Topic: Quadratic Equation

Sub: Mathematics   **Assignment: Solution**   Prof. Chetan Sir

1. The value of p and q ($p \neq 0, q \neq 0$) for which p, q are the roots of the equation $x^2 + px + q = 0$ are:

(A) $p = 1, q = 2$ (B) $p = -1, q = 2$
(C) $p = -1, q = -2$ (D) $p = 1, q = -2$

Solution:
Given the equation is $x^2 + px + q = 0$.
The roots of the equation are p and q.

From the properties of quadratic equations (Vieta's formulas):

- Sum of roots: $p + q = -p \implies 2p + q = 0 \quad \cdots (i)$

- Product of roots: $p \cdot q = q \quad \cdots (ii)$

From equation (ii), since it is given that $q \neq 0$, we can divide both sides by q.

$$pq = q$$
$$p = 1.$$

Now, substitute the value $p = 1$ into equation (i).

$$2(1) + q = 0$$
$$q = -2.$$

The values are $p = 1$ and $q = -2$.
The correct option is **(D)**.

2. If the roots of the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ are equal in magnitude and opposite in sign, then:

(A) $a - b = 0$ (B) $a + b = 1$
(C) $a - b = 1$ (D) $a + b = 0$

Solution:
First, we simplify the given equation into the standard quadratic form $Ax^2 + Bx + C = 0$.

$$a(x - b) + b(x - a) = (x - a)(x - b)$$
$$ax - ab + bx - ab = x^2 - bx - ax + ab$$

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Now, group the terms and move them to one side.

$$x^2 - ax - bx + ab - (ax + bx - 2ab) = 0$$
$$x^2 - (a+b)x - (a+b)x + ab + 2ab = 0$$
$$x^2 - 2(a+b)x + 3ab = 0$$

The condition that roots are equal in magnitude and opposite in sign (e.g., α and $-\alpha$) means their sum is zero.

$$\text{Sum of roots} = \alpha + (-\alpha) = 0.$$

From our derived quadratic equation, the sum of the roots is $-B/A$.

$$\text{Sum of roots} = -\frac{-2(a+b)}{1} = 2(a+b).$$

Equating the two expressions for the sum of roots:

$$2(a+b) = 0 \implies a + b = 0.$$

The correct option is **(D)**.

3. If $a_1, a_2, ..., a_n \in R$ then $(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2$ assumes its least value at $x =$

(A) $a_1 + a_2 + \cdots + a_n$
(B) $2(a_1 + a_2 + \cdots + a_n)$
(C) $n(a_1 + a_2 + \cdots + a_n)$
(D) none of these

Solution:
Let the given expression be $S(x) = \sum_{i=1}^{n}(x - a_i)^2$. To find the minimum value of this expression, we can expand it to see that it is a quadratic function of x.

$$S(x) = \sum_{i=1}^{n}(x^2 - 2xa_i + a_i^2)$$
$$= \sum_{i=1}^{n}x^2 - \sum_{i=1}^{n}2xa_i + \sum_{i=1}^{n}a_i^2$$
$$= nx^2 - 2x\left(\sum_{i=1}^{n}a_i\right) + \left(\sum_{i=1}^{n}a_i^2\right)$$

This is a quadratic equation in the form $Ax^2 + Bx + C$, where $A = n$, $B = -2\sum a_i$, and $C = \sum a_i^2$.

Since $A = n > 0$, the parabola opens upwards, and its minimum value occurs at the vertex, $x = -B/(2A)$.

$$x = -\frac{-2\left(\sum_{i=1}^{n}a_i\right)}{2n} = \frac{\sum_{i=1}^{n}a_i}{n} = \frac{a_1 + a_2 + ... + a_n}{n}.$$

The least value occurs when x is the arithmetic mean of $a_1, a_2, \ldots, a_n$. This result does not match options (A), (B), or (C).
The correct option is **(D)**.

4. Number of values of k for which roots of equation $x^2 - 3x + k = 0$ lie in the interval (0, 1) is:

(A) only one (B) no value
(C) finite but more than one (D) $k \leq \frac{9}{4}$

Solution:

For both roots of a quadratic $f(x) = Ax^2 + Bx + C$ to lie in an interval (k_1, k_2), three conditions must be met:

1. The discriminant must be non-negative: $D = B^2 - 4AC \geq 0$.

2. The vertex of the parabola must lie between k_1 and k_2: $k_1 < -B/(2A) < k_2$.

3. The value of the function at the interval boundaries must have the same sign as A.

Here, $f(x) = x^2 - 3x + k$, so $A = 1$, $B = -3$, $C = k$. The interval is $(0, 1)$. Let's check the necessary conditions.

Condition 2: Location of Vertex

The x-coordinate of the vertex is given by $x = -B/(2A)$.

$$x = -\frac{-3}{2(1)} = \frac{3}{2}.$$

For the roots to lie in $(0, 1)$, the vertex must also lie in $(0, 1)$. We must check if $0 < 3/2 < 1$. The inequality $3/2 < 1$ is false. Since a necessary condition is not satisfied, it is impossible for both roots to lie in the interval $(0, 1)$. Therefore, there are no values of k for which this condition holds.

The correct option is **(B)**.

5. If the roots of the equation $2x^2 - (a^3 + 1)x + (a^2 - 2a) = 0$ are of opposite signs, then the set of possible value of a is:

(A) (0,2) (B) [0,2]
(C) (-∞,0) ∪ (2,∞) (D) (-2,0)

Solution:

The condition for the roots of a quadratic equation to be of opposite signs is that their product is negative. For a quadratic $Ax^2 + Bx + C = 0$, the product of roots is given by C/A.

$$\text{Product of roots} < 0$$
$$\frac{a^2 - 2a}{2} < 0$$
$$a^2 - 2a < 0$$
$$a(a - 2) < 0$$

Thus, the solution is $0 < a < 2$.

The correct option is **(A)**.

