Trigonometry

Lecture notes

World of Trigonometry

JEE Main

2025 → 6

Jan (10) + April (10) = 20 papers

2024 → 7

2023 → 6

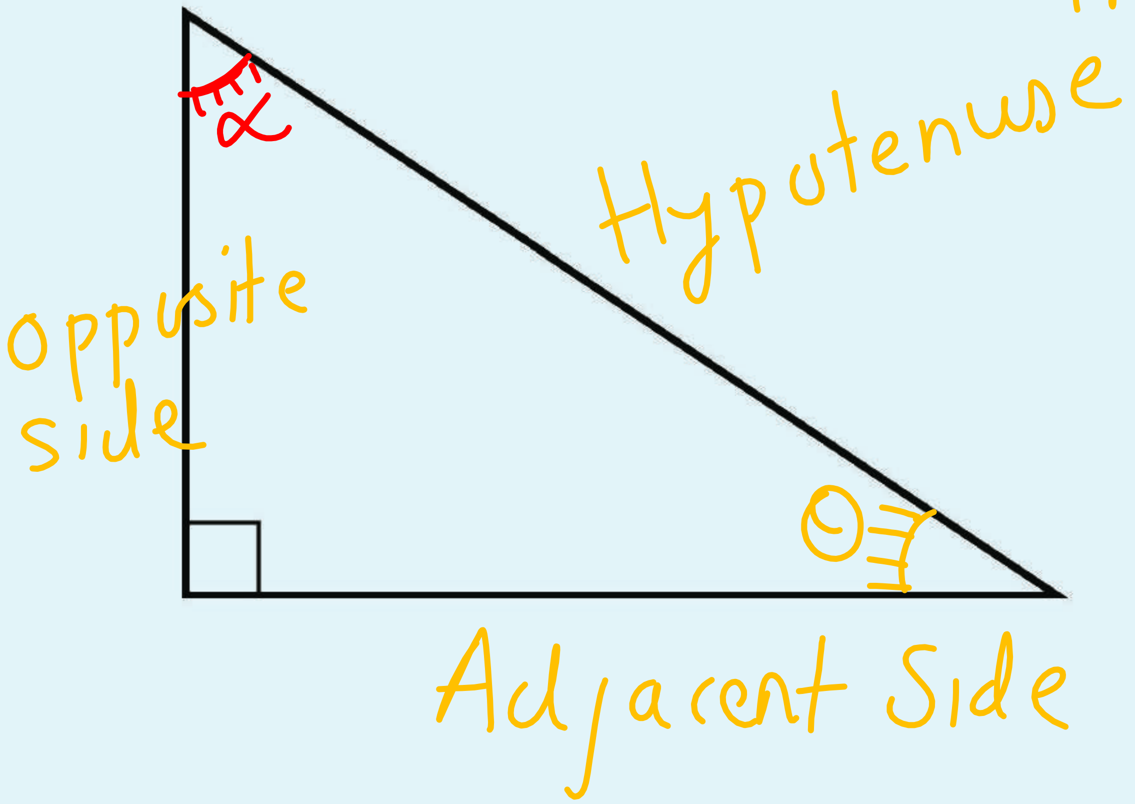
2022 → 7

1. Trigonometric Ratios and Identities (Compound Angle) → 45-50 formula + Problem Solving Tech
2. Trigonometric Equation
3. Solution of Triangle
4. Inverse Trigonometric Function (ITF) (12th Class)

Definition of T-Ratios

Trigonometry

Tri → Three, gono → Side, metry (metron) → Measurement



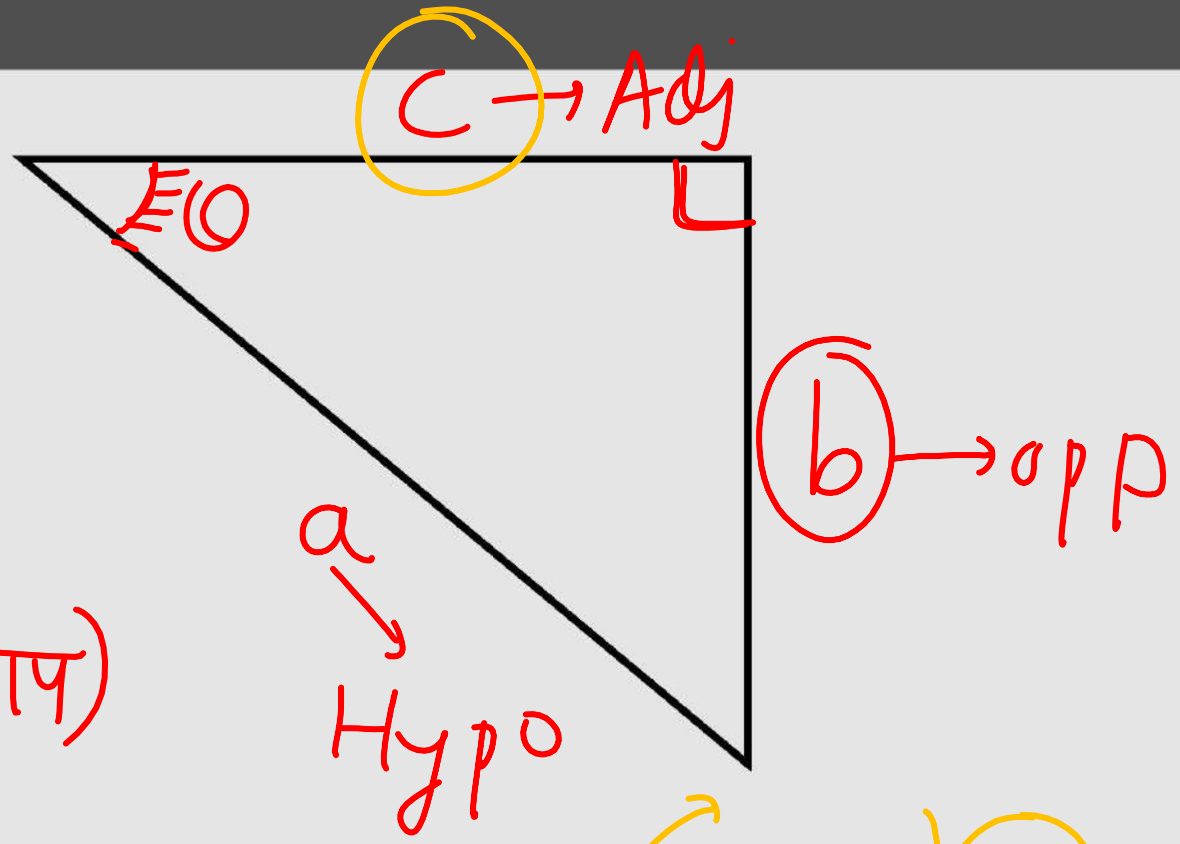
In the right-angled triangle,

The trigonometric ratios are defined as:

- $\sin\theta = \dfrac{\text{Opposite Side}}{\text{Hypotenuse}}$ ✓ — $\operatorname{cosec}\theta = \dfrac{\text{Hypo}}{\text{opp}}$
- $\cos\theta = \dfrac{\text{Adjacent Side}}{\text{Hypotenuse}}$ ✓ — $\sec\theta = \dfrac{\text{Hypo}}{\text{Adj}}$
- $\tan\theta = \dfrac{\text{Opposites side}}{\text{Adjacent Side}}$ ✓ — $\cot\theta = \dfrac{\text{Adj}}{\text{Opp}}$

Example: Finding Trigonometric Ratios

Example 1



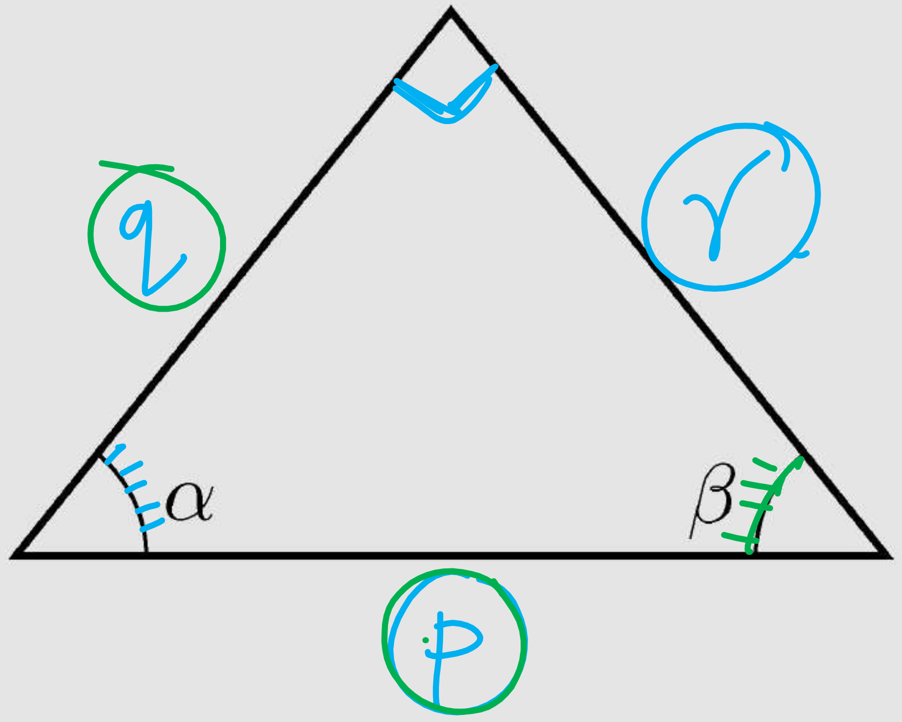
Sine (पाप)

$\cos\theta = \dfrac{c}{a}$

Find:

- $\sin\theta = \dfrac{b}{a}$
- $\cos\theta =$
- $\tan\theta = \dfrac{b}{c}$

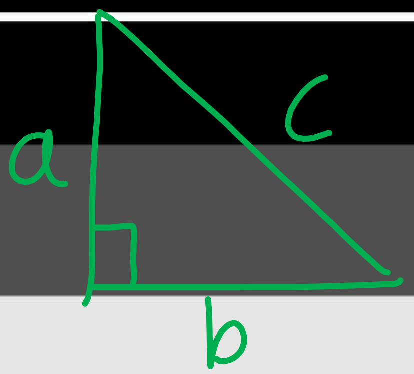
Example 2



Find:

- $\sin\alpha = \dfrac{r}{p}$
- $\cos\alpha = \dfrac{q}{p}$
- $\tan\alpha = \dfrac{r}{q}$
- $\sin\beta = \dfrac{q}{p}$
- $\cos\beta = \dfrac{r}{p}$
- $\tan\beta = \dfrac{q}{r}$

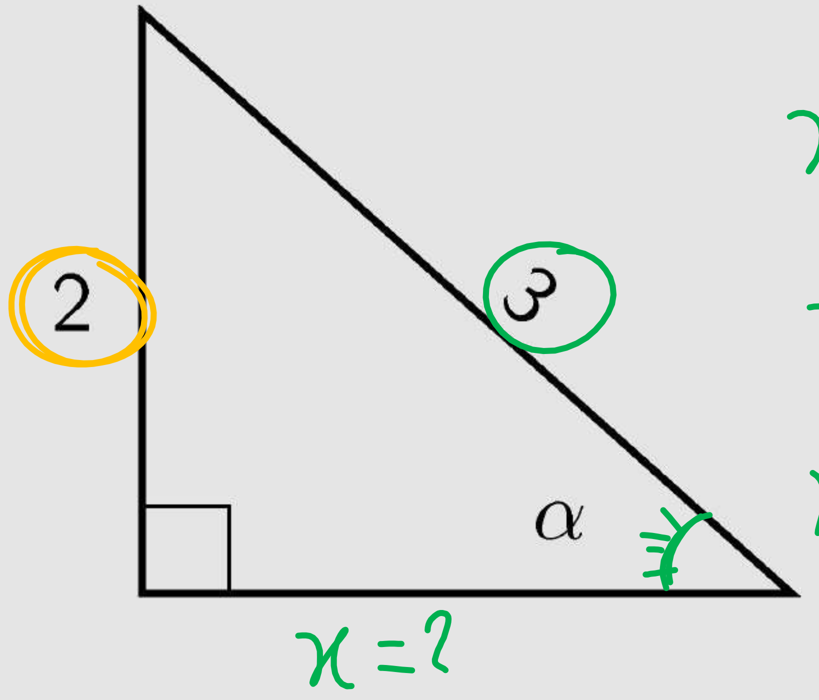
Example: Finding Trigonometric Ratios



Pythagorus Thm

$a^2 + b^2 = c^2$

Example 3



$x^2 + 2^2 = 3^2$

$x^2 = 3^2 - 2^2$

$x = \sqrt{3^2 - 2^2}$

$= \sqrt{9 - 4}$

$\boxed{x = \sqrt{5}}$

Find:

- $\sin\alpha = \frac{2}{3}$
- $\cos\alpha = \frac{\sqrt{5}}{3}$
- $\tan\alpha = \frac{2}{\sqrt{5}}$

Example 4

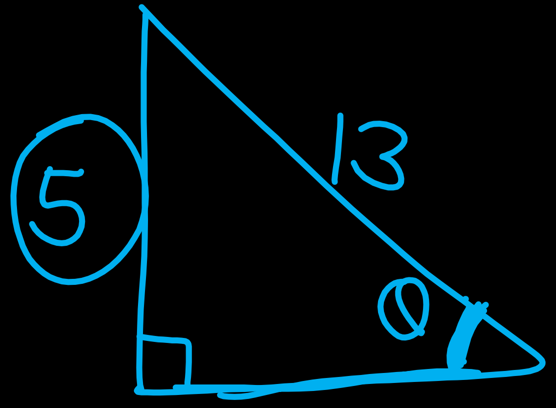
If $\boxed{\sin\theta = \frac{5}{13}}$, find value of $\cos\theta$, $\tan\theta$:

$5 \to$ opp

$13 \to$ Hypo

✓ Triangle Method:

M-2 Using Identities



$\sqrt{13^2 - 5^2} = \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

Basic Identities: Set-1

Basic Identities

(1) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$; $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sin\theta \operatorname{cosec}\theta = 1$

(2) $\cos\theta = \frac{1}{\sec\theta}$; $\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \sec\theta = 1$

(3) $\tan\theta = \frac{1}{\cot\theta}$; $\cot\theta = \frac{1}{\tan\theta}$; $\tan\theta . \cot\theta = 1$

(4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Basic Identities: Set-2

Basic Identities

(5) $\sin^2\theta + \cos^2\theta = 1$

$\hookrightarrow \cos^2\theta = 1 - \sin^2\theta$

$\hookrightarrow \sin^2\theta = 1 - \cos^2\theta$

$\hookrightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\hookrightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

(6) $1 + \tan^2\theta = \sec^2\theta$

$\hookrightarrow \tan^2\theta = \sec^2\theta - 1$

$\hookrightarrow \sec^2\theta - \tan^2\theta = 1$

$a^2 - b^2 = (a-b)(a+b)$

$\hookrightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\hookrightarrow \sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$

$\hookrightarrow \sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

(7) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\hookrightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\cot\theta = \pm\sqrt{\operatorname{cosec}^2\theta - 1}$

$\hookrightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\hookrightarrow (\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$\hookrightarrow \operatorname{cosec}\theta + \cot\theta = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$

Expressing T-ratios in terms of each other

$1 + \tan^2\theta = \sec^2\theta$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

Table of Interrelations

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	1 2 $\sqrt{1-\cos^2\theta}$	1 3 $\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	1 4 $\dfrac{1}{\sqrt{1+\cot^2\theta}}$	1 5 $\sqrt{1-\dfrac{1}{\sec^2\theta}}$	$\dfrac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	2 3 $\dfrac{1}{\sqrt{1+\tan^2\theta}}$	2 4 $\dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\dfrac{1}{\sec\theta}$	$\sqrt{1-\dfrac{1}{\operatorname{cosec}^2\theta}}$
$\tan\theta$	$\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\dfrac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\dfrac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$

1.2

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

1.3

$$\sin\theta = \frac{\sin\theta}{\cos\theta}\cos\theta$$

$$= \tan\theta \cos\theta$$

$$= \tan\theta \cdot \frac{1}{\sec\theta}$$

$$= \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

* $1+\tan^2\theta = \sec^2\theta$

$$\sqrt{1+\tan^2\theta} = \sec\theta$$

** $1+\cot^2\theta = \operatorname{cosec}^2\theta$

$$\sqrt{1+\cot^2\theta} = \operatorname{cosec}\theta$$

$\cot\theta = 2$

1.4

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$= \frac{1}{\sqrt{1+\cot^2\theta}}$$

1.5

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\frac{1}{\sec^2\theta}}$$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = ?$

2.3 $\cos\theta = \dfrac{1}{\sec\theta}$

$= \dfrac{1}{\sqrt{1+\tan^2\theta}}$

$1+\tan^2\theta = \sec^2\theta$

$\sqrt{1+\tan^2\theta} = \sec\theta$

2 6 $\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\dfrac{1}{\operatorname{cosec}^2\theta}}$

2 4 $\cos\theta = \dfrac{\cos\theta}{\sin\theta}\,\sin\theta$ ✓

$= \cot\theta\,\dfrac{1}{\operatorname{cosec}\theta}$

$= \dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$

31 $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ ✓

$= \dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

32 $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ ✓

$= \dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

36 $\tan\theta = \dfrac{1}{\cot\theta}$

$= \dfrac{1}{\sqrt{\operatorname{cosec}^2\theta - 1}}$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$

Question:1 on Identities

Ans: 1

Find the value of $(0^\circ < A < 90^\circ)$

$$\frac{\tan^2 A \cdot \sin^2 A}{\tan^2 A - \sin^2 A}$$

common leneka mukey आए to common lena chahiye!

$$\text{LHS} = \frac{\tan^2 A \sin^2 A}{\tan^2 A - \sin^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A} \cdot \sin^2 A}{\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 A}{1}}$$

$$=$$

$$= \frac{\cancel{\sin^2 A}\ \frac{\sin^2 A}{\cos^2 A}}{\cancel{\sin^2 A}\left(\frac{1}{\cos^2 A} - 1\right)}$$

$$= \frac{\frac{\sin^2 A}{\cancel{\cos^2 A}}}{\frac{1-\cos^2 A}{\cancel{\cos^2 A}}}$$

$$= \frac{\cancel{\sin^2 A}}{\cancel{\sin^2 A}}$$

$$= 1$$

Question:2 on Identities

Prove that:

$$\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\sec\alpha}{1+\operatorname{cosec}\alpha}\right)=\tan\alpha$$

$\sec\alpha = \frac{1}{\cos\alpha}$, $\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$

सबको Badlu sin/cos me

$$LHS = \left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\frac{1}{\cos\alpha}}{1+\frac{1}{\sin\alpha}}\right)$$

$$= \left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\frac{\left(\frac{\cos\alpha+1}{\cos\alpha}\right)}{\left(\frac{\sin\alpha+1}{\sin\alpha}\right)}$$

$$= \cancel{\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)}\left(\frac{\cancel{\cos\alpha+1}}{\cos\alpha}\right)\left(\frac{\sin\alpha}{\cancel{\sin\alpha+1}}\right)$$

$$LHS = \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= RHS //$$

Question:3 on Identities

Prove that:

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

Rationlisation

$$LHS = \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$$

$$(a+b)(a-b) = a^2-b^2$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \qquad \sqrt{\frac{a^2}{b^2}} = \frac{a}{b}$$

$$= \sqrt{\left(\frac{1-\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

split

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

$$= RHS$$

Question:4 on Identities

Prove that:

$$(\sec\theta + \operatorname{cosec}\theta)(\sin\theta + \cos\theta) - \sec\theta \cdot \operatorname{cosec}\theta = 2$$

सबको badla sin/cos

$$LHS = \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)\left(\sin\theta + \cos\theta\right) - \frac{1}{\cos\theta}\,\frac{1}{\sin\theta}$$

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\,\cos\theta}\right)\frac{(\sin\theta + \cos\theta)}{1} - \frac{1}{\sin\theta\,\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\,\cos\theta}$$

$$= \frac{\cancel{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cos\theta \cancel{- 1}}{\sin\theta.\cos\theta}$$

$$LHS = \frac{\cancel{2\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$$

$$= 2$$

$$= RHS$$

H.P

Question:5

If $\tan\theta + \sec\theta = 1.5$, find the values of $\sin\theta$, $\tan\theta$, and $\sec\theta$.

$$\sec\theta + \tan\theta = \frac{3}{2} \quad \text{--- (1)}$$

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{3/2} = 2/3$$

$$\sec\theta - \tan\theta = \frac{2}{3} \quad \text{--- (2)}$$

$$2\sec\theta = \frac{3}{2} + \frac{2}{3}$$

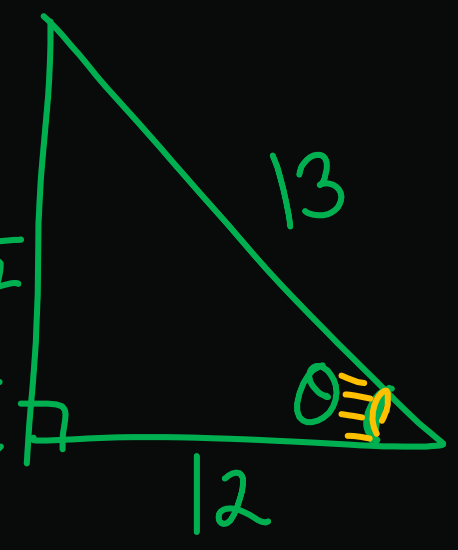
$$2\sec\theta = \frac{13}{6}$$

$$\boxed{\sec\theta = \frac{13}{12}}$$

13 → Hypo

12 → Adj

Triangle Method



$\sqrt{13^2 - 12^2}$

$\sqrt{169 - 144}$

$\sqrt{25}$

5

$$\sin\theta = \frac{5}{13}$$

$$\tan\theta = \frac{5}{12}$$

M-2

Using Identites

$$1 + \tan^2\theta = \sec^2\theta$$

Question:6

If $2\sin\theta = 2 - \cos\theta$, then find the value(s) of $\sin\theta$.

$\rightarrow 2\sin\theta = 2 - \cos\theta$ $\cancel{\rightarrow} \sqrt{1-\sin^2\theta}$

$\cancel{\rightarrow} (2\sin\theta)^2 = (2-\cos\theta)^2$

$\cos\theta = 2 - 2\sin\theta$

Squaring

$\cos^2\theta = 4 + 4\sin^2\theta - 8\sin\theta$

$1 - \sin^2\theta = 4 + 4\sin^2\theta - 8\sin\theta$

$\boxed{5\sin^2\theta - 8\sin\theta + 3 = 0}$

15

$-5 \quad -3$

$5\sin^2\theta - 5\sin\theta - 3\sin\theta + 3 = 0$

$5\sin\theta(\sin\theta - 1) - 3(\sin\theta - 1) = 0$

$(\sin\theta - 1)(5\sin\theta - 3) = 0$

$\boxed{\sin\theta = 1}$ ✓ $\boxed{\sin\theta = \frac{3}{5}}$ ✓

Question:7

If $\sin\theta + \sin^2\theta = 1$, then prove that:

$$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^{8}\theta + \cos^{6}\theta - 1 = 0$$

$\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta$

$\boxed{\sin\theta = \cos^2\theta}$

$a^3 b^3 = (ab)^3$

LHS $= (\cos^2\theta)^6 + 3(\cos^2\theta)^5 + 3(\cos^2\theta)^4 + (\cos^2\theta)^3 - 1$

$= \underbrace{(\sin\theta)^6 + 3(\sin\theta)^5 + 3(\sin\theta)^4 + (\sin\theta)^3}_{\text{Common}} - 1$

$= \sin^3\theta\left[1\sin^3\theta + 3\sin^2\theta + 3\sin\theta + 1\right] - 1$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= \sin^3\theta\left[(\sin\theta+1)^3\right] - 1$

$= \left[\sin\theta(\sin\theta+1)\right]^3 - 1$

$= \left[\sin^2\theta + \sin\theta\right]^3 - 1$

$= 1^3 - 1$

$= 0$

$=$ RHS

Question:8 JEE Main 2025 (January)

If $\sin x + \sin^2 x = 1$, where $x \in (0, \frac{\pi}{2})$, then the expression:

$$(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$$

is equal to: $= \cos^{12} x + \cos^{12} x + 3\left(\cos^{10} x + \cos^{10} x + \cos^8 x + \cos^8 x\right) + \cos^6 x + \cos^6 x$

(A) 3 (B) 4 (C) 2 (D) 1

$\sin x + \sin^2 x = 1$

$\sin x = 1 - \sin^2 x$

$\boxed{\sin x = \cos^2 x}$ — (*)

$\frac{\sin x}{\cos x} = \cos x$

$\boxed{\tan x = \cos x}$ — (**)

$= 2\cos^{12} x + 3\left(2\cos^{10} x + 2\cos^8 x\right) + 2\cos^6 x$

$= 2\sin^6 x + 6 \cdot \sin^5 x + 6\sin^4 x + 2 \cdot \sin^3 x$

\# common

$= 2\sin^3 x\left[\sin^3 x + 3\sin^2 x + 3\sin x + 1\right]$

$= 2\sin^3 x \cdot (\sin x + 1)^3$

$= 2\left(\sin x(\sin x + 1)\right)^3$

$= 2\left(\sin^2 x + \sin x\right)^3$

$= 2(1)^3$

$= 2$

Question:9

If $3\sec^4\theta + 8 = 10\sec^2\theta$, then find the positive value(s) of $\tan\theta$.

$3\sec^4\theta + 8 = 10\sec^2\theta$

let $\boxed{\sec^2\theta = t} \rightarrow \sec^4\theta = t^2$

$3t^2 + 8 = 10t$

$3t^2 - 10t + 8 = 0$ 24

$3t^2 - 6t - 4t + 8 = 0$ -6 -4

$3t(t-2) - 4(t-2) = 0$

$t = 2 \quad t = \frac{4}{3}$

$\sec^2\theta = 2 \quad \sec^2\theta = \frac{4}{3}$

Using identity

$1 + \tan^2\theta = 2$

$\tan^2\theta = 2 - 1$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$1 + \tan^2\theta = \frac{4}{3}$

$\tan^2\theta = \frac{4}{3} - 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

Ans: positive values $\tan\theta = 1, \frac{1}{\sqrt{3}}$

M-2

$3\sec^4\theta + 8 = 10\sec^2\theta$

* $\sec^2\theta = 1 + \tan^2\theta$

$3(1+\tan^2\theta)^2 + 8 = 10(1+\tan^2\theta)$

$3[1 + \tan^4\theta + 2\tan^2\theta] + 8$

$= 10 + 10\tan^2\theta$

let $\tan^2\theta = t$

$\therefore$ Quadratic

Question:10

If $3\sin^4 x + 2\cos^4 x = \dfrac{6}{5}$, then find the value of $\tan^2 x$.

$3\sin^4 x + 2(\cos^2 x)^2 = \frac{6}{5}$

$3\sin^4 x + 2(1-\sin^2 x)^2 = \frac{6}{5}$

$3\sin^4 x + 2[1+\sin^4 x - 2\sin^2 x] = \frac{6}{5}$

$** \; 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$

let $\sin^2 x = t$

$\therefore \sin^4 x = t^2$

$5t^2 - 4t + \frac{4}{5} = 0$

$25t^2 - 20t + 4 = 0$

100

$25t^2 - 10t - 10t + 4 = 0 \quad -10 \quad -10$

$5t(5t-2) - 2(5t-2) = 0$

$(5t-2)(5t-2) = 0$

$(5t-2)^2 = 0$

$t = 2/5$

$\boxed{\sin^2 x = \frac{2}{5}}$ — ①

$* \; \sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \frac{2}{5}$

$\cos^2 x = \frac{3}{5}$ — ②

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{2/5}{3/5}$

$= \boxed{2/3}$

Question:11 JEE Main 2021

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6\alpha + 8\,\mathrm{cosec}^6\alpha$ is equal to:

(A) 350 (B) 500 (C) 400 (D) 250 ✓

$15\sin^4\alpha + 10(\cos^2\alpha)^2 = 6$

$15\sin^4\alpha + 10(1-\sin^2\alpha)^2 = 6$

let $\sin^2\alpha = t$

$15t^2 + 10(1-t)^2 - 6 = 0$

$15t^2 + 10(1+t^2-2t) - 6 = 0$

$\boxed{25t^2 - 20t + 4 = 0}$

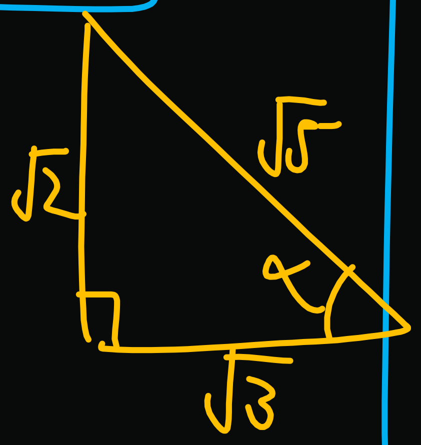
$25t^2 - 20t + 4 = 0$

$(5t-2)^2 = 0$

$t = \frac{2}{5}$

$\boxed{\sin^2\alpha = \frac{2}{5}}$

$\sin\alpha = \pm\frac{\sqrt{2}}{\sqrt{5}}$



Req $= 27\sec^6\alpha + 8\,\mathrm{cosec}^6\alpha$

$= 27\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^6 + 8\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^6$

$= \cancel{27}\cdot\frac{125}{\cancel{27}} + \cancel{8}\,\frac{125}{\cancel{8}}$

$= 125 + 125$

$= 250$

Question:12

If the following equation holds true:

$$\sec^4 x - \operatorname{cosec}^4 x - 2\sec^2 x + 2\operatorname{cosec}^2 x = \frac{15}{4}$$

then find the value of $\tan x$.

