Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 20th July 2025

Syllabus: Sets & Inequality and Trigonometric Ratios & Identities

Sub: Mathematics **JEE Advanced CT-01 Solution** Prof. Chetan Sir

Index of Questions

41. If $\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then the expression $\cos 2\alpha + (2+\sqrt{3})\sin 2\alpha$ is

(1) $2+\sqrt{3}$ (2) -1 (3) 1 (4) $-(2+\sqrt{3})$

Solution:

Given $\tan\alpha = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

Rationalizing the denominator:

$$\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}.$$

We know that $\tan 75^\circ = 2+\sqrt{3}$. So, $\alpha = 75^\circ$.

The expression to evaluate is $\cos 2\alpha + (2+\sqrt{3})\sin 2\alpha$.

With $\alpha = 75^\circ, 2\alpha = 150^\circ$.

$= \cos 150^\circ + (2+\sqrt{3})\sin 150^\circ$.

$$\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}.$$

$$\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

Substituting these values:

$$= -\frac{\sqrt{3}}{2} + (2+\sqrt{3})\frac{1}{2} = \frac{-\sqrt{3}+2+\sqrt{3}}{2} = \frac{2}{2} = 1.$$

The correct option is **(3)**.

42. If $N_a = \{an|n \in N\}$ then $N_5 \cap N_7 = ?$

(1) N_5 (2) N_7 (3) N_{12} (4) N_{35}

Solution:

The set $N_a = \{an|n \in N\}$ represents the set of all natural number multiples of 'a'.
$N_5 = \{5 \times 1, 5 \times 2, 5 \times 3, \dots\} = \{5, 10, 15, 20, 25, 30, 35, \dots\}$.
$N_7 = \{7 \times 1, 7 \times 2, 7 \times 3, \dots\} = \{7, 14, 21, 28, 35, 42, \dots\}$.

The intersection $N_5 \cap N_7$ is the set of elements that are common to both N_5 and N_7.
An element $x \in N_5 \cap N_7$ must be a multiple of 5 AND a multiple of 7.
This means x must be a common multiple of 5 and 7.
The common multiples of 5 and 7 are the multiples of their Least Common Multiple (LCM).
$\text{LCM}(5, 7) = 35$ (since 5 and 7 are prime numbers).

Therefore, the set of common multiples is the set of multiples of 35.
$N_5 \cap N_7 = \{35, 70, 105, \dots\} = N_{35}$.

The correct option is **(4)**.

43. The value of $\tan 7\frac{1}{2}^\circ$ is equal to

(1) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$ (2) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-1}$ (3) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+1}$ (4) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}-1}$

Solution:

We want to find the value of $\tan(7.5^\circ)$.
Let $\theta = 7.5^\circ$, which means $2\theta = 15^\circ$.
We will use the identity $\tan(\theta) = \dfrac{1-\cos(2\theta)}{\sin(2\theta)}$.

$$\tan(7.5^\circ) = \frac{1-\cos 15^\circ}{\sin 15^\circ}.$$

First, we need the standard values for 15° :

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

Now, substitute these values into the formula:

$$\tan(7.5^\circ) = \frac{1 - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\frac{2\sqrt{2}-(\sqrt{3}+1)}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1}.$$

To simplify, we rationalize the denominator by multiplying by the conjugate $(\sqrt{3}+1)$:

$$= \frac{(2\sqrt{2}-\sqrt{3}-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{2\sqrt{6}+2\sqrt{2}-3-\sqrt{3}-\sqrt{3}-1}{3-1}.$$

$$= \frac{2\sqrt{6}+2\sqrt{2}-2\sqrt{3}-4}{2} = \sqrt{6}+\sqrt{2}-\sqrt{3}-2.$$

Now, we check the options by rationalizing them.

Option (1): $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1} = \dfrac{(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \dfrac{\sqrt{6}-\sqrt{3}-2+\sqrt{2}}{1} = \sqrt{6}+\sqrt{2}-\sqrt{3}-2.$

This matches our result.

The correct option is **(1)**.