6. The value of a for which the equations $x^2 - 3x + a = 0$ and $x^2 + ax - 3 = 0$ have a common root is:

(A) 3 (B) 1
(C) -2 (D) 2

Solution:

Let the common root be α. Then α must satisfy both equations.

$$\alpha^2 - 3\alpha + a = 0 \quad \cdots (1)$$
$$\alpha^2 + a\alpha - 3 = 0 \quad \cdots (2)$$

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Subtracting equation (2) from (1) to eliminate the α^2 term:

$$(\alpha^2 - 3\alpha + a) - (\alpha^2 + a\alpha - 3) = 0$$
$$-3\alpha - a\alpha + a + 3 = 0$$
$$-(3 + a)\alpha + (a + 3) = 0$$
$$(a + 3)(1 - \alpha) = 0$$

This gives two possibilities: $a + 3 = 0$ or $1 - \alpha = 0$.

- **Case 1:** $a + 3 = 0 \implies a = -3$. If $a = -3$, both equations become $x^2 - 3x - 3 = 0$. They are identical and thus have two common roots. This is a valid solution.

- **Case 2:** $1 - \alpha = 0 \implies \alpha = 1$. If the common root is 1, substitute it into either equation. Using equation (1):

$$(1)^2 - 3(1) + a = 0$$
$$1 - 3 + a = 0 \implies a = 2.$$

Both $a = -3$ and $a = 2$ are possible values. Since $a = 2$ is one of the options, we choose it.
The correct option is **(D)**.

7. The value of a for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and other is less than 2, are given by:

(A) $3 < a < 10$ (B) $a \geq 10$
(C) $-2 < a < 3$ (D) $a \leq -2$

Solution:
Let $f(x) = x^2 - (a+1)x + a^2 + a - 8$.

The condition that one root is greater than 2 and the other is less than 2 means that the number 2 lies between the roots.

For a quadratic $Ax^2 + Bx + C = 0$ with $A > 0$ (an upward-opening parabola), a number k lies between the roots if and only if $f(k) < 0$. Here, $A = 1 > 0$, so the condition is $f(2) < 0$.

$$f(2) = (2)^2 - (a+1)(2) + a^2 + a - 8 < 0$$
$$= 4 - 2a - 2 + a^2 + a - 8 < 0$$
$$= a^2 - a - 6 < 0$$

We factor the quadratic inequality:

$$(a - 3)(a + 2) < 0$$

The critical points are $a = 3$ and $a = -2$. The expression is negative between these points. So, the solution is $-2 < a < 3$.
The correct option is **(C)**.

8. If x be real and $y = \frac{x^2 + 6x + 1}{2x}$ then which of the following is true:

(A) $2 < y < 4$ (B) $\frac{5}{2} < y < \frac{7}{2}$
(C) either $y \leq 2$ or $y \geq 4$ (D) none of these

Solution:

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We rearrange the given equation to form a quadratic in the variable x.

$$2xy = x^2 + 6x + 1$$
$$x^2 + 6x - 2xy + 1 = 0$$
$$x^2 + (6 - 2y)x + 1 = 0$$

Since x is a real number, the discriminant (D) of this quadratic equation must be greater than or equal to zero $(D \geq 0)$.

$$
\begin{aligned}
D = B^2 - 4AC &\geq 0 \\
= (6 - 2y)^2 - 4(1)(1) &\geq 0 \\
= 4(3 - y)^2 - 4 &\geq 0 \\
= (3 - y)^2 - 1 &\geq 0 \\
= (3 - y - 1)(3 - y + 1) &\geq 0 \qquad \text{[Using } a^2 - b^2 = (a - b)(a + b)\text{]} \\
= (2 - y)(4 - y) &\geq 0 \\
= (y - 2)(y - 4) &\geq 0
\end{aligned}
$$

This inequality holds when both factors are non-negative or both are non-positive. This occurs when $y \leq 2$ or $y \geq 4$. This defines the range of possible values for y.
The correct option is **(C)**.

9. Number of real solutions of the equation $(12x - 1)(6x - 1)(4x - 1)(3x - 1) = 5$ **are:**

(A) 0   (B) 2
(C) 1   (D) 4

Solution:
Rearrange the multiplication.

$$
\begin{aligned}
\left[(12x - 1)(3x - 1)\right] \cdot \left[(6x - 1)(4x - 1)\right] &= 5 \\
(36x^2 - 15x + 1) \cdot (24x^2 - 10x + 1) &= 5
\end{aligned}
$$

Factor out common terms to reveal a hidden structure.

$$\left[3(12x^2 - 5x) + 1\right] \cdot \left[2(12x^2 - 5x) + 1\right] = 5$$

Let $y = 12x^2 - 5x$. The equation becomes a quadratic in y.

$$
\begin{aligned}
(3y + 1)(2y + 1) &= 5 \\
6y^2 + 5y + 1 &= 5 \\
6y^2 + 5y - 4 &= 0
\end{aligned}
$$

Solving for y by factoring: $6y^2 + 8y - 3y - 4 = 0 \implies 2y(3y + 4) - 1(3y + 4) = 0 \implies (2y - 1)(3y + 4) = 0$.
This gives two possible values for y: $y = 1/2$ or $y = -4/3$.

- **Case 1:** $y = 1/2$.

  $12x^2 - 5x = 1/2 \implies 24x^2 - 10x - 1 = 0$. Discriminant $D = (-10)^2 - 4(24)(-1) = 100 + 96 = 196 > 0$. This gives two distinct real solutions for x.

- **Case 2:** $y = -4/3$.

  $12x^2 - 5x = -4/3 \implies 36x^2 - 15x + 4 = 0$. Discriminant $D = (-15)^2 - 4(36)(4) = 225 - 576 = -351 < 0$. This gives no real solutions for x.

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In total, there are two real solutions.

The correct option is **(B)**.

10. If α, β are the roots of $ax^2 - 2bx + c = 0$ then $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2$ is:

(A) $\frac{c^2(c+2b)}{a^3}$

(B) $\frac{bc^3}{a^3}$

(C) $\frac{c^2}{a^3}$

(D) none of these

Solution:

From the given equation, we find the sum and product of roots.

- Sum of roots: $\alpha + \beta = -(-2b)/a = 2b/a$.

- Product of roots: $\alpha\beta = c/a$.