M-1

$$\left(\sec^4 x - 2\sec^2 x\right) - \left(\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x\right) = \frac{15}{4}$$

$(t^2 - 2t + 1) = (t-1)^2$

$$\left(\sec^4 x - 2\sec^2 x + 1\right) - \left(\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x + 1\right) = \frac{15}{4}$$

$$(\sec^2 x - 1)^2 - (\operatorname{cosec}^2 x - 1)^2 = \frac{15}{4}$$

$$(\tan^2 x)^2 - (\cot^2 x)^2 = \frac{15}{4}$$

$$\tan^4 x - \cot^4 x = \frac{15}{4}$$

$$\tan^4 x - \frac{1}{\tan^4 x} = \frac{15}{4}$$

Quad

let $\tan^4 x = t$

$$t - \frac{1}{t} = \frac{15}{4}$$

$$\frac{t^2 - 1}{t} = \frac{15}{4}$$

$$\tan^4 x - \frac{1}{\tan^4 x} = \frac{16-1}{4}$$

$$\boxed{\tan^4 x} - \frac{1}{\boxed{\tan^4 x}} = 4 - \frac{1}{4}$$

$$\tan^4 x = 4$$

Method-2

$$\sec^4 x - \operatorname{cosec}^4 x - 2\sec^2 x + 2\operatorname{cosec}^2 x = \frac{15}{4}$$

$$(1+\tan^2 x)^2 - (1+\cot^2 x)^2 - 2(1+\tan^2 x) + 2(1+\cot^2 x) = \frac{15}{4}$$

let $\tan^2 x = t$ $\therefore$ $\cot^2 x = \frac{1}{t}$

$$(1+t)^2 - \left(1+\frac{1}{t}\right)^2 - 2(1+t) + 2\left(1+\frac{1}{t}\right) = \frac{15}{4}$$

$$t^2 + \cancel{2t} + 1 - \left(\frac{t+1}{t}\right)^2 - \cancel{2} - \cancel{2t} + \cancel{2} + \frac{2}{t} = \frac{15}{4}$$

$$\frac{(t^2+1)}{1} - \frac{(t^2+2t+1)}{t^2} + \frac{2}{t} = \frac{15}{4}$$

$$\frac{t^4 + \cancel{t^2} - \cancel{t^2} - \cancel{2t} - 1 + \cancel{2t}}{t^2} = \frac{15}{4}$$

$$\frac{t^4-1}{t^2} = \frac{15}{4}$$

$$\frac{t^2-1}{t} = \frac{15}{4}$$

$$4t^2 - 4 = 15t$$

$$4t^2 - 15t - 4 = 0 \qquad -16$$

$$4t^2 - 16t + t - 4 = 0 \qquad -16 \quad +1$$

$$4t(t-4) + 1(t-4) = 0$$

$$(t-4)(4t+1) = 0$$

$t = 4$	$t = -\frac{1}{4}$
$\tan^4 x = 4$	$\tan^4 x = -\frac{1}{4}$ × Reject
$\tan^2 x = 2$	
$\tan x = \pm\sqrt{2}$	

$$\frac{t^4-1}{t^2}=\frac{15}{4}$$

$\tan^2 x = t$

$4t^4-4=15t^2$

$4t^4-15t^2-4=0$ $\quad -16$

$\underbrace{4t^4-16t^2}+t^2-4=0$ $\quad -16+1$

$4t^2(t^2-4)+1(t^2-4)=0$

$(t^2-4)(4t^2+1)=0$

$t^2=4$ $\qquad t^2=-\frac{1}{4}$ Reject

$t^2=4$

$(\tan^2 x)^2=4$

$\tan^4 x=4$

$\tan^2 x=2$

$$\boxed{\tan x=\pm\sqrt{2}}$$

Question:13

The expression $4(\sin^6\theta + \cos^6\theta) - 6(\sin^4\theta + \cos^4\theta)$ is equal to:

(A) 0

(B) 1

(C) -2

(D) 2

$4(1-3\sin^2\theta\cos^2\theta)-6(1-2\sin^2\theta\cdot\cos^2\theta) = 4 - \cancel{12 s^2c^2} - 6 + \cancel{12 s^2 c^2}$

$= 4-6$

$= \boxed{-2}$

$\sin^6\theta+\cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

* $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$= (\sin^2\theta+\cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$\boxed{\#\ \sin^6\theta+\cos^6\theta = 1-3\sin^2\theta\cos^2\theta}$

$\sin^4\theta+\cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2$ * $a^2+b^2 = (a+b)^2-2ab$

$= (\sin^2\theta+\cos^2\theta)^2 - 2(\sin^2\theta)(\cos^2\theta)$

$\boxed{\#\ \sin^4\theta+\cos^4\theta = 1-2\sin^2\theta\cos^2\theta}$

Question:14

Prove that:

$$\sin^8 x - \cos^8 x = (\sin^2 x - \cos^2 x)(1 - 2\sin^2 x \cos^2 x)$$

$LHS = \sin^8 x - \cos^8 x$

$= \left(\boxed{\sin^4 x}\right)^2 - \left(\cos^4 x\right)^2$

$= \left(\sin^4 x - \cos^4 x\right)\left(\sin^4 x + \cos^4 x\right)$

$= \left(\left(\boxed{\sin^2 x}\right)^2 - \left(\cos^2 x\right)^2\right)\left(1 - 2\sin^2 x \cdot \cos^2 x\right)$

$= \left(\sin^2 x - \cos^2 x\right)\cancel{\left(\sin^2 x + \cos^2 x\right)}\left(1 - 2\sin^2 x \cos^2 x\right)$

$= \left(\sin^2 x - \cos^2 x\right)\left(1 - 2\sin^2 x \cos^2 x\right)$

$= RHS$

Question:15

If $T_n = \sin^n x + \cos^n x$, prove that:

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

$T_1 = \sin^1 x + \cos^1 x$

$T_2 = \sin^2 x + \cos^2 x$

$T_3 = \sin^3 x + \cos^3 x$

$$LHS = \frac{T_3 - T_5}{T_1} = \frac{(\sin^3 x + \cos^3 x) - (\sin^5 x + \cos^5 x)}{(\sin x + \cos x)}$$

$$= \frac{(\sin^3 x - \sin^5 x) + (\cos^3 x - \cos^5 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^3 x (1 - \sin^2 x) + \cos^3 x (1 - \cos^2 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^3 x (\cos^2 x) + \cos^3 x (\sin^2 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^2 x \cos^2 x \cancel{(\sin x + \cos x)}}{\cancel{(\sin x + \cos x)}}$$

$$= \sin^2 x \cdot \cos^2 x$$

$$RHS = \frac{T_5 - T_7}{T_3}$$

$$= \frac{(\sin^5 x + \cos^5 x) - (\sin^7 x + \cos^7 x)}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^5 x(1 - \sin^2 x) + \cos^5 x(1 - \cos^2 x)}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^5 x \cos^2 x + \cos^5 x \sin^2 x}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^2 x \cos^2 x \cancel{(\sin^3 x + \cos^3 x)}}{\cancel{(\sin^3 x + \cos^3 x)}}$$

$RHS = \sin^2 x \cos^2 x$

$\therefore$ LHS = RHS

HP //

Question:16 JEE Main 2019

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \ldots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to:

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{12}$ (D) $\frac{5}{12}$

$$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$$

$$\text{Req} = f_4(x) - f_6(x)$$

$$= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4}(1 - 2\sin^2 x \cdot \cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x \cos^2 x)$$

$$= \frac{1}{4} - \cancel{2\left(\frac{1}{4}\right)\sin^2 x \cos^2 x} - \frac{1}{6} + \cancel{\frac{1}{6}(3)\sin^2 x \cdot \cos^2 x}$$

$$= \frac{1}{4} - \frac{1}{6}$$

$$= \frac{3-2}{12}$$

$$= \boxed{\frac{1}{12}}$$

Question:17

Prove that:

$$\# \quad (\operatorname{cosec} A + \cot A - 1)(\operatorname{cosec} A - \cot A + 1) = 2\cot A$$

$$= \left[(\operatorname{cosec} A) + (\cot A - 1) \right] \left[(\operatorname{cosec} A) - (\cot A - 1) \right]$$

$$[a + b] \qquad [a - b]$$

$$= (\operatorname{cosec} A)^2 - (\cot A - 1)^2$$

$$= \operatorname{cosec}^2 A - \left[\cot^2 A + 1 - 2\cot A \right]$$

$$= \cancel{\operatorname{cosec}^2 A} - \cancel{\operatorname{cosec}^2 \theta} + 2\cot A$$

$$= 2\cot A$$

$$= \text{RHS}$$

Question:18 (Que.116)

Prove that:

$$\frac{(1-\sin\theta+\cos\theta)^2}{(1+\cos\theta)(1-\sin\theta)}=2$$

$(a+b+c)^2 = (a+b)^2 + c^2 + 2(a+b)(c)$

$(x_1 + x_2)^2$

$$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc}$$

$(1-\sin\theta+\cos\theta)^2$

$(\underbrace{1}_{a}+\underbrace{(-\sin\theta)}_{b}+\underbrace{\cos\theta}_{c})^2 = 1^2+\sin^2\theta+\cos^2\theta+2(-\sin\theta)+2(\cos\theta)+2(-\sin\theta)(\cos\theta)$

$= 2-2\sin\theta+2\cos\theta-2\sin\theta\cdot\cos\theta$

$= 2\left(1-\sin\theta+\underbrace{\cos\theta-\sin\theta\cdot\cos\theta}_{\text{common}}\right) = 2\left[(1-\sin\theta)+\cos\theta(1-\sin\theta)\right]$

$$(1+(-\sin\theta)+\cos\theta)^2 = 2\left[\underbrace{(1-\sin\theta)} + \cos\theta\underbrace{(1-\sin\theta)}\right]$$

$$= 2\left[(1-\sin\theta)(1+\cos\theta)\right]$$

$$LHS = \frac{2\cancel{(1-\sin\theta)}\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\cancel{(1-\sin\theta)}}$$

$$= 2$$

$$= RHS \,//$$

Question:19 (Que.115)

If $\operatorname{cosec}\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, prove that:

$$a^2b^2(a^2+b^2) = 1$$

$a = ?$

$$a^3 = \operatorname{cosec}\theta - \sin\theta$$

$$a^3 = \frac{1}{\sin\theta} - \sin\theta$$

$$a^3 = \frac{1-\sin^2\theta}{\sin\theta}$$

$$a^3 = \frac{\cos^2\theta}{\sin\theta}$$

$$(\cancel{a^3})^{\cancel{1/3}} = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$$

$$a = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$$

$$b^3 = \sec\theta - \cos\theta$$

$$b^3 = \frac{1}{\cos\theta} - \cos\theta$$

$$b^3 = \frac{1-\cos^2\theta}{\cos\theta}$$

$$b^3 = \frac{\sin^2\theta}{\cos\theta}$$

$$b = \left(\frac{\sin^2\theta}{\cos\theta}\right)^{1/3}$$

$$\text{Req} = a^2b^2(a^2+b^2)$$

$$= a^4b^2 + a^2b^4$$

$$\text{Req} = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{4/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{4/3}$$

$$= \frac{(\cos\theta)^{\frac{8}{3}-\frac{2}{3}} \cdot \cancel{\sin^{4/3}\theta}}{\cancel{\sin^{4/3}\theta}} + \frac{\cancel{\cos^{4/3}\theta}\,(\sin\theta)^{\frac{8}{3}-\frac{2}{3}}}{\cancel{\cos^{4/3}\theta}}$$

$$= (\cos\theta)^2 + (\sin\theta)^2$$

$$= 1$$

$$= \text{RHS}$$

Question:20

Prove that:

$$\frac{\sin x + \cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$LHS = \dfrac{\sin x + \cos x}{\cos^3 x}$

Divide N^r & D^r by $\cos x$

$= \dfrac{\dfrac{\sin x}{\cos x} + \cancel{\dfrac{\cos x}{\cos x}}}{\dfrac{\cos^3 x}{\cancel{\cos x}}}$

$= \dfrac{\tan x + 1}{\cos^2 x}$

$LHS = (\tan x + 1)\left(\dfrac{1}{\cos^2 x}\right)$

$= (\tan x + 1)(\sec^2 x)$

$= (\tan x + 1)(1 + \tan^2 x)$

$= \tan^3 x + \tan^2 x + \tan x + 1$

$= RHS$

T-Ratios of Standard Angles

$\sin 30 < \cos 30$

Remember this Values

$\sin 60 > \cos 60$

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
$\cot\theta$	N D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N D
$\text{cosec}\,\theta$	N D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

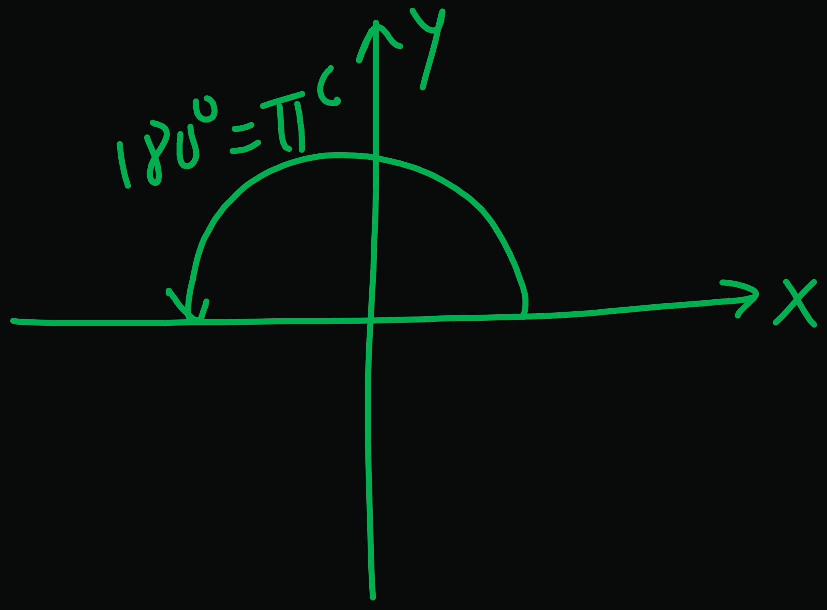
$\frac{1}{0} =$ N.D

Angle Measurement: Degrees & Radians

Conversion Formulas

$$\text{degree} \times \frac{\pi}{180^\circ} = \text{radian} \qquad \longleftrightarrow \qquad \text{radian} \times \frac{180^\circ}{\pi} = \text{degree}$$

$$\boxed{180^\circ = \pi^c}$$



① $30^\circ = (\ ?\ )^c$

$180^\circ \longrightarrow \pi^c$

$30^\circ \longrightarrow x$

$x \; 180 = 30\,\pi$

$x = \cancel{30} \times \frac{\pi}{\cancel{180}} = \frac{\pi}{6}$

② $60^\circ = \cancel{60} \times \frac{\pi}{\cancel{180}} = \frac{\pi}{3}$

③ $90^\circ = \cancel{90} \times \frac{\pi}{\cancel{180}} = \frac{\pi}{2}$

Examples:

④ $330^\circ = 33\cancel{0}^\circ \times \dfrac{\pi}{18\cancel{0}} = \dfrac{11}{6}\pi$

$$\boxed{\text{Radian} \times \frac{180}{\pi} = \text{Degree}}$$

① $\dfrac{\pi}{4} = \dfrac{\cancel{\pi}}{4} \times \dfrac{180}{\cancel{\pi}} = 45^\circ$

② $\dfrac{2\pi}{3} = \dfrac{2\cancel{\pi}}{3} \times \dfrac{180}{\cancel{\pi}} = 120^\circ$

③ $\dfrac{5\pi}{3} = \dfrac{5 \times \overset{60}{\cancel{180}}}{\cancel{3}} = 300^\circ$

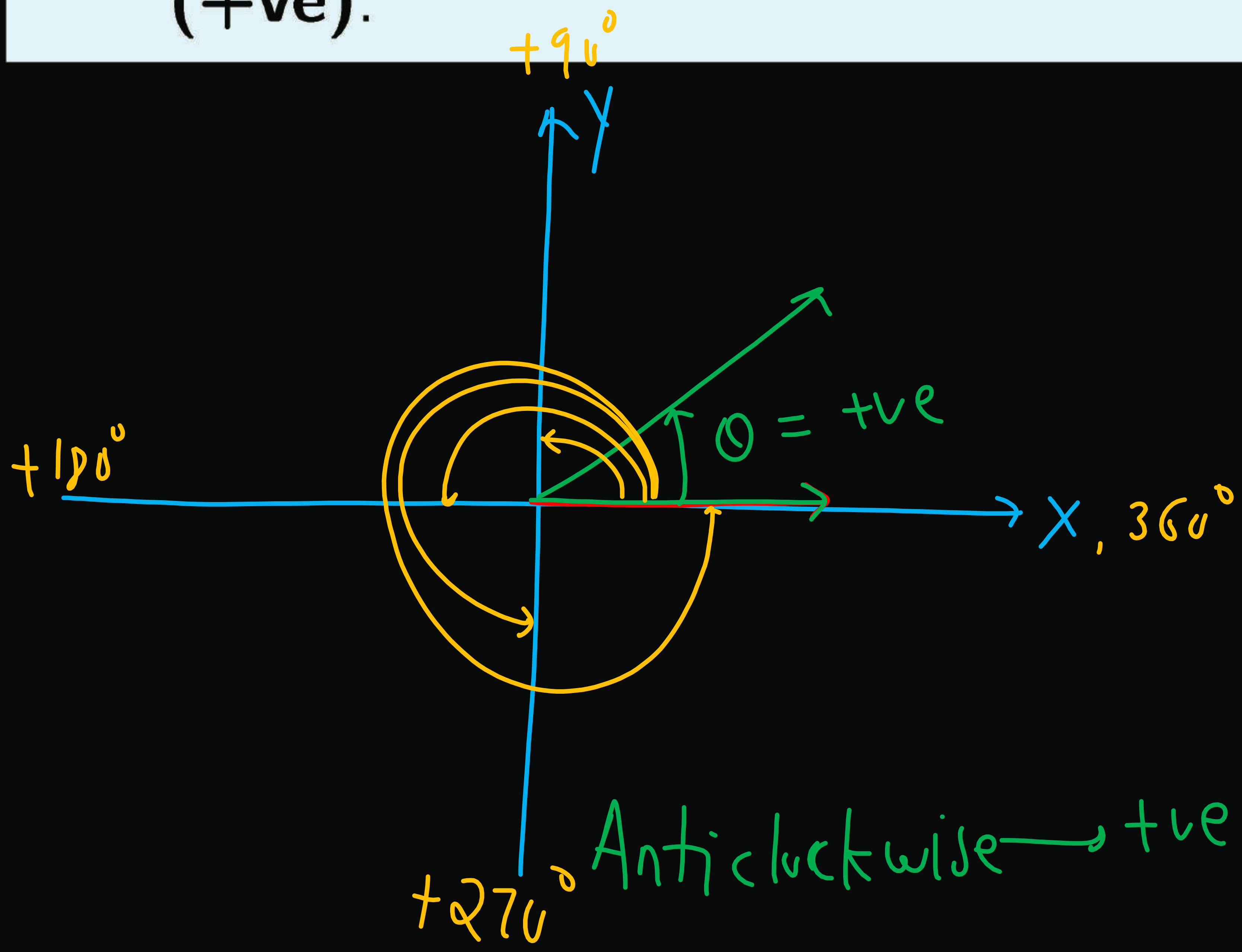
$$45 \longrightarrow \frac{\pi}{4}$$
$$90 \longrightarrow \frac{\pi}{2}$$
$$135 \longrightarrow \frac{3\pi}{4}$$
$$180 \longrightarrow \pi$$

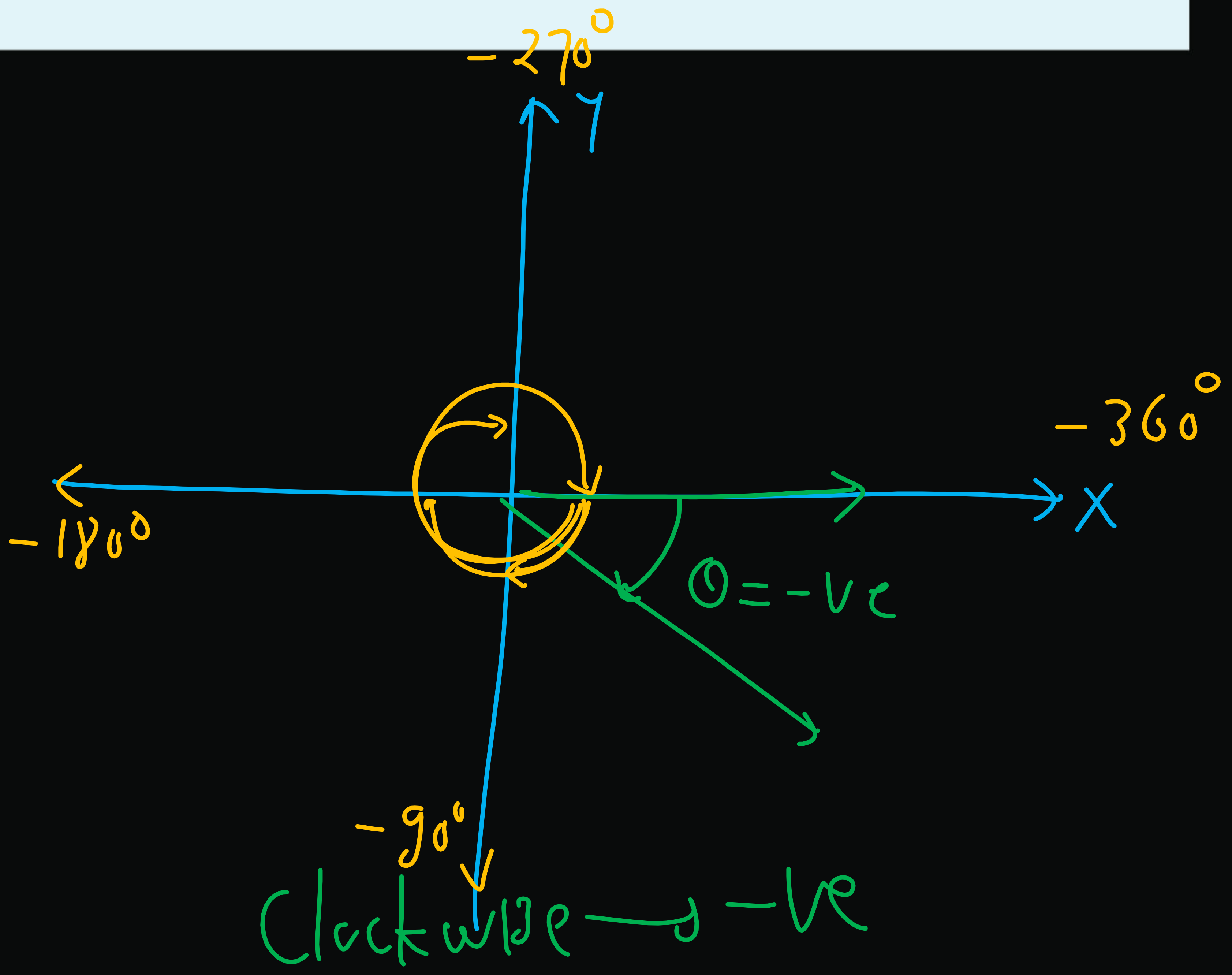
Some Important Angles

- $15^\circ = 15\times\frac{\pi}{180} = \frac{\pi}{12}$
- $18^\circ = 18\times\frac{\pi}{180} = \frac{\pi}{10}$
- $22.5^\circ = \frac{45^\circ}{2} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$
- $36^\circ = 2\times 18^\circ = 2\times\frac{\pi}{10} = \frac{\pi}{5}$
- $75^\circ = 90^\circ - 15^\circ = \frac{\pi}{2} - \frac{\pi}{12} = \frac{6\pi-\pi}{12} = \frac{5\pi}{12}$
- $72^\circ = \left(\frac{\pi}{10}\right)\times 4 = \frac{2\pi}{5}$
- $67.5^\circ = 90^\circ - 22.5^\circ = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}$
- $54^\circ = 3\times 18^\circ = 3\times\frac{\pi}{10} = \frac{3\pi}{10}$

Angle Convention

- **Anticlockwise** rotation from the positive x-axis is considered **positive (+ve)**.
- **Clockwise** rotation from the positive x-axis is considered **negative (-ve)**.

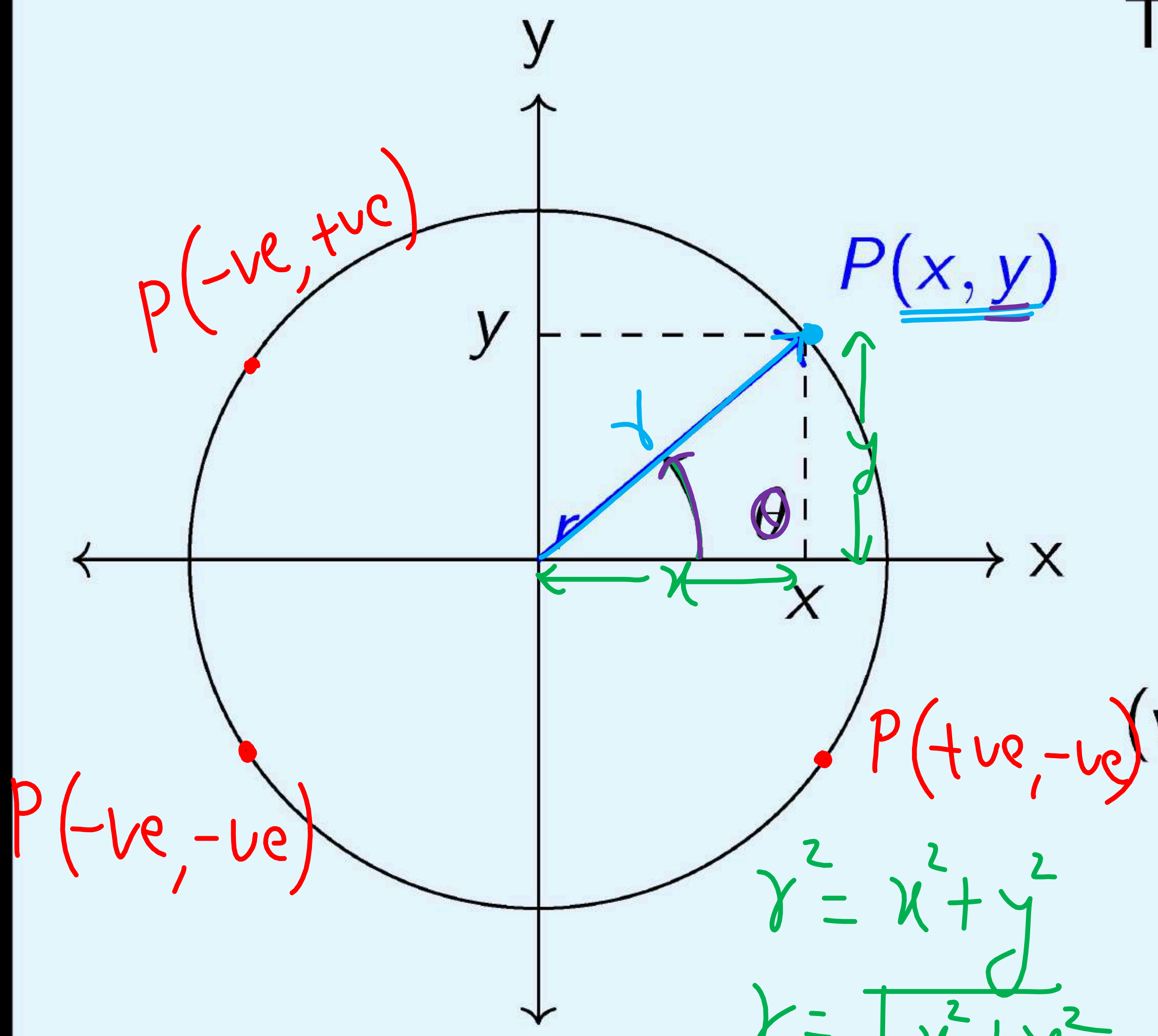




New Definition of Trigonometric Ratios

old defination $\sin\theta = \frac{Opp}{Hypo}$ $\cos\theta = \frac{Adj}{Hyp}$ $\tan\theta = \frac{Opp}{Adj}$

Let $P(x, y)$ be any point on a circle of radius r centered at the origin, and let θ be the angle formed with the positive x-axis.



The ratios are defined as:

- $\sin\theta = \dfrac{y\text{-coordinate}}{r}$
- $\cos\theta = \dfrac{x\text{-coordinate}}{r}$
- $\tan\theta = \dfrac{y\text{-coordinate}}{x\text{-coordinate}}$

(where $r = \sqrt{x^2 + y^2}$)

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

Limiting Cases:

ASTC Rule: Signs of T-Ratios in Quadrants

The sign of a trigonometric ratio depends on the quadrant in which the angle θ terminates.

