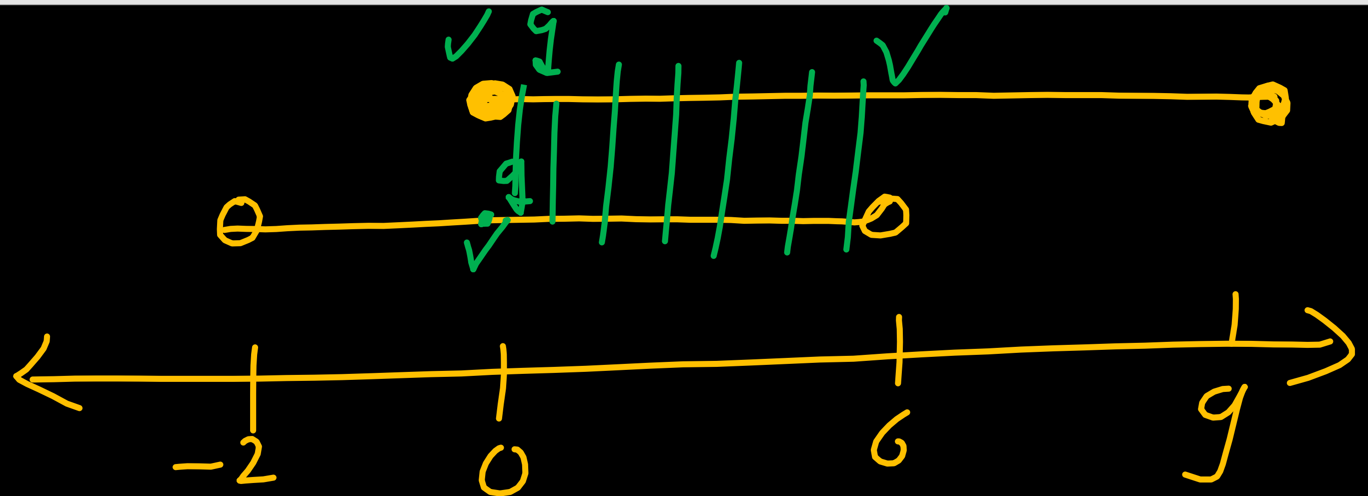
Ans: $(-\infty, 0) \cup (2, \infty)$

OR $\mathbb{R} - [0,2]$

Question 3

Find the result of:

$$(-2, 6) \cap [0, 9]$$

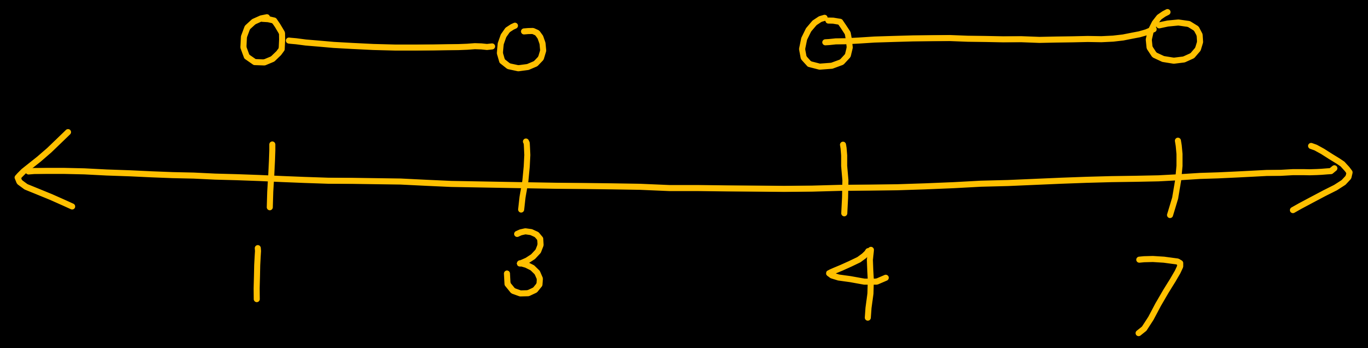


Ans: $[0, 6)$

Question 4

Find the result of:

$$(1, 3) \cap (4, 7)$$



$x \in \phi$

Question 5

Find the result of:

$$[-4,4] \cup [-1,1]$$

Question 6

Find the result of:

$$(-\infty,10] \cap [-2,5]$$

Question 7

Find the result of:

$$(-\infty, 3] \cup [3, \infty)$$

Question 8

Find the result of:

$$[-5, 2] \cap [2, 7)$$

Question 9

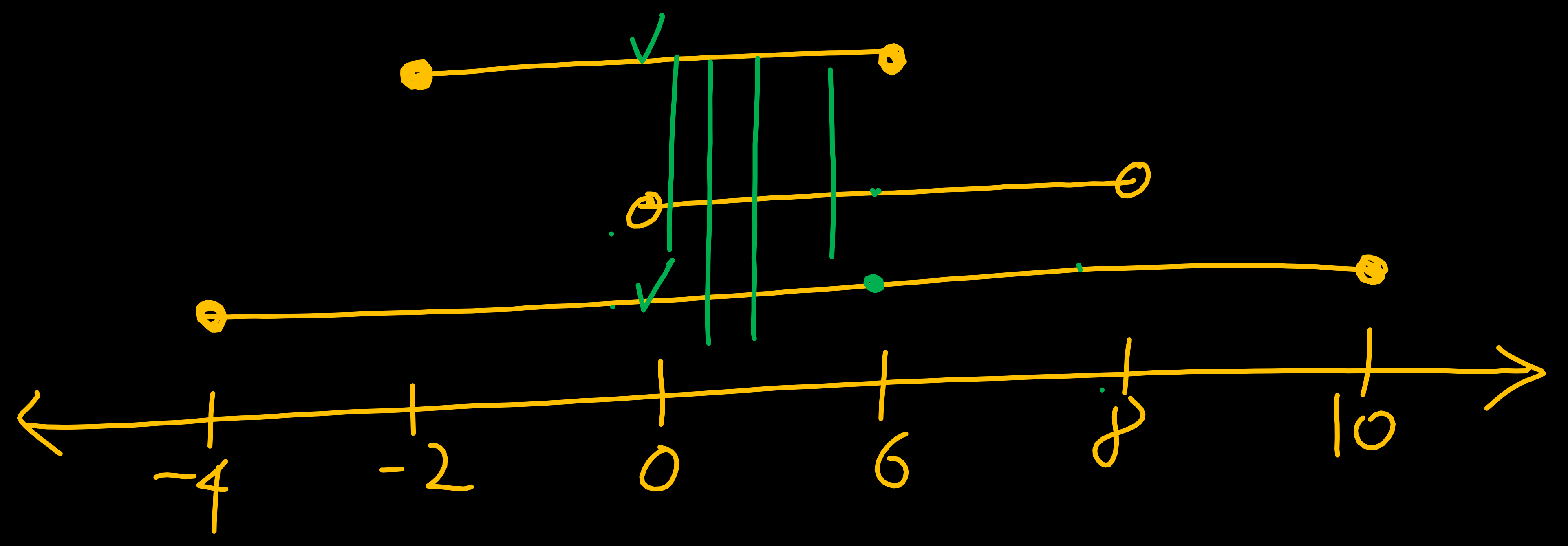
Find the result of:

$$(-5, 0) \cup [2, 6] \cup (8, 10]$$

Question 10

Find the result of:

$$[-4, 10] \cap (0, 8) \cap [-2, 6]$$



$(0, 6]$

Rules of Inequalities: Rule 1

$a > b$

$a + k > b + k \quad (k > 0 \text{ or } k < 0)$

Rule 1

If $a > b$, then $a \pm k > b \pm k$

$5 > -2$

$5 + 3 > -2 + 3$ (True)