Now, we simplify the given expression by factoring out common terms.

$$\text{Expression} = \alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2$$
$$= (\alpha\beta)^3 + \alpha^2\beta^2(\beta + \alpha)$$
$$= (\alpha\beta)^3 + (\alpha\beta)^2(\alpha + \beta)$$

Substitute the values for the sum and product of roots.

$$= \left(\frac{c}{a}\right)^3 + \left(\frac{c}{a}\right)^2\left(\frac{2b}{a}\right)$$
$$= \frac{c^3}{a^3} + \frac{2bc^2}{a^3} = \frac{c^3 + 2bc^2}{a^3} = \frac{c^2(c + 2b)}{a^3}.$$

The correct option is **(A)**.

11. The equation $x^{\frac{3}{4}(log_2 x)^2 + log_2 x - \frac{5}{4}} = \sqrt{2}$ has:

(A) atleast one real solution

(B) exactly two irrational solutions

(C) exactly one rational solution

(D) complex roots

Solution:

To solve this exponential equation, we take the logarithm with base 2 on both sides.

$$\log_2\left(x^{\frac{3}{4}(log_2 x)^2 + log_2 x - \frac{5}{4}}\right) = \log_2(\sqrt{2})$$

Using the log property $\log(a^b) = b\log(a)$:

$$\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right)\log_2 x = \log_2(2^{1/2}) = \frac{1}{2}.$$

Let $y = \log_2 x$. The equation becomes a polynomial in y.

$$\left(\frac{3}{4}y^2 + y - \frac{5}{4}\right)y = \frac{1}{2}$$
$$(3y^2 + 4y - 5)y = 2 \qquad \text{[Multiplying both sides by 4]}$$
$$3y^3 + 4y^2 - 5y - 2 = 0$$

We test for integer roots that are divisors of -2 (i.e., $\pm 1, \pm 2$).

For $y = 1$: $3(1)^3 + 4(1)^2 - 5(1) - 2 = 3 + 4 - 5 - 2 = 0$. So, $(y - 1)$ is a factor.

Factoring the cubic polynomial:

$$(y - 1)(3y^2 + 7y + 2) = 0$$
$$(y - 1)(3y + 1)(y + 2) = 0$$

The solutions for y are $y = 1$, $y = -2$, and $y = -1/3$.

Now we find the corresponding values of $x = 2^y$.

- If $y = 1 \implies \log_2 x = 1 \implies x = 2^1 = 2$ (Rational).

- If $y = -2 \implies \log_2 x = -2 \implies x = 2^{-2} = 1/4$ (Rational).

- If $y = -1/3 \implies \log_2 x = -1/3 \implies x = 2^{-1/3} = 1/\sqrt[3]{2}$ (Irrational).

The equation has three distinct real solutions, including rational and irrational ones. Therefore, it has at least one real solution.

The correct option is **(A)**.

12. If the equation $x^2 + 2|a|x + 4 = 0$ has integral roots, then minimum value of 'a' is:

(A) 4   (B) $-\frac{5}{2}$
(C) 0   (D) -4

Solution:
Let the integral roots be r_1 and r_2. From Vieta's formulas:

- Product of roots: $r_1 r_2 = 4$.

- Sum of roots: $r_1 + r_2 = -2|a|$.

Since the roots are integers, the possible pairs for (r_1, r_2) are: $(1, 4), (-1, -4), (2, 2), (-2, -2)$.

We check the sum for each pair:

- $(1, 4) \implies r_1 + r_2 = 5$. Then $5 = -2|a| \implies |a| = -5/2$. Not possible as $|a| \geq 0$.

- $(-1, -4) \implies r_1 + r_2 = -5$. Then $-5 = -2|a| \implies |a| = 5/2$.

- $(2, 2) \implies r_1 + r_2 = 4$. Then $4 = -2|a| \implies |a| = -2$. Not possible.

- $(-2, -2) \implies r_1 + r_2 = -4$. Then $-4 = -2|a| \implies |a| = 2$.

The possible values for $|a|$ are $5/2$ and 2.

This means the possible values for a are $\pm 5/2$ and ± 2.

The set of possible values for a is $\{-5/2, -2, 2, 5/2\}$.

The minimum value in this set is $-5/2$.

The correct option is **(B)**.

13. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has:

(A) both roots in [a, b]   (B) both roots in $(-\infty, a)$
(C) both roots in (b, ∞) (D) one root in $(-\infty, a)$ and other in (b, ∞).

Solution:

Let $f(x) = (x - a)(x - b) - 1$. This is an upward-opening parabola since the coefficient of x^2 is 1. We can analyze the location of the roots by evaluating the function at the points $x = a$ and $x = b$.

$$f(a) = (a - a)(a - b) - 1 = 0 - 1 = -1.$$
$$f(b) = (b - a)(b - b) - 1 = 0 - 1 = -1.$$

Since the parabola opens upwards and its value is negative at both $x = a$ and $x = b$, it must cross the x-axis at two points.

Because $f(a) = -1 < 0$, the function must cross the x-axis at a point $x_1 < a$.

Because $f(b) = -1 < 0$, the function must cross the x-axis at a point $x_2 > b$.

Therefore, one root is in $(-\infty, a)$ and the other root is in (b, ∞).

The correct option is **(D)**.

14. The number of integral values of 'a' for which $(a+2)x^2 + 2(a+1)x + a = 0$ will have both roots integers is:

(A) 3   (B) 5

(C) 7   (D) none of these

Solution:
Let's check for an obvious root by substituting simple values for x. Let's try $x = -1$.

$$(a + 2)(-1)^2 + 2(a + 1)(-1) + a = (a + 2) - 2(a + 1) + a$$
$$= a + 2 - 2a - 2 + a = 0.$$

So, one root is always $\alpha = -1$, which is an integer.

Let the other integer root be β.