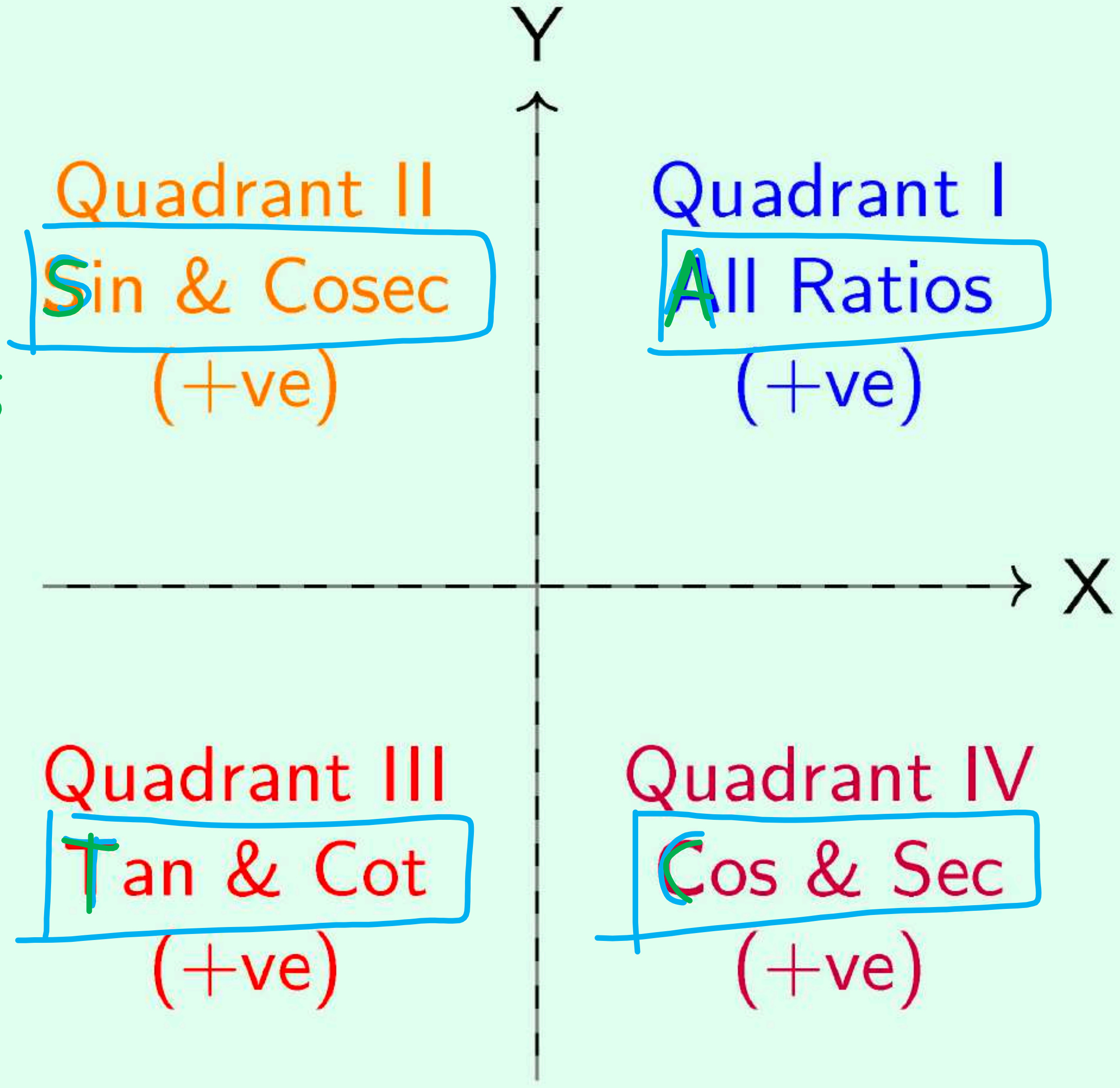
ASTC

A → All

S → students

T → Take

C → coffee



Proof: using new defination

I^{st} quad: x and y both co-ordinates are positive

$$\sin\theta = \frac{Y\text{-cordi}}{r} = \frac{+ve}{+ve} = +ve$$

II^{nd} quad

x-coordinate → $-ve$

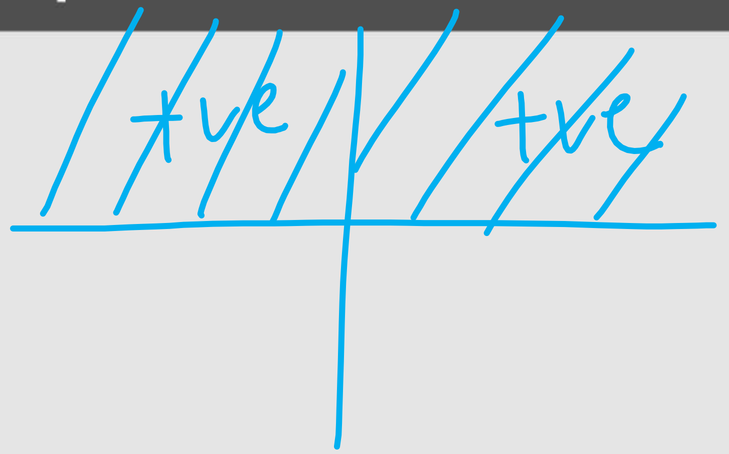
Y-coordinate → $+ve$

$$\sin\theta = \frac{Y\text{-cor}}{r} = \frac{+}{+} = +$$

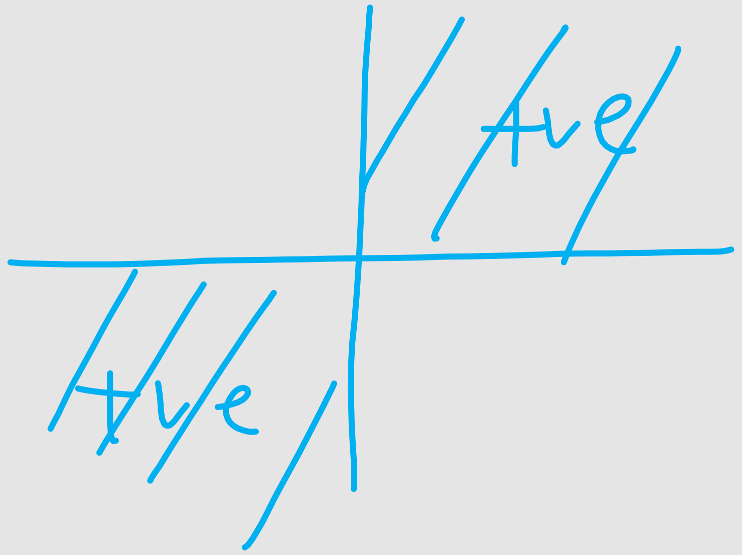
$$\cos\theta = \frac{X\text{-cor}}{r} = \frac{-ve}{+ve} = -ve$$

ASCT: Different Perspective

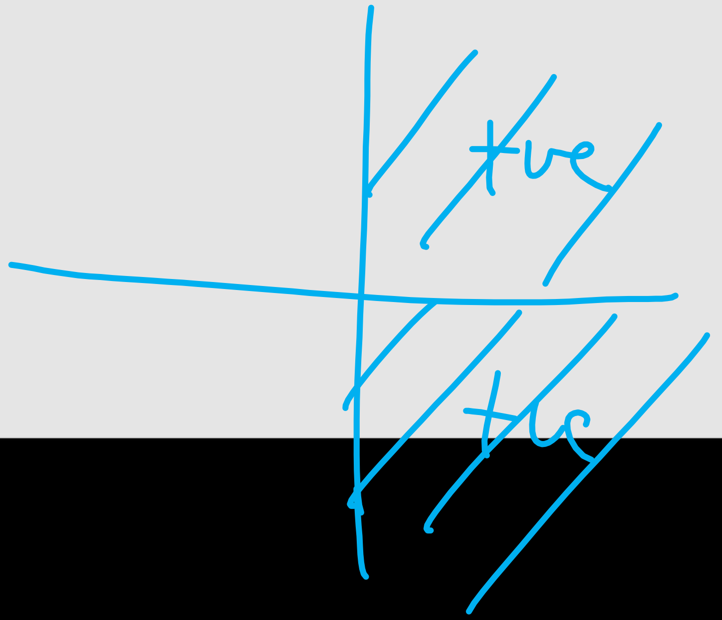
1. sin & cosec



2. tan & cot



3. cos & sec



Trigonometric Function of Allied Angle

1st Question: Change or NO Change?

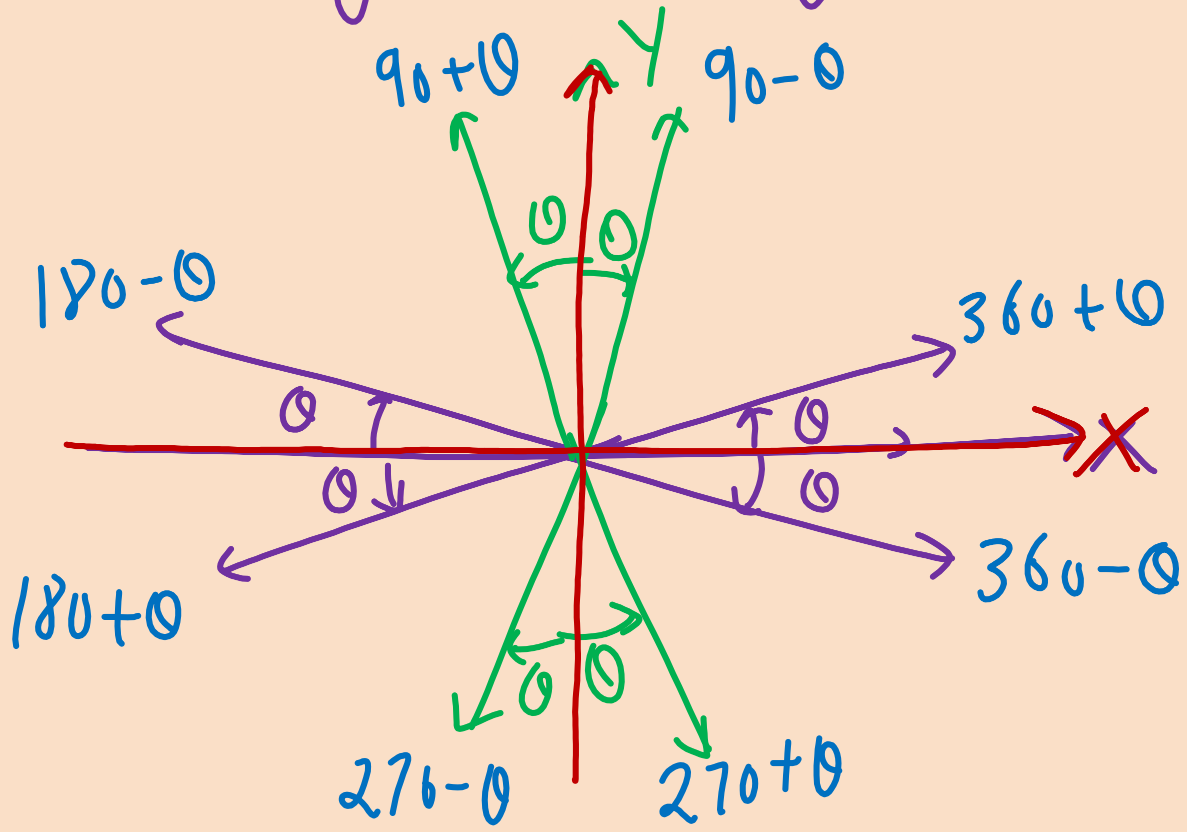
Change kya hoga!

sin ⇌ cos

tan ⇌ cot

sec ⇌ cosec

Change kab hoga!



$90 \pm \theta$, $270 \pm \theta$ } Change hoga

$180 \pm \theta$, $360 \pm \theta$ } Change nhi hoga.

अगर θ X-axis ke saath juda hai ⇒ No change

अगर θ X-axis ke saath juda hai ⇒ Change hoga!

2nd Question: Sign?

ASTC rule will decide the sign

$(90^\circ - \theta) \longrightarrow$ Ist Quad.

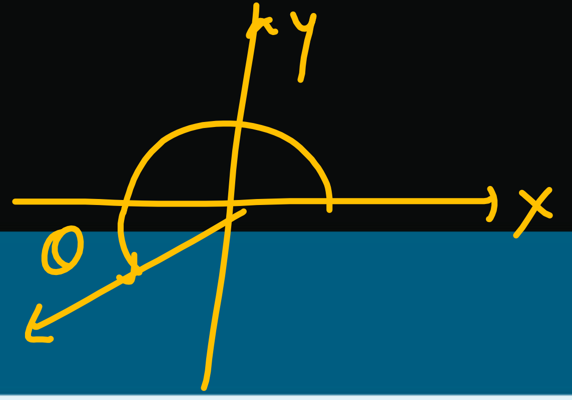
- $\sin(90^\circ - \theta) = + \cos\theta$

  Angle

- $\cos(90^\circ - \theta) = + \sin\theta$
- $\tan(90^\circ - \theta) = + \cot\theta$
- $\cot(90^\circ - \theta) = + \tan\theta$
- $\sec(90^\circ - \theta) = + \operatorname{cosec}\theta$
- $\operatorname{cosec}(90^\circ - \theta) = + \sec\theta$

$(90^\circ + \theta) \longrightarrow$ IInd Quad

- $\sin(90^\circ + \theta) = + \cos\theta$
- $\cos(90^\circ + \theta) = - \sin\theta$
- $\tan(90^\circ + \theta) = - \cot\theta$
- $\cot(90^\circ + \theta) = - \tan\theta$
- $\sec(90^\circ + \theta) = - \operatorname{cosec}\theta$
- $\operatorname{cosec}(90^\circ + \theta) = + \sec\theta$



$(180^\circ - \theta) \longrightarrow$ IInd quad

- $\sin(\underbrace{180^\circ - \theta}_{\text{II}^{nd}\text{ quad}}) = +\sin\theta$
- $\cos(\underbrace{180^\circ - \theta}) = -\cos\theta$
- $\tan(180^\circ - \theta) = -\tan\theta$
- $\cot(180^\circ - \theta) = -\cot\theta$
- $\sec(180^\circ - \theta) = -\sec\theta$
- $\operatorname{cosec}(180^\circ - \theta) = +\operatorname{cosec}\theta$

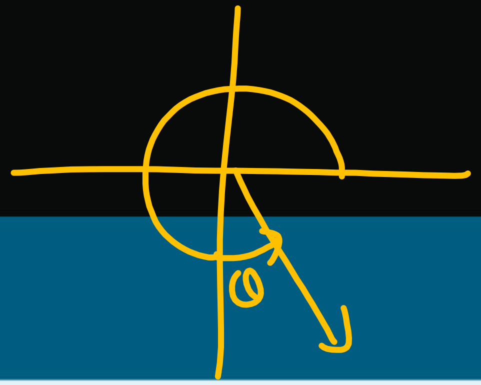
$(180^\circ + \theta) \longrightarrow$ IIIrd quad.

- $\sin(\underbrace{180^\circ + \theta}_{\text{III}^{rd}\text{ quad}}) = -\sin\theta$
- $\cos(180^\circ + \theta) = -\cos\theta$
- $\tan(180^\circ + \theta) = +\tan\theta$
- $\cot(180^\circ + \theta) = +\cot\theta$
- $\sec(180^\circ + \theta) = -\sec\theta$
- $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$

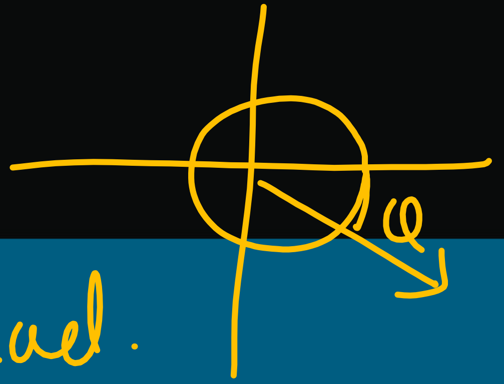
$(270^\circ - \theta) \longrightarrow$ IIIrd Quad

- $\sin(\underbrace{270^\circ - \theta}_{\text{III}^{rd}\text{ Quad}}) = -\cos\theta$
- $\cos(270^\circ - \theta) = -\sin\theta$
- $\tan(270^\circ - \theta) = +\cot\theta$
- $\cot(270^\circ - \theta) = +\tan\theta$
- $\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$
- $\operatorname{cosec}(270^\circ - \theta) = -\sec\theta.$

$(270^\circ + \theta) \longrightarrow$ IVth Quad



- $\sin(\underbrace{270^\circ + \theta}_{\text{IV}^{th}\text{ Quad}}) = -\cos\theta$
- $\cos(270^\circ + \theta) = +\sin\theta$
- $\tan(270^\circ + \theta) = -\cot\theta$
- $\cot(270^\circ + \theta) = -\tan\theta$
- $\sec(270^\circ + \theta) = +\operatorname{cosec}\theta$
- $\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$



$(360^\circ - \theta) \longrightarrow$ IVth Quad.

- $\sin(\underbrace{360^\circ - \theta}_{\text{IV}^{th}\text{ quad}}) = -\sin\theta$
- $\cos(360^\circ - \theta) = +\cos\theta$
- $\tan(360^\circ - \theta) = -\tan\theta$
- $\cot(360^\circ - \theta) = -\cot\theta$
- $\sec(360^\circ - \theta) = +\sec\theta$
- $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$

$(360^\circ + \theta) \longrightarrow$ Ist Quad

- $\sin(\underbrace{360^\circ + \theta}_{\text{I}^{st}\text{ quad}}) = +\sin\theta$
- $\cos(360^\circ + \theta) = +\cos\theta$
- $\tan(360^\circ + \theta) = +\tan\theta$
- $\cot(360^\circ + \theta) = +\cot\theta$
- $\sec(360^\circ + \theta) = +\sec\theta$
- $\operatorname{cosec}(360^\circ + \theta) = +\operatorname{cosec}\theta$.

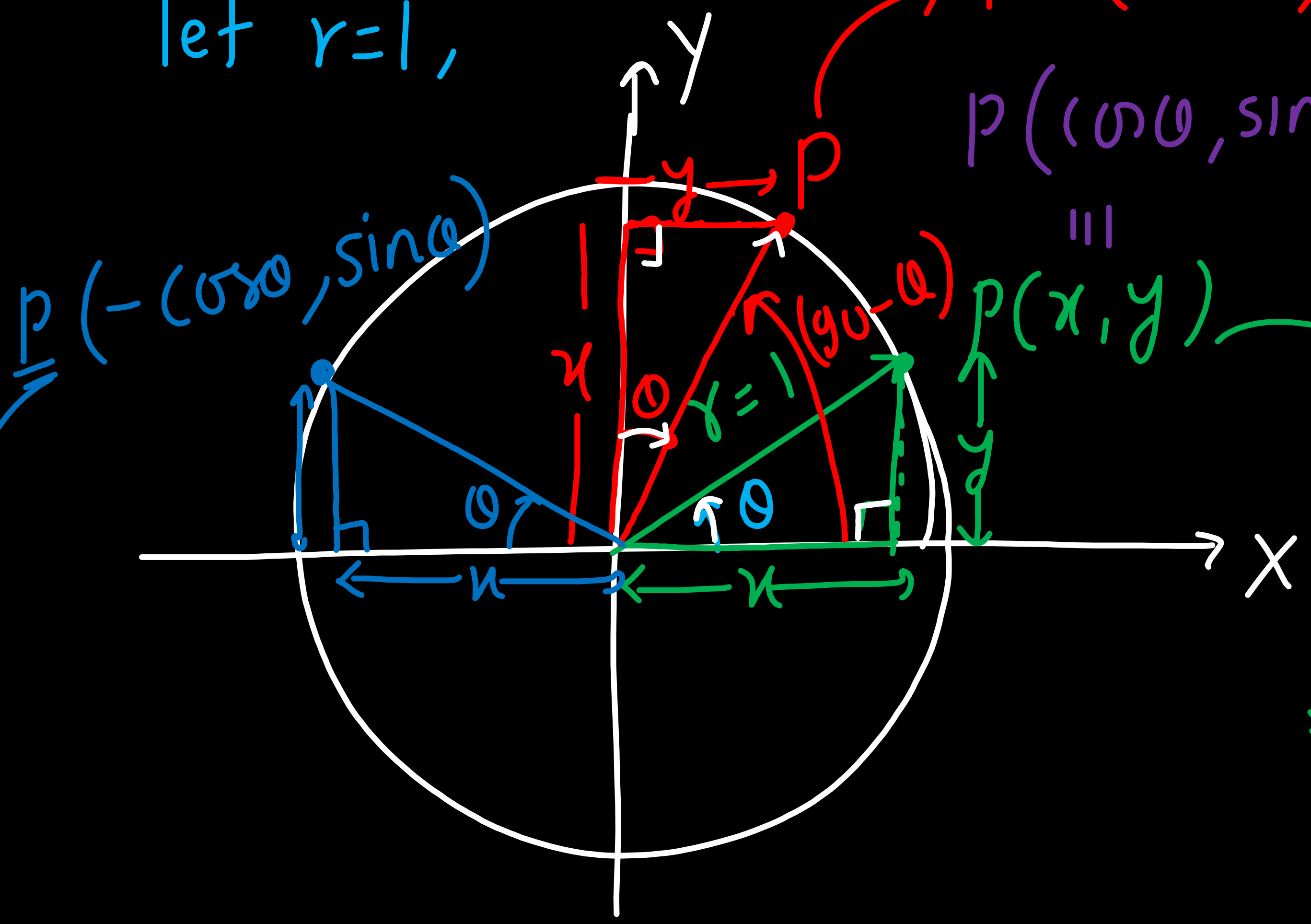
Not necessary for JEE / Proof not important

Why ? $\sin(90-\theta) = \cos\theta$

$P \equiv (\sin\theta, \cos\theta) \longrightarrow * \sin(90-\theta) = \dfrac{y\text{-coordinate}}{r=1}$

$\boxed{\sin(90-\theta) = \cos\theta}$

let $r=1$,



$p(x,y) \longrightarrow * \sin\theta = \dfrac{y\text{-coordinate}}{r=1}$

$\boxed{\sin\theta = y}$

$* \cos(90-\theta) = \dfrac{x\text{-coordinate}}{r=1}$

$\boxed{\cos(90-\theta) = \sin\theta}$

$* \cos\theta = \dfrac{x\text{-coordinate}}{r=1}$

$\boxed{\cos\theta = x}$

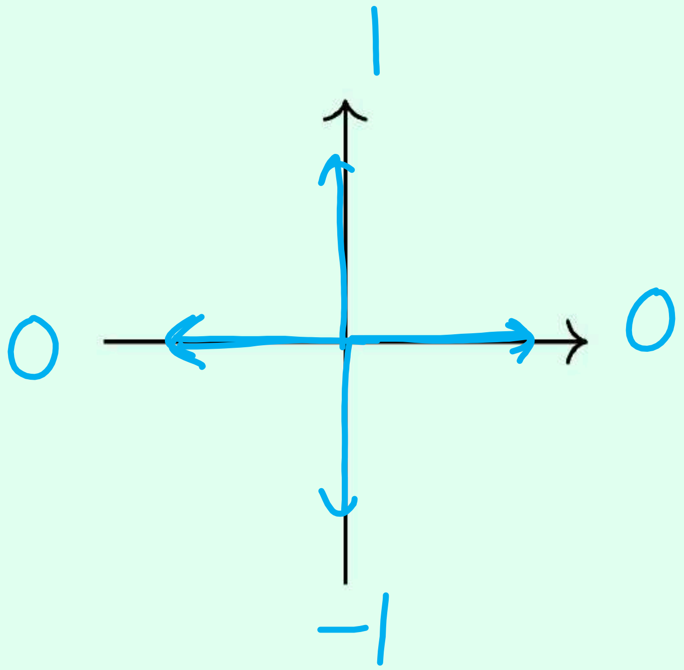
Both green & red triangles are congruent

$* \sin(180-\theta) = \sin\theta$

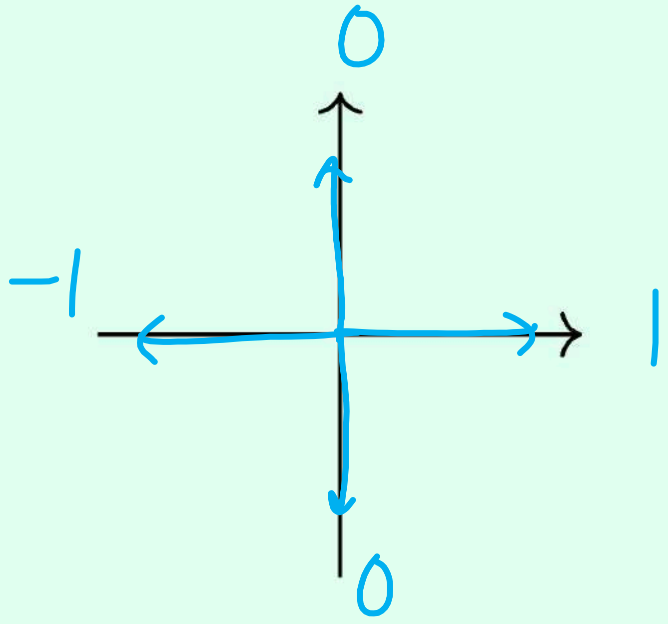
$* \cos(180-\theta) = -\cos\theta$

Note: T-Ratio Values at Quadrantal Angles (0°, 90°, 180°, 270°)

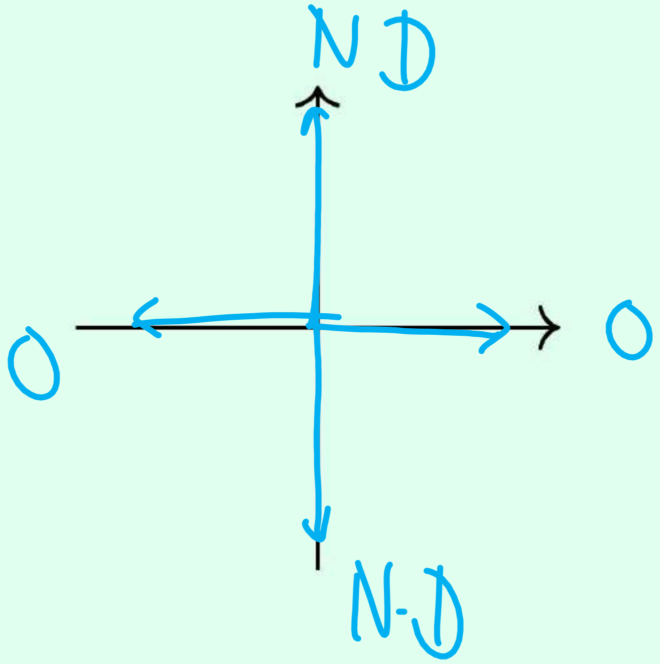
1) $\sin\theta$



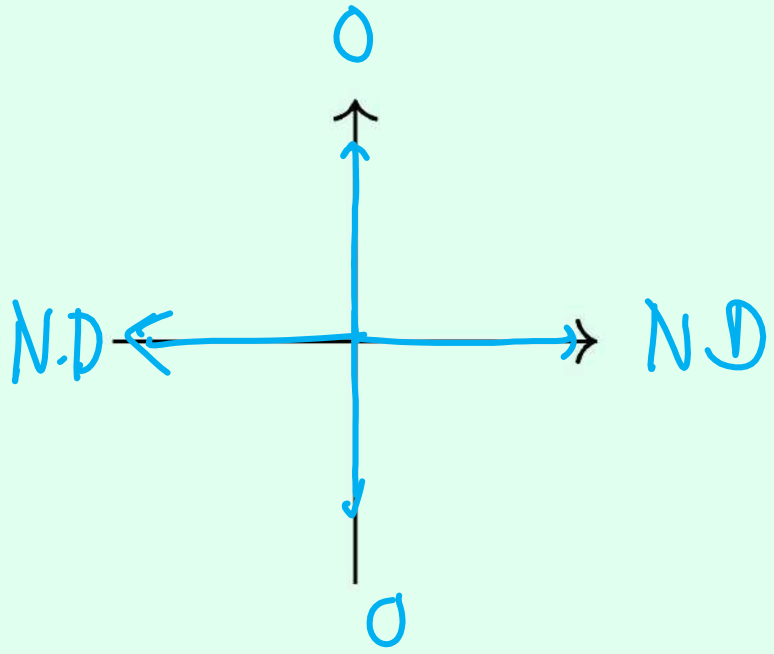
2) $\cos\theta$



3) $\tan\theta$



4) $\cot\theta$



*

$\sin\theta = 0 \Rightarrow$ on X-axis

$\cos\theta = 0 \Rightarrow$ on Y-axis

$\tan\theta = 0 \Rightarrow$ on X-axis

$\cot\theta = 0 \Rightarrow$ on Y-axis

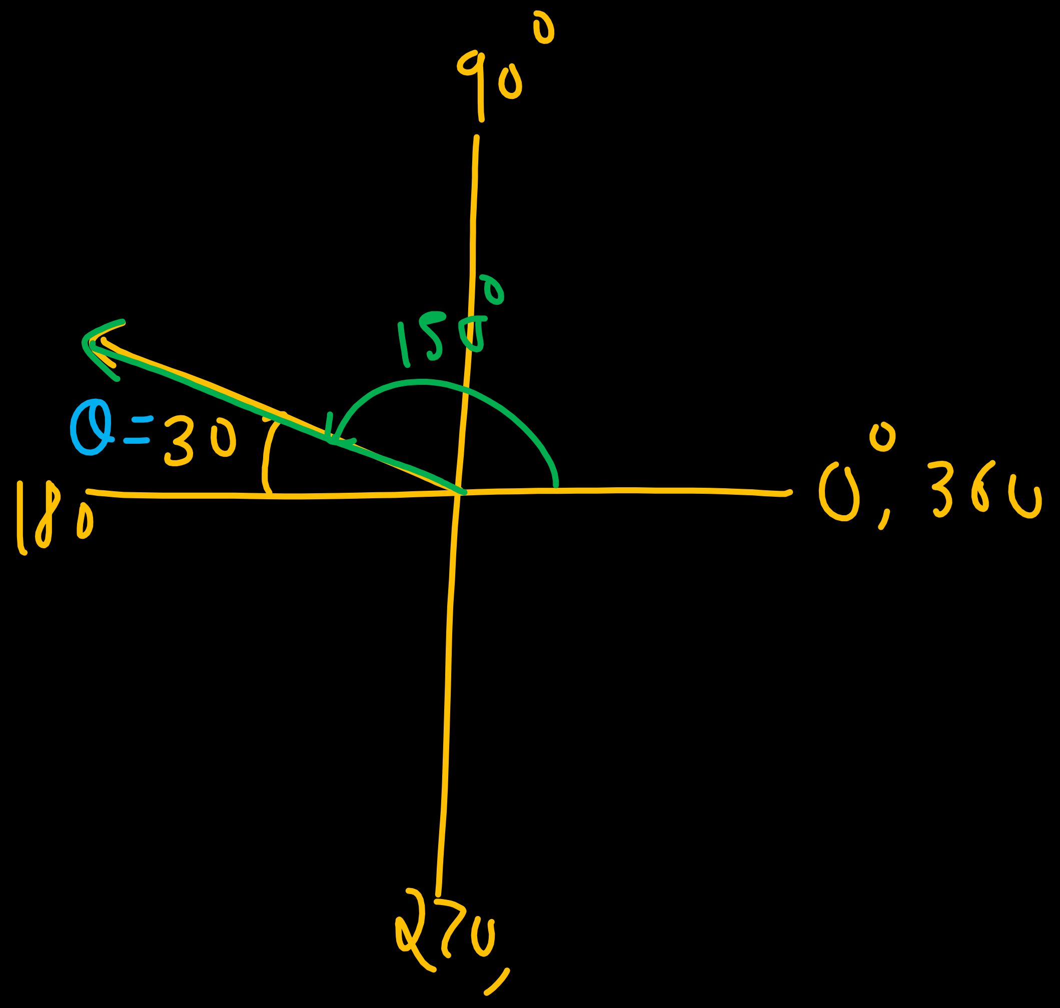
Example: 1

$\sin(150^\circ) =$

M-1

$$\sin 150^\circ = \sin(180^\circ - \underset{\theta}{\underset{\parallel}{30^\circ}}) = + \sin 30$$