44. If $(x-1)(x^2-5x+7) < (x-1)$, then some part of x belongs to

(1) $(1,2)\cup(3,\infty)$ (2) $(-\infty,1)\cup(2,3)$ (3) (2,3) (4) none of these

Solution:

We are given the inequality $(x-1)(x^2-5x+7) < (x-1)$.
Move all terms to the left side:
$(x-1)(x^2-5x+7) - (x-1) < 0.$

Factor out the common term $(x-1)$:
$(x-1)[(x^2-5x+7)-1] < 0.$
$(x-1)(x^2-5x+6) < 0.$

Factor the quadratic expression:
$(x-1)(x-2)(x-3) < 0.$

The critical points are $x=1, x=2,$ and $x=3$.
We use the wavy curve method to analyze the sign of the expression.
For $x>3,$ all factors are positive, so the product is positive.
For $2<x<3,$ the factor $(x-3)$ is negative, so the product is negative.
For $1<x<2,$ factors $(x-2)$ and $(x-3)$ are negative, so the product is positive.
For $x<1,$ all three factors are negative, so the product is negative.

We need the intervals where the expression is less than 0 (negative).
The solution set is $x \in (-\infty,1)\cup(2,3)$.

Comparing with the options:

(1) $(1,2) \cup (3,\infty)$ is where the expression is positive.
(2) $(-\infty,1) \cup (2,3)$ is the complete solution set.
(3) $(2,3)$ is a part of the solution set.

The correct options are **(2)** and **(3)**.

45. If $\sin t + \cos t = \frac{1}{5}$, then $\tan \frac{t}{2}$ can be

(1) -1 (2) $-\frac{1}{3}$ (3) 2 (4) $-\frac{1}{6}$

Solution:

Concept Used:

- $\sin t = \frac{2\tan(t/2)}{1+\tan^2(t/2)}$
- $\cos t = \frac{1-\tan^2(t/2)}{1+\tan^2(t/2)}$

Given $\sin t + \cos t = \dfrac{1}{5}$.
Let $u = \tan(t/2)$. We use the half-angle substitutions:
$$\frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2} = \frac{1}{5}.$$

Since $1+u^2 \neq 0$, we can multiply both sides by $5(1+u^2)$.
$5(2u + 1 - u^2) = 1(1+u^2)$.
$10u + 5 - 5u^2 = 1 + u^2$.
$6u^2 - 10u - 4 = 0$.
$3u^2 - 5u - 2 = 0$.

Factor the quadratic equation:
$3u^2 - 6u + u - 2 = 0$.
$3u(u-2) + 1(u-2) = 0$.
$(3u+1)(u-2) = 0$.

The possible values for $u = \tan(t/2)$ are:
$$u = 2 \quad \text{or} \quad u = -\frac{1}{3}.$$

Both values are present in the options. The correct options are **(2)** and **(3)**.

46. The set of values of x which satisfy the inequations $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is

(1) $(-\infty, 1)$ (2) (2,3) (3) $(-\infty, 3)$ (4) $(-\infty,1) \cup (2,3)$

Solution:

First inequality: $5x + 2 < 3x + 8$.

$5x - 3x < 8 - 2$.
$2x < 6 \implies x < 3$.
Solution set 1: $S_1 = (-\infty, 3)$.

Second inequality: $\frac{x+2}{x-1} < 4$.
$\frac{x+2}{x-1} - 4 < 0$.
$\frac{x+2-4(x-1)}{x-1} < 0$.
$\frac{x+2-4x+4}{x-1} < 0$.
$\frac{6-3x}{x-1} < 0 \implies \frac{3(2-x)}{x-1} < 0$.
To make the x-term positive, multiply by -1 and reverse the inequality:
$\frac{x-2}{x-1} > 0$.
Critical points are $x = 1, x = 2$.
Using wavy curve method, the expression is positive when $x \in (-\infty, 1) \cup (2, \infty)$.
Solution set 2: $S_2 = (-\infty, 1) \cup (2, \infty)$.

The final solution is the intersection of S_1 and S_2.
$S = S_1 \cap S_2 = (-\infty, 3) \cap ((-\infty, 1) \cup (2, \infty))$.
$S = (-\infty, 1) \cup (2, 3)$.