$5 - 3 > -2 - 3$ (True)

Examples

- Given $7 > -2$:
  - Adding 5: $7 + 5 > -2 + 5 \implies 12 > 3$ (True)
  - Subtracting 10: $7 - 10 > -2 - 10 \implies -3 > -12$ (True)

- Given $-10 + \cancel{6} > -20 + \cancel{6}$:
  - We can subtract 6 from both sides to get $-10 > -20$ (True)

$-10 + \underline{6 - 6} > -20 + \underline{6 - 6}$

$-10 > -20$

Rules of Inequalities: Rule 2

Rule 2

- If $a > b$ and $k > 0$, then $ka > kb$
- If $a > b$ and $k < 0$, then $ka < kb$

$a>b \xrightarrow{k>0} ka>kb$

$a>b \xrightarrow{k<0} ka<kb$

Examples

Given the inequality $5 > 3$:

- Multiplying by a positive number (4):

$$5 \times 4 > 3 \times 4 \implies 20 > 12 \quad \text{(Sign remains)}$$

- Multiplying by a negative number (-4):

$$5 \times (-4) < 3 \times (-4) \implies -20 < -12 \quad \text{(Sign reverses)}$$

Rules of Inequalities: Rule 3

Rule 3

Never multiply both sides of an inequality by an expression whose sign is unknown.

Examples

$8(x^2+1) > 3(x^2+1)$ (True)

$8(-(x^2+1)) < 3(-(x^2+1))$ (True)

$-ve$ $\quad$ $-ve$

Given the inequality $\boxed{8 > 3}$:

- Multiply by x: $8x > 3x \implies$ Cannot be determined as sign of x is unknown. ✗

  Ye galat hai.

- Multiply by (x^2+1): Since x^2+1 is always positive, the sign is unchanged.

  $+ve$

  $$8(x^2+1) > 3(x^2+1) \quad \text{(True)}$$

- Multiply by $(x+2)$: $8(x+2) > 3(x+2) \implies$ Cannot be determined as sign of $(x+2)$ is unknown. ✗

Rules of Inequalities: Rule 4

Rule 4: Cancelling a common factor

- If $A \cdot B > A \cdot C$ and $A > 0$, then $B > C$. (Sign is unchanged)
- If $A \cdot B > A \cdot C$ and $A < 0$, then $B < C$. (Sign is reversed)

Examples

- Given $\cancel{2}(x - 3) > \cancel{2}(2x - 8)$:
  - Since 2 is positive, we can cancel it: $x - 3 > 2x - 8$

    $(x-3) > (2x-8)$

- Given $\cancel{-3}(x^2) > \cancel{-3}(6x^2)$:
  - Since -3 is negative, we cancel it and reverse the sign: $x^2 < 6x^2$

    $x^2 < 6x^2$

Rules of Inequalities: Rule 5

③ $2 > -3$

$\frac{1}{2} > \frac{-1}{3}$

Rule 5: Reciprocal Property

- Given $5 > 2$:
  - Both sides are positive. Taking the reciprocal reverses the sign:

① $5 > 2$

$0.2 = \frac{1}{5} < \frac{1}{2} = 0.5$

$$\frac{1}{5} < \frac{1}{2} \implies 0.2 < 0.5 \quad \text{(True)}$$

- Given $-3 > -7$:
  - Both sides are negative. Taking the reciprocal reverses the sign:

② $-3 > -7$

$-0.3 = \frac{-1}{3} < \frac{-1}{7} = 0.14$

$$-\frac{1}{3} < -\frac{1}{7} \quad \text{(True, since } -0.33 < -0.14\text{)}$$

① sign of $(x-3)$

$-$ $\quad$ $+$

$x=3$

② sign of $(5x-7)$

$-$ $\quad$ $+$

$x=\frac{7}{5}$

③ sign of $(2x+5)$

$-$ $\quad$ $+$

$x=\frac{-5}{2}=-2.5$

$2x-3=0$

$x=\frac{3}{2}$

④ sign of $(5-x)$

$+$ $\quad$ $-$

$x=5$

$5-x=0$

$x=5$

⑤ sign of $(4-2x)$

$4-2x=0$
$4=2x$
$x=2$

$+ \qquad\qquad -$

$x=2$

⑥ sign of $(x-2)^2$

$+ \qquad\qquad +$

$x=2$

⑦ sign of $(2x+8)^3$

$2x+8=0$
$x=-4$

$- \qquad\qquad +$

$x=-4$

⑧ sign of $(x+3)^4$

$+ \qquad\qquad +$

$x=-3$

⑨ sign of $(x+3)^5$

$- \qquad\qquad +$

$x=-3$

(10) sign of $(2-x)^2$

$2-x=0$

$x=2$

$+ \qquad +$

$x=2$

(12) sign of $(2x-6)^{100}$

$+ \qquad +$

$x=3$

(11) sign of $(2-x)^3$

$+ \qquad -$

$x=2$

(13) sign of $(4-2x)^{99}$

$4-2x=0$

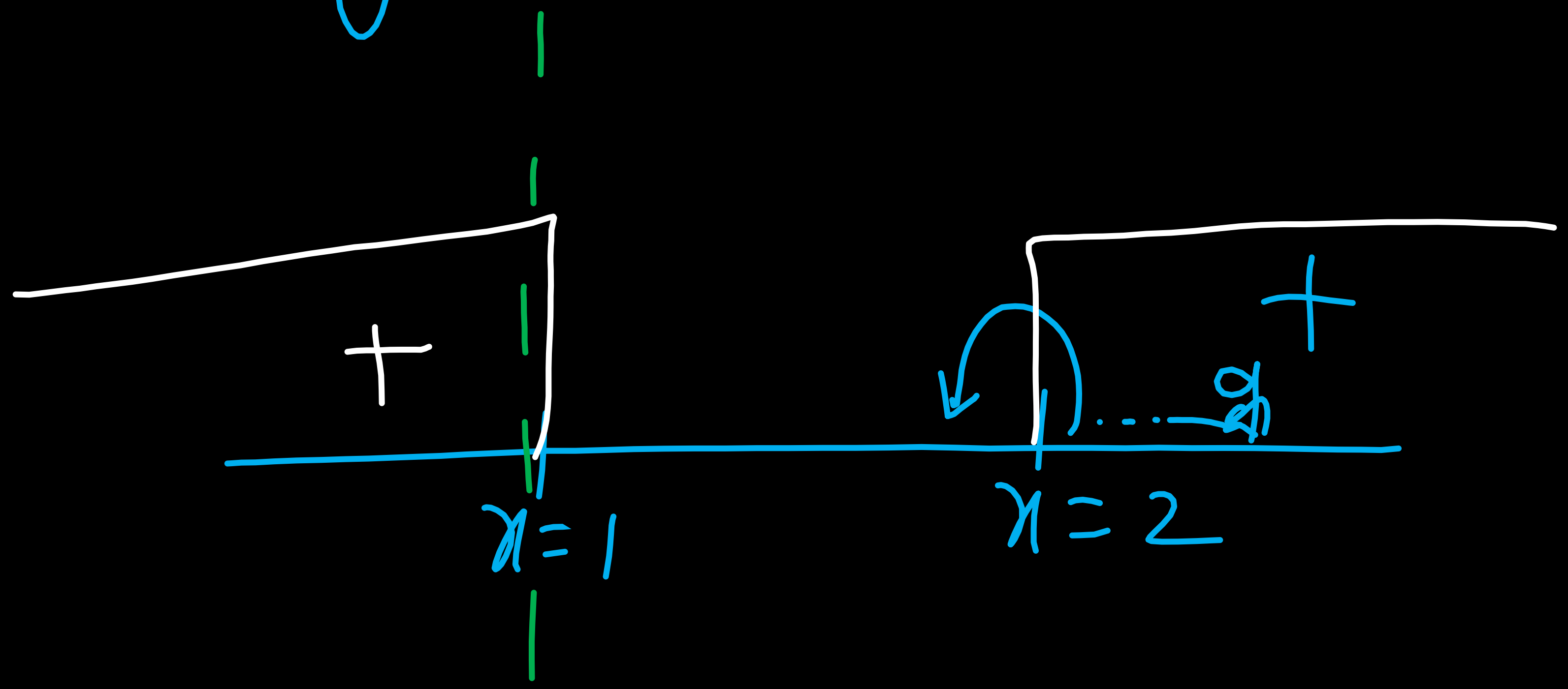
$4=2x$

$x=2$

$+ \qquad -$

$x=2$

(14) Sign of $(x-1)(x-2)$



$$(x-1)(x-2) \geq 0 \Rightarrow x \in (-\infty, 1] \cup [2, \infty)$$

$$(x-1)(x-2) < 0 \Rightarrow x \in (1, 2)$$

$$\frac{(x-1)}{(x-2)} \geq 0 \quad x \in (-\infty, 1] \cup (2, \infty)$$

Practice: Sign of Linear Expressions

1. Sign of $(x - 3)$
2. Sign of $(2x + 5)$
3. Sign of $(x + 8)$
4. Sign of $(13 - x)$

Practice: Sign of Expressions with Powers

1. Sign of $(10 - 2x)$
2. Sign of $(8 - 2x)^3$
3. Sign of $(x + 1)^2$
4. Sign of $(x - 3)(5 - x)$

Practice: Sign of Combined Expressions

1. Sign of $(x-2)(x-5)$
2. Sign of $\frac{x-1}{x+4}$
3. Sign of $(x-3)(x-6)(x-9)$
4. Sign of $(x-7)^2(x+2)$

The Wavy Curve Method

1. **Step 1:** Shift all the terms to the LHS and make the RHS equal to zero.

2. **Step 2:** Factorize the expression into linear factors $(ax + b)$
   ("x ka coefficient positive banao")

3. **Step 3:** Find all the roots (critical points) and plot them on a number line in increasing order.

   $(2x-4)^{4}$

4. **Step 4:** Identify roots that come from factors with an **even exponent**. Mark these roots on the number line with a **cross sign**
   (to remember that the sign will *not* change when crossing them.)

5. **Step 5:** Put a positive sign $(+)$ to the right of the greatest root.
   Now, move from right to left. Change the sign as you cross each root, **except** for the marked roots with cross sign where the sign remains the same.

Example 1

Solve the inequality:

$$(5 - x)(x + 3)^2(2x - 6) \geq 0$$

Example 2

Solve the inequality:

$$\frac{(x-3)(x+5)^2}{(x-2)(x+11)^3} > 0$$

Example 3

Solve the inequality:

$$\frac{(x+1)^2(2x-3)}{(7-x)^3} \geq 0$$

Example 4

Solve the inequality:

$$\frac{(x^2 - 5x + 6)(x^2 - 8x + 15)}{(x^2 - 16)} > 0$$

Example 5

Solve the inequality:

$$\frac{x^3 - 6x^2 + 11x - 6}{(x^2 + 2)(x^2 - 8x + 12)} \leq 0$$

Example 6

Find the intervals where $y \geq 0$ for the function:

$$y = \frac{(x^2 - 3)\cancel{(x^4 + x^2 + 1)}(2^x - 1)}{(3x + 1)^7 (x + 5)^2} \geq 0$$

$(x^4 + x^2 + 1) > 0$

always +ve

Example 7

Find the number of positive integral values of x satisfying the inequality:

$$\frac{(x-4)^{2017}(x+8)^{2016}(x+1)}{x^{2016}(x-1)^3(x+3)^5(x-6)(x+9)^{2018}} \leq 0$$

Example 8

Find the number of positive integral values of x satisfying the inequality:

$$\frac{(5^x - 3^x)(x-2)}{x^2+5x+2} \geq 0$$

Example 9

Solve the inequality:

$$\frac{0.5}{x - x^2 - 1} < 0$$

Example 10

Solve the inequality:

$$\boxed{x^4 - 5x^2 + 4 < 0}$$

Bi-quadratic

let $x^2 = t$

$t^2 - 5t + 4$

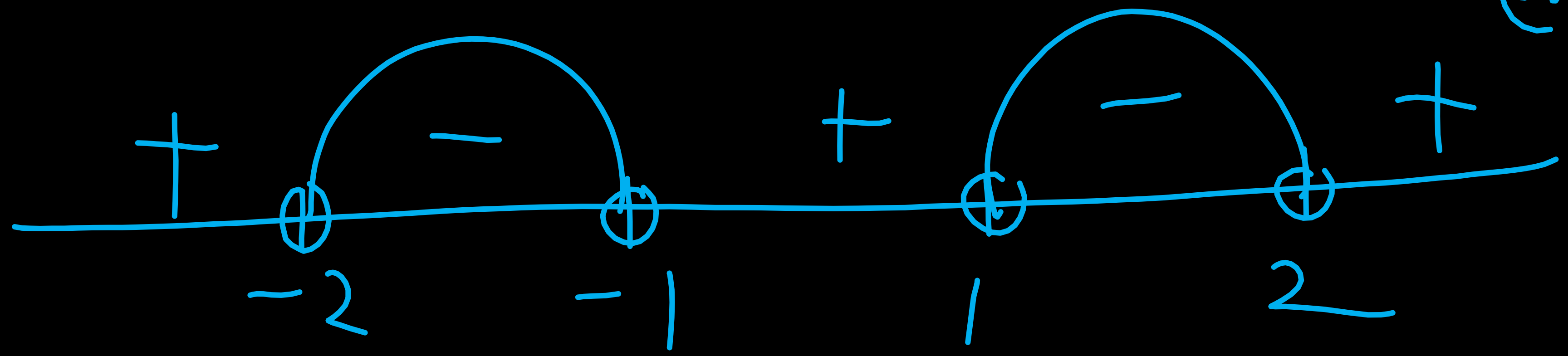
$(t-4)(t-1)$

$(x^2-4)(x^2-1)$

4: -4, -1

$(x^2-4)(x^2-1) < 0$

$(x-2)(x+2)(x-1)(x+1) < 0$



$x \in (-2,-1) \cup (1,2)$

Example 11

Solve the inequality:

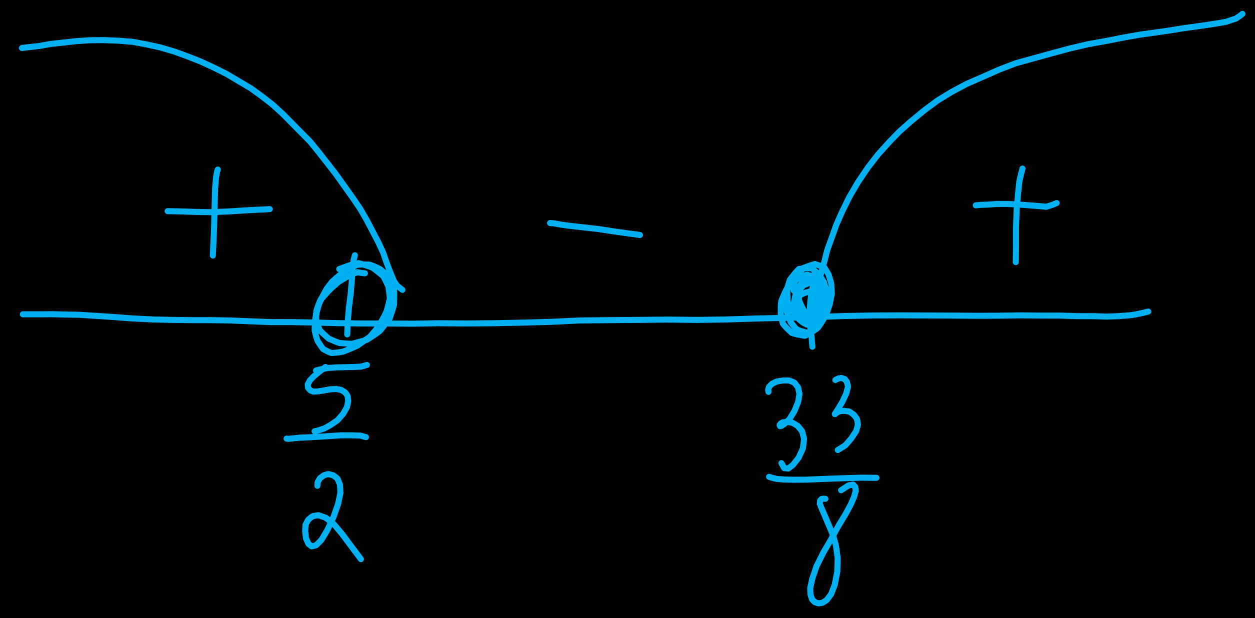
$$\frac{4x+3}{2x-5} \leq 6$$

$$\frac{4x+3}{2x-5} - 6 \leq 0$$

$$\frac{4x+3-12x+30}{2x-5} \leq 0$$

$$\frac{-8x+33}{2x-5} \leq 0$$

$$\frac{8x-33}{2x-5} \geq 0$$



$$x \in \left(-\infty, \frac{5}{2}\right) \cup \left[\frac{33}{8}, \infty\right)$$

Example 12

Solve the inequality:

$$\frac{x+1}{(x-1)^2} \leq 1$$

$$\frac{x+1}{(x-1)^2} - 1 \leq 0$$

$$\frac{(x+1) - (x^2 - 2x + 1)}{(x-1)^2} \leq 0$$

$$\frac{-x^2 + 3x}{(x-1)^2} \leq 0$$

$$\frac{x^2 - 3x}{(x-1)^2} \geq 0$$

$$\frac{x(x-3)}{(x-1)^2} \geq 0$$

$$x \in (-\infty, 0] \cup [3, \infty)$$

Example 13

Solve the inequality:

$$(\boxed{x^2+3x}+1)(\boxed{x^2+3x}-3) \geq 5$$

let $x^2+3x=t$

$(t+1)(t-3)-5 \geqslant 0$

$t^2-2t-3-5 \geq 0$

$t^2-2t-8 \geq 0$

$(t-4)(t+2) \geq 0$

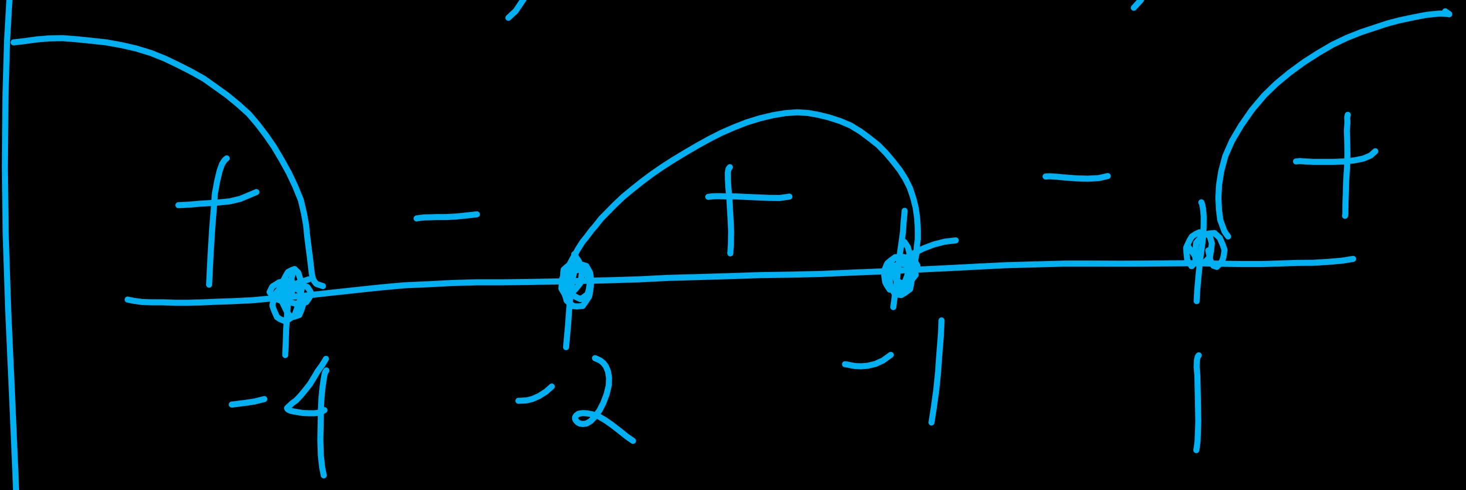
$(x^2+3x-4)(x^2+3x+2) \geq 0$

-8: -4, $+2$

-4: $+4$, -1

2: 2, 1

$(x+4)(x-1)(x+2)(x+1) \geqslant 0$



$x \in (-\infty,-4] \cup [-2,-1] \cup [1,\infty)$

Example 14

Find x for the inequality:

$$1 < \frac{x-1}{x+2} < 7$$

$x = -\frac{15}{6}$ $\quad$ $x = -2$

Ans: $x \in \left(-\infty, -\frac{15}{6}\right)$

$\boxed{1 < \square < 7}$ → Intersection

$1 < \square$ and $\square < 7$

$1 < \frac{x-1}{x+2}$ $\qquad$ $\frac{x-1}{x+2} < 7$

$\frac{x-1}{x+2} - 1 > 0$ $\qquad$ $\frac{x-1}{x+2} - 7 < 0$

$\frac{-3}{x+2} > 0$ $\qquad$ $\frac{-6x-15}{x+2} < 0$

$\frac{1}{(x+2)} < 0$

$-$ $\quad$ $+$

$x = -2$

$x \in (-\infty, -2)$

$\frac{6x+15}{x+2} > 0$

$+$ $\quad$ $-$ $\quad$ $+$

$-\frac{15}{6}$ $\quad$ -2

Example 15

Find the number of integral solutions of the inequality:

$$x^2 + 9 < (x+3)^2 < 8x + 25$$

$$x^2+9<(x+3)^2 \quad \text{and} \quad (x+3)^2<8x+25$$

$$\cancel{x^2}+6x+\cancel{9}-\cancel{x^2}-\cancel{9}>0$$

$$6x>0$$

$$x>0$$

$$x\in(0,\infty)$$

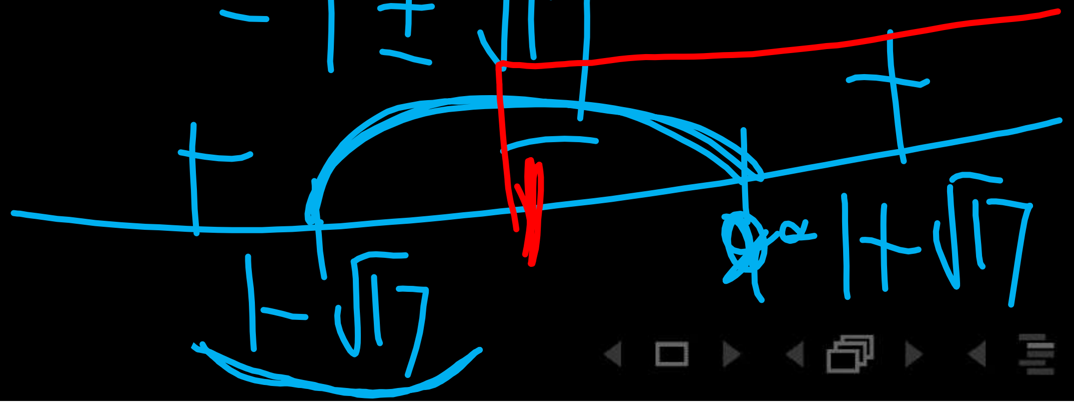
$$x^2-2x-16<0$$

-16

$$x=\frac{2\pm\sqrt{4+64}}{2}$$

$$=\frac{2\pm2\sqrt{17}}{2}$$

$$=1\pm\sqrt{17}$$



$$\text{I} \cap \text{II}$$

$$x\in(0,\,1+\sqrt{17})$$

Domain of a Function

Rules

1. $\log_{\triangle}(\square)$ $\quad$ $\square > 0$, $\triangle > 0$, $\triangle \neq 1$ $\quad \Longrightarrow \quad \square > 0 \quad \& \quad \underline{\triangle > 0} \quad \& \quad \triangle \neq \cancel{0}\,1$

2. $\dfrac{N}{\square} \quad \Longrightarrow \quad \square \neq 0$

3. $\sqrt{\square} \quad \Longrightarrow \quad \square \geq 0$

4. $\dfrac{N}{\sqrt{\square}} \quad \Longrightarrow \quad \square > 0$

5. $\sin^{-1}(\square)$ or $\cos^{-1}(\square) \quad \Longrightarrow \quad -1 \leq \square \leq 1$

6. $\tan^{-1}(\square)$ or $\cot^{-1}(\square) \quad \Longrightarrow \quad \square \in (-\infty, \infty)$

7. $\sec^{-1}(\square)$ or $\csc^{-1}(\square) \quad \Longrightarrow \quad |\square| \geq 1$ $\quad$ $\square \in (-\infty, -1] \cup [1, \infty)$

Que.1

Find the domain of the function:

$$y = \sqrt{\boxed{2x - x^2}}$$

$\sqrt{\square}\ ;\ \square \geq 0$

$$2x - x^2 \geq 0$$

$$x^2 - 2x \leq 0$$

$$x(x-2) \leq 0$$

$+ \quad - \quad +$

$x = 0 \qquad x = 2$

$$x \in [0, 2]$$

Que.2

Find the domain of the function:

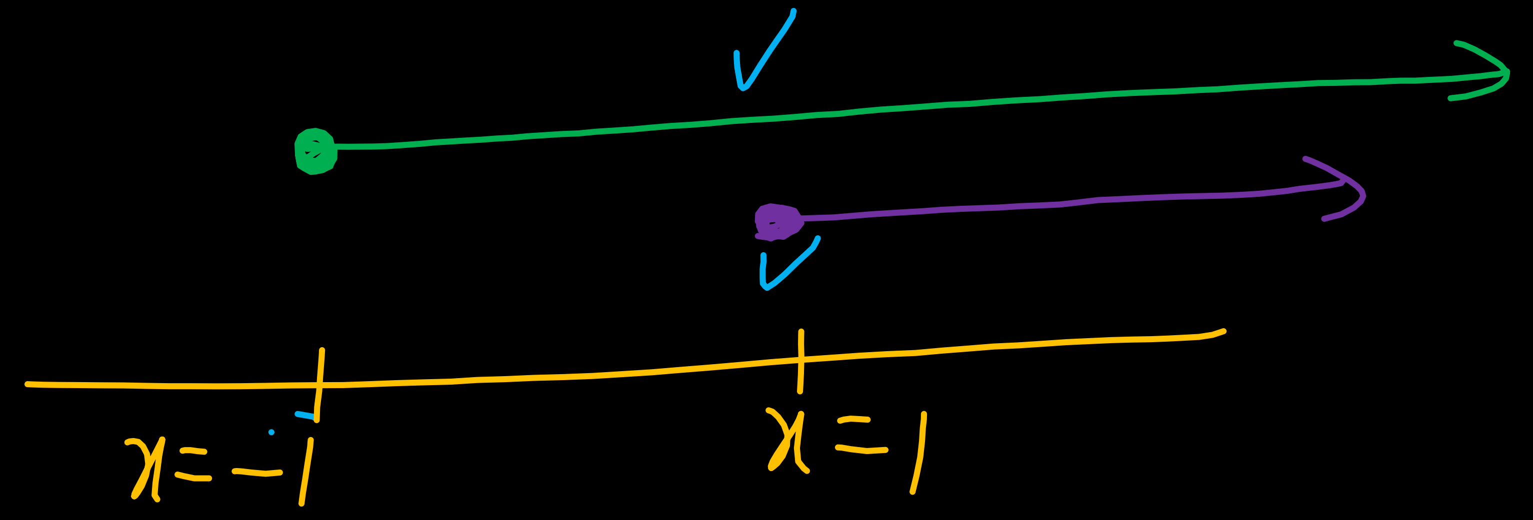
$$y = \sqrt{x-1} + \sqrt{x+1}$$

C-1 $\quad x - 1 \geq 0$

$x \geq 1$

C-2 $\quad x + 1 \geq 0$

$x \geq -1$



$x \in [1, \infty)$

Que.3

Find the domain of the function:

$$y = \frac{1}{x-1} + \sqrt{2+x}$$

$\underline{\underline{C-1}}$ $x-1 \neq 0$

$x \neq 1$

$\underline{\underline{C-2}}$ $2+x \geq 0$

$x \geq -2$

$x \in [-2, \infty) - \{1\}$

Que.4

Find the domain of the function:

$$y = \frac{1}{\sqrt{\square}} \qquad y = \frac{1}{\sqrt{2x^2 - 5x + 3}}$$

$$\square > 0$$

6

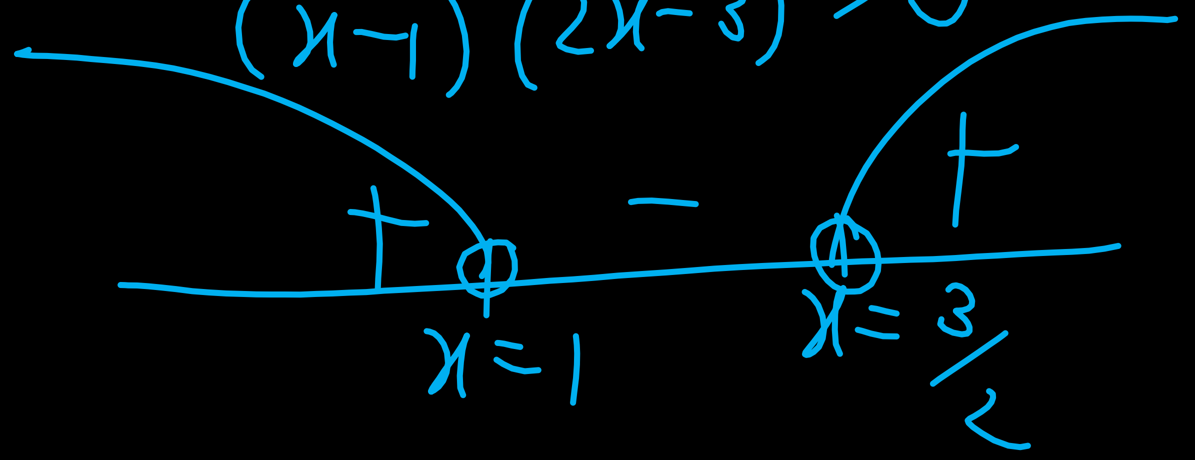
$-2 \quad -3$

$$2x^2 - 5x + 3 > 0$$

$$2x^2 - 2x - 3x + 3 > 0$$

$$2x(x-1) - 3(x-1) > 0$$

$$(x-1)(2x-3) > 0$$



$$x \in (-\infty, 1) \cup (3/2, \infty)$$

Que.5 JEE Main 2019

Find the domain of the function:

$$f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$$

C-1

$4 - x^2 \neq 0$

$x^2 \neq 4$

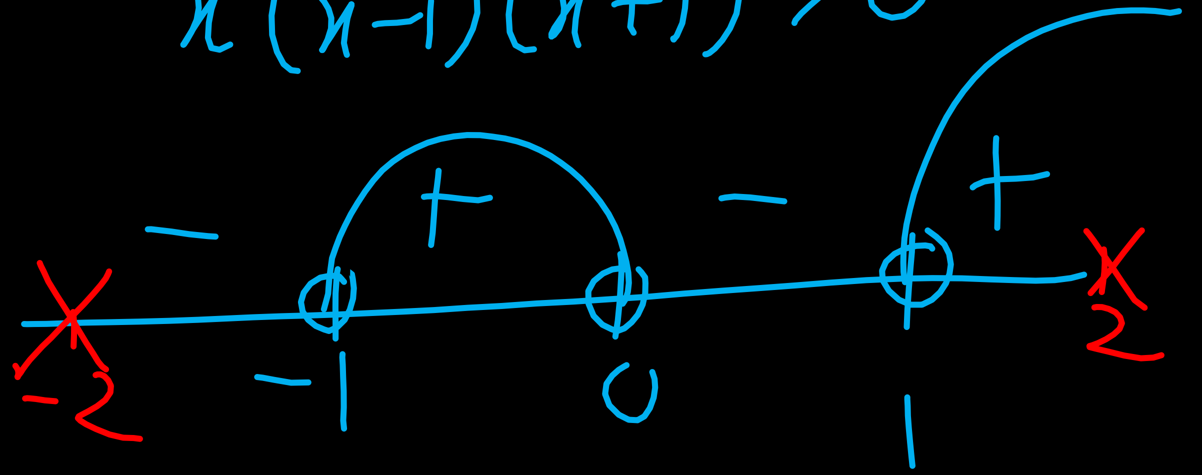
$\boxed{x \neq \pm 2}$

C-2

$x^3 - x > 0$

$x(x^2 - 1) > 0$

$x(x-1)(x+1) > 0$



$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

$① \cap ② \cap ③$

Que.6 JEE Main 2025 (03 Apr Morning) Ans:(A)

If the domain of the function

$$f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$$

$\left[-3, \frac{-5}{4}\right)$ $\quad 9 + \cancel{4}\left(\frac{-5}{\cancel{4}}\right) = 4$

is $[\alpha, \beta)$, then $\alpha^2 + 4\beta$ is equal to:

(A) 4 ✓ (B) 3 (C) 7 (D) 5

C-1 $\frac{2x-3}{5+4x} > 0$

$\frac{2x-3}{4x+5} > 0$

$+ \quad - \quad +$ — ①

$x = -\frac{5}{4}$, $x = 3/2$

C-2 $-1 \leq \frac{4+3x}{2-x} \leq 1$

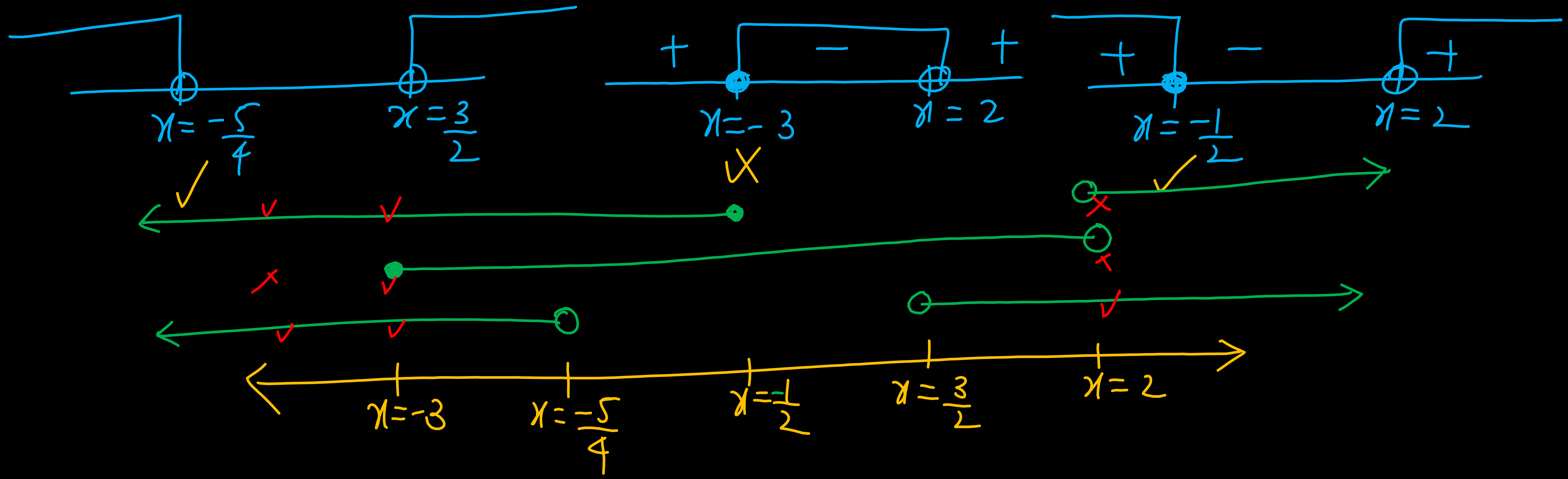
$-1 \leq \frac{4+3x}{2-x}$ and $\frac{4+3x}{2-x} \leq 1$

Intersection

$\frac{2x+6}{x-2} \leq 0$ $\quad$ $\frac{4x+2}{x-2} \geq 0$

② — $+ \quad - \quad +$ $\qquad$ $+ \quad - \quad +$ — ③

-3, $x = 2$ $\qquad$ $-1/2$, 2



$x \in \left[-3, -\frac{5}{4}\right)$

Que.7 JEE Main 2025 (29 Jan Shift 2) Ans:(C)

$\alpha^2+\beta^2+\gamma^2 = (7)^2+(11)^2+(4)^2$

If the domain of the function $\log_5(18x - x^2 - 77)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2+\beta^2+\gamma^2$ is equal to:

(A) 195 (B) 179 (C) 186 ✓ (D) 174

$$\begin{array}{r} 121 \\ 49 \\ 16 \\ \hline 186 \end{array}$$

$f(x) = \log_5(18x - x^2 - 77)$

$18x - x^2 - 77 > 0$

$x^2 - 18x + 77 < 0$

$(x-7)(x-11) < 0$

$x=7$, $x=11$

$(\alpha, \beta) \equiv (7, 11)$

$g(x) = \log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right) \rightarrow (4, \infty) \equiv (\gamma, \delta)$

C-1: $\frac{2x^2+3x-2}{x^2-3x-4} > 0$

$\frac{(x+2)(2x-1)}{(x-4)(x+1)} > 0$

-2, -1, $\frac{1}{2}$, 1, 4

C-2: $x-1>0$, $x>1$

C-3: $x-1 \neq 1$, $x \neq 2$

Que.8 JEE Main 2024 (04 Apr Shift 1) Ans:(C)

If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to:

(A) 100 (B) 95 (C) 97 (D) 98

C-1

$$-1 \le \frac{3x-22}{2x-19} \le 1$$

$$-1 \le \frac{3x-22}{2x-19} \qquad \frac{3x-22}{2x-19} \le 1$$

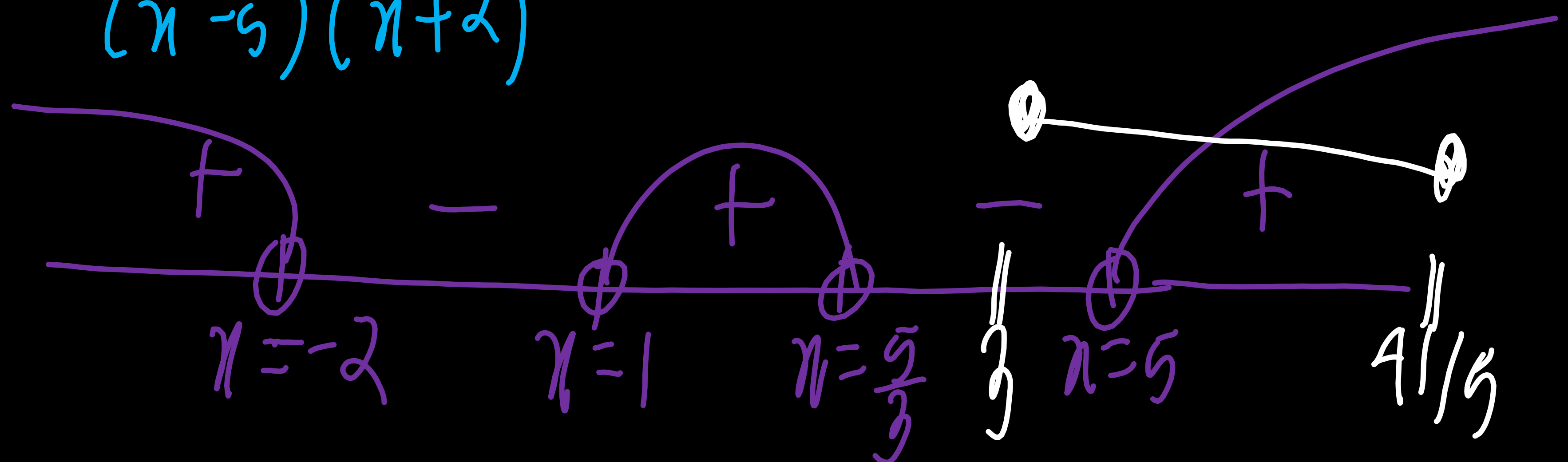
C-2

$$\frac{3x^2-8x+5}{x^2-3x-10} > 0$$

15: -5, -3 -10: -5, $+2$

$$\frac{3x^2-5x-3x+5}{(x-5)(x+2)} > 0$$

$$\frac{(3x-5)(x-1)}{(x-5)(x+2)} > 0$$



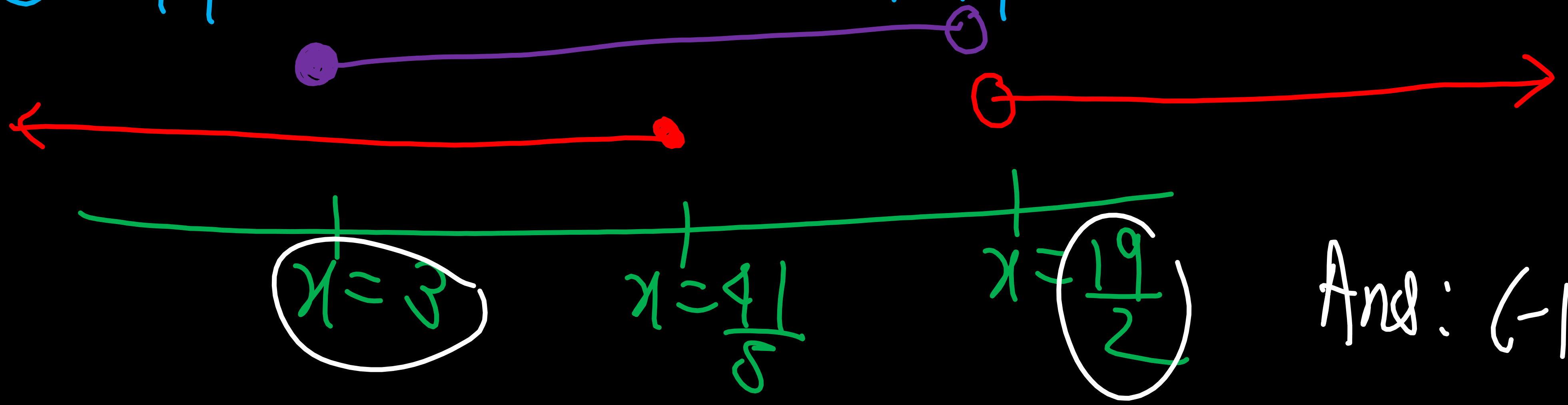
C-1

$$-1 \le \frac{3x-22}{2x-19} \le 1$$

$$-1 \le \frac{3x-22}{2x-19} \quad \text{and} \quad \frac{3x-22}{2x-19} \le 1$$

$$\frac{3x-22}{2x-19} + 1 \ge 0 \qquad\qquad \frac{3x-22}{2x-19} - 1 \le 0$$

$$\frac{5x-41}{2x-19} \ge 0 \qquad\qquad \frac{x-3}{2x-19} \le 0$$



Ans: C-1 $x \in \left[3, \frac{41}{5}\right]$

$C_1 \cap C_2$

$$x \in \left(5, \frac{41}{5}\right] \equiv (\alpha, \beta]$$

$$3\alpha + 10\beta = 3(5) + \cancel{10}^{2}\left(\frac{41}{\cancel{5}}\right)$$

$$= 15 + 82$$

$$= 97$$

Que.9 JEE Main 2024 (01 Feb Shift 2) Ans:(C)

If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{4-x^2} + \log_{10}(x^2+2x-15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to:

(A) 140 (B) 175 (C) 150 (D) 125

$\alpha^2+\beta^3$

$(-5)^2+(5)^3$

$25+125$

150

C-1 $x^2-25 \geq 0 \longrightarrow (x-5)(x+5) \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

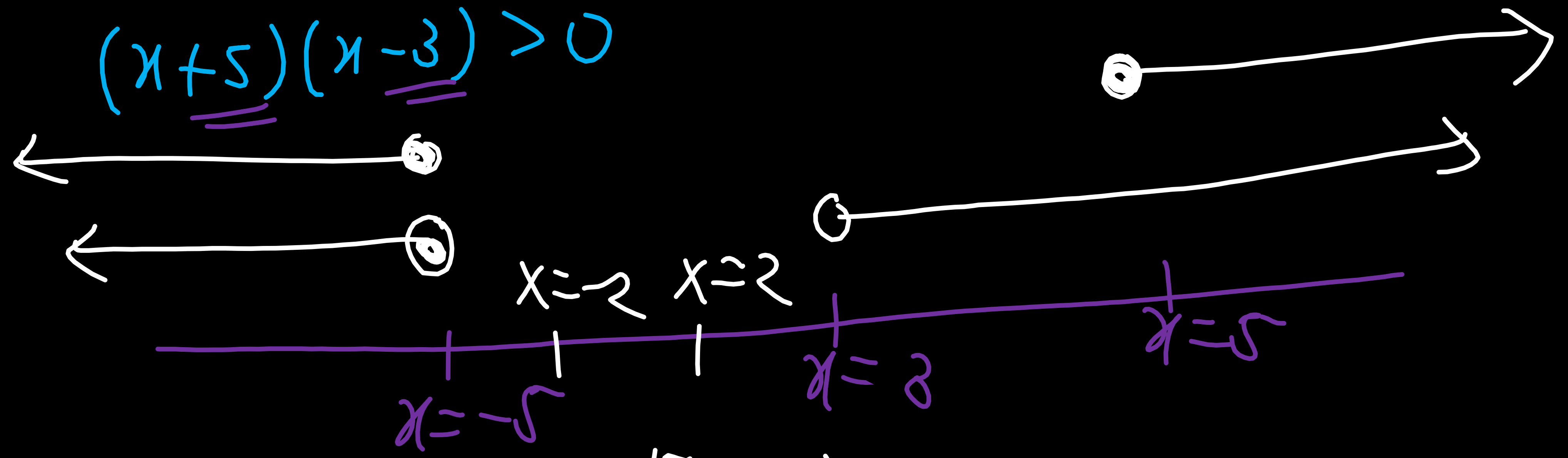
C-2 $4-x^2 \neq 0 \longrightarrow x^2 \neq 4 \Rightarrow x \neq \pm 2$

C-3 $x^2+2x-15 > 0$

$(x+5)(x-3) > 0$

-15

$+5$ -3



Ans: $x \in (-\infty, -5) \cup [5, \infty)$

Que.10 JEE Main 2024 (30 Jan Shift 2) Ans:(B)

If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to:

(A) 10 (B) 12 (C) 11 (D) 9

Que.11 JEE Main 2023 (08 Apr Shift 2) Ans:1

If the domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right) + \cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to:

C-1 $\quad \frac{6x^2+5x+1}{2x-1} > 0$

C-2 $\quad -1 \le \frac{2x^2-3x+4}{3x-5} \le 1$

Que.12 JEE Main 2023 Ans:(C)

If the domain of the function

$$f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x + 6}{3}\right)$$

is $(\alpha, \beta]$, then $36|\alpha + \beta|$ is:

(A) 72 (B) 54 (C) 45 (D) 63