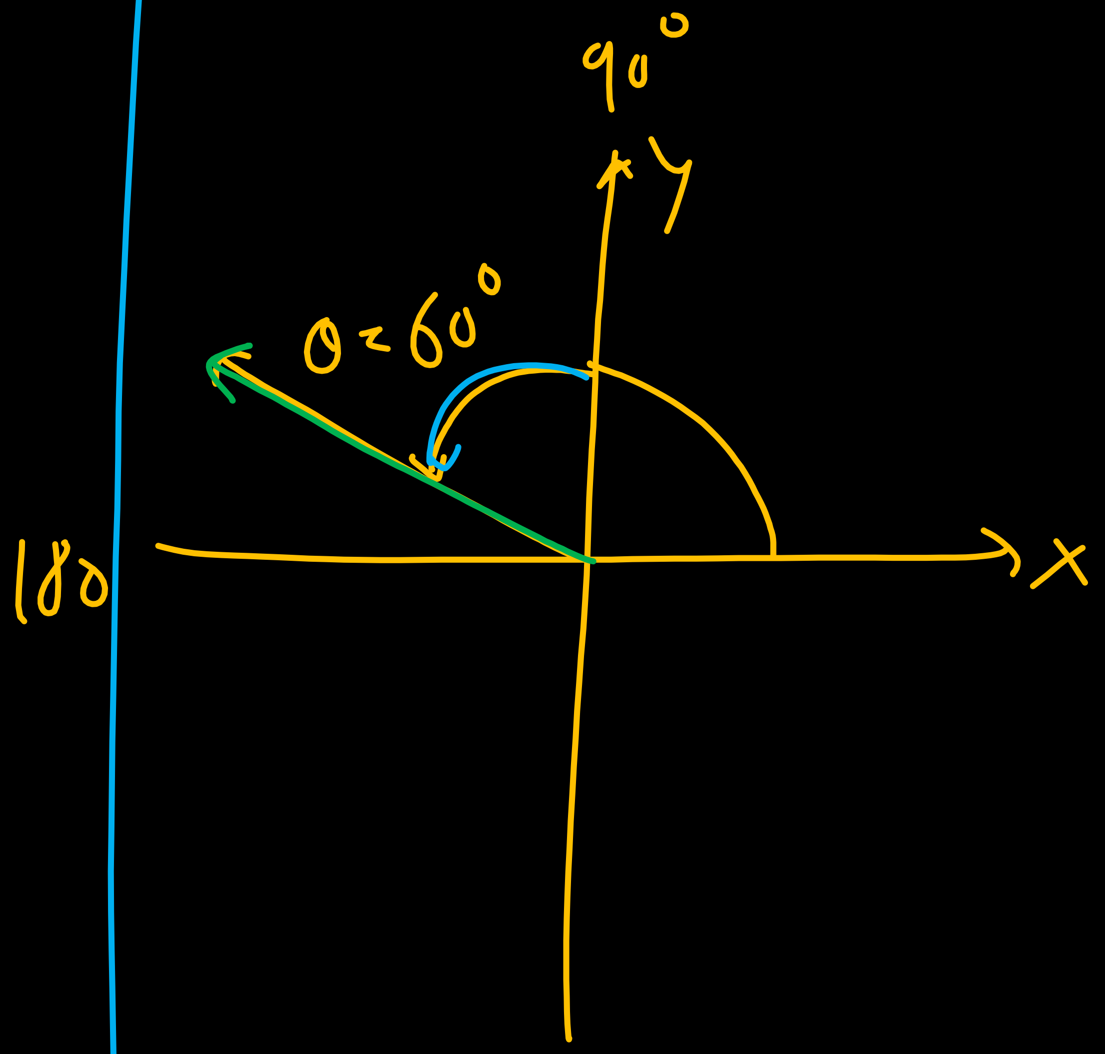
$$= \frac{1}{2}$$



M-2

$$\sin(150^\circ) = \sin(90^\circ + \underset{\theta}{\underset{\parallel}{60^\circ}}) = + \cos 60$$

$$= + \frac{1}{2}$$



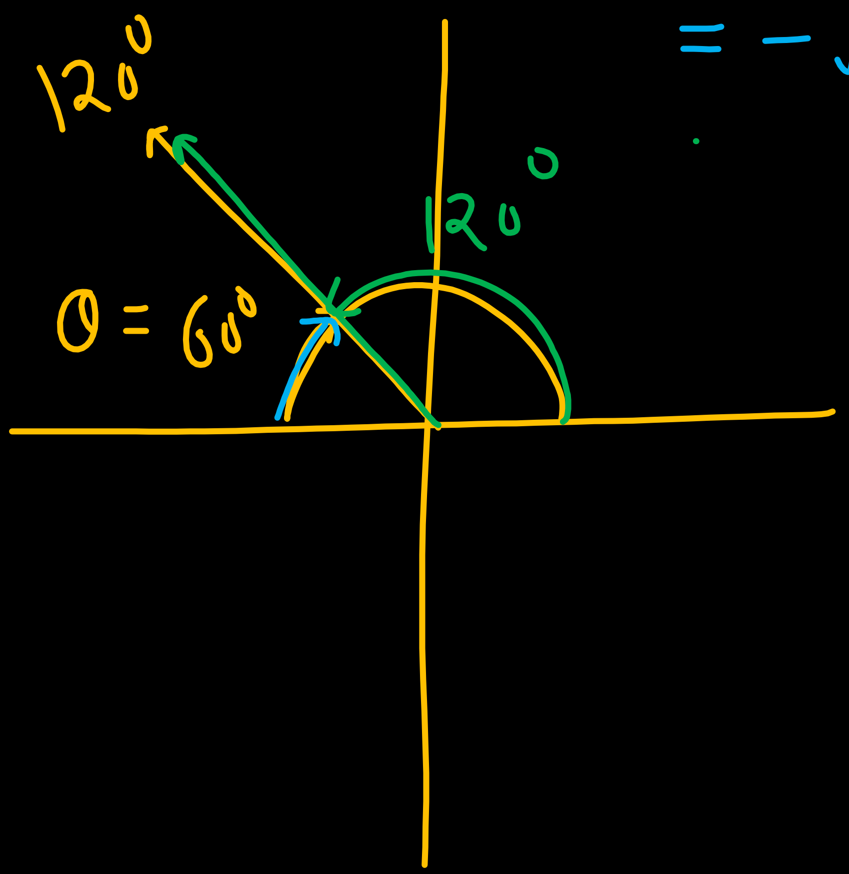
Example: 2

$\tan(120^\circ) =$

M-1 $\tan 120^\circ = \tan(180^\circ - 60^\circ)$

$= -\tan 60$

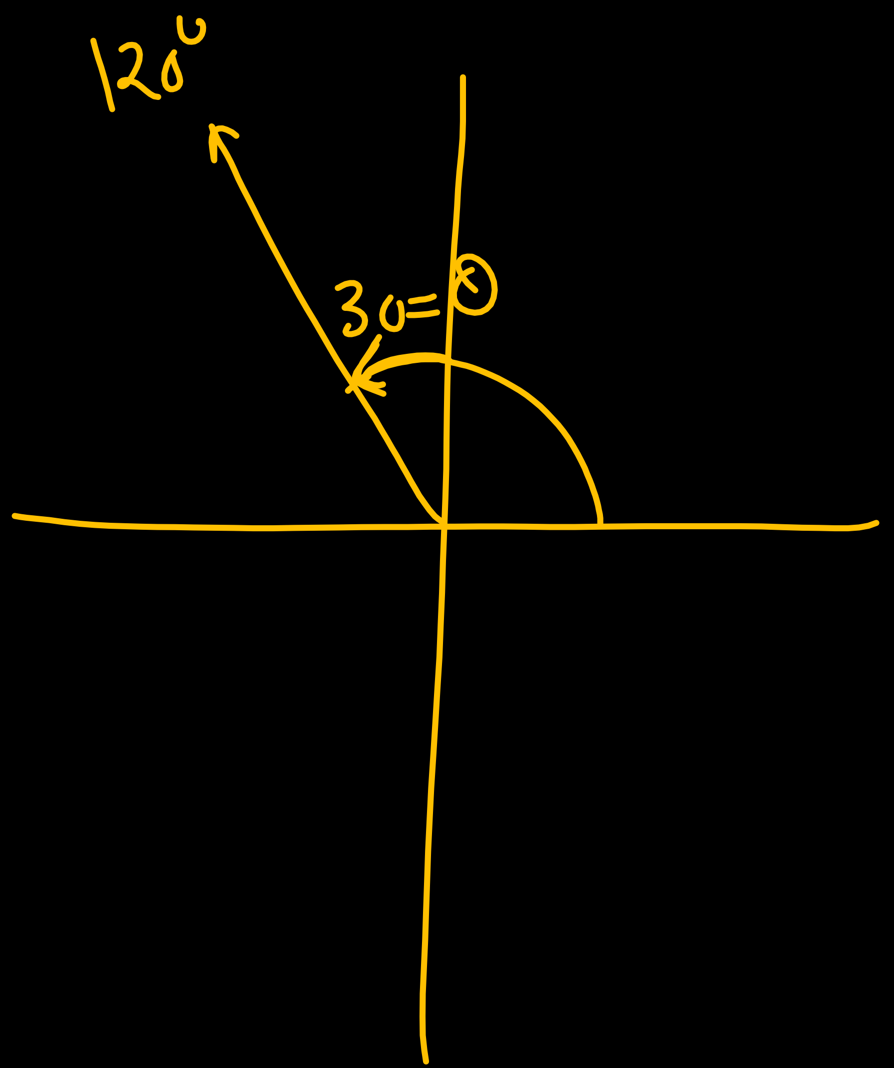
$= -\sqrt{3}$



M-2 $\tan 120^\circ = \tan(90^\circ + 30^\circ)$

$= -\cot 30$

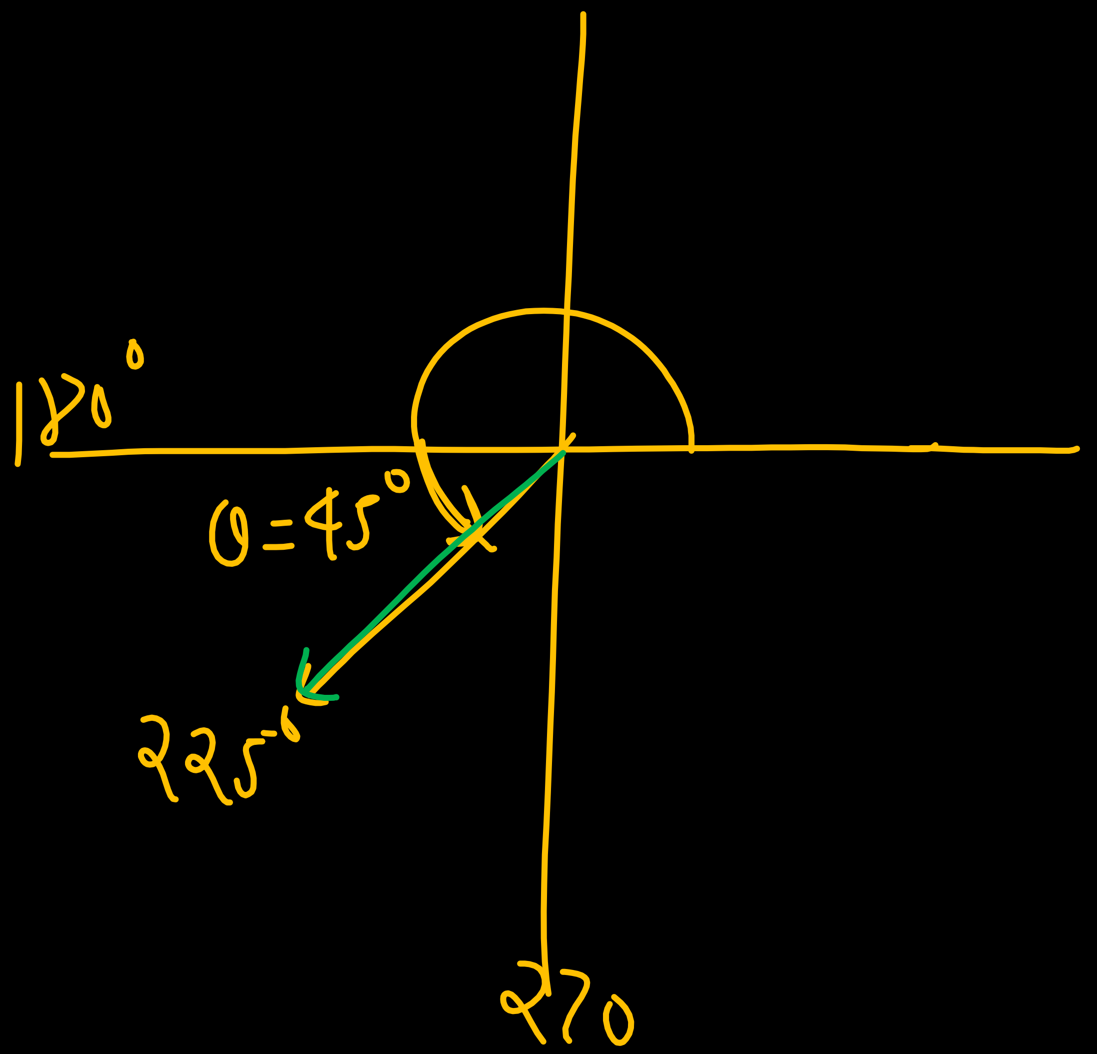
$= -\sqrt{3}$



Example: 3

$\cos(225^\circ) =$

$$\cos(225^\circ) = \cos(\underbrace{180^\circ + 45^\circ}_{\text{III}^{rd}\text{ quad}}) = -\cos 45 = -\frac{1}{\sqrt{2}}$$

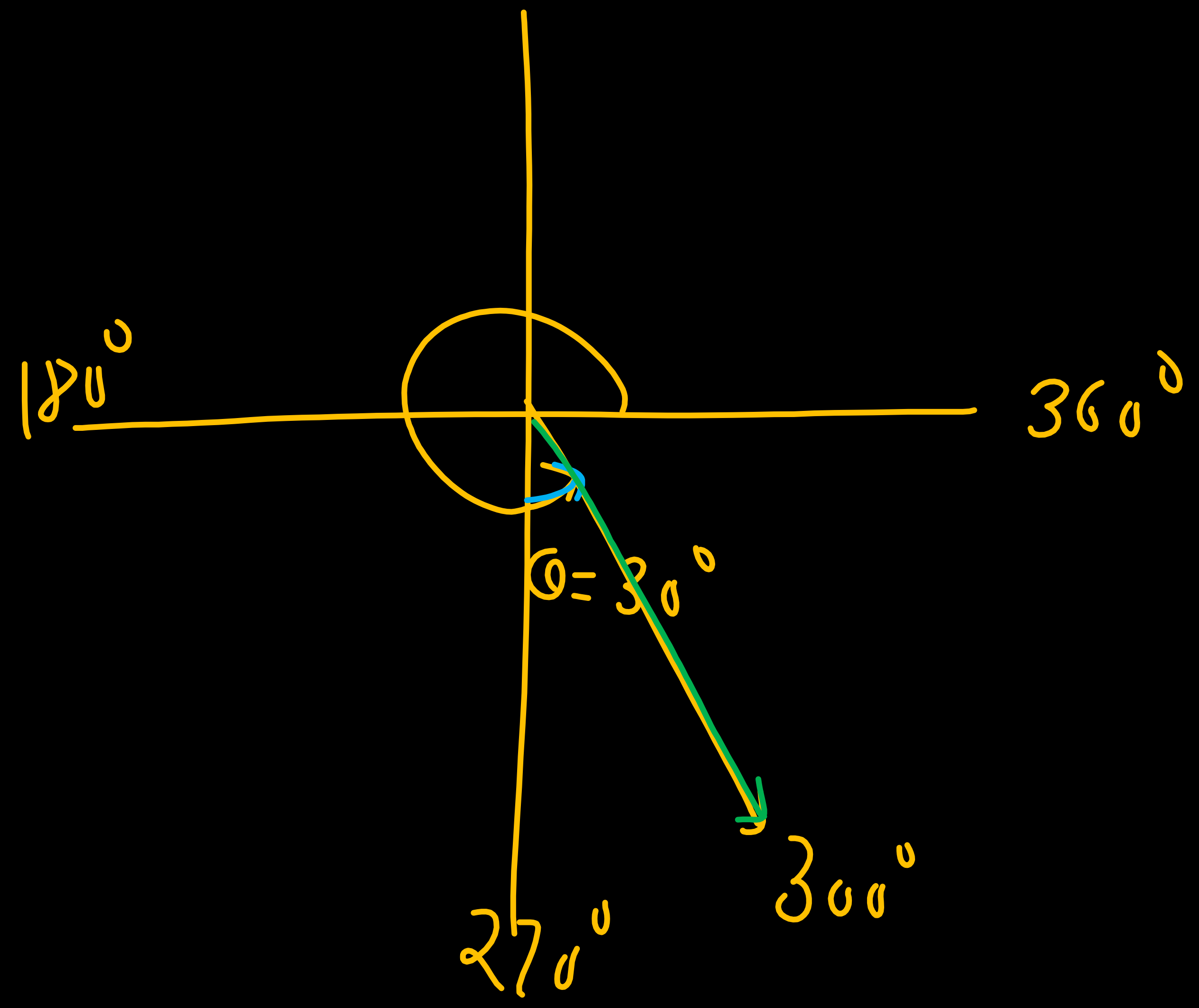


Example: 4

$\tan(300^\circ) =$

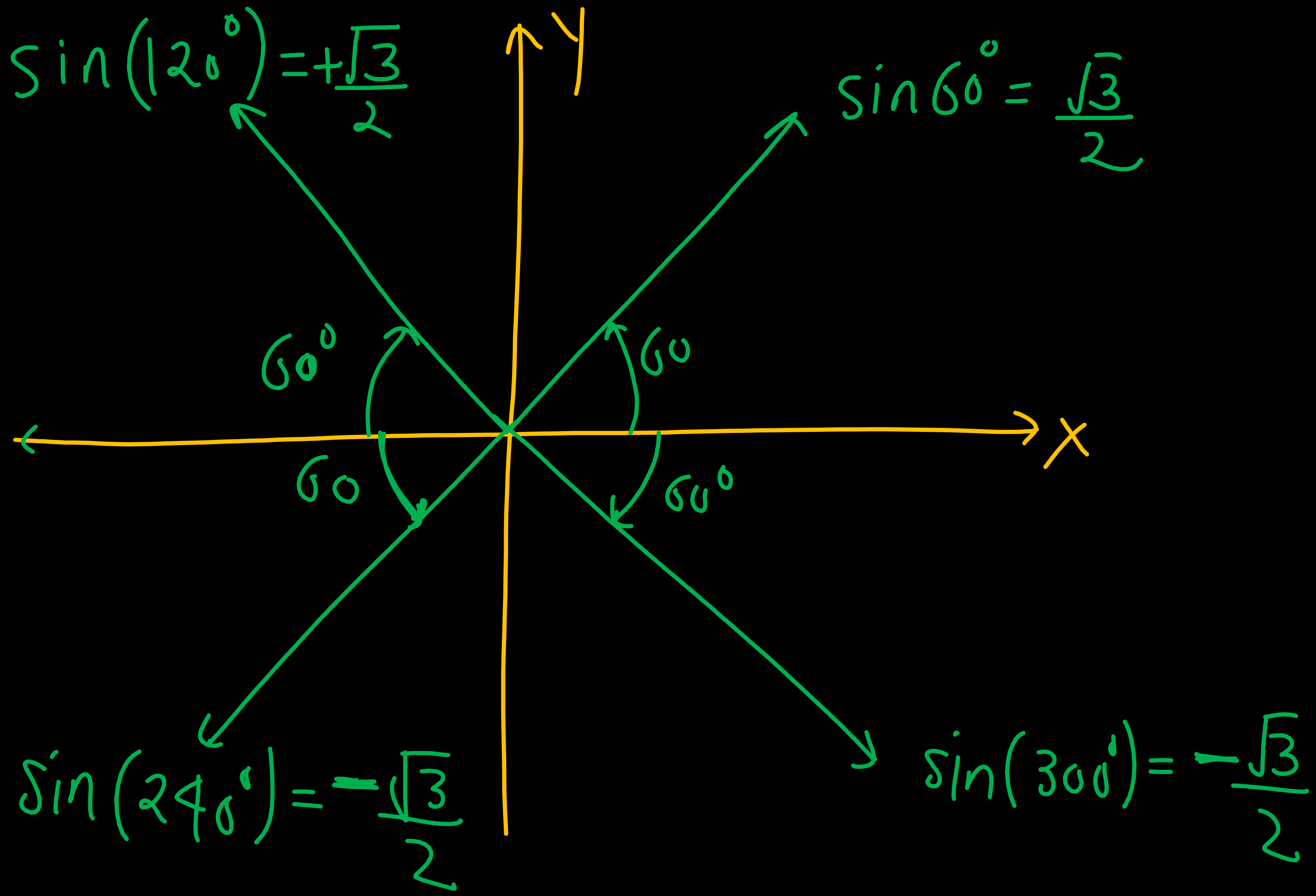
$$\tan 300^\circ = \tan(\underbrace{270^\circ + 30^\circ}_{\text{IV}^{th}\text{ quad}}) = -\cot 30$$