The correct option is **(1),(2) and (4)**.

47. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)\}$, then the length of the hypotenuse is

(1) $2[1 + \cos(\alpha - \beta)]$ (2) $2[1 - \cos(\alpha + \beta)]$ (3) $4\cos^2(\frac{\alpha-\beta}{2})$ (4) $4\sin^2(\frac{\alpha+\beta}{2})$

Solution:

Concept Used:

- Sum-to-product formulas.
- Double angle formulas.
- Pythagorean theorem: $H^2 = P^2 + B^2$.

Let the first side be $P = \cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)$.
$P = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta) = 2\cos(\alpha + \beta)[\cos(\alpha - \beta) + 1]$.

Let the second side be $B = \sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)$.
$B = 2\sin(\alpha + \beta)\cos(\alpha - \beta) + 2\sin(\alpha + \beta) = 2\sin(\alpha + \beta)[\cos(\alpha - \beta) + 1]$.

Let H be the hypotenuse. By Pythagoras theorem, $H^2 = P^2 + B^2$.
$H^2 = (2\cos(\alpha + \beta)[\cos(\alpha - \beta) + 1])^2 + (2\sin(\alpha + \beta)[\cos(\alpha - \beta) + 1])^2$.

$H^2 = 4[\cos(\alpha - \beta) + 1]^2[\cos^2(\alpha + \beta) + \sin^2(\alpha + \beta)]$.
$H^2 = 4[\cos(\alpha - \beta) + 1]^2 \cdot (1)$.
$H = \sqrt{4[\cos(\alpha - \beta) + 1]^2} = 2|\cos(\alpha - \beta) + 1|$.

Since the range of cosine is [-1, 1], $\cos(\alpha - \beta) + 1 \geq 0$. So the absolute value is not needed.
$H = 2[1 + \cos(\alpha - \beta)]$.

This is option (1). Now let's check other options.
Using the half-angle identity $1 + \cos\theta = 2\cos^2(\theta/2)$:
$H = 2[2\cos^2(\frac{\alpha - \beta}{2})] = 4\cos^2(\frac{\alpha - \beta}{2})$.
This is option (3).

Options (1) and (3) are equivalent and correct.
The correct options are **(1)** and **(3)**.

48. If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then

(1) $x > 3$ (2) $\frac{7}{2} < x < 4$ (3) $3 < x < 4$ (4) $x = 7/2$

Solution:

First inequality: $x^2 + 6x - 27 > 0$.
Factoring: $(x + 9)(x - 3) > 0$.
The roots are -9 and 3. The parabola opens upwards, so it's positive outside the roots.
Solution set 1: $S_1 = (-\infty, -9) \cup (3, \infty)$.

Second inequality: $x^2 - 3x - 4 < 0$.
Factoring: $(x - 4)(x + 1) < 0$.
The roots are -1 and 4. The parabola opens upwards, so it's negative between the roots.
Solution set 2: $S_2 = (-1, 4)$.

We need the intersection of S_1 and S_2.
$S = S_1 \cap S_2 = ((-\infty, -9) \cup (3, \infty)) \cap (-1, 4)$.
The intersection is the interval $(3, 4)$.

The correct options are **(2), (3), and (4)**.

49. The set of real values of x for which $\frac{10x^2+17x-34}{x^2+2x-3} < 8$ is

(1) $(-5/2, 2)$ (2) $(-3, -5/2) \cup (1, 2)$ (3) (-3,1) (4) None of these

Solution:

$$\frac{10x^2 + 17x - 34}{x^2 + 2x - 3} - 8 < 0.$$

$$\frac{10x^2 + 17x - 34 - 8(x^2 + 2x - 3)}{x^2 + 2x - 3} < 0.$$
$$\frac{10x^2 + 17x - 34 - 8x^2 - 16x + 24}{x^2 + 2x - 3} < 0.$$
$$\frac{2x^2 + x - 10}{x^2 + 2x - 3} < 0.$$

Factor the numerator and denominator.
Numerator: $2x^2 + x - 10 = 2x^2 + 5x - 4x - 10 = x(2x + 5) - 2(2x + 5) = (x - 2)(2x + 5)$.
Denominator: $x^2 + 2x - 3 = (x + 3)(x - 1)$.