From Vieta's formulas, the product of roots is $\alpha\beta = \frac{a}{a+2}$.

$$(-1)\beta = \frac{a}{a + 2}$$
$$\beta = -\frac{a}{a + 2}$$

Since β must be an integer, we can rewrite the expression for β.

$$\beta = -\frac{a + 2 - 2}{a + 2} = -\left(1 - \frac{2}{a + 2}\right) = \frac{2}{a + 2} - 1.$$

So, $\beta + 1 = \frac{2}{a+2}$. For β to be an integer, $\beta + 1$ must also be an integer. This means $(a + 2)$ must be an integer divisor of 2. The integer divisors of 2 are $\{1, -1, 2, -2\}$.

- If $a + 2 = 1 \implies a = -1$. (Integer)

- If $a + 2 = -1 \implies a = -3$. (Integer)

- If $a + 2 = 2 \implies a = 0$. (Integer)

- If $a + 2 = -2 \implies a = -4$. (Integer)

The possible integral values for 'a' are $\{-1, -3, 0, -4\}$. This is a total of 4 values.

The correct option is **(D)**.

15. If the equation $(x^2 + x + 3)^2 - (\lambda - 4)(x^2 + x + 3)(x^2 + x + 2) + (\lambda - 5)(x^2 + x + 2)^2 = 0$ **has at least one real solution. Then find** $[\lambda]$.

Solution:

Let $y = x^2 + x + 2$. Then the expression $x^2 + x + 3 = y + 1$.

First, we find the range of possible values for y. The quadratic $x^2 + x + 2$ is an upward-opening parabola with its minimum value at the vertex $x = -1/2$.

$$\text{Minimum value of } y = (-1/2)^2 + (-1/2) + 2 = 1/4 - 1/2 + 2 = 7/4.$$

So, for any real solution x, we must have $y \geq 7/4$. Substitute y into the given equation:

$$(y + 1)^2 - (\lambda - 4)y(y + 1) + (\lambda - 5)y^2 = 0$$

Expand and group terms by powers of y.

$$(y^2 + 2y + 1) - (\lambda - 4)(y^2 + y) + (\lambda - 5)y^2 = 0$$
$$y^2(1 - (\lambda - 4) + (\lambda - 5)) + y(2 - (\lambda - 4)) + 1 = 0$$
$$y^2(1 - \lambda + 4 + \lambda - 5) + y(2 - \lambda + 4) + 1 = 0$$
$$y^2(0) + y(6 - \lambda) + 1 = 0$$
$$y(6 - \lambda) = -1$$

If $\lambda = 6$, we get $0 = -1$, which is impossible. So $\lambda \neq 6$.

$$y = \frac{-1}{6 - \lambda} = \frac{1}{\lambda - 6}$$

For the original equation to have a real solution in x, we need a valid solution for y. We established that $y \geq 7/4$.

$$\frac{1}{\lambda - 6} \geq \frac{7}{4}$$

For the fraction to be positive (since $7/4$ is positive), the denominator must be positive. So, $\lambda - 6 > 0 \implies \lambda > 6$. Now we can cross-multiply without flipping the inequality sign.

$$4 \geq 7(\lambda - 6)$$
$$4 \geq 7\lambda - 42$$
$$46 \geq 7\lambda$$
$$\lambda \leq \frac{46}{7}$$

Combining the conditions, we get $6 < \lambda \leq 46/7$. Since $46/7 \approx 6.57$, the range for λ is $(6, 6.57]$. The question asks for the value of $[\lambda]$, the greatest integer function. For any value of λ in this range, $[\lambda] = 6$. The answer is **6**.

16. The number of real solutions of $1 + |e^x - 1| = e^x(e^x - 2)$ **is:**

Solution:

Let $y = e^x$. Since x is real, the range of y is $y > 0$. The equation becomes:

$$1 + |y - 1| = y(y - 2) = y^2 - 2y$$

We consider two cases based on the absolute value expression.

- **Case 1:** Assume $y - 1 \geq 0 \implies y \geq 1$. In this case, $|y - 1| = y - 1$. The equation becomes:

$$1 + (y - 1) = y^2 - 2y$$
$$y = y^2 - 2y$$
$$y^2 - 3y = 0 \implies y(y - 3) = 0$$

  The potential solutions are $y = 0$ and $y = 3$. We must check these against the conditions for this case ($y \geq 1$). $y = 0$ is not valid. $y = 3$ is valid. If $y = 3$, then $e^x = 3 \implies x = \ln 3$. This is one real solution.

- **Case 2:** Assume $y - 1 < 0 \implies 0 < y < 1$. In this case, $|y - 1| = -(y - 1) = 1 - y$. The equation becomes:

$$1 + (1 - y) = y^2 - 2y$$
$$2 - y = y^2 - 2y$$
$$y^2 - y - 2 = 0 \implies (y - 2)(y + 1) = 0$$

  The potential solutions are $y = 2$ and $y = -1$. Neither of these values satisfy the condition for this case ($0 < y < 1$). So there are no solutions from this case.

Combining both cases, there is only one real solution, $x = \ln 3$.
The answer is **1**.

17. If α is a positive integer and the roots of the equation $6x^2 - 11x + \alpha = 0$ are rational numbers, then the smallest value of α is:

Solution:
For the roots of a quadratic equation to be rational, the discriminant (D) must be a perfect square of a rational number. Since the coefficients are integers, D must be a perfect square of an integer.

$$D = B^2 - 4AC = (-11)^2 - 4(6)(\alpha) = 121 - 24\alpha.$$

Let $D = k^2$ for some non-negative integer k.

$$121 - 24\alpha = k^2$$

Since α is a positive integer, 24α is positive. Therefore, $k^2 < 121$, which means $k < 11$. Also, $121 - k^2 = 24\alpha$. This implies that $121 - k^2$ must be a positive multiple of 24. We test integer values for k from 0 to 10.

- $k = 0 \implies 121 - 0 = 121$ (Not divisible by 24)

- $k = 1 \implies 121 - 1 = 120$. $120 = 24 \times 5$. This is a valid multiple. Here, $24\alpha = 120 \implies \alpha = 5$.

- $k = 2 \implies 121 - 4 = 117$ (Not divisible by 24)

- $k = 3 \implies 121 - 9 = 112$ (Not divisible by 24)

- $k = 4 \implies 121 - 16 = 105$ (Not divisible by 24)

- $k = 5 \implies 121 - 25 = 96$. $96 = 24 \times 4$. This is a valid multiple. Here, $24\alpha = 96 \implies \alpha = 4$.