$$= -\sqrt{3}$$



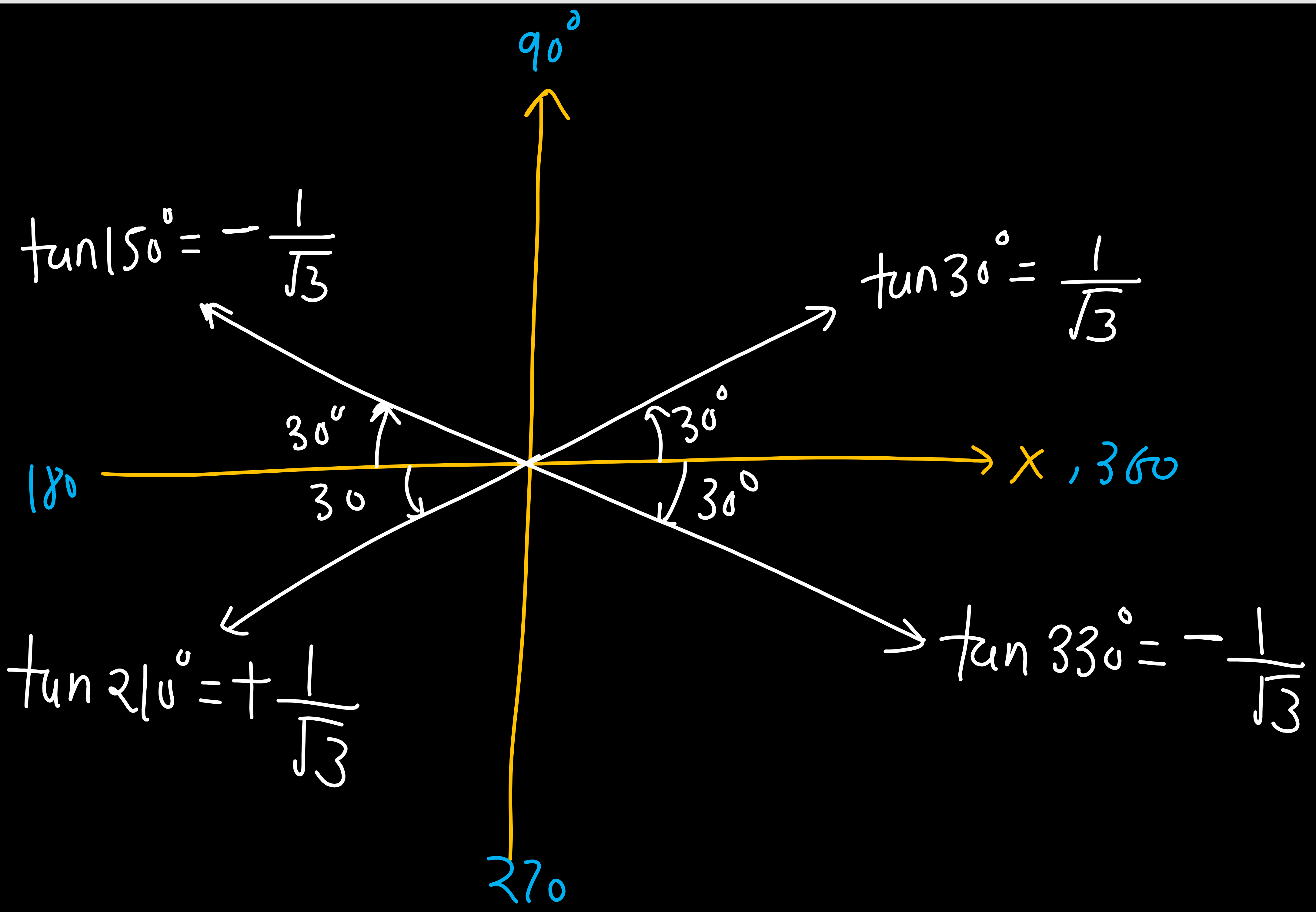
Example: 5

$\sin(120^\circ) =$



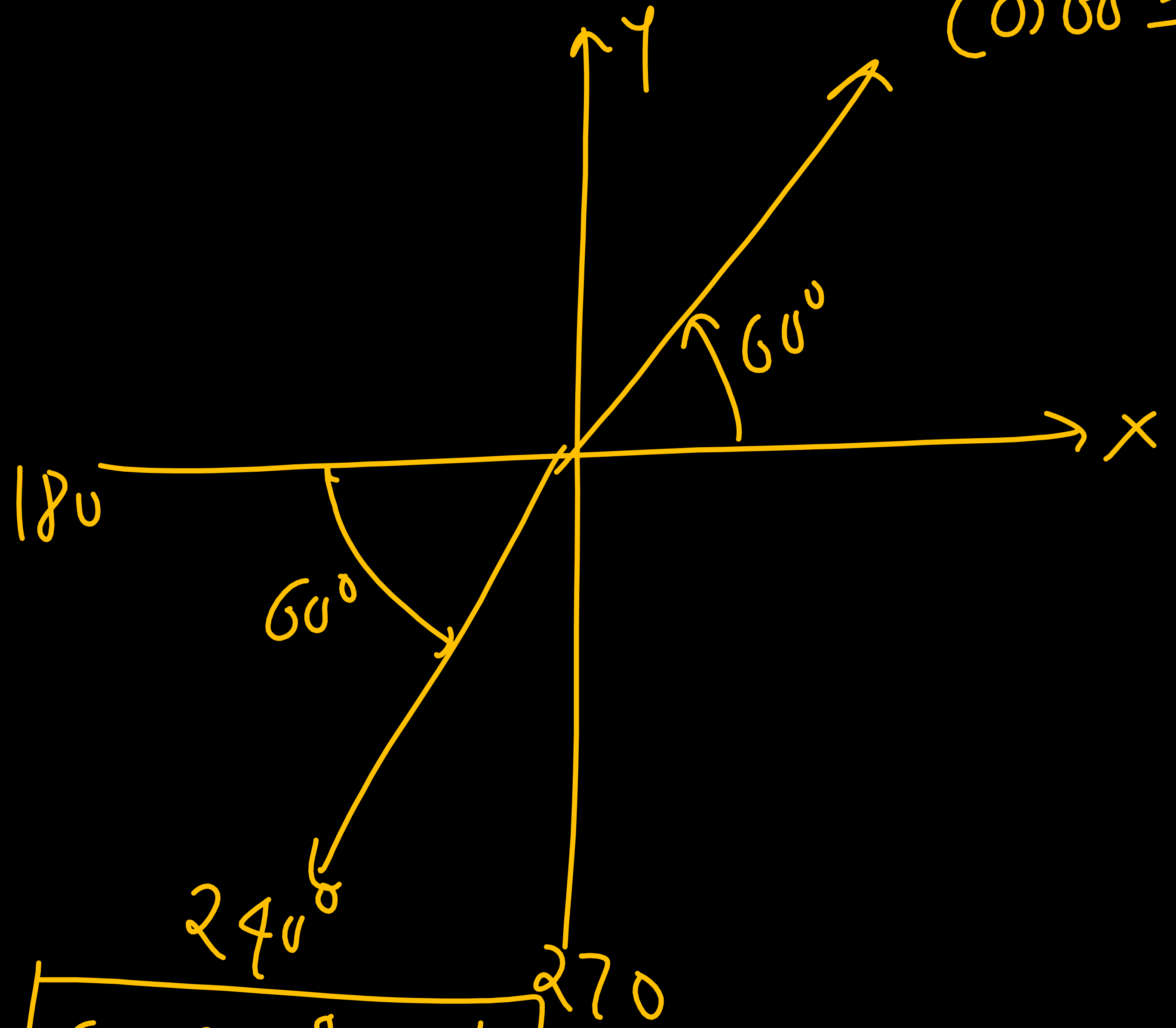
Example: 6

$\tan(150^\circ) =$



Example: 7

$\cos(240^\circ) =$

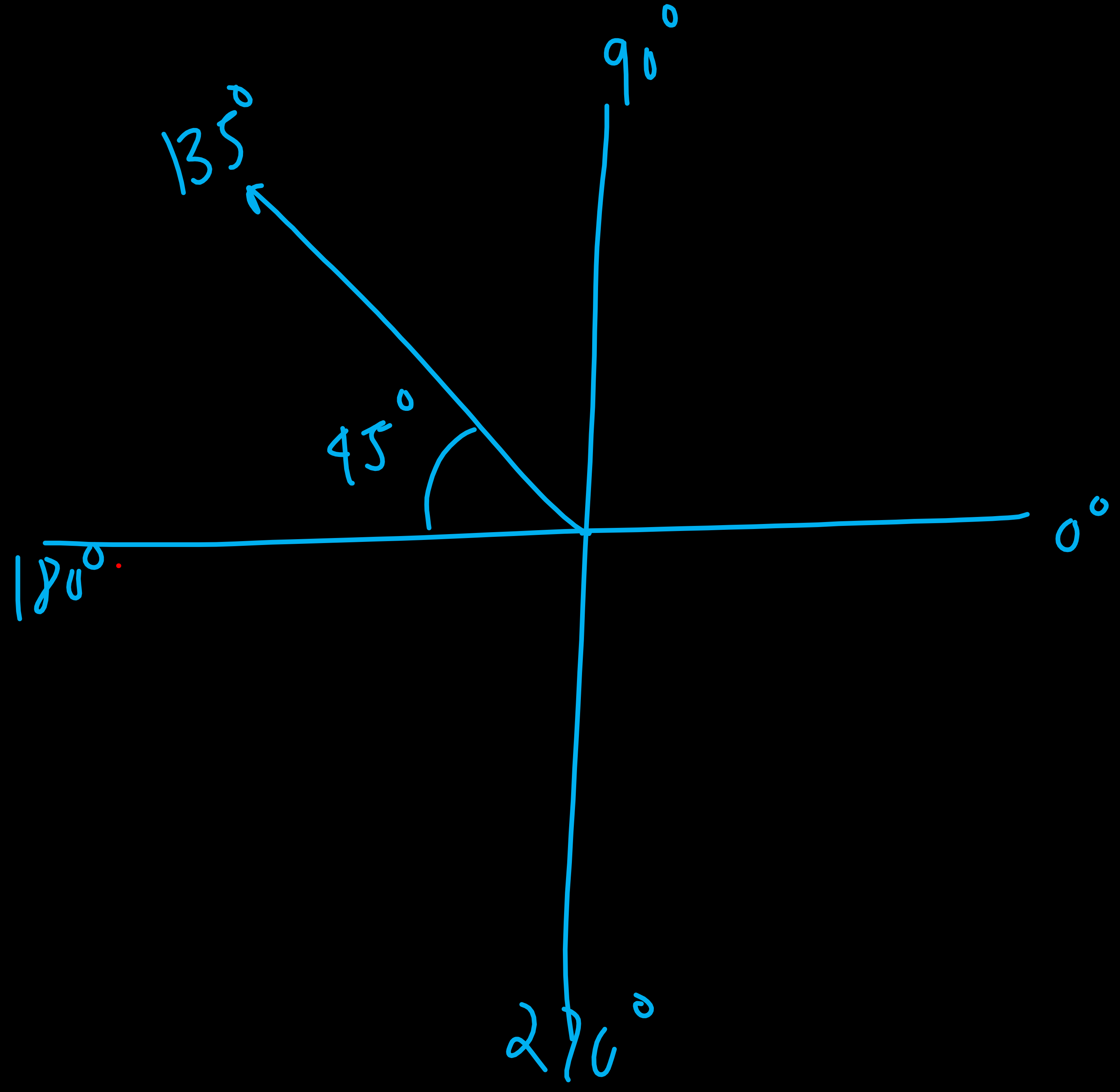


$\cos 60^\circ = \frac{1}{2}$

$\boxed{\cos 240^\circ = -\frac{1}{2}}$

Example: 8

$\sec(135^\circ) =$

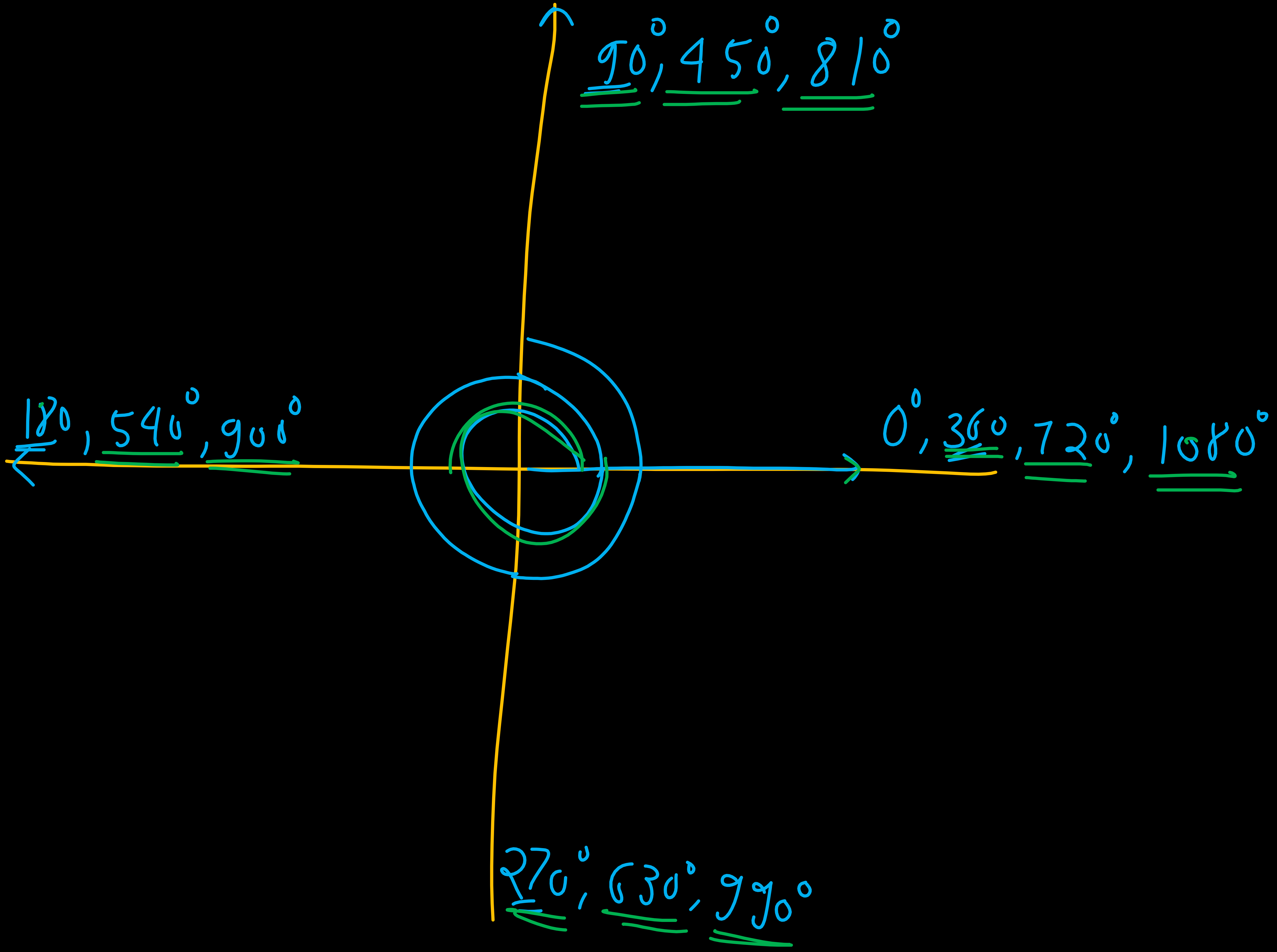


$$\sec(135^\circ) = \sec(180^\circ - 45^\circ)$$

$$= -\sec 45$$

$$= -\sqrt{2}$$

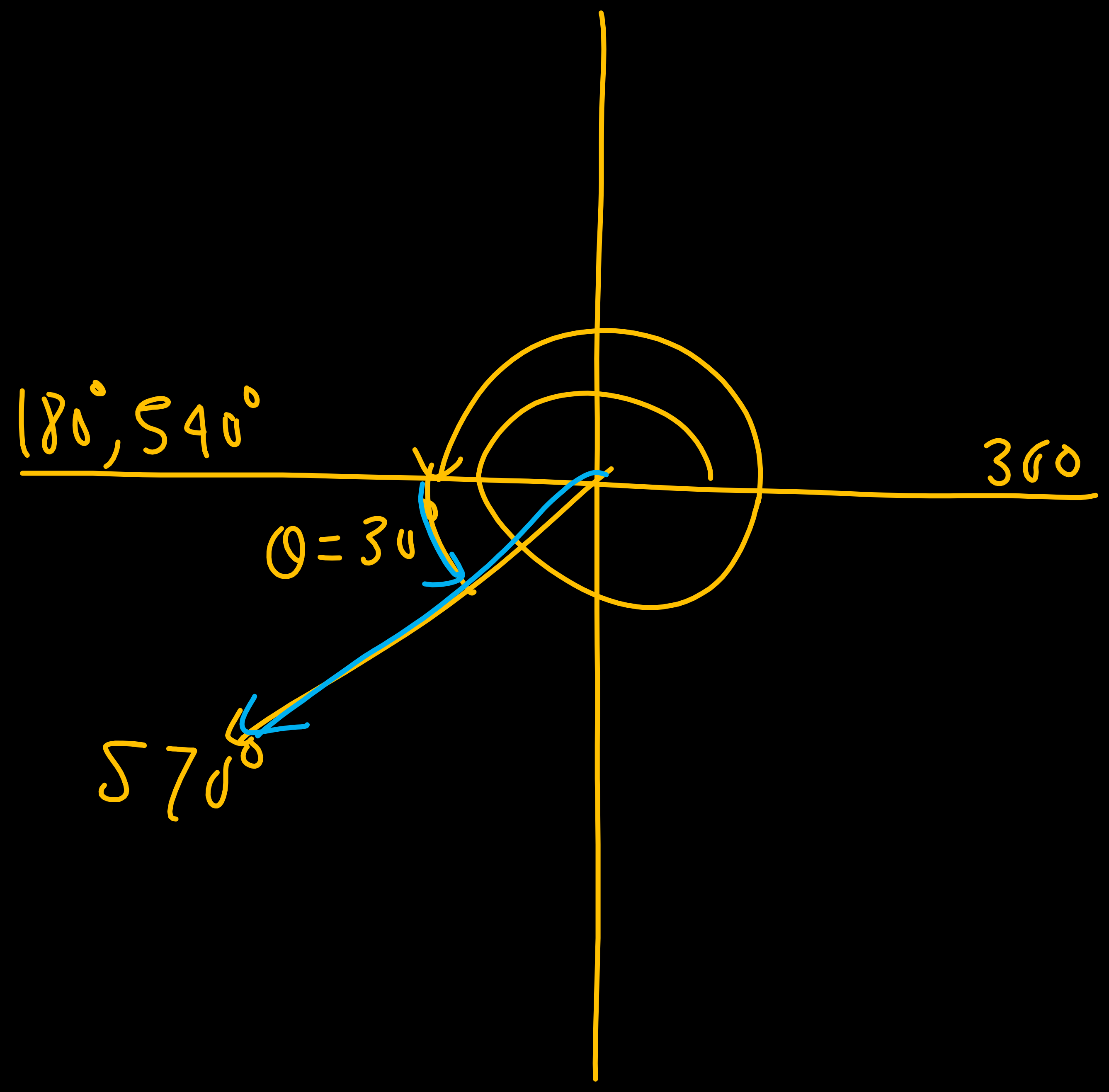
Note



Example: 9

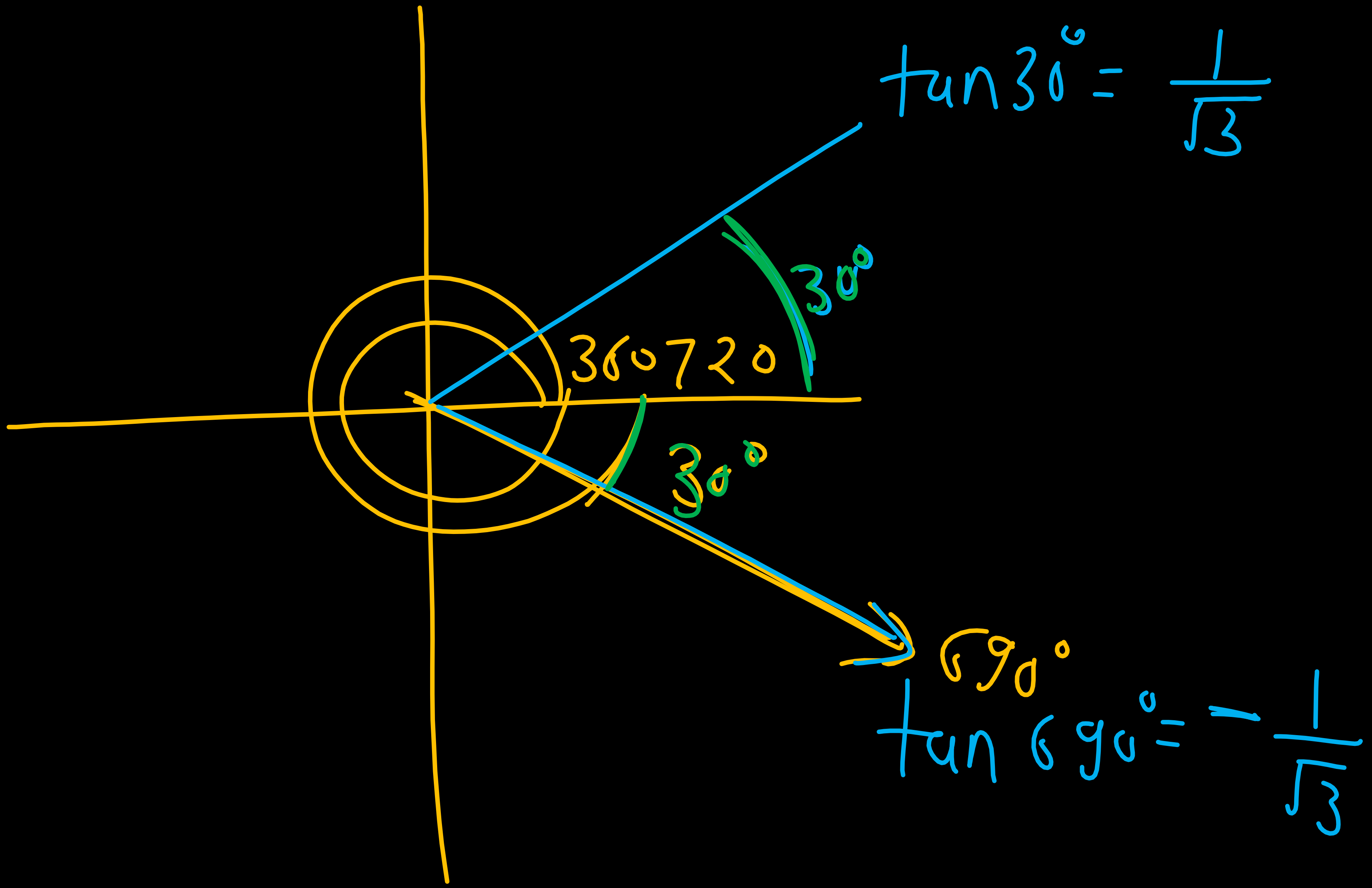
$\cos(570^\circ) =$

$$\cos(570^\circ) = \cos(540^\circ + 30^\circ) = -\cos 30^\circ$$



Example: 10

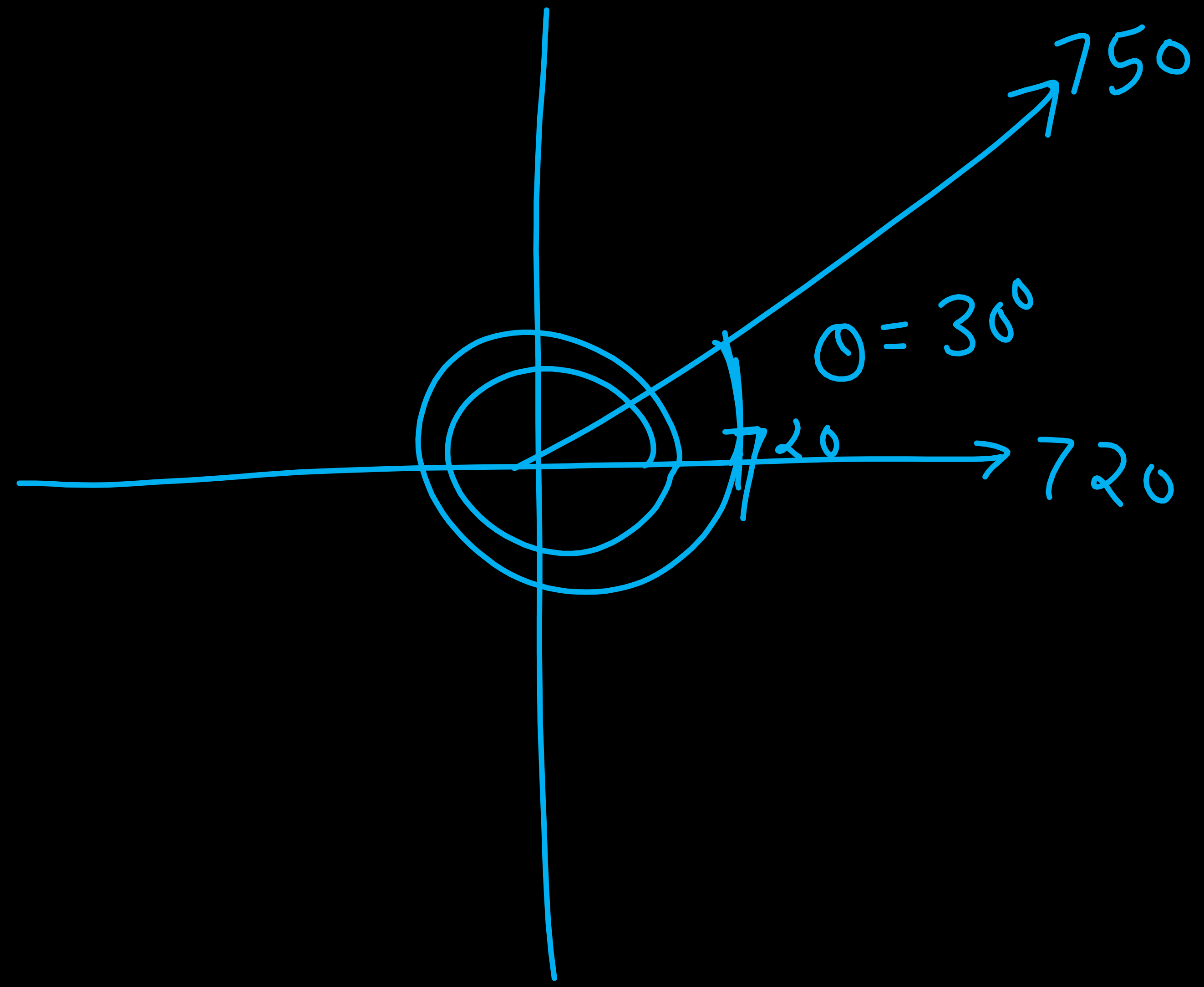
$\tan(690^\circ) =$



Example: 11

$\text{cosec}(750^\circ) =$

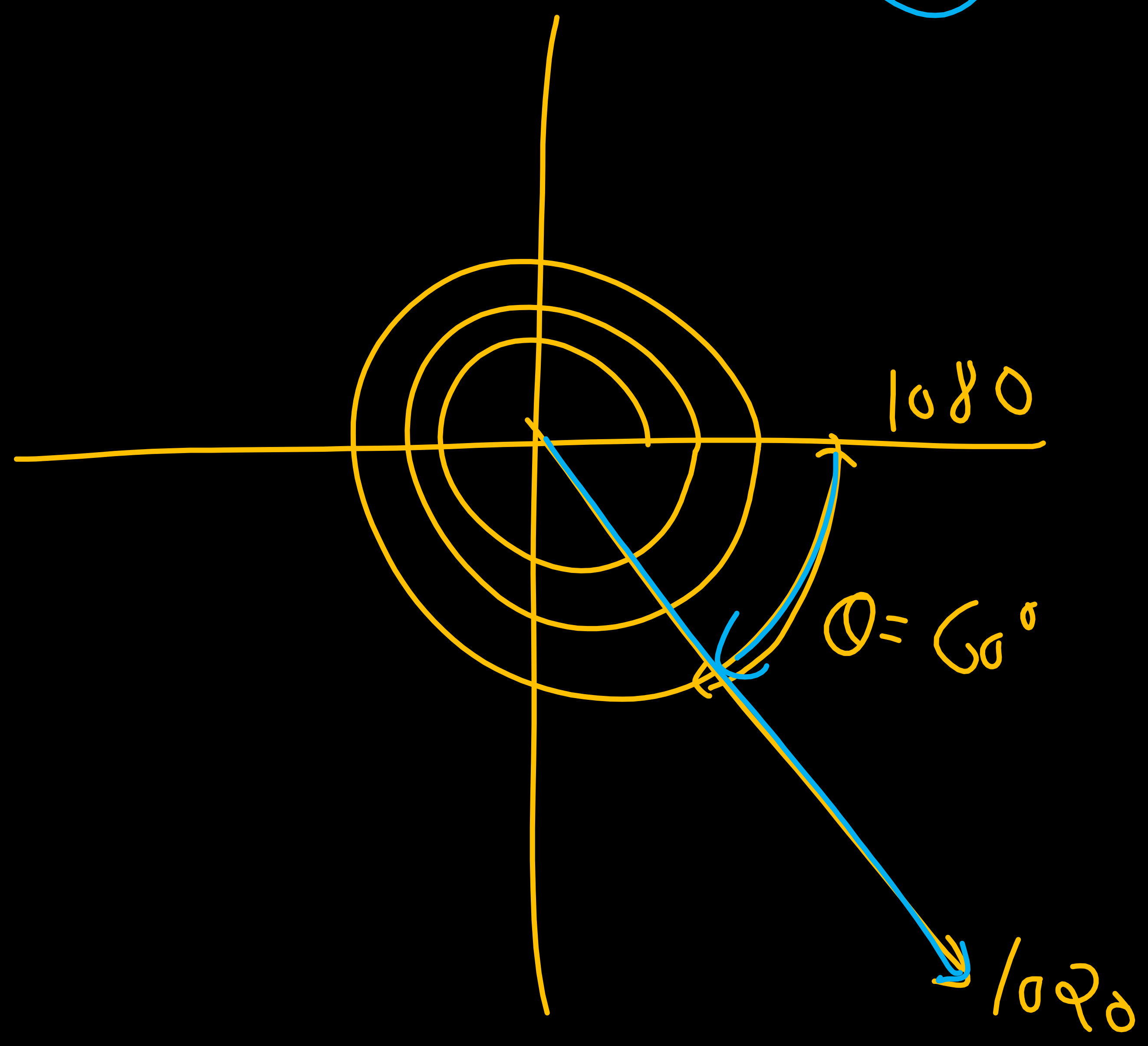
$\text{cosec}(750^\circ) = \text{cosec}(720^\circ + 30^\circ) = + \text{cosec}\,30^\circ$



Example: 12

$\sin(1020^\circ) =$

$$\sin(1020^\circ) = \sin(\underbrace{1080^\circ - 60^\circ}_{\text{IV}^{\text{th}}\text{ quad}}) = -\sin 60 = -\frac{\sqrt{3}}{2}$$



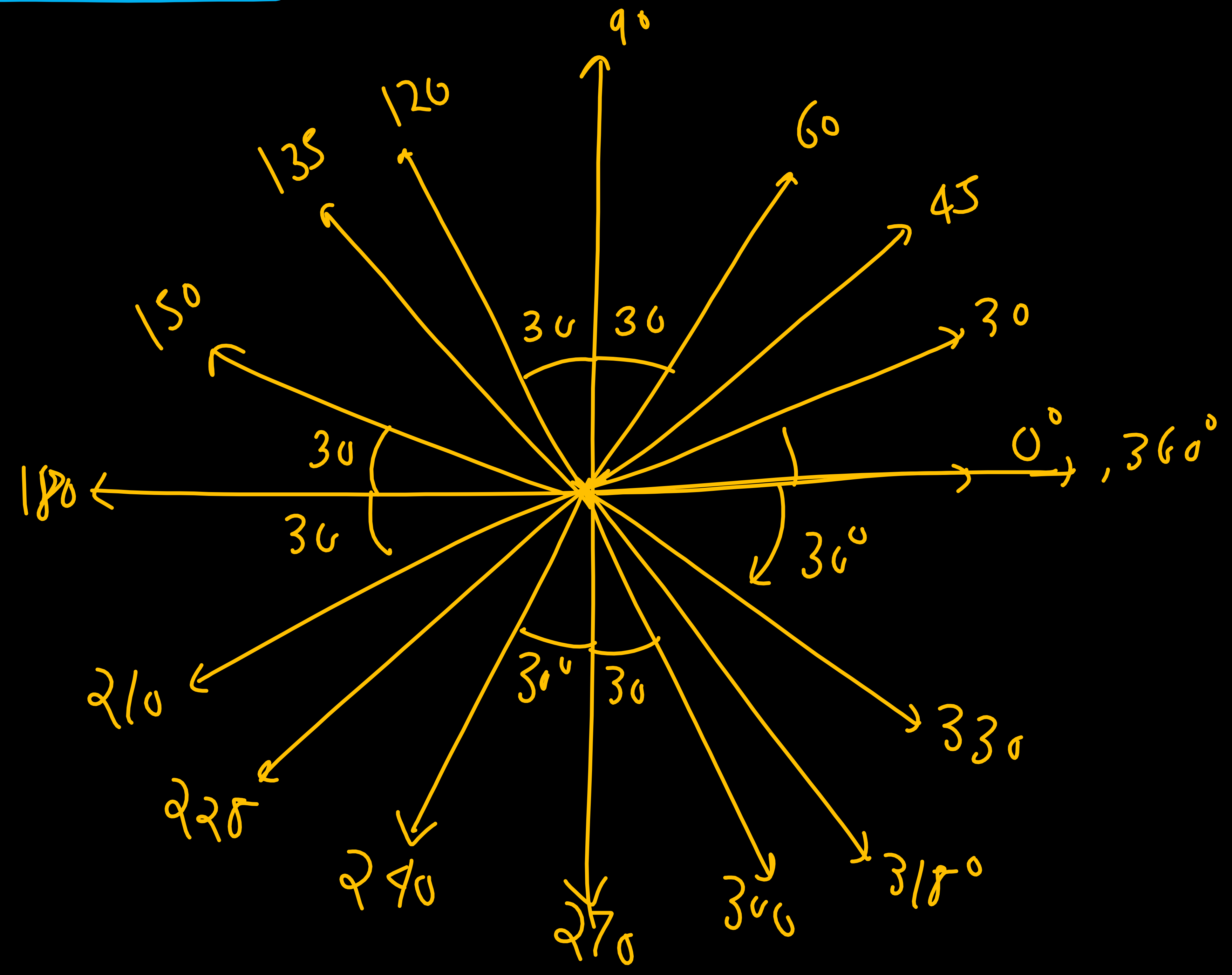
Write down this table in classnotes by finding each value

Trigonometric Ratios Table (0 to 2π)

Ratio	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$\csc\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	ND
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND

$$\sin(210^\circ) = \sin(180+30) = -\sin 30 = -\frac{1}{2}$$

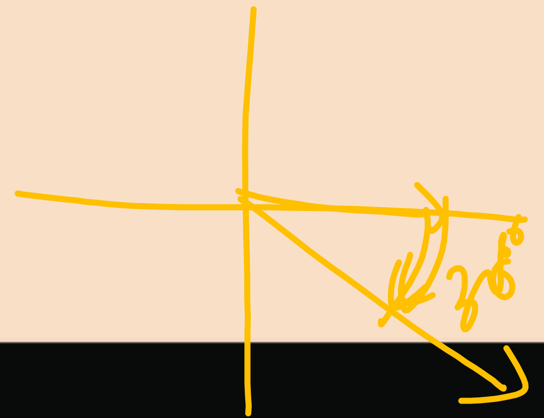
IMP Angles for tables

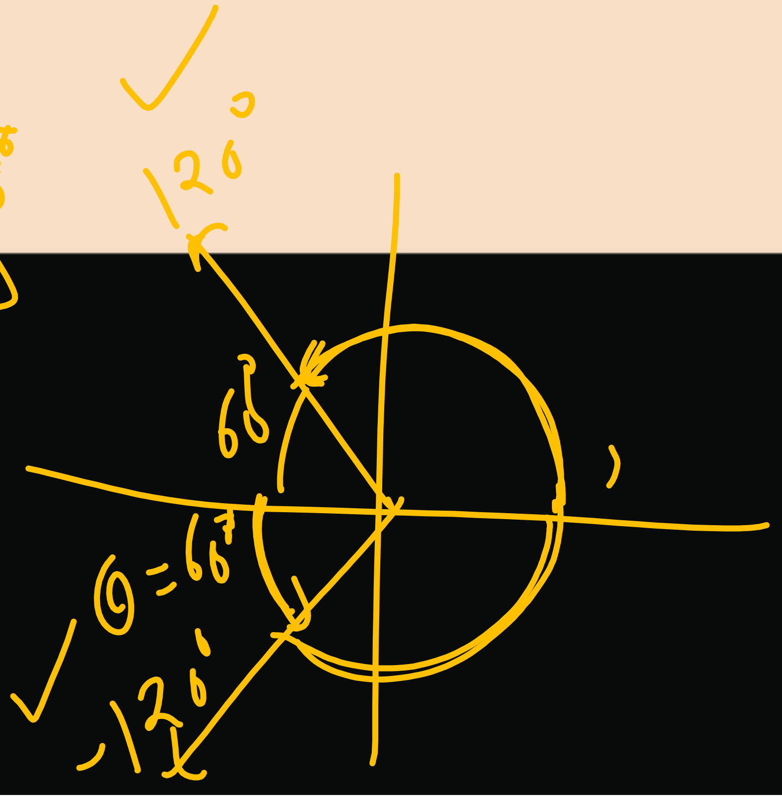


Formulas for $(-\theta)$

- $\sin(-\theta) = -\sin\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\text{cosec}(-\theta) = -\text{cosec}\,\theta$
- $\cos(-\theta) = \cos\theta$
- $\sec(-\theta) = \sec\theta$