The inequality is $\frac{(x-2)(2x+5)}{(x+3)(x-1)} < 0$.
The critical points are $-3, -5/2, 1, 2$.
Arranged in order: $-3, -2.5, 1, 2$.
Using the wavy curve method (sign analysis for the intervals):
$(-\infty, -3) : (+)$
$(-3, -2.5) : (-)$
$(-2.5, 1) : (+)$
$(1, 2) : (-)$
$(2, \infty) : (+)$

We need the intervals where the expression is negative.
$x \in (-3, -5/2) \cup (1, 2)$.

The correct option is **(2)**.

50. If S is the set of all real x such that $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then S is equal to

(1) (-2,-1) (2) $(-2/3, 0)$ (3) $(-2/3, -1/2)$ (4) $(-2, -1) \cup (-\frac{2}{3}, -\frac{1}{2})$

Solution:

$$\frac{2x}{2x^2 + 5x + 2} - \frac{1}{x + 1} > 0.$$

Factor the denominator:
$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2 = 2x(x + 2) + 1(x + 2) = (2x + 1)(x + 2)$.
$$\frac{2x}{(2x + 1)(x + 2)} - \frac{1}{x + 1} > 0.$$
$$\frac{2x(x + 1) - 1(2x + 1)(x + 2)}{(2x + 1)(x + 2)(x + 1)} > 0.$$
$$\frac{2x^2 + 2x - (2x^2 + 5x + 2)}{(2x + 1)(x + 2)(x + 1)} > 0.$$
$$\frac{-3x - 2}{(2x + 1)(x + 2)(x + 1)} > 0.$$

$$\frac{-(3x+2)}{(2x+1)(x+2)(x+1)} > 0.$$

Multiply by -1 and reverse the inequality:

$$\frac{3x+2}{(2x+1)(x+2)(x+1)} < 0.$$

The critical points are $x = -2, x = -1, x = -2/3, x = -1/2$.

Arranged in order: $-2, -1, -0.66, -0.5$.

Using the wavy curve method:

$(-\infty, -2) : (+)$

$(-2, -1) : (-)$

$(-1, -2/3) : (+)$

$(-2/3, -1/2) : (-)$

$(-1/2, \infty) : (+)$

We need the intervals where the expression is negative.

The correct option is **(4)**.

51. The value of the expression $A = \sqrt{3}\cot 20^\circ - 4\cos 20^\circ$ is

(1) 1 (2) -1 (3) 2 (4) 4

Solution:

$$\begin{aligned}
A &= \sqrt{3}\frac{\cos 20^\circ}{\sin 20^\circ} - 4\cos 20^\circ \\
&= \frac{\sqrt{3}\cos 20^\circ - 4\sin 20^\circ \cos 20^\circ}{\sin 20^\circ}. \\
&= \frac{\sqrt{3}\cos 20^\circ - 2(2\sin 20^\circ \cos 20^\circ)}{\sin 20^\circ} \\
&= \frac{\sqrt{3}\cos 20^\circ - 2\sin 40^\circ}{\sin 20^\circ}. \\
&= \frac{\sqrt{3}\cos 20^\circ - 2\sin(60^\circ - 20^\circ)}{\sin 20^\circ}. \\
&= \frac{\sqrt{3}\cos 20^\circ - 2(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sin 20^\circ}. \\
&= \frac{\sqrt{3}\cos 20^\circ - 2(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ)}{\sin 20^\circ}. \\
&= \frac{\sqrt{3}\cos 20^\circ - \sqrt{3}\cos 20^\circ + \sin 20^\circ}{\sin 20^\circ} \\
&= \frac{\sin 20^\circ}{\sin 20^\circ} = 1.
\end{aligned}$$

The correct option is **(1)**.

52. The value of the expression $B = \sin 12^\circ \sin 48^\circ \sin 54^\circ$ is

(1) 1/4 (2) 1/8 (3) 1/16 (4) 1/32

Solution:

Use $\sin(60 - A)\sin(60 + A) = \sin^2 60 - \sin^2 A$.