- $k = 6 \implies 121 - 36 = 85$ (Not divisible by 24)

- $k = 7 \implies 121 - 49 = 72$. $72 = 24 \times 3$. This is a valid multiple. Here, $24\alpha = 72 \implies \alpha = 3$.

The possible positive integer values for α are 5, 4, and 3. The smallest of these values is 3.
The answer is **3**.

18. **If $\alpha \neq 0$ then the number of values of the pair (α, β) such that $\alpha + \beta + \frac{\beta}{\alpha} = \frac{1}{2}$ and $(\alpha + \beta)\frac{\beta}{\alpha} = -\frac{1}{2}$ is:**

Solution:
Let $X = \alpha + \beta$ and $Y = \beta/\alpha$. The given equations are:

$$X + Y = 1/2 \quad \cdots (1)$$
$$X \cdot Y = -1/2 \quad \cdots (2)$$

Consider a quadratic equation with roots X and Y: $t^2 - (\text{Sum of roots})t + (\text{Product of roots}) = 0$.

$$t^2 - (1/2)t - 1/2 = 0$$
$$2t^2 - t - 1 = 0$$
$$(2t + 1)(t - 1) = 0$$

The solutions are $t = 1$ and $t = -1/2$. This gives two possible cases for the pair (X, Y).

- **Case 1:** $X = 1$ and $Y = -1/2$.

  $\alpha + \beta = 1$ and $\beta/\alpha = -1/2 \implies \beta = -\alpha/2$. Substituting β into the first equation: $\alpha - \alpha/2 = 1 \implies \alpha/2 = 1 \implies \alpha = 2$. Then $\beta = -2/2 = -1$. This gives one pair $(\alpha, \beta) = (2, -1)$.

- **Case 2:** $X = -1/2$ and $Y = 1$.

  $\alpha + \beta = -1/2$ and $\beta/\alpha = 1 \implies \beta = \alpha$. Substituting β into the first equation: $\alpha + \alpha = -1/2 \implies 2\alpha = -1/2 \implies \alpha = -1/4$. Then $\beta = -1/4$. This gives a second pair $(\alpha, \beta) = (-1/4, -1/4)$.

There are two possible pairs for (α, β).
The answer is **2**.

19. **Suppose a and b are real numbers with $ab \neq 0$. If the three quadratic equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$, and $x^2 + (a+b)x + 36 = 0$ have a common negative root then $|a| + |b| = \_..$**

Solution:
Let the common negative root be α. Then α satisfies all three equations.

$$\alpha^2 + a\alpha + 12 = 0 \quad \cdots (1)$$
$$\alpha^2 + b\alpha + 15 = 0 \quad \cdots (2)$$
$$\alpha^2 + (a+b)\alpha + 36 = 0 \quad \cdots (3)$$

Add equation (1) and (2):

$$2\alpha^2 + (a+b)\alpha + 27 = 0 \quad \cdots (4)$$

Now we have two equations involving $(a+b)$: equation (3) and equation (4). Subtracting (4) from (3):

$$(\alpha^2 + (a+b)\alpha + 36) - (2\alpha^2 + (a+b)\alpha + 27) = 0$$
$$-\alpha^2 + 9 = 0$$
$$\alpha^2 = 9 \implies \alpha = \pm 3.$$

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Since the common root is negative, we must have $\alpha = -3$. Now substitute $\alpha = -3$ back into equations (1) and (2) to find a and b.

$$\text{From (1): } (-3)^2 + a(-3) + 12 = 0 \implies 9 - 3a + 12 = 0 \implies 3a = 21 \implies a = 7.$$
$$\text{From (2): } (-3)^2 + b(-3) + 15 = 0 \implies 9 - 3b + 15 = 0 \implies 3b = 24 \implies b = 8.$$

We need to find the value of $|a| + |b|$.

$$|a| + |b| = |7| + |8| = 7 + 8 = 15.$$

The answer is **15**.

20. The number of integral values of C for which $\dfrac{x^2 + 2x + c}{x^2 + 4x + 3c}$ can take all real values is:

Solution:
Let $y = \frac{x^2+2x+c}{x^2+4x+3c}$.

For y to take all real values, the quadratic equation formed by cross-multiplication must have real roots for all $y \in \mathbb{R}$ (except possibly one value if the coefficient of x^2 becomes zero).

$$y(x^2 + 4x + 3c) = x^2 + 2x + c$$
$$yx^2 + 4yx + 3yc = x^2 + 2x + c$$
$$(y-1)x^2 + (4y-2)x + (3yc - c) = 0$$

For x to be real, the discriminant $D \geq 0$.

$$(4y-2)^2 - 4(y-1)(3yc - c) \geq 0$$
$$4(2y-1)^2 - 4c(y-1)(3y-1) \geq 0$$
$$(4y^2 - 4y + 1) - c(3y^2 - 4y + 1) \geq 0$$
$$(4 - 3c)y^2 + (4c - 4)y + (1 - c) \geq 0$$

For this quadratic in y to be always non-negative, two conditions must hold:

1. The leading coefficient must be positive: $4 - 3c > 0 \implies 3c < 4 \implies c < 4/3$.

2. The discriminant of this quadratic in y must be non-positive: $D_y \leq 0$.

$$D_y = (4c - 4)^2 - 4(4 - 3c)(1 - c) \leq 0$$
$$16(c-1)^2 - 4(4 - 7c + 3c^2) \leq 0$$
$$4(c^2 - 2c + 1) - (4 - 7c + 3c^2) \leq 0$$
$$4c^2 - 8c + 4 - 4 + 7c - 3c^2 \leq 0$$
$$c^2 - c \leq 0$$
$$c(c - 1) \leq 0$$

This inequality holds for $0 \leq c \leq 1$. We must satisfy both conditions: $c < 4/3$ and $0 \leq c \leq 1$. The intersection is $0 \leq c \leq 1$. The integral values of c in this range are 0 and 1. The number of integral values is 2.
The answer is **2**.