Proof: $\cos(-30^\circ) = \cos(0^\circ - 30^\circ) = +\cos 30$





$\cos(120^\circ) = \cos(-120^\circ)$

Example: 13

$\sin(-150^\circ) =$

$$\sin(-150^\circ) = -\left[\underline{\underline{\sin 150^\circ}}\right]$$

$$= -\left[\sin(\underline{180^\circ - 30^\circ})\right]$$

$$= -\left[+\sin 30\right]$$

$$= -\frac{1}{2}$$ //

Example: 14

$\cos(-225^\circ) =$

$$\cos(-225^\circ) = \cos 225^\circ$$
$$= \cos(\underline{\underline{180}} + 45)$$
$$= -\cos 45$$
$$= -\frac{1}{\sqrt{2}}$$

Example: 15

$\tan(-330^\circ) =$

$$\tan(-330^\circ) = -\tan(330^\circ)$$
$$= -\left[\tan(360^\circ - 30^\circ)\right]$$
$$= -\left[-\tan 30^\circ\right]$$
$$= \tan 30^\circ$$
$$= \frac{1}{\sqrt{3}}$$

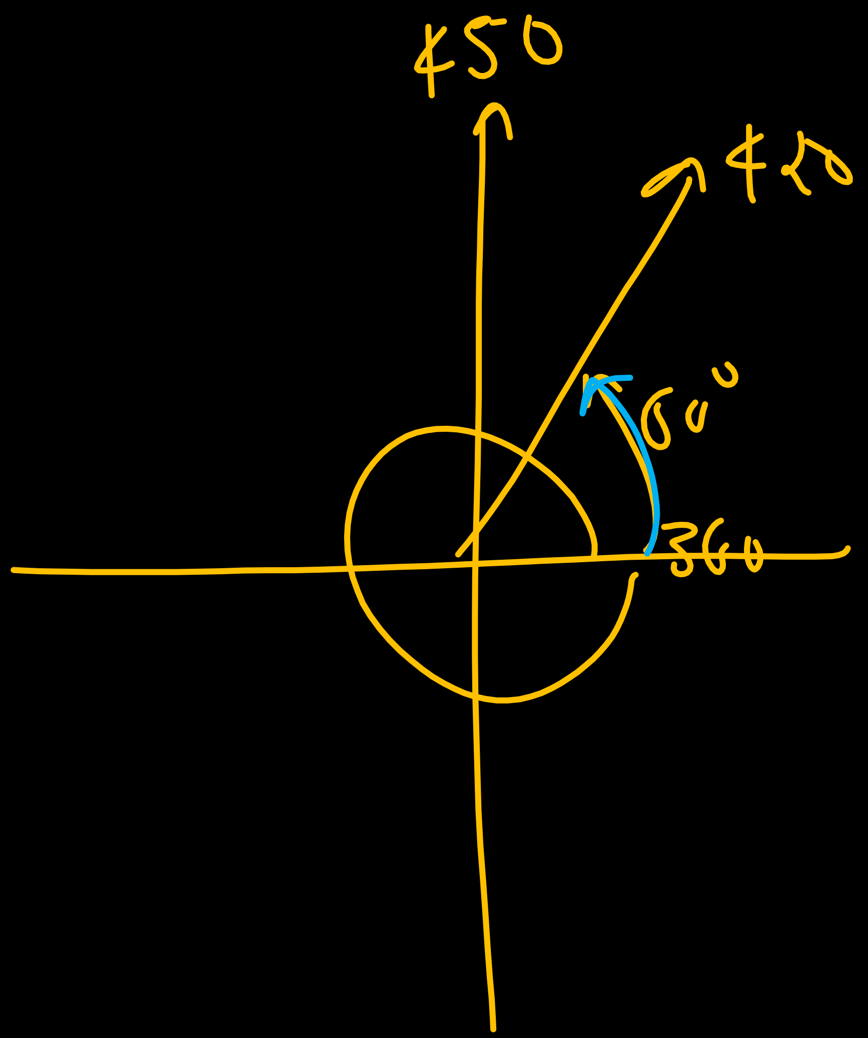
Example: 16

$\sec(-420^\circ) =$

$$\sec(-420^\circ) = \sec(420^\circ)$$

$$= \sec(\underline{360^\circ + 60^\circ}) = +\sec 60$$

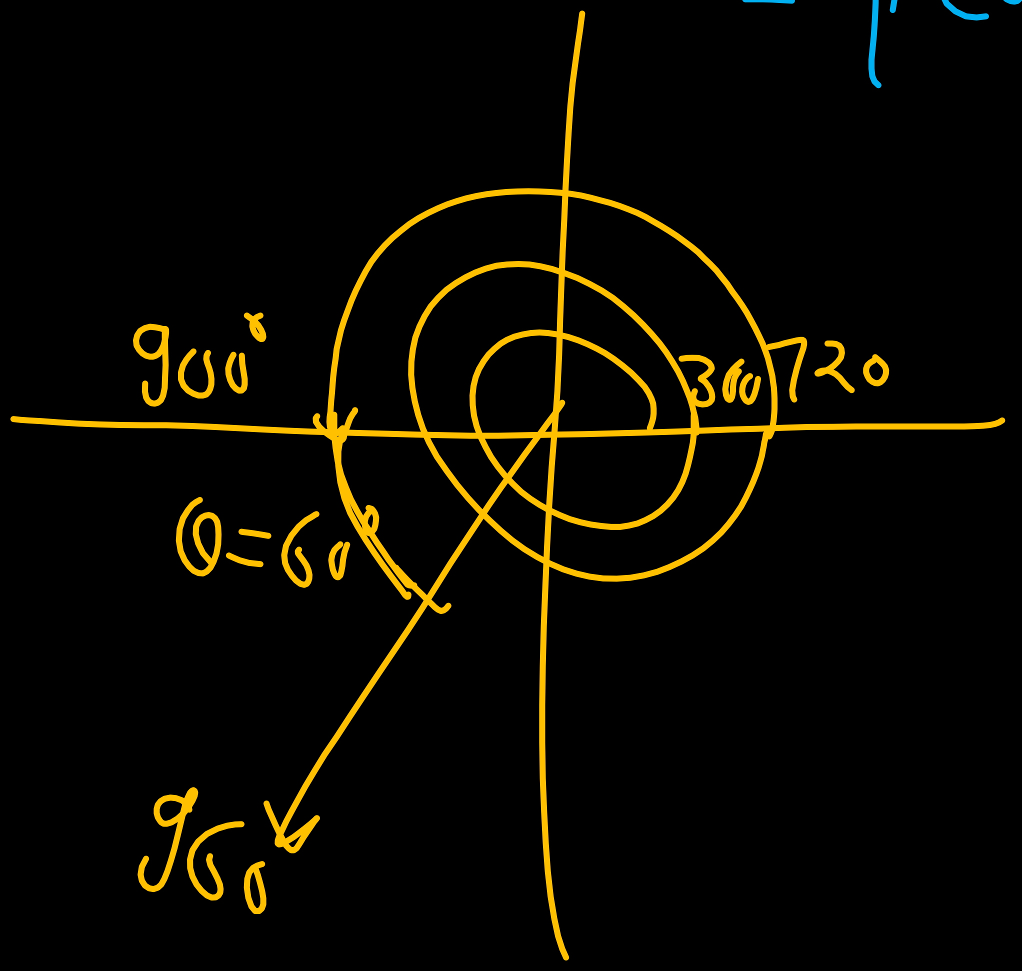
$$= 2$$



Example: 17

$\cot(-960^\circ) =$

$$\cot(-960^\circ) = -\cot(960^\circ)$$
$$= -\left[\cot(900+60)\right]$$
$$= -\left[+\cot 60\right] = -\frac{1}{\sqrt{3}}$$



Example: 18

$\cos(\frac{7\pi}{6}) =$

M-1 $\cos\left(\frac{7\pi}{6}\right) = \cos\left(\frac{7\times \cancel{180}^{30}}{\cancel{6}}\right) = \cos(210^\circ)$

M-2 $\cos\left(\frac{7\pi}{6}\right) = \cos\left(\frac{6\pi+\pi}{6}\right)$

$= \cos\left(\frac{\cancel{6}\pi}{\cancel{6}} + \frac{\pi}{6}\right)$

$= \cos\left(\pi + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

Example: 19

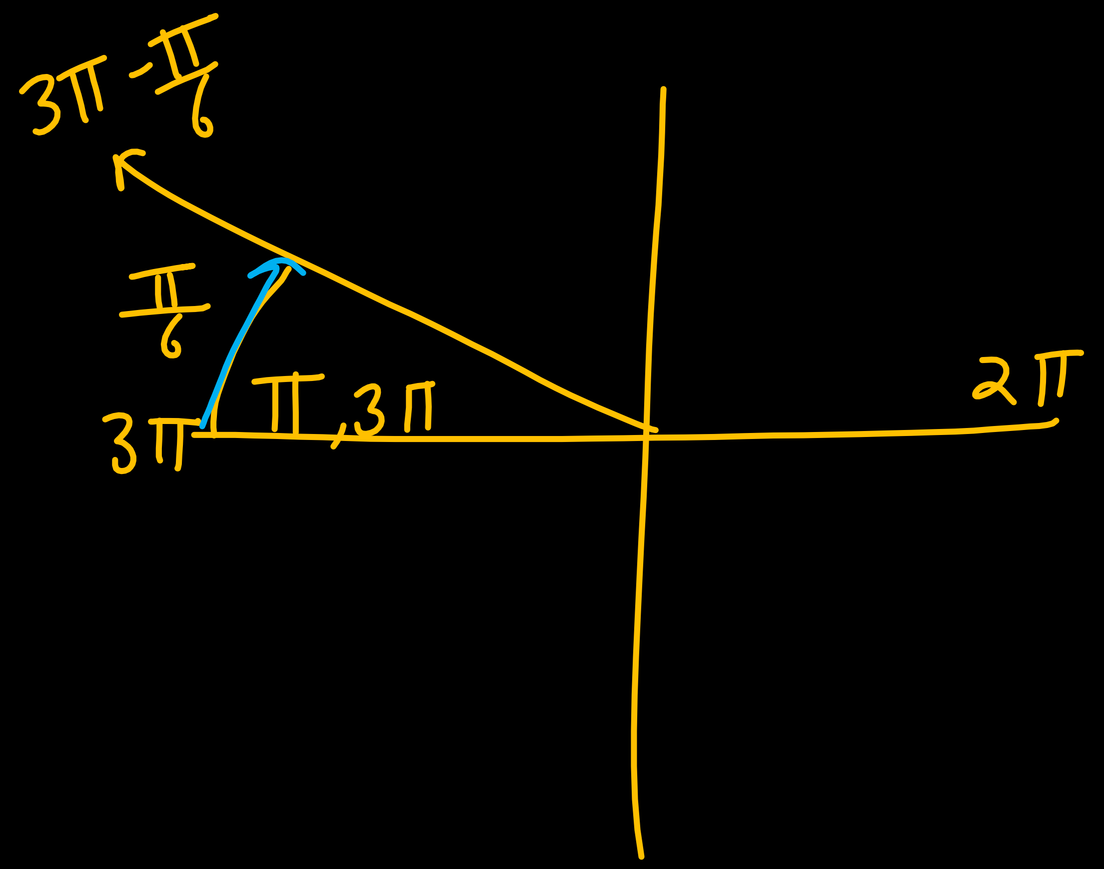
$\tan(\frac{17\pi}{6}) =$

$$\tan\left(\frac{17\pi}{6}\right) = \tan\left(\frac{18\pi - \pi}{6}\right)$$

$$= \tan\left(3\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{6}$$

$$= -\frac{1}{\sqrt{3}}$$



Example: 20

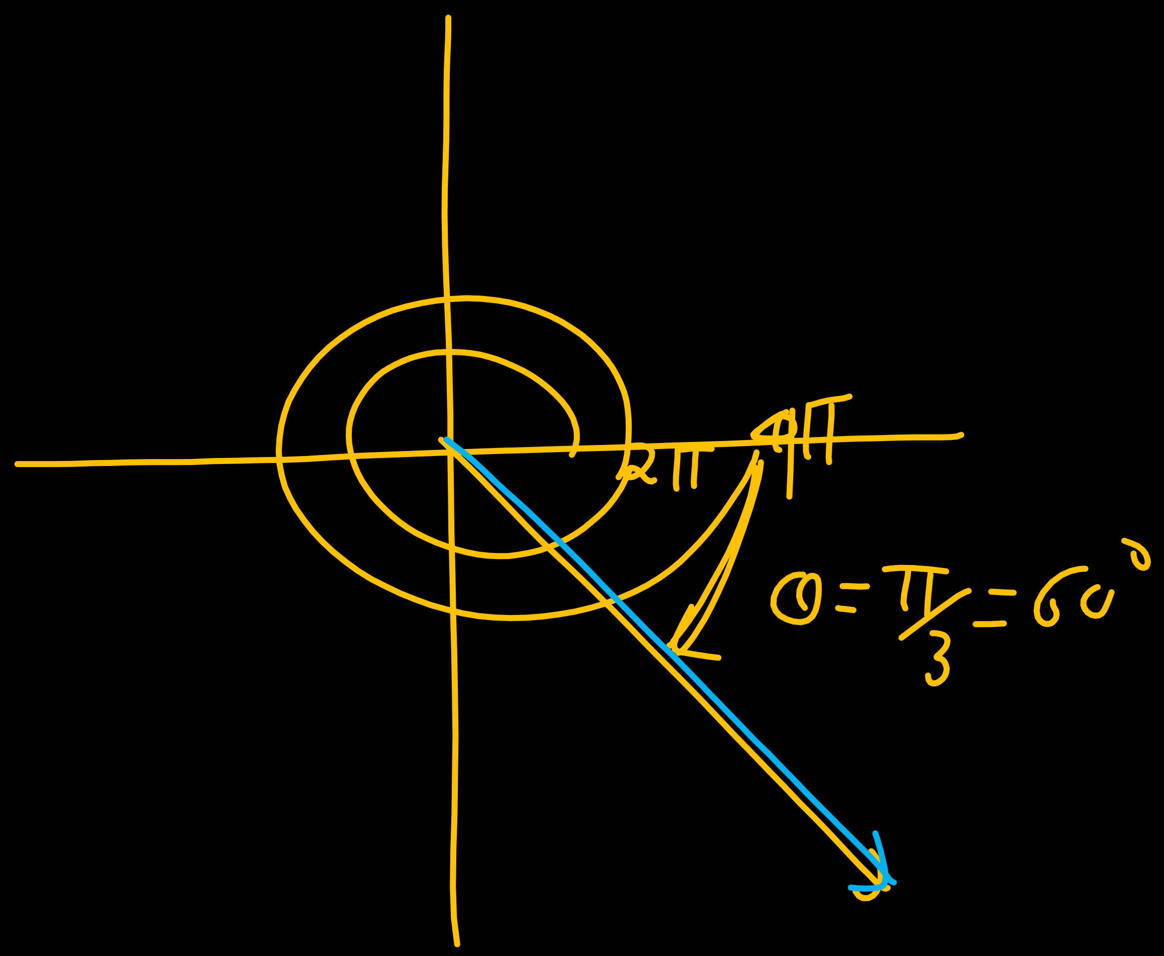
$\sin(\frac{11\pi}{3}) =$

$$\sin\left(\frac{11\pi}{3}\right) = \sin\left(\frac{12\pi - \pi}{3}\right)$$

$$= \sin\left(4\pi - \frac{\pi}{3}\right)$$

$$= -\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{2}$$



Example: 21

$\tan(\frac{-4\pi}{3}) =$

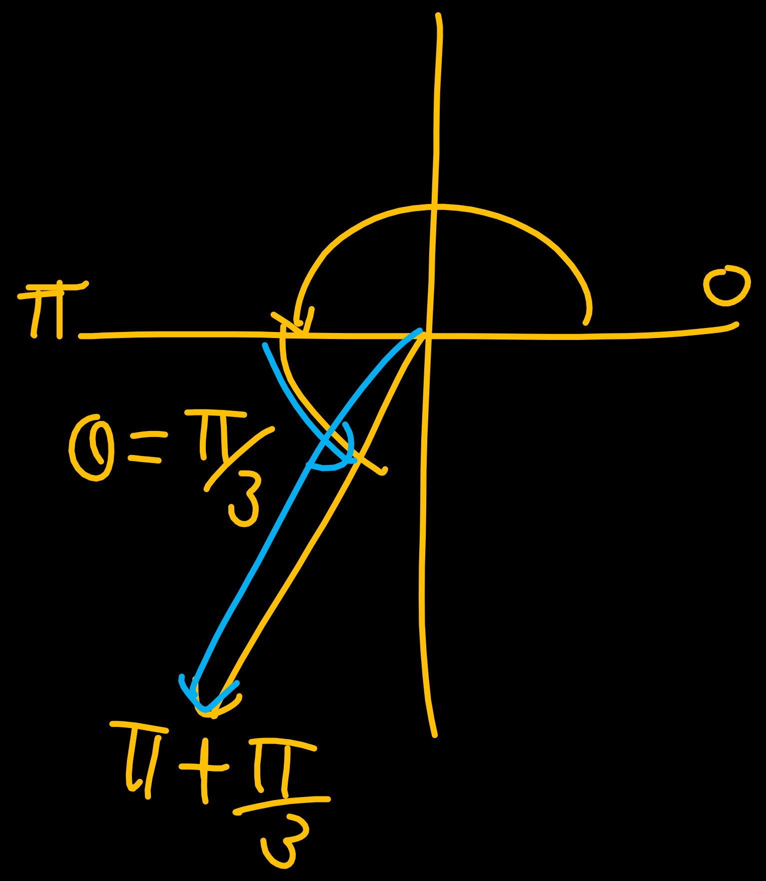
$$\tan\left(-\frac{4\pi}{3}\right) = -\tan\left(\frac{4\pi}{3}\right)$$

$$= -\tan\left(\frac{3\pi+\pi}{3}\right)$$

$$= -\left[\tan\left(\pi+\frac{\pi}{3}\right)\right]$$

$$= -\left[+\tan\frac{\pi}{3}\right]$$

$$= -\sqrt{3}$$



Example: 22

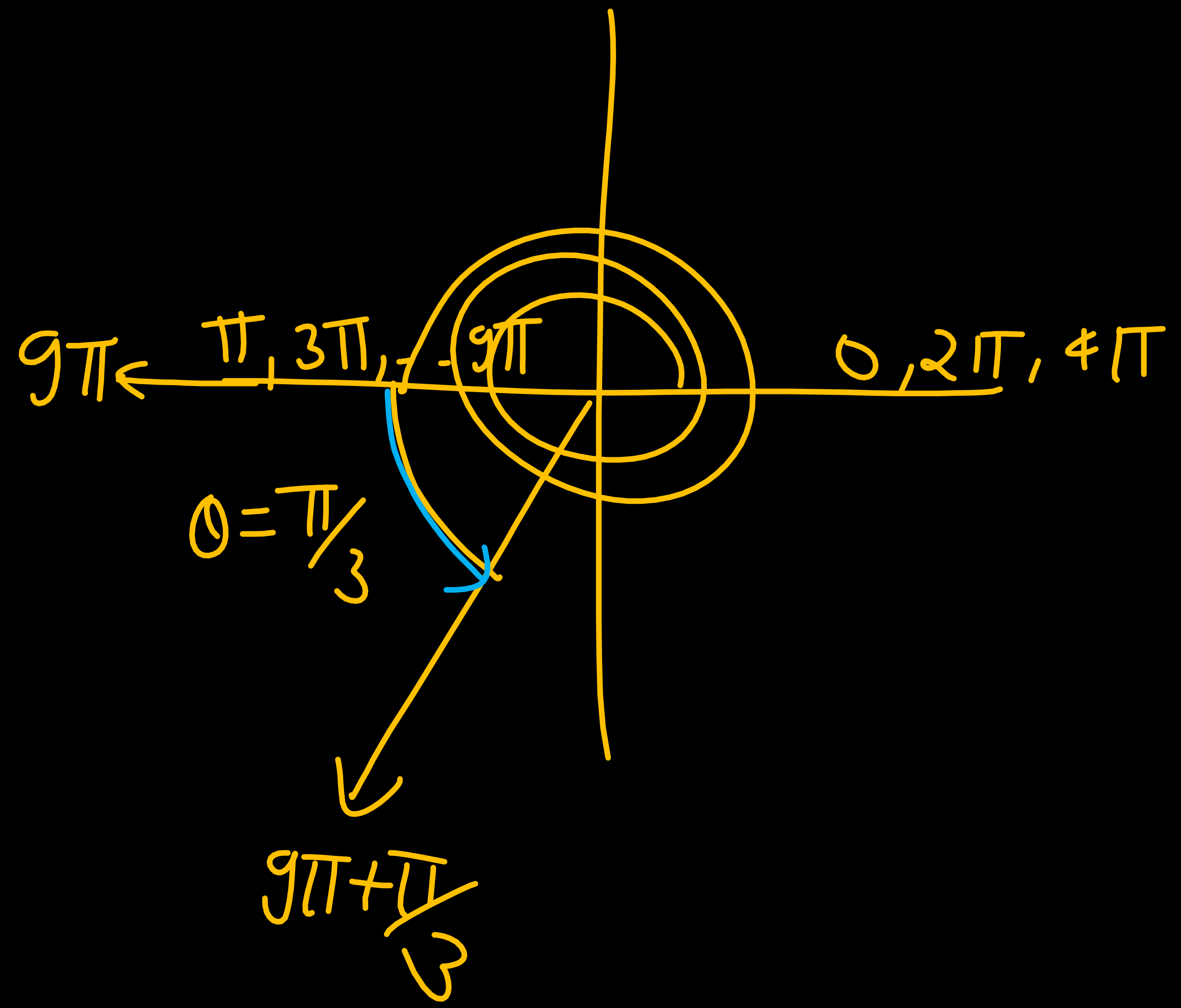
$\sin(\frac{28\pi}{3}) =$

$$\sin\left(\frac{28\pi}{3}\right) = \sin\left(\frac{27\pi+\pi}{3}\right)$$

$$= \sin\left(9\pi + \frac{\pi}{3}\right)$$

$$= -\sin\frac{\pi}{3}$$

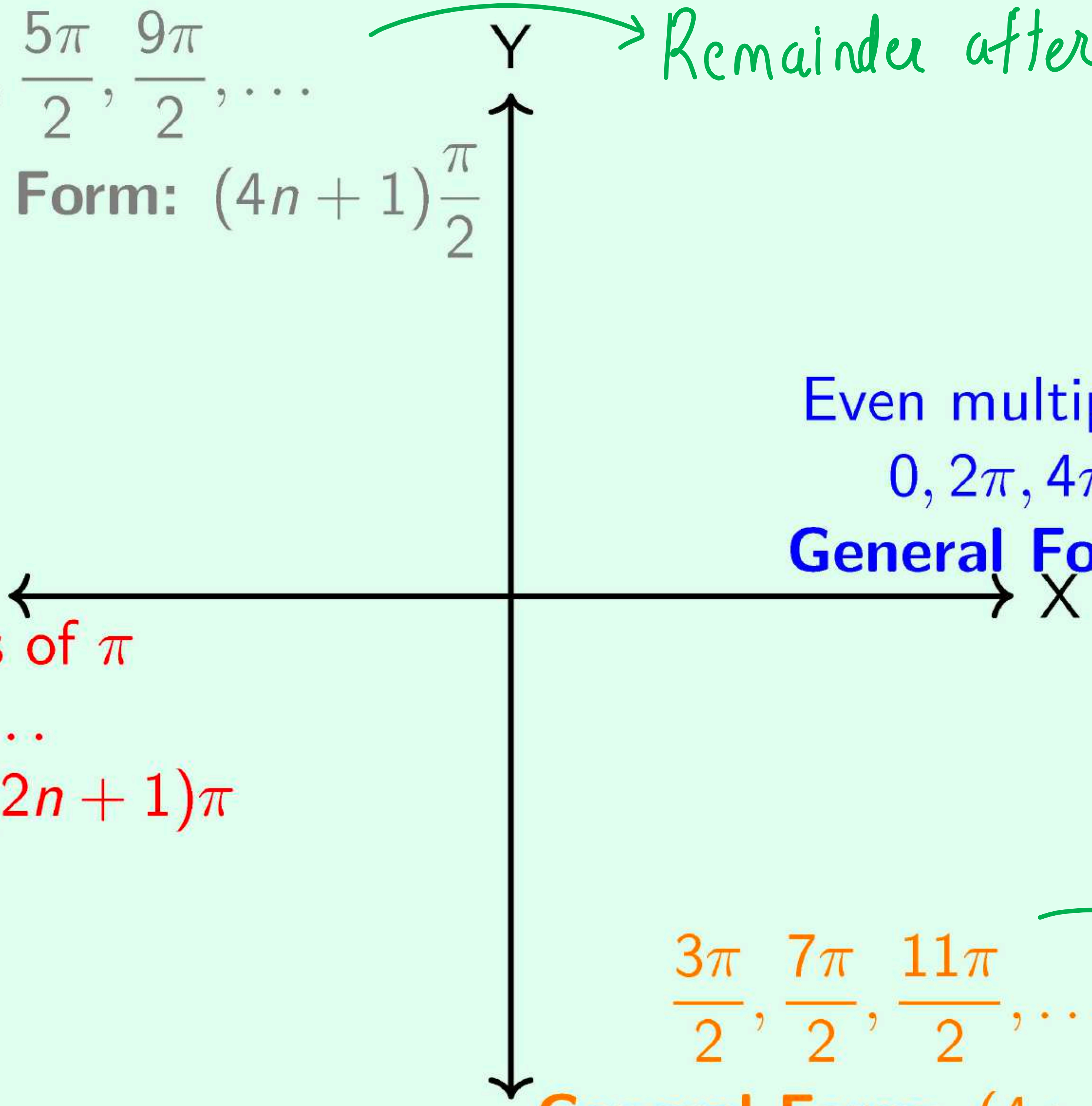
$$= -\frac{\sqrt{3}}{2}$$



Note

1 $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$ → Remainder after dividing by 4 = 1

General Form: $(4n+1)\frac{\pi}{2}$



Even multiples of π

$0, 2\pi, 4\pi, \dots$

General Form: $2n\pi$

Odd multiples of π

$\pi, 3\pi, 5\pi, \dots$

General Form: $(2n+1)\pi$

$\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots$ → Remainder after dividing by 4 $= \boxed{3}$

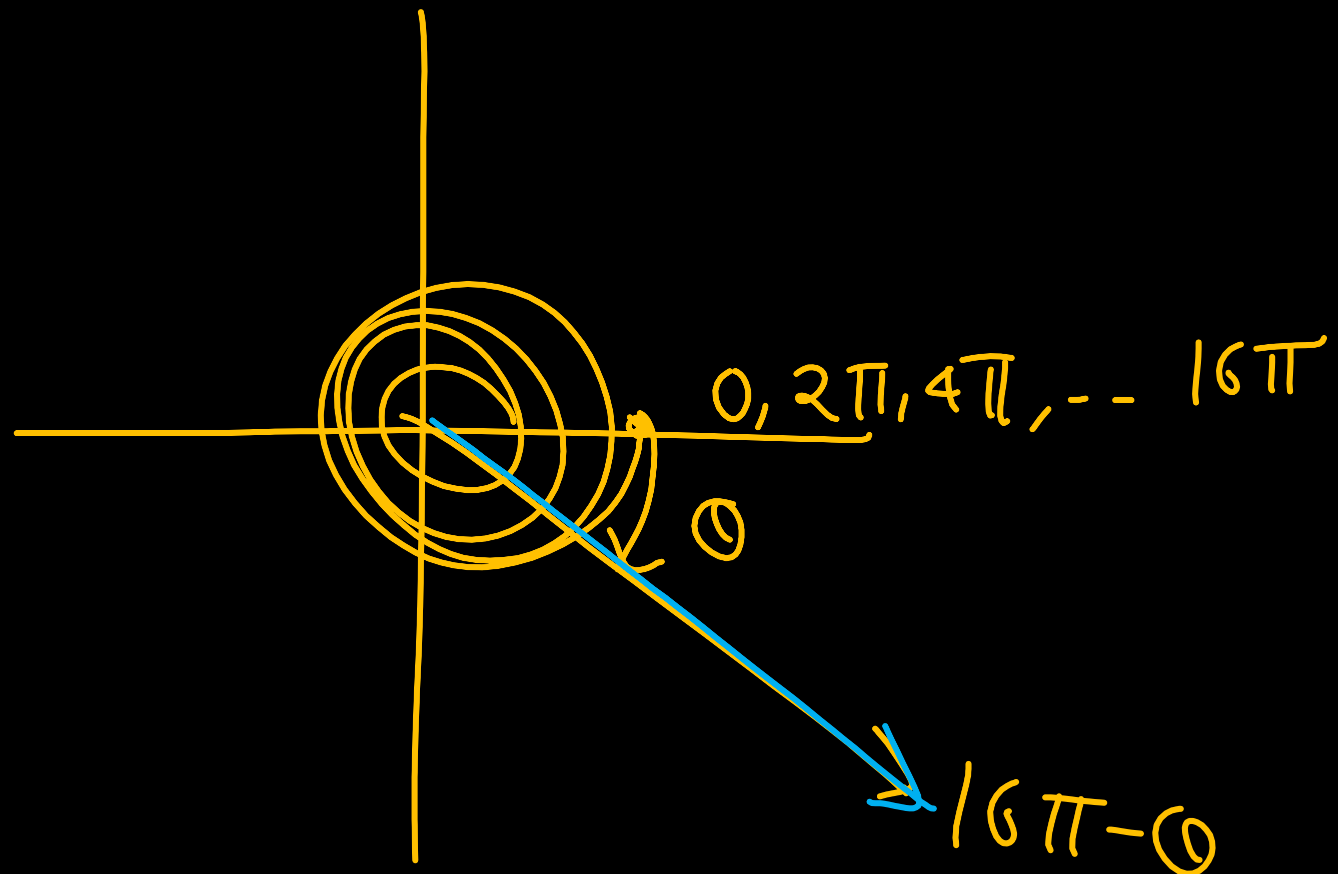
General Form: $(4n-1)\frac{\pi}{2}$

- Complete X-axis: $n\pi$
- Complete Y-axis: $(2n+1)\frac{\pi}{2}$ or $n\pi + \frac{\pi}{2}$ → odd multiple of $\frac{\pi}{2}$