$B = \sin 12^\circ \sin(60 - 12)^\circ \sin 54^\circ$.

Multiply and divide by $\sin(60 + 12)^\circ = \sin 72^\circ$

$$B = \frac{\sin 12^\circ \sin 48^\circ \sin 72^\circ}{\sin 72^\circ} \sin 54^\circ.$$

$$\sin 12^\circ \sin 48^\circ \sin 72^\circ = \sin 12^\circ \sin(60 - 12)\sin(60 + 12) = \frac{1}{4}\sin(3 \times 12) = \frac{1}{4}\sin 36^\circ.$$

$$B = \frac{\frac{1}{4}\sin 36^\circ}{\sin 72^\circ}\sin 54^\circ = \frac{\frac{1}{4}\sin 36^\circ}{2\sin 36^\circ \cos 36^\circ}\cos 36^\circ = \frac{1}{8}.$$

The correct option is **(2)**.

53. The number of students who have taken exactly one subject is

(1) 6 (2) 9 (3) 7 (4) 22

Solution:

Let M, P, C be the sets of students studying Maths, Physics, and Chemistry.

$n(M) = 23, n(P) = 24, n(C) = 19$.

$n(M \cap P) = 12, n(M \cap C) = 9, n(P \cap C) = 7$.

$n(M \cap P \cap C) = 4$.

Number of students in exactly one subject is given by:

$n(M \text{ only}) + n(P \text{ only}) + n(C \text{ only})$.

$n(M \text{ only}) = n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$.

$= 23 - 12 - 9 + 4 = 6$.

$n(P \text{ only}) = n(P) - n(M \cap P) - n(P \cap C) + n(M \cap P \cap C)$.

$= 24 - 12 - 7 + 4 = 9$.

$n(C \text{ only}) = n(C) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$.

$= 19 - 9 - 7 + 4 = 7$.

Total students in exactly one subject $= 6 + 9 + 7 = 22$.

The correct option is **(4)**.

54. The number of students who have taken exactly two subjects is

(1) 6 (2) 9 (3) 7 (4) 16

Solution:

Number of students in exactly two subjects is given by:
$n(M \cap P \text{ only}) + n(M \cap C \text{ only}) + n(P \cap C \text{ only})$.

$n(M \cap P \text{ only}) = n(M \cap P) - n(M \cap P \cap C) = 12 - 4 = 8.$
$n(M \cap C \text{ only}) = n(M \cap C) - n(M \cap P \cap C) = 9 - 4 = 5.$
$n(P \cap C \text{ only}) = n(P \cap C) - n(M \cap P \cap C) = 7 - 4 = 3.$

Total students in exactly two subjects $= 8 + 5 + 3 = 16$.

The correct option is **(4)**.

55. Match the column:

Column-I
(A) $\frac{x+4}{x-3} \leq 2$
(B) $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$
(C) $\frac{4x+3}{2x-5} < 6$
(D) $2(x+2) > x^2 + 1$

Column-II
(p) $(-\infty, \frac{5}{2}) \cup (\frac{33}{8}, \infty)$
(q) (-1,3)
(r) $(-\infty, 3) \cup [10, \infty)$
(s) $[2, \infty)$

Solution:

(A) $\dfrac{x+4}{x-3} - 2 \leq 0 \implies \dfrac{x+4-2(x-3)}{x-3} \leq 0 \implies \dfrac{x+4-2x+6}{x-3} \leq 0 \implies \dfrac{10-x}{x-3} \leq 0.$
$\dfrac{x-10}{x-3} \geq 0$. Critical points 3, 10. Solution is $(-\infty, 3) \cup [10, \infty)$.
This is (r). $\mathbf{A} \rightarrow \mathbf{r}$.