21. If α and β are the roots of the equation $x^2 - ax + b = 0$ where $a = 2020$ and $A_n = \alpha^n + \beta^n$, then $(A_{n+1} + bA_{n-1})/A_n$ is:

Solution:

Since α and β are roots of $x^2 - ax + b = 0$, they satisfy the equation.

$$\alpha^2 - a\alpha + b = 0 \implies \alpha^2 = a\alpha - b$$
$$\beta^2 - a\beta + b = 0 \implies \beta^2 = a\beta - b$$

We can establish a recurrence relation for A_n. Multiply the first equation by α^{n-1} and the second by β^{n-1}.

$$\alpha^{n+1} = a\alpha^n - b\alpha^{n-1}$$
$$\beta^{n+1} = a\beta^n - b\beta^{n-1}$$

Adding these two equations:

$$(\alpha^{n+1} + \beta^{n+1}) = a(\alpha^n + \beta^n) - b(\alpha^{n-1} + \beta^{n-1})$$
$$A_{n+1} = aA_n - bA_{n-1}$$

Rearranging this recurrence relation to match the expression we need to find:

$$A_{n+1} + bA_{n-1} = aA_n$$
$$\frac{A_{n+1} + bA_{n-1}}{A_n} = a$$

We are given that $a = 2020$. The value of the expression is 2020.
The answer is **2020**.

22. If α and β are the roots of the equation $x^2 - 2x + 3 = 0$ then the sum of roots of the equation having roots as $\alpha^3 - 3\alpha^2 + 5\alpha - 2$ and $\beta^3 - \beta^2 + \beta + 5$ is:

(A) 1   (B) 3

(C) 5   (D) 7

Solution:

Since α is a root of $x^2 - 2x + 3 = 0$, we have $\alpha^2 - 2\alpha + 3 = 0$, which means $\alpha^2 = 2\alpha - 3$. Let the new roots be P and Q. Let's simplify P and Q.

$$\begin{aligned}
P &= \alpha^3 - 3\alpha^2 + 5\alpha - 2 \\
&= \alpha(\alpha^2) - 3\alpha^2 + 5\alpha - 2 \\
&= \alpha(2\alpha - 3) - 3(2\alpha - 3) + 5\alpha - 2 \\
&= 2\alpha^2 - 3\alpha - 6\alpha + 9 + 5\alpha - 2 \\
&= 2\alpha^2 - 4\alpha + 7 = 2(\alpha^2 - 2\alpha) + 7
\end{aligned}$$

From the original equation, $\alpha^2 - 2\alpha = -3$.

$$P = 2(-3) + 7 = 1.$$

Now, simplify Q. Since β is also a root, $\beta^2 - 2\beta + 3 = 0$, so $\beta^2 = 2\beta - 3$.

$$\begin{aligned}
Q &= \beta^3 - \beta^2 + \beta + 5 \\
&= \beta(\beta^2) - \beta^2 + \beta + 5 \\
&= \beta(2\beta - 3) - (2\beta - 3) + \beta + 5 \\
&= 2\beta^2 - 3\beta - 2\beta + 3 + \beta + 5 \\
&= 2\beta^2 - 4\beta + 8 = 2(\beta^2 - 2\beta) + 8
\end{aligned}$$

From the original equation, $\beta^2 - 2\beta = -3$.

$$Q = 2(-3) + 8 = 2.$$

The new roots are $P = 1$ and $Q = 2$. The sum of the new roots is $P + Q = 1 + 2 = 3$.
The correct option is **(B)**.

23. A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common is:

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$

(C) $i\sqrt{5}$ (D) $\sqrt{2}$

Solution:
Let the common root be α.

$$\alpha^2 + b\alpha - 1 = 0 \quad \cdots (1)$$
$$\alpha^2 + \alpha + b = 0 \quad \cdots (2)$$

Subtracting (2) from (1):

$$(b\alpha - 1) - (\alpha + b) = 0$$
$$b\alpha - \alpha - 1 - b = 0$$
$$\alpha(b - 1) = b + 1$$

If $b = 1$, the equations are $x^2 + x - 1 = 0$ and $x^2 + x + 1 = 0$, which have no common root. So $b \neq 1$.

$$\alpha = \frac{b + 1}{b - 1}$$

Substitute this value of α into equation (2):

$$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$$
$$(b + 1)^2 + (b + 1)(b - 1) + b(b - 1)^2 = 0$$
$$(b^2 + 2b + 1) + (b^2 - 1) + b(b^2 - 2b + 1) = 0$$
$$2b^2 + 2b + b^3 - 2b^2 + b = 0$$
$$b^3 + 3b = 0$$
$$b(b^2 + 3) = 0$$

This gives $b = 0$ or $b^2 = -3$.

If $b = 0$, the equations are $x^2 - 1 = 0$ and $x^2 + x = 0$. Roots are (± 1) and $(0, -1)$. The common root is -1. So $b = 0$ is a possible value.

If $b^2 = -3$, then $b = \pm i\sqrt{3}$.

The option given is $-i\sqrt{3}$.
The correct option is **(B)**.

24. If α, β are real and α^2, β^2 are the roots of the equation $a^2x^2 - x + 1 - a^2 = 0$ and $\beta^2 \neq 1$ then $\beta^2 =$

(A) a^2 (B) $\frac{1-a^2}{a^2}$

(C) $1 - a^2$ (D) $1 + a^2$

Solution:
Let the roots of $a^2x^2 - x + (1 - a^2) = 0$ be $r_1 = \alpha^2$ and $r_2 = \beta^2$. From Vieta's formulas:

$$\text{Sum of roots: } \alpha^2 + \beta^2 = -(-1)/a^2 = 1/a^2$$
$$\text{Product of roots: } \alpha^2\beta^2 = (1 - a^2)/a^2$$

Let's check if $x = 1$ is a root of the given equation.

$$a^2(1)^2 - (1) + 1 - a^2 = a^2 - 1 + 1 - a^2 = 0.$$

So, one of the roots is 1. Let $\alpha^2 = 1$. Since $\beta^2 \neq 1$, we have found one root. The other root, β^2, can be found from the product of roots.

$$\alpha^2\beta^2 = \frac{1 - a^2}{a^2}$$
$$(1)\beta^2 = \frac{1 - a^2}{a^2}$$
$$\beta^2 = \frac{1 - a^2}{a^2}.$$

The correct option is **(B)**.

25. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m?

(A) $-\frac{9}{8}$ \hspace{4cm} (B) $\frac{9}{8}$

(C) $-\frac{8}{9}$ \hspace{4cm} (D) $\frac{8}{9}$

Solution:
Let the roots be α and 2α. From Vieta's formulas:

$$\text{Sum of roots: } \alpha + 2\alpha = 3\alpha = -\frac{l}{l - m} \quad \cdots (1)$$
$$\text{Product of roots: } \alpha(2\alpha) = 2\alpha^2 = \frac{1}{l - m} \quad \cdots (2)$$

From (1), $\alpha = -\frac{l}{3(l-m)}$. Substitute this into (2).

$$2\left(-\frac{l}{3(l - m)}\right)^2 = \frac{1}{l - m}$$
$$2\frac{l^2}{9(l - m)^2} = \frac{1}{l - m}$$

Since l is real, the roots are real, so $l - m \neq 0$. We can cancel one factor of $(l - m)$.

$$\frac{2l^2}{9(l - m)} = 1$$
$$2l^2 = 9(l - m)$$
$$2l^2 = 9l - 9m$$

We need to find the greatest value of m. Let's express m in terms of l.

$$9m = 9l - 2l^2$$
$$m = l - \frac{2}{9}l^2$$

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This is a downward-opening parabola in terms of l. Its maximum value occurs at the vertex. The vertex of $f(l) = -\frac{2}{9}l^2 + l$ is at $l = -B/(2A)$.

$$l = -\frac{1}{2(-2/9)} = \frac{1}{4/9} = \frac{9}{4}.$$

The maximum value of m is found by substituting this value of l back into the expression for m.

$$m_{max} = \left(\frac{9}{4}\right) - \frac{2}{9}\left(\frac{9}{4}\right)^2 = \frac{9}{4} - \frac{2}{9}\left(\frac{81}{16}\right) = \frac{9}{4} - \frac{9}{8} = \frac{18-9}{8} = \frac{9}{8}.$$

The correct option is **(B)**.

26. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is:

(A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 + 19x - 3 = 0$
(C) $3x^2 - 19x - 3 = 0$ (D) $x^2 - 5x + 3 = 0$

Solution:
The conditions $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ mean that α and β are the distinct roots of the quadratic equation $x^2 - 5x + 3 = 0$. From this equation, we have:

- Sum of roots: $\alpha + \beta = 5$.

- Product of roots: $\alpha\beta = 3$.

The new equation has roots $r_1 = \alpha/\beta$ and $r_2 = \beta/\alpha$. We find the sum and product of these new roots.

$$\text{Sum of new roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$
$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(5)^2 - 2(3)}{3} = \frac{25-6}{3} = \frac{19}{3}.$$

$$\text{Product of new roots} = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1.$$

The new quadratic equation is $x^2 - (\text{Sum})x + (\text{Product}) = 0$.

$$x^2 - \frac{19}{3}x + 1 = 0$$
$$3x^2 - 19x + 3 = 0.$$

The correct option is **(A)**.

27. The real roots of the equation $x^2 + 5|x| + 4 = 0$ are:

(A) $\{-1, -4\}$ (B) $\{1, 4\}$
(C) $\{-4, 4\}$ (D) None of these

Solution:
Let $y = |x|$. Since $|x|$ is always non-negative, we have $y \geq 0$. The equation can be written in terms of y:

$$y^2 + 5y + 4 = 0$$

We can factor this quadratic equation.

$$(y + 4)(y + 1) = 0$$

The solutions for y are $y = -4$ and $y = -1$. However, we have the condition that $y = |x| \geq 0$. Neither of these solutions satisfy this condition. Therefore, there are no real values of x that can solve the equation.

The correct option is **(D)**.

28. Consider $f(x) = x^2 - 3x + a + \frac{1}{a}$, $a \in R - \{0\}$, such that $f(3) > 0$ and $f(2) \leq 0$. If α and β are the roots of equation $f(x) = 0$ then the value of $\alpha^2 + \beta^2$ is equal to:

(A) greater than 11   (B) less than 5

(C) 5   (D) depends upon a and cannot be determined.

Solution:

Let's analyze the given conditions.

$$f(3) = 3^2 - 3(3) + a + 1/a = 9 - 9 + a + 1/a = a + 1/a > 0.$$
$$f(2) = 2^2 - 3(2) + a + 1/a = 4 - 6 + a + 1/a = a + 1/a - 2 \leq 0.$$

From $f(2) \leq 0$, we have $a + 1/a \leq 2$.

The only way $a + 1/a \leq 2$ for $a \in \mathbb{R}$ is if a is positive and $a + 1/a = 2$, or if a is negative.

If $a > 0$, by AM-GM, $a + 1/a \geq 2$. So the only possibility is $a + 1/a = 2$, which means $a = 1$.

If $a < 0$, let $a = -k$ where $k > 0$. Then $-k - 1/k = -(k + 1/k) \leq -2$. This satisfies $a + 1/a \leq 2$.

The condition $f(3) > 0$ means $a + 1/a > 0$, which is only possible if $a > 0$. So, the only value of a that satisfies both conditions is $a = 1$.

The equation becomes $f(x) = x^2 - 3x + 1 + 1/1 = x^2 - 3x + 2 = 0$.

The roots α, β are from $(x - 1)(x - 2) = 0$, so the roots are 1 and 2.

We need to find $\alpha^2 + \beta^2$.

$$\alpha^2 + \beta^2 = 1^2 + 2^2 = 5.$$

The correct option is **(C)**.