Example: 23

$\cos(16\pi - \theta) =$

$\cos(16\pi - \theta) = +\cos\theta$

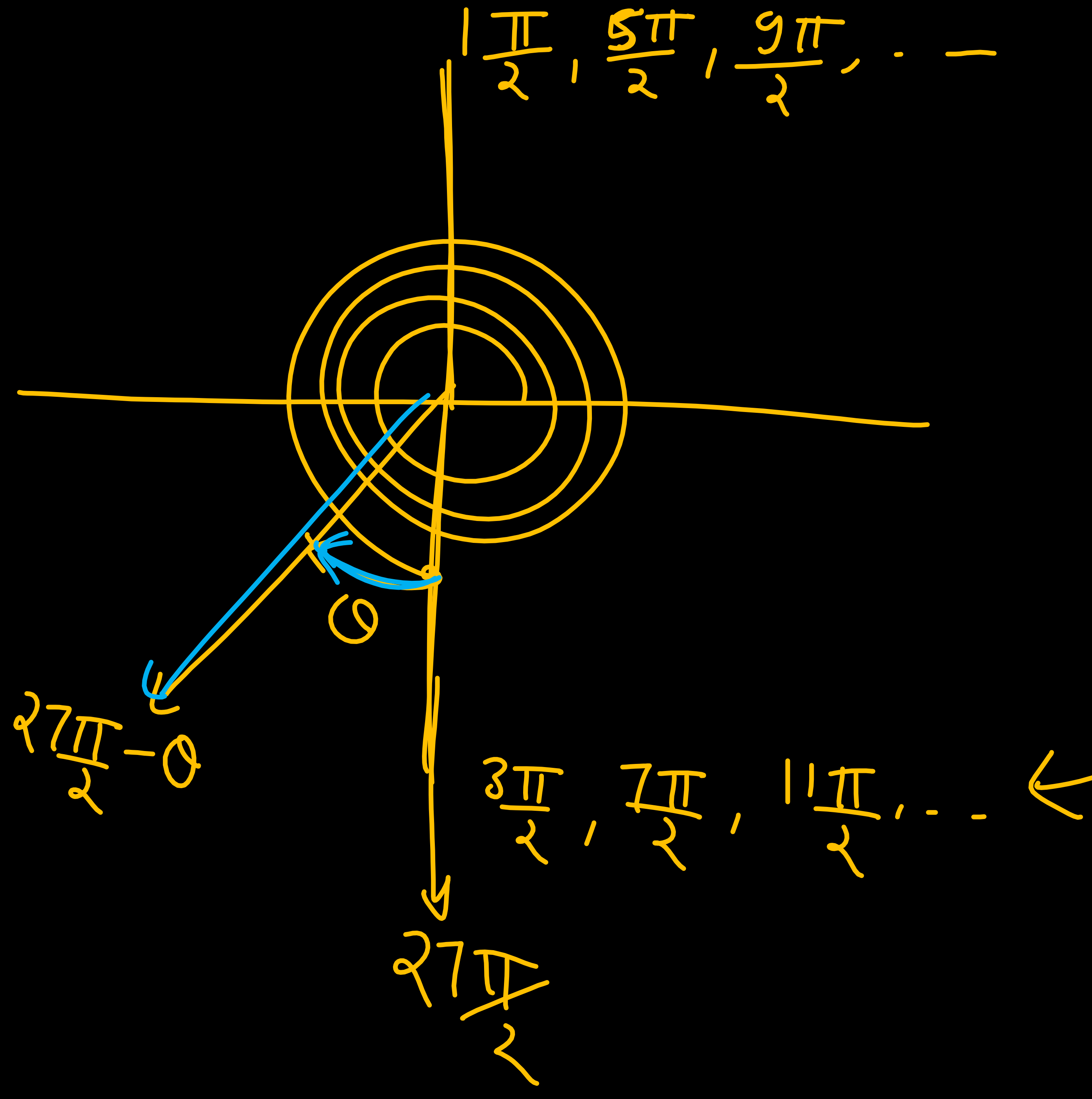


Example: 24

$\sin(\frac{27\pi}{2} - \theta) =$

$\sin\left(\underbrace{\frac{27\pi}{2}-\theta}_{3^{rd}\text{ quad}}\right) = -\cos\theta$

$27 \frac{\pi}{2}$



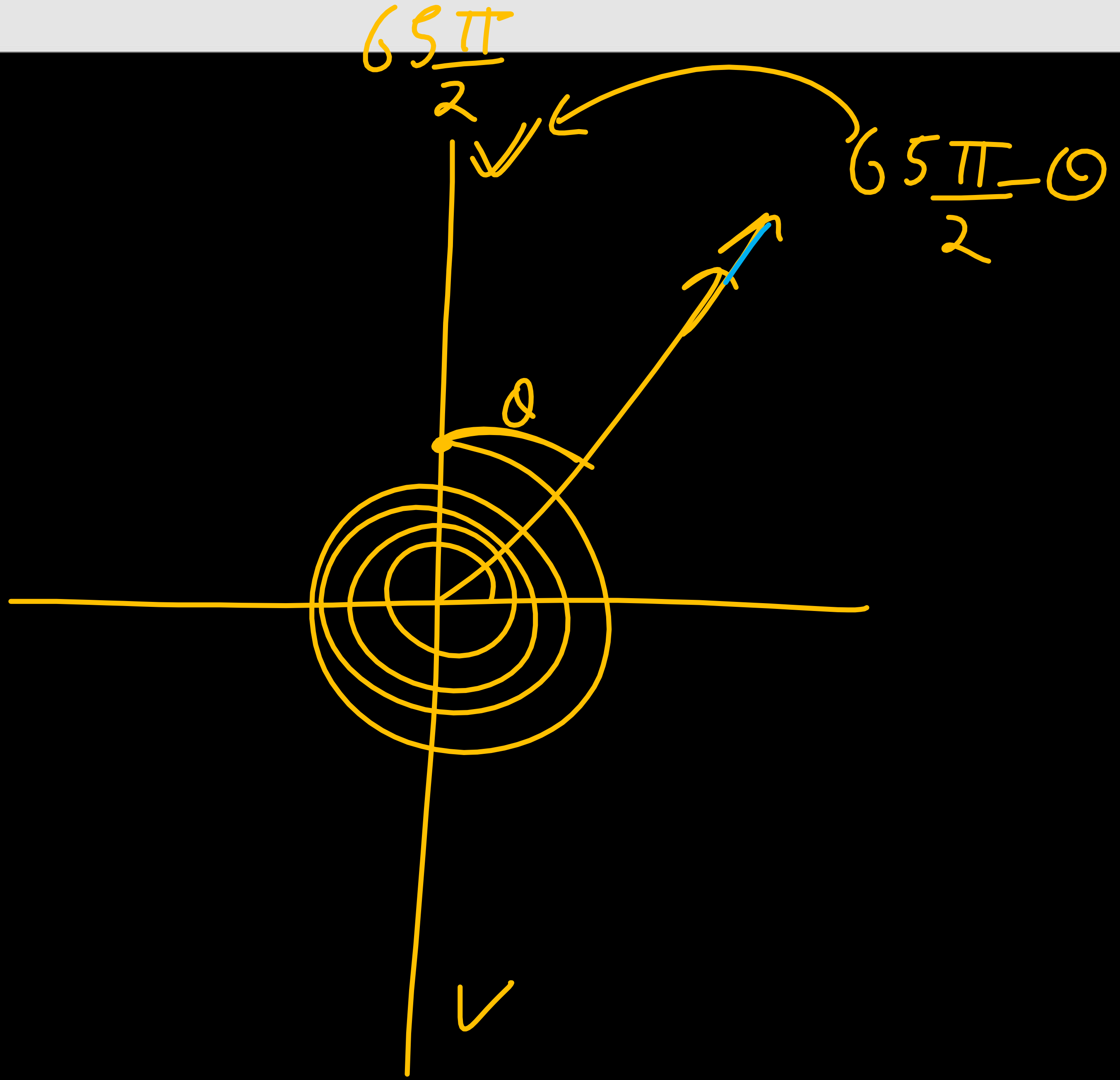
$$\begin{array}{r} 6 \\ 4\overline{)27} \\ -24 \\ \hline \boxed{3} \end{array}$$

Rem 3

Example: 25

$\tan(\frac{65\pi}{2} - \theta) =$

$\tan\left(\frac{65\pi}{2} - \theta\right) = +\cot\theta$



$$\begin{array}{r} 16 \\ 4\,\overline{)\,65} \\ -\,64 \\ \hline \end{array}$$

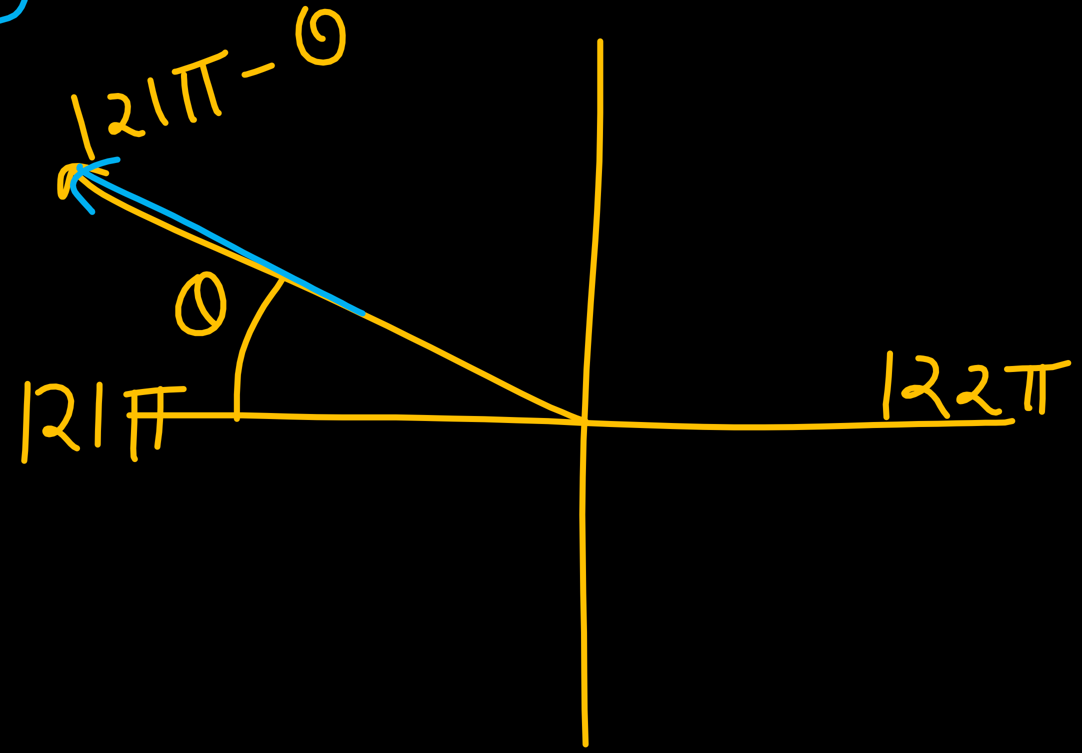
Rem: $\boxed{1}$

Example: 26

$\tan(\theta - 121\pi) =$

$$\tan(\theta - 121\pi) = \tan\left(-(121\pi - \theta)\right) = -\left[\tan(121\pi - \theta)\right]$$

take out "−"



$$= -\left[-\tan\theta\right]$$

$$= \tan\theta$$

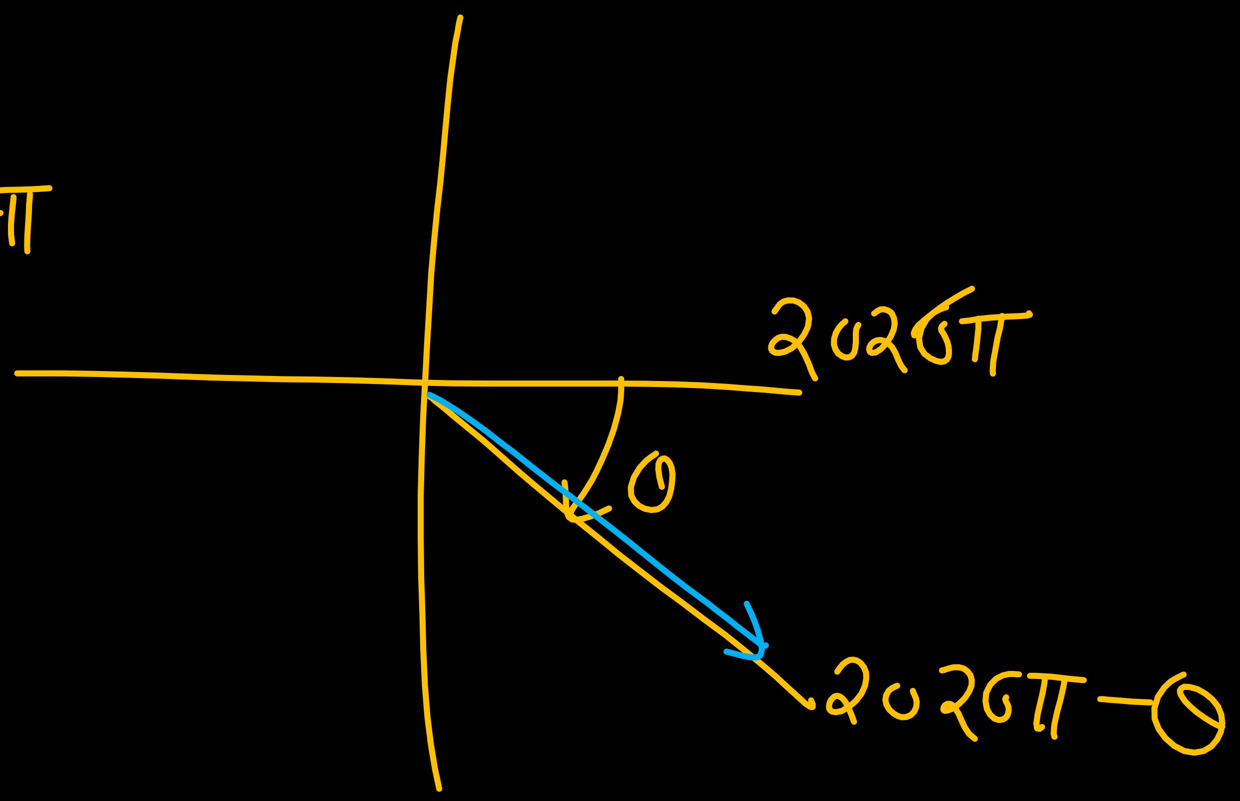
Example: 27

$\cos(\theta - 2026\pi) =$

$$\cos(\theta - 2026\pi) = \cos(-(2026\pi - \theta))$$

$$= \cos(\underbrace{2026\pi}_{\text{Even multiple of } \pi} - \theta)$$

$$= +\cos\theta$$



Questions on Allied Angles

Question: 1

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \cdots + \sin^2 90^\circ$$

Complementary angles ko saath me rakho

$$\left(\sin^2 5^\circ + \underbrace{\sin^2 85^\circ}_{\cos^2 5^\circ}\right) + \left(\sin^2 10^\circ + \underbrace{\sin^2 80^\circ}_{\cos^2 10^\circ}\right) + \left(\sin^2 15^\circ + \underbrace{\sin^2 75^\circ}_{\cos^2 15^\circ}\right) + \cdots$$

$$+ \left(\sin^2 40^\circ + \underbrace{\sin^2 50^\circ}_{\cos^2 40^\circ}\right) + \left(\sin^2 45^\circ\right) + \left(\sin^2 90^\circ\right)$$

$$\Rightarrow \underbrace{1 + 1 + 1 + \cdots + 1}_{8\text{-times}} + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2 = 8 + \frac{1}{2} = \boxed{\frac{19}{2}}$$

Question: 2

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot \cdots \cdot \tan 89^\circ$$

Complementary angles ko saath me rakho

$$(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \cdot \tan 88^\circ)(\tan 3^\circ \tan 87^\circ)\ldots(\tan 44^\circ \tan 46^\circ)(\tan 45^\circ)$$

$$(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ)(\tan 3^\circ \cot 3^\circ)\ldots(\tan 44^\circ \cot 44^\circ)(1)$$

$$\tan\theta \cot\theta = 1$$

$$\cancel{\tan\theta}\,\frac{1}{\cancel{\tan\theta}} = 1$$

$$1 \times 1 \times 1 \times \ldots \times 1 = \boxed{1}$$

Question: 3

CCMM

$$\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$$

Complementary angles ko saath me rakho

$$\frac{\pi}{16}+\frac{7\pi}{16}=\frac{8\pi}{16}=\frac{\pi}{2}$$

$$\frac{3\pi}{16}+\frac{5\pi}{16}=\frac{8\pi}{16}=\frac{\pi}{2}$$

$$\left(\cos^2\frac{\pi}{16}+\cos^2\frac{7\pi}{16}\right)+\left(\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}\right)$$

$$\left(\cos^2\frac{\pi}{16}+\sin^2\frac{\pi}{16}\right)+\left(\cos^2\frac{3\pi}{16}+\sin^2\left(\frac{3\pi}{16}\right)\right)$$

$$1+1$$

$$\boxed{2}$$

Question: 4

$$\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$$

$\frac{\pi}{20}+\frac{9\pi}{20}=\frac{10\pi}{20}=\frac{\pi}{2}$

$\cos\left(\frac{\pi}{4}\right)$

$\left(\cot\frac{\pi}{20}\ \cot\frac{9\pi}{20}\right)\left(\cot\frac{3\pi}{20}\ \cot\frac{7\pi}{20}\right)\ \cot\left(\frac{\pi}{4}\right)$

$\tan\theta\ \cot\theta=1$ $\quad\left(\cot\frac{\pi}{20}\ \tan\left(\frac{\pi}{20}\right)\right)\left(\cot\frac{3\pi}{20}\ \tan\left(\frac{3\pi}{20}\right)\right)\quad 1$

$1\times1\times1=\boxed{1}$

Question: 5

$$\cos 10^\circ + \cos 20^\circ + \cos 30^\circ + \cdots + \cos 180^\circ$$

Supplementary angles ko saath me rakho

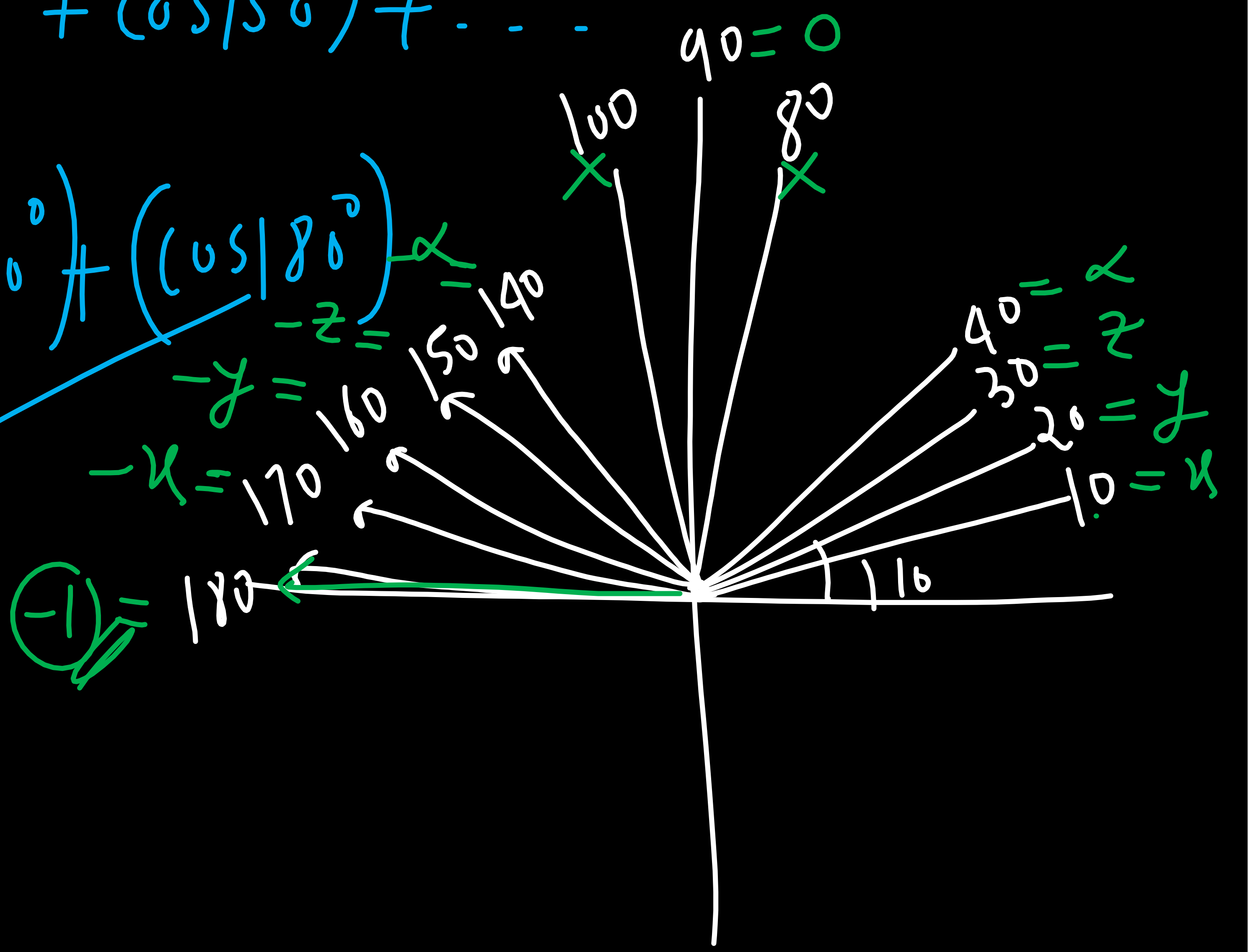
$$(\cos 10^\circ + \cos 170^\circ) + (\underbrace{\cos 20^\circ + \cos 160^\circ}) + (\cos 30^\circ + \cos 150^\circ) + \dots$$

$$+ (\underbrace{\cos 80^\circ + \cos 100^\circ}) + (\cos 90^\circ) + (\cos 180^\circ)$$

$$A+B=180^\circ \Rightarrow \cos A + \cos B = 0$$

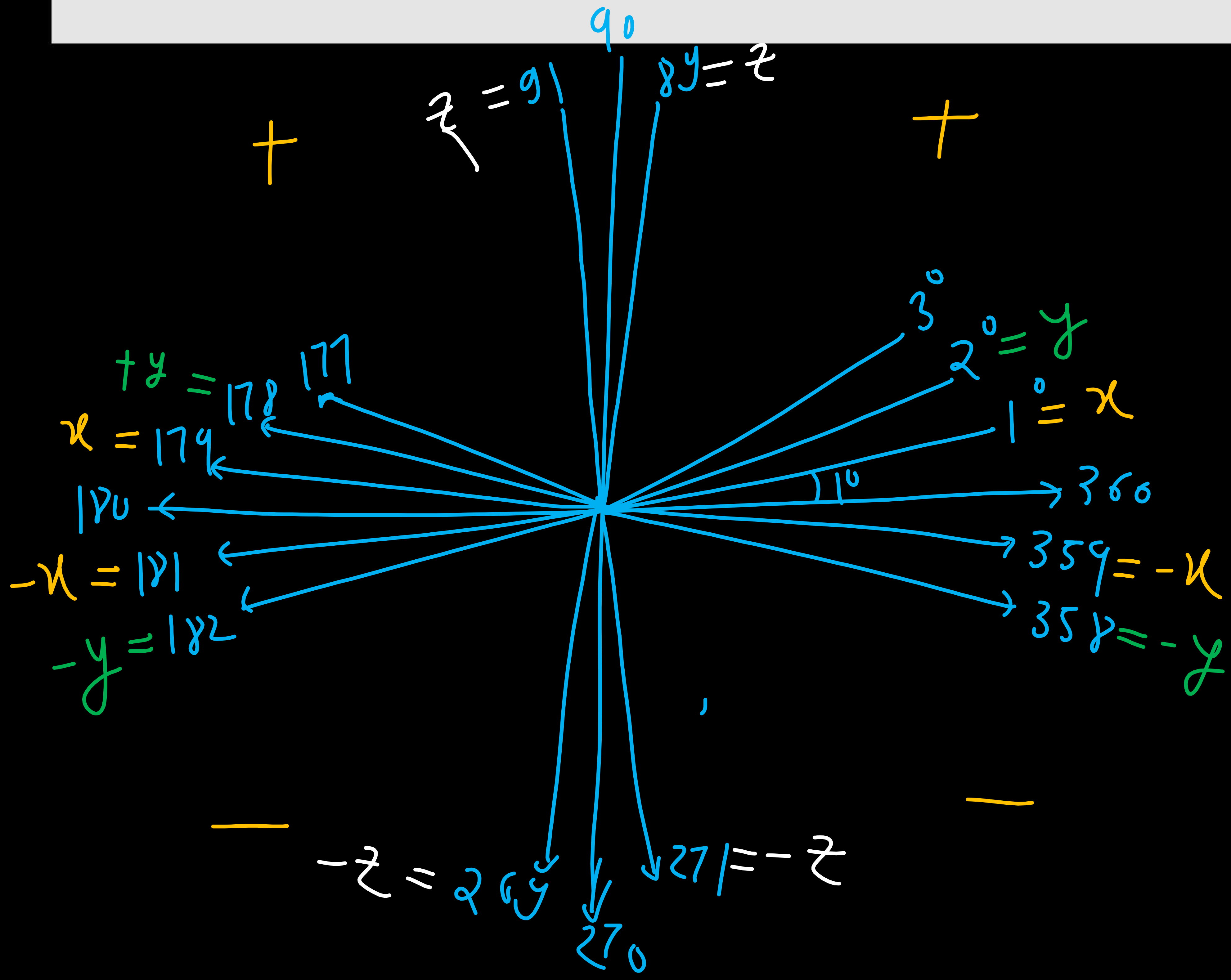
$$0+0+0+\dots+0+0-1$$

$$\boxed{-1}$$



Question: 6

$$\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 360^\circ$$



$$\left(\sin 1^\circ + \sin 359^\circ\right) = 0$$

$$+\left(\sin 2^\circ + \sin 358\right)$$

$$+\left(\sin 3^\circ + \sin 357^\circ\right)$$

$$\vdots$$

$$+\left(\sin(89^\circ) + \sin 271\right)$$

$$+\left(\underset{1}{\sin 90} + \underset{-1}{\sin 270}\right)$$

$$+\left(\sin 91 + \sin 269\right)$$

$$+\left(\sin 92 + \sin 268\right)$$

$$+$$

$$+\left(\sin 179 + \sin 181\right)$$

$$+ \sin 180^\circ$$

$$+ \sin 360^\circ$$

Ans zero

Question: 7

$$\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}$$

$$\frac{\pi}{11}+\frac{10\pi}{11}=\frac{11\pi}{11}=\pi$$

$\tan A+\tan B=0 \Rightarrow A+B=\pi$

$$\left(\tan\frac{\pi}{11}+\tan\frac{10\pi}{11}\right)+\left(\tan\frac{2\pi}{11}+\tan\frac{9\pi}{11}\right)+\left(\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}\right)$$

zero + zero + zero

$\boxed{\text{zero}}$

Question: 8 Evaluate the following expression:

$$\frac{\sin\frac{11\pi}{17}\cos\frac{10\pi}{13}\tan\frac{\pi}{7}}{\cos\frac{3\pi}{13}\sin\frac{6\pi}{17}\tan\frac{6\pi}{7}} = (-1)\times(-1) = \boxed{1}$$

$A+B=\pi$

$\sin A = \sin B$

$\tan A = -\tan B$

$\cos A = -\cot B$

$\frac{11\pi}{17}+\frac{6\pi}{17}=\pi$

$\frac{10\pi}{13}+\frac{3\pi}{13}=\pi$

$\frac{\pi}{7}+\frac{6\pi}{7}=\pi$

Question: 9 (HW) CC MM Ans:2

$$\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}$$

$\sin 10 = \cos 80$

$$\left(\sin^2\frac{\pi}{18}+\sin^2\frac{4\pi}{9}\right)+\left(\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}\right)$$

$\frac{\pi}{2}$

$$\left(\sin^2\frac{\pi}{18}+\cos^2\frac{\pi}{18}\right)+\left(\sin^2\frac{\pi}{9}+\cos^2\left(\frac{\pi}{9}\right)\right)$$

$1+1$

2 //

$\frac{\pi}{18}+\frac{7\pi}{18}=\frac{8\pi}{18}=\frac{4\pi}{9}$ ✗

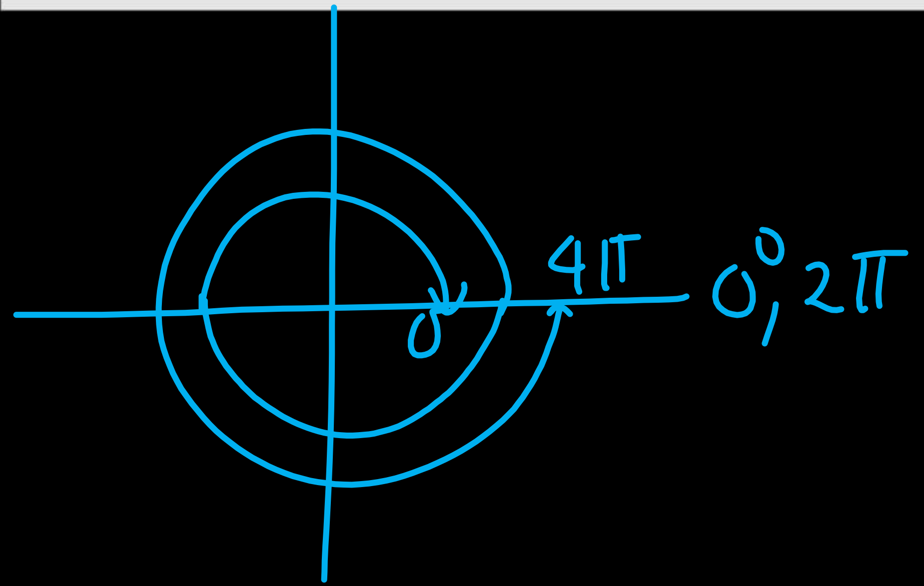
CC : $\frac{\pi}{18}+\frac{4\pi}{9}=\frac{9\pi}{18}=\frac{\pi}{2}$ ✓

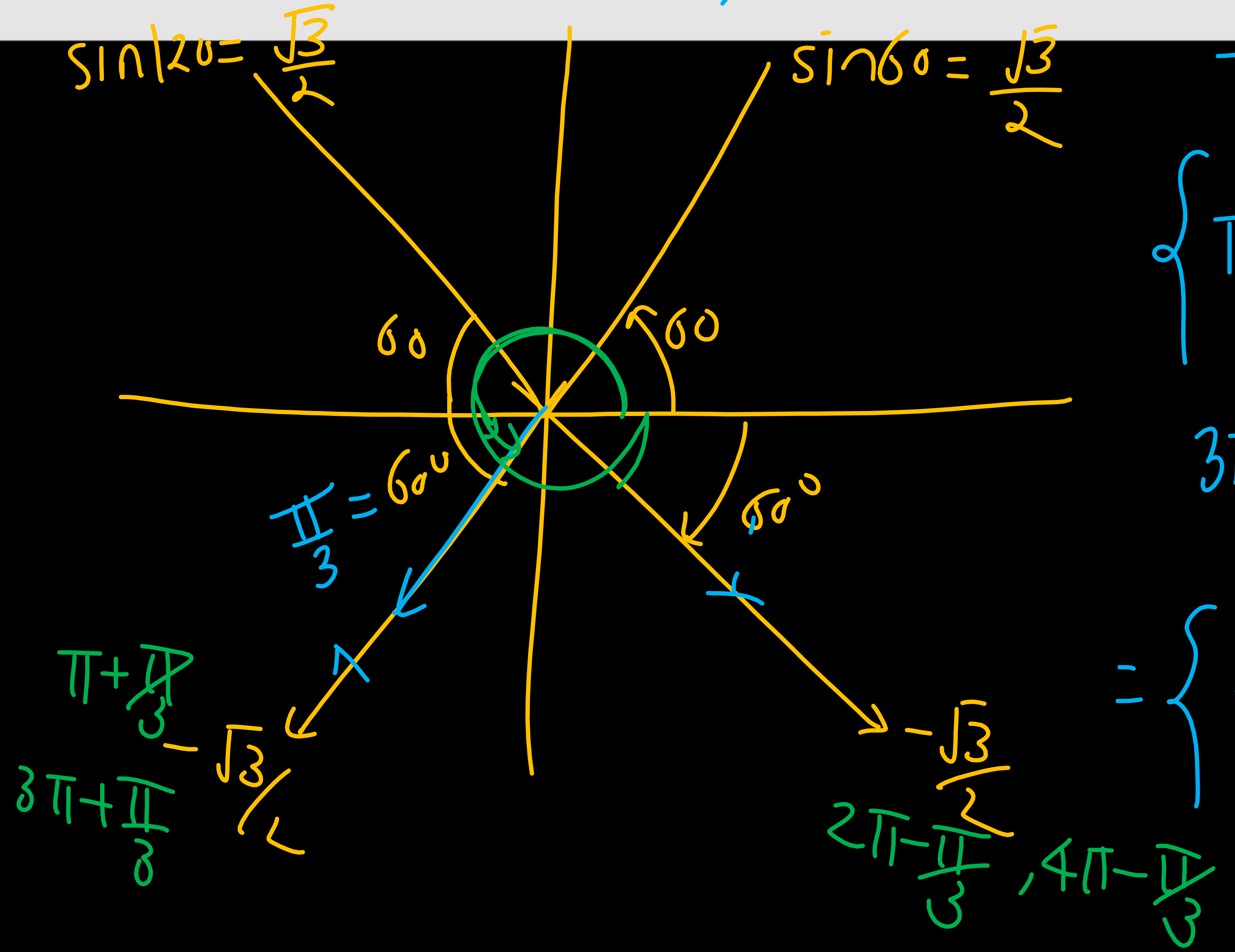
MM $\frac{\pi}{9}+\frac{7\pi}{18}=\frac{9\pi}{18}=\frac{\pi}{2}$ ✓

Question: 10 ~~(Triangle Method Question)~~

If $\theta \in (0, 4\pi)$, find all possible values of θ for which:

$$\sin\theta = -\frac{\sqrt{3}}{2}$$





Total values 4

$$\left\{ \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3},\ 3\pi + \frac{\pi}{3},\ 4\pi - \frac{\pi}{3} \right\}$$

$$= \left\{ \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \right\}$$

Question: 11 (Triangle Method Question)

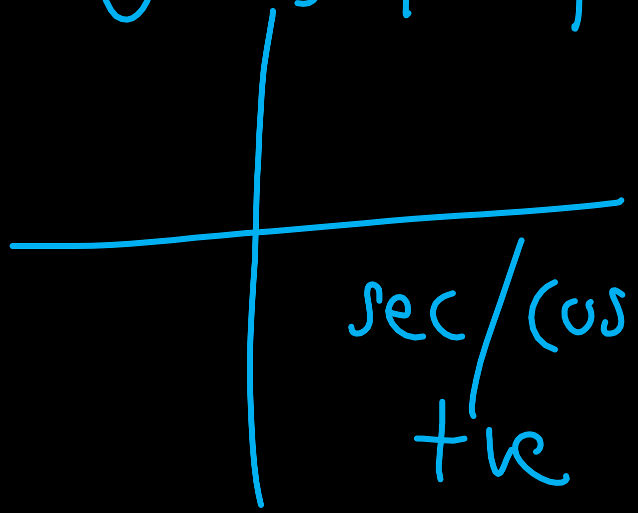
If $\boxed{\sec\theta = \sqrt{2}}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the value of the expression:

$$X = \frac{1 + \tan\theta + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$$

(A) 1 (B) -1 (C) 0 (D) 2

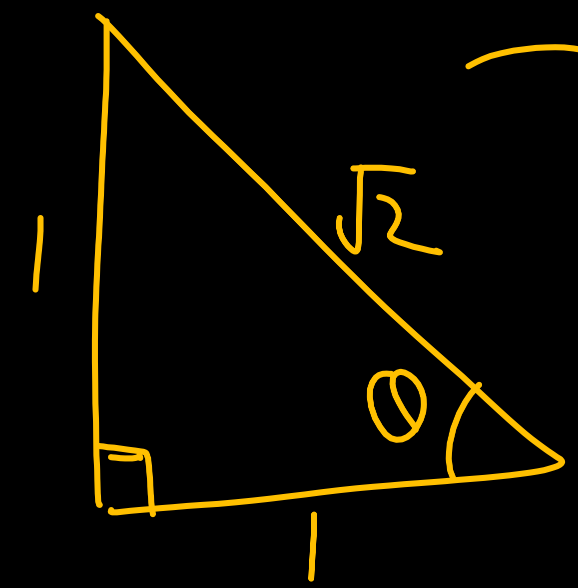
$270^\circ < \theta < 360^\circ$

$\theta \longrightarrow 4^{th}$ Quad



$\sec\theta = \frac{\sqrt{2}}{1}$

$\cos\theta = \frac{1}{\sqrt{2}}$



$\tan\theta = -1$

$\cot\theta = -1$

$\operatorname{cosec}\theta = \frac{-\sqrt{2}}{1}$

$$X = \frac{\cancel{1} + \cancel{(-1)} + (-\sqrt{2})}{\cancel{1} + \cancel{(-1)} - (-\sqrt{2})}$$

$$= \frac{-\cancel{\sqrt{2}}}{+\cancel{\sqrt{2}}}$$

$$\boxed{X = -1}$$

Question: 12 JEE Mains 2024 (Triangle Method Question)

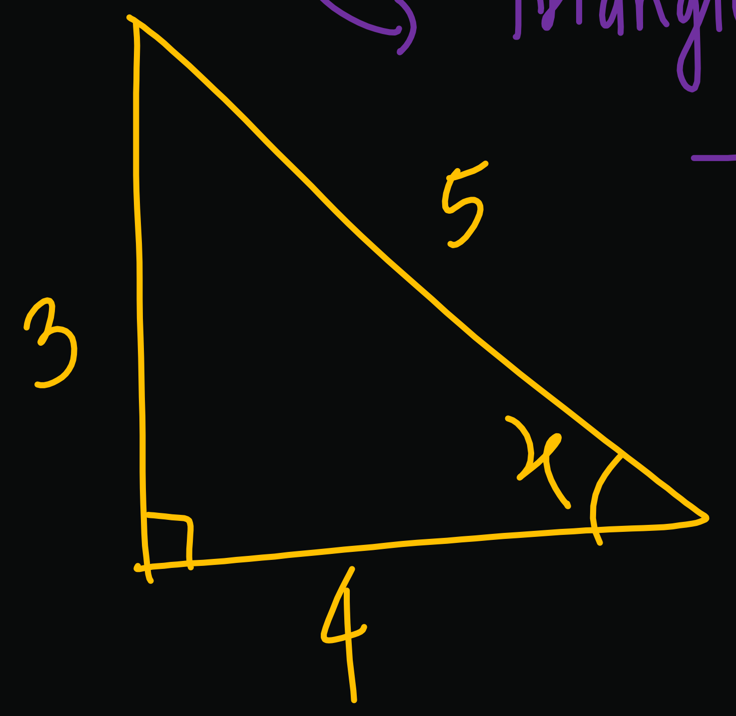
If $\boxed{\sin x = -\frac{3}{5},}$ where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to:

(A) 109 (B) 108 (C) 19 (D) 18

$x \longrightarrow 3^{rd}$ Quad $\tan x = ?$ $\cos x = ?$

$\sin x = -\frac{3}{5}$

triangle banate samay −ve sign ko consider na karo.



$\tan x = +\frac{3}{4}$, $\cos x = -\frac{4}{5}$

$$\text{Req} = 80(\tan^2 x - \cos x)$$
$$= 80\left(\left(\frac{3}{4}\right)^2 - \left(-\frac{4}{5}\right)\right)$$
$$= 80\left(\frac{9}{16} + \frac{4}{5}\right)$$
$$= \cancel{80}\left(\frac{45+64}{\cancel{80}}\right)$$
$$= 109$$

True or False?

- $\sin 1 > 0 \longrightarrow$ True $\quad \sin 1 = \sin 1^c \approx \underline{\sin 57.3^\circ} > 0$
- $\sin 2 > 0 \longrightarrow$ True $\quad \sin 2 = \sin 114.6^\circ > 0$
- $\sin 3 > 0 \longrightarrow$ True
- $\sin 4 > 0 \longrightarrow$ False
- $\sin 7 > 0 \longrightarrow$ True $\quad 7 \times 60 = 420 = 360 + 60$

  $\longrightarrow$ 1st Quad
- $\cos 1 > 0 \longrightarrow$ True
- $\cos 2 > 0 \longrightarrow$ False
- $\cos 3 > 0 \longrightarrow$ False
- $\tan 5 > 0 \longrightarrow$ False

Note:

- If no unit (like the degree symbol $^\circ$) is specified for an angle, it is assumed to be measured in **radians**.
- 1 **radian** $\approx 57.3^\circ$.

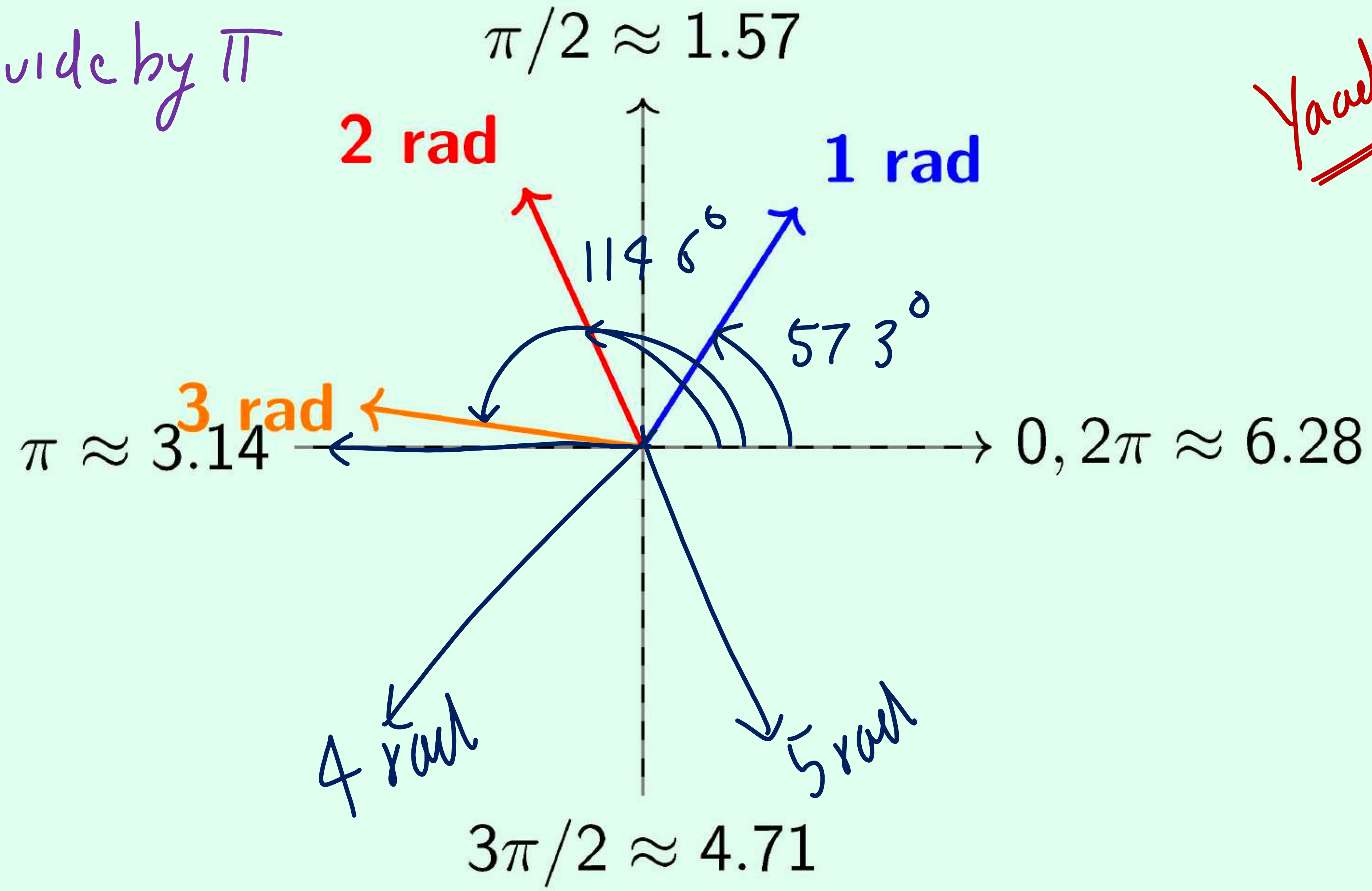
$\sin 1^\circ \neq \sin 1$ → no unit

$\pi^c = 180^\circ$ ∴ divide by π

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$1^c = \left(\frac{180}{3.14}\right)^\circ$

$\boxed{1^c \approx 57.3^\circ}$



Yaad Rakhe

$$\boxed{\begin{array}{l} \pi \approx 3.14 \\ \frac{\pi}{2} \approx 1.57 \end{array}}$$

Important Concept: Complementary Angles

$A + B = 90^\circ$ or $\frac{\pi}{2}$

If $A + B = 90^\circ$ (or $\pi/2$ radians)

- $\sin A = \cos B$
- $\cos A = \sin B$
- $\boxed{\tan A = \cot B}$ $\implies$ $\tan A \tan B = 1$

$\sin\left(\frac{\pi}{9}\right) = \cos\left(\frac{7\pi}{18}\right)$ $\qquad \frac{\pi}{2} - \frac{\pi}{9} = \frac{7\pi}{18}$

$\cos\left(\frac{3\pi}{10}\right) = \sin\left(\frac{2\pi}{10}\right)$

$A + B = 90^\circ$

$\sin A = \sin(90 - B) = \cos B$

$\tan A = \tan(90 - B) = \cot B$

$\sin 30^\circ = \cos 60^\circ$

$\sin 60^\circ = \cos 30^\circ$

$\sin 15^\circ = \cos 75^\circ$

$\sin 10^\circ = \cos 80^\circ$

$\sin 1^\circ = \cos 89^\circ$

$\cos 70^\circ = \sin 20^\circ$

$\cos(3^\circ) = \sin(87^\circ)$

$\tan 30 = \cot 60$

$\cot 50 = \tan 40$

$^*\tan\left(\frac{5\pi}{12}\right) =$

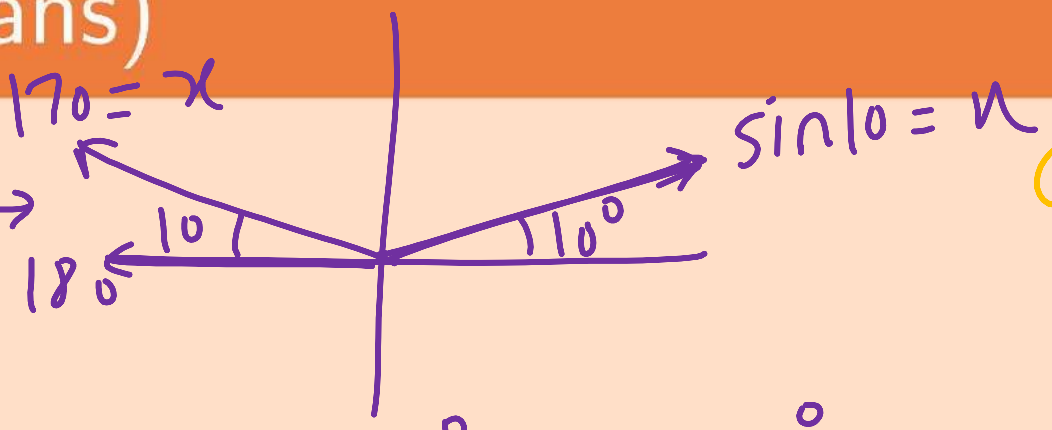
$\cot\left(\frac{\pi}{12}\right)$

Important Concept: Supplementary Angles

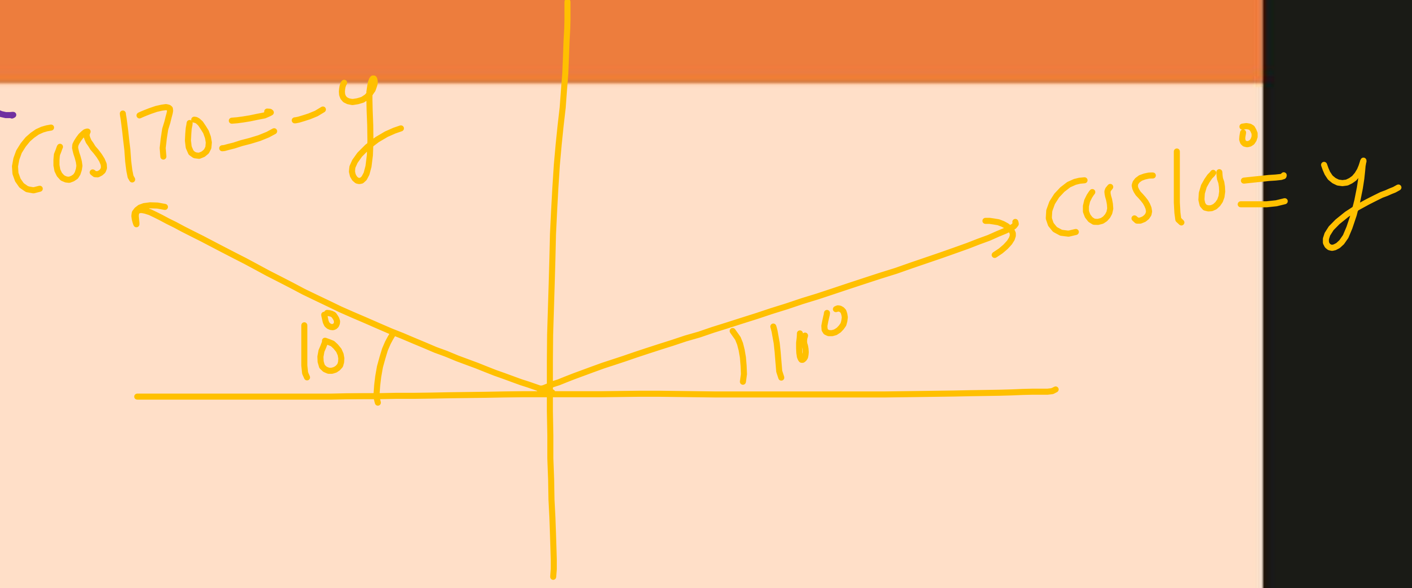
$A + B = 180^\circ / \pi^c$

If $A + B = 180^\circ$ (or π radians)

- $\sin A = \sin B$
- $\cos A + \cos B = 0$
- $\tan A + \tan B = 0$



$\sin 10^\circ = \sin 170^\circ$



$\cos 10^\circ + \cos 170^\circ = 0$

$y + (-y) = 0$

$\sin \underline{A} = \sin(\underline{180 - B}) = \sin \underline{B}$

$\cos A = \cos(180 - B) = -\cos B$

$\tan A = \tan(180 - B) = -\tan B$

eg

$\sin 10^\circ = \sin 170^\circ$

$\sin 60 = \sin 120^\circ$

$\sin 89^\circ = \sin 91^\circ$

$\cos 20 + \cos 1\cancel{8}60 = 0$

$\cos 35^\circ + \cos(145^\circ) = 0$

$\sin\left(\frac{3\pi}{16}\right) = \sin\left(\frac{13\pi}{16}\right)$

$\cos\left(\frac{7\pi}{9}\right) + \cos\left(\frac{2\pi}{9}\right) = 0$

$\tan\left(\frac{5\pi}{12}\right) + \tan\left(\frac{7\pi}{12}\right) = 0$

Trigonometric Ratios & Identities: Section 3

Formulas

Section-2 Allied Angle

HW: Assignment-02

Section-1 Basic Identities

HW Assignment-01

Formula Set-1: Compound Angle Formulas

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Formula Set-1: Compound Angle Formulas (Continued)

5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

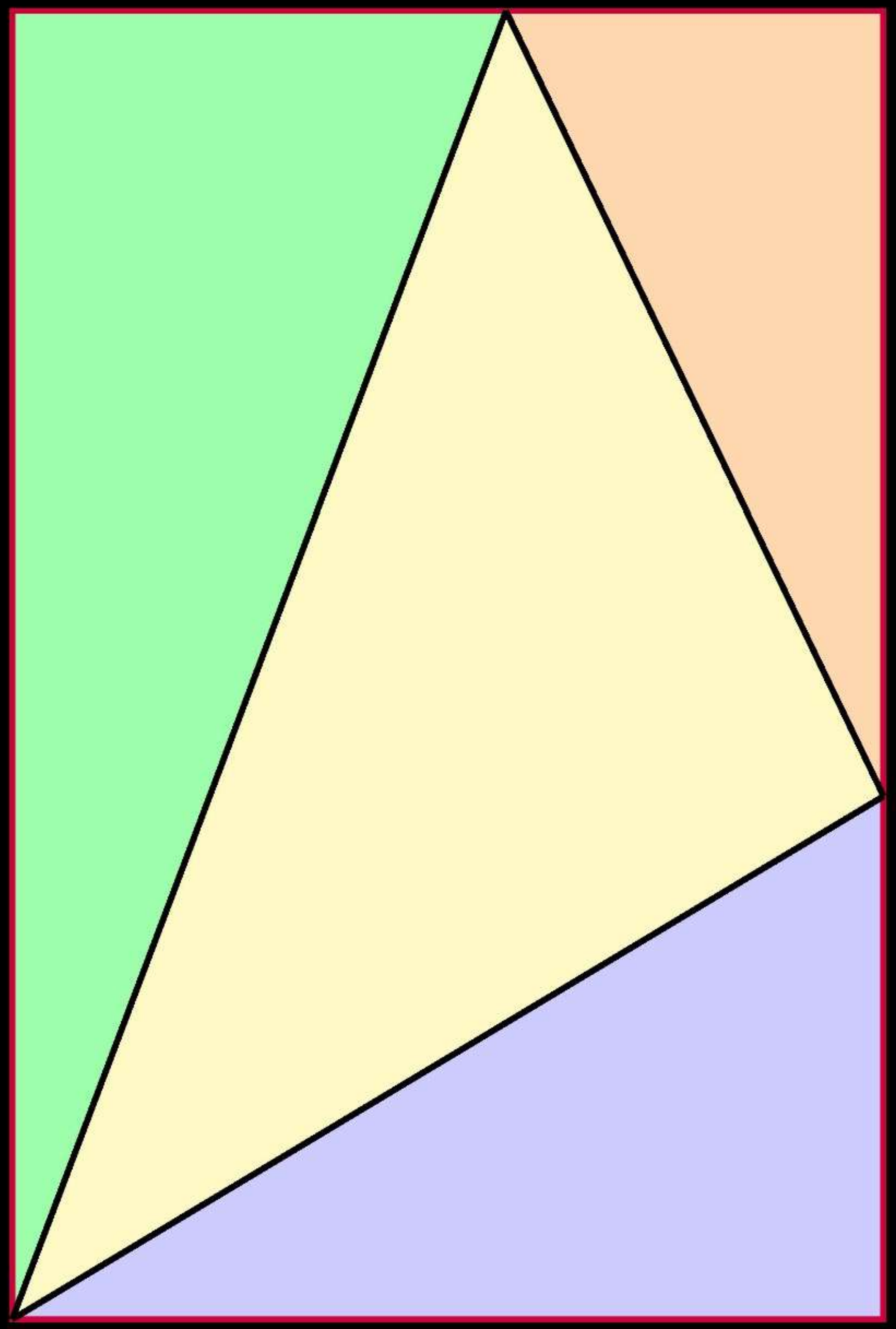
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Proof of $\tan(A+B)$ & $\cot(A+B)$

Compound Angle Formula Proof:



Simplify

1. $\sin(45^\circ + \theta)\cos(15^\circ + \theta) - \cos(45^\circ + \theta)\sin(15^\circ + \theta)$

2. $\cos(45^\circ - A)\cos(45^\circ - B) - \sin(45^\circ - A)\sin(45^\circ - B)$

3. $\sin 99^\circ \cos 21^\circ + \cos 99^\circ \sin 21^\circ$

4. $\cos 50^\circ \cos 10^\circ - \sin 50^\circ \sin 10^\circ$

Prove that: 1

$$\tan A \cot\left(\frac{A}{2}\right) - 1 = \sec A$$

Prove that: 2 (HOMEWORK)

$$1 + \tan A \tan\left(\frac{A}{2}\right) = \sec A$$

Question: Find the value

1. $\sin 15^\circ$ 2. $\cos 15^\circ$ 3. $\tan 15^\circ$ 4.$\sin 75^\circ$ 5.$\sin 75^\circ$ 6. $\cos 75^\circ$

Important Values

Ratio	$15^\circ = \frac{\pi}{12}$	$75^\circ = \frac{5\pi}{12}$
$\sin\theta$	$\dfrac{\sqrt{3}-1}{2\sqrt{2}}$	$\dfrac{\sqrt{3}+1}{2\sqrt{2}}$
$\cos\theta$	$\dfrac{\sqrt{3}+1}{2\sqrt{2}}$	$\dfrac{\sqrt{3}-1}{2\sqrt{2}}$
$\tan\theta$	$2-\sqrt{3}$	$2+\sqrt{3}$

Question: 1 [JEE Main 2023] Ans:(A)

If $\tan 15^\circ + \dfrac{1}{\tan 75^\circ} + \dfrac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \dfrac{1}{a}\right)$ is:

(A) 4

(B) $4 - 2\sqrt{3}$

(C) 2

(D) $5 - \dfrac{3}{2}\sqrt{3}$

Question: 2

$\sin\alpha = \dfrac{3}{5}$ and $\cos\beta = \dfrac{9}{41}$, where α and β are acute angles (in the first quadrant).
Find the values of:

1. $\sin(\alpha - \beta)$
2. $\cos(\alpha - \beta)$

Question: 3 [JEE Main 2022] Ans:(A)

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are:

(A) $-\frac{1}{7}$ and IV^{th} quadrant

(B) 7 and I^{st} quadrant

(C) -7 and IV^{th} quadrant

(D) $\frac{1}{7}$ and I^{st} quadrant

Remark-1

Yaad Rakho!

1. $\tan(45^\circ + \theta) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

2. $\tan(45^\circ - \theta) = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

Question: 4

If $A + B = \frac{\pi}{2}$,then prove that:

$$\tan A \tan B = 1$$

Question: 5

If $A + B = \dfrac{\pi}{4}$ then prove that:

$$(1 + \tan A)(1 + \tan B) = 2$$

Question: 6 HOMEWORK

If $A + B = 45^\circ$, then prove that:

$$(\cot A - 1)(\cot B - 1) = 2$$

Question: 7 [MHT-CET 2019] HOMEWORK

If $A - B = \frac{\pi}{4}$, then the value of $(1 + \tan A)(1 - \tan B)$?

Question: 8

Prove that:

$$\tan 5\theta \cdot \tan 3\theta \cdot \tan 2\theta = \tan 5\theta - \tan 3\theta - \tan 2\theta$$

Question: 9

Find the value of the following expression:

$$\tan 125^\circ + \tan 100^\circ + \tan 100^\circ \tan 125^\circ$$

Question: 10 HOMEWORK

Prove that:

$$\tan 80^\circ = \tan 10^\circ + 2\tan 70^\circ$$

Question: 11

If $\tan\theta = \dfrac{1}{2}$ and $\tan\phi = \dfrac{1}{3}$, find the value of $\tan(2\theta + \phi)$.

Question: 12 [AIEEE 2010]

If $\cos(\alpha+\beta) = \dfrac{4}{5}$ and $\sin(\alpha-\beta) = \dfrac{5}{13}$, where α and β both lie between 0 and $\dfrac{\pi}{4}$, then find the value of $\tan(2\alpha)$.

Question: 13 HOMEWORK

If $\cos(\theta - \alpha) = a$ and $\sin(\theta - \beta) = b$, then prove that:

$$\cos^2(\alpha - \beta) + 2ab\sin(\alpha - \beta) = a^2 + b^2$$