29. If $y = 2 + \cfrac{1}{4 + \frac{1}{4 + \dots \infty}}$, then:

(A) $y = 6$ (B) $y = 5$

(C) $y = \sqrt{6}$ (D) $y = \sqrt{5}$

Solution:

Let the continued fraction part be z.

$$z = 4 + \cfrac{1}{4 + \frac{1}{4 + \dots \infty}}$$

We can see that the expression repeats, so we can write a recursive equation.

$$z = 4 + \frac{1}{z}$$

Multiply by z to form a quadratic equation.

$$z^2 = 4z + 1$$
$$z^2 - 4z - 1 = 0$$

Using the quadratic formula to solve for z:

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}.$$

Since the continued fraction is a sum of positive terms, z must be positive. So, $z = 2 + \sqrt{5}$. Now substitute this back into the expression for y.

$$y = 2 + \frac{1}{z} = 2 + \frac{1}{2 + \sqrt{5}}$$

Rationalize the denominator.

$$y = 2 + \frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}} = 2 + \frac{2 - \sqrt{5}}{4 - 5} = 2 + \frac{2 - \sqrt{5}}{-1} = 2 - (2 - \sqrt{5}) = \sqrt{5}.$$

The correct option is **(D)**.

30. If the roots of $ax^2 + bx + c = 0$ are the reciprocals of those of $lx^2 + mx + n = 0$ then

(A) $n : m : l$ (B) $l : m : n$
(C) $m : n : l$ (D) $n : l : m$

Solution:
Let the roots of $lx^2 + mx + n = 0$ be α and β. Then the roots of $ax^2 + bx + c = 0$ are $1/\alpha$ and $1/\beta$. From the second equation: $\alpha + \beta = -m/l$ and $\alpha\beta = n/l$. From the first equation:

$$\text{Sum of roots: } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{b}{a}$$

$$\text{Product of roots: } \frac{1}{\alpha}\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{c}{a}$$

Substitute the expressions from the second equation into the first.

$$\frac{-m/l}{n/l} = -\frac{b}{a} \implies \frac{-m}{n} = -\frac{b}{a} \implies \frac{m}{n} = \frac{b}{a}.$$

$$\frac{1}{n/l} = \frac{c}{a} \implies \frac{l}{n} = \frac{c}{a}.$$

From these two relations, we get $b = \frac{am}{n}$ and $c = \frac{al}{n}$. So the ratio $a : b : c$ is:

$$a : \frac{am}{n} : \frac{al}{n}$$

Multiplying by n/a (assuming $a, n \neq 0$):

$$n : m : l$$

The correct option is **(A)**.

31. For the equation $|x^2| + |x| - 6 = 0$ the roots are:

(A) One and only one real number   (B) Real with sum one
(C) Real with sum zero   (D) Real with product zero

Solution:

Since x^2 is always non-negative, $|x^2| = x^2$. Let $y = |x|$, so $y \geq 0$. The equation becomes a quadratic in y.

$$y^2 + y - 6 = 0$$

Factoring the quadratic:

$$(y + 3)(y - 2) = 0$$

The solutions for y are $y = -3$ and $y = 2$. Since $y = |x|$ must be non-negative, we discard $y = -3$. We are left with $y = 2$.

$$|x| = 2 \implies x = 2 \text{ or } x = -2.$$

The equation has two real roots: 2 and -2. Let's check the options: (A) False, there are two roots. (B) Sum is $2 + (-2) = 0$. So, false. (C) Sum is 0. This is true. (D) Product is $2(-2) = -4$. So, false. The correct option is **(C)**.

32. If α, β be the roots of the equation $x^2 - px + q = 0$ then the equation whose roots are $\alpha^2(\frac{\alpha^2}{\beta} - \beta)$ and $\beta^2(\frac{\beta^2}{\alpha} - \alpha)$ is:

Solution:
This question seems to have a typo in option (A). Let's solve it from first principles. Let the new roots be r_1 and r_2.

$$r_1 = \alpha^2 \left(\frac{\alpha^2 - \beta^2}{\beta} \right) = \frac{\alpha^2(\alpha - \beta)(\alpha + \beta)}{\beta}.$$

$$r_2 = \beta^2 \left(\frac{\beta^2 - \alpha^2}{\alpha} \right) = \frac{\beta^2(\beta - \alpha)(\beta + \alpha)}{\alpha} = -\frac{\beta^2(\alpha - \beta)(\alpha + \beta)}{\alpha}.$$

From the original equation, $\alpha + \beta = p$ and $\alpha\beta = q$. Also, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = p^2 - 4q$. So, $\alpha - \beta = \sqrt{p^2 - 4q}$.

$$r_1 = \frac{\alpha^2 p\sqrt{p^2 - 4q}}{\beta} = \frac{\alpha^3 p\sqrt{p^2 - 4q}}{q}.$$

$$r_2 = -\frac{\beta^2 p\sqrt{p^2 - 4q}}{\alpha} = -\frac{\beta^3 p\sqrt{p^2 - 4q}}{q}.$$

Sum of new roots:

$$S = r_1 + r_2 = \frac{p\sqrt{p^2 - 4q}}{q}(\alpha^3 - \beta^3) = \frac{p\sqrt{p^2 - 4q}}{q}(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$$

$$= \frac{p\sqrt{p^2 - 4q}}{q}\sqrt{p^2 - 4q}((\alpha + \beta)^2 - \alpha\beta) = \frac{p(p^2 - 4q)}{q}(p^2 - q).$$

Product of new roots:

$$P = r_1 r_2 = -\frac{\alpha^2 p\sqrt{p^2 - 4q}}{\beta} \cdot \frac{\beta^2 p\sqrt{p^2 - 4q}}{\alpha} = -p^2(p^2 - 4q)(\alpha\beta) = -p^2 q(p^2 - 4q).$$

The new equation is $x^2 - Sx + P = 0$. $x^2 - \frac{p(p^2-4q)(p^2-q)}{q}x - p^2 q(p^2 - 4q) = 0$. Multiplying by q: $qx^2 - p(p^2 - 4q)(p^2 - q)x - p^2 q^2(p^2 - 4q) = 0$. Comparing this with the options, option (A) seems closest but has typos. Let's re-read the option (A): $qx^2 - p(p^2 - q)(p^2 - 4q)x - p^2 q^2(p^2 - 4q) = 0$. This matches our derived equation exactly. The correct option is **(A)**.

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