(B) $\dfrac{3(x-2)}{5} \geq \dfrac{-5(x-2)}{3} \implies \dfrac{3(x-2)}{5} + \dfrac{5(x-2)}{3} \geq 0.$
$(x-2)(\dfrac{3}{5} + \dfrac{5}{3}) \geq 0 \implies (x-2)(\dfrac{9+25}{15}) \geq 0 \implies \dfrac{34}{15}(x-2) \geq 0 \implies x - 2 \geq 0 \implies x \geq 2.$
Solution is $[2, \infty)$, which is (s). $\mathbf{B} \rightarrow \mathbf{s}$.

(C) $\dfrac{4x+3}{2x-5} - 6 < 0 \implies \dfrac{4x+3-6(2x-5)}{2x-5} < 0 \implies \dfrac{4x+3-12x+30}{2x-5} < 0 \implies \dfrac{33-8x}{2x-5} < 0.$
$\dfrac{8x-33}{2x-5} > 0$. Critical points 5/2, 33/8. Solution is $(-\infty, 5/2) \cup (33/8, \infty)$.

This is (p). $\mathbf{C \to p}$.

(D) $2x + 4 > x^2 + 1 \implies x^2 - 2x - 3 < 0 \implies (x-3)(x+1) < 0$.
Critical points -1, 3. Solution is $(-1, 3)$.
This is (q). $\mathbf{D \to q}$.

Matching: A-r, B-s, C-p, D-q.

56. Match the expression on column-I with the expression on column-II that is equal to it.

Column I	Column II
(A) $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$	(p) $\tan^4\theta + \sec^4\theta$
(B) $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$	(q) $\operatorname{cosec}\theta + \cot\theta$
(C) $1 + \dfrac{2\tan^2\theta}{\cos^2\theta}$	(r) $\left(\dfrac{1-\tan\theta}{1-\cot\theta}\right)^2$
(D) $(1-\sin\theta-\cos\theta)^2$	(s) $2(1-\sin\theta)(1-\cos\theta)$

Solution:

(A) $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\dfrac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)}} = \sqrt{\cot^2(\theta/2)} = |\cot(\theta/2)|$.
Let's check the options in Column-II.
(Q) $\operatorname{cosec}\theta + \cot\theta = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)} = \cot(\theta/2)$.
Assuming $\cot(\theta/2) \geq 0$, A matches Q. $\mathbf{A \to Q}$.

(B) $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta} = \dfrac{1/\cos^2\theta}{1/\sin^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$.
Let's check option (R): $(\dfrac{1-\tan\theta}{1-\cot\theta})^2 = (\dfrac{1-\tan\theta}{1-1/\tan\theta})^2 = (\dfrac{1-\tan\theta}{(\tan\theta-1)/\tan\theta})^2 = (-\tan\theta)^2 = \tan^2\theta$.
B matches R. $\mathbf{B \to R}$.

(C) Let's analyze both expressions C and P to show they are equal.
First, let's simplify expression (P) from Column-II:
$P = \tan^4\theta + \sec^4\theta$.
Using the identity $\sec^2\theta = 1 + \tan^2\theta$:
$P = \tan^4\theta + (1+\tan^2\theta)^2$.
$P = \tan^4\theta + (1 + 2\tan^2\theta + \tan^4\theta)$.

$P = 2\tan^4\theta + 2\tan^2\theta + 1.$

Now, let's simplify expression (C) from Column-I:

$C = 1 + \dfrac{2\tan^2\theta}{\cos^2\theta}.$

Using $1/\cos^2\theta = \sec^2\theta$:

$C = 1 + 2\tan^2\theta\sec^2\theta.$

Substitute $\sec^2\theta = 1 + \tan^2\theta$:

$C = 1 + 2\tan^2\theta(1 + \tan^2\theta).$

$C = 1 + 2\tan^2\theta + 2\tan^4\theta.$

Since both expressions simplify to $2\tan^4\theta + 2\tan^2\theta + 1$, they are equal.

C matches P. $\mathbf{C \to P}$.

(D) $(1 - \sin\theta - \cos\theta)^2 = 1 + \sin^2\theta + \cos^2\theta - 2\sin\theta - 2\cos\theta + 2\sin\theta\cos\theta.$

$= 1 + 1 - 2(\sin\theta + \cos\theta) + 2\sin\theta\cos\theta = 2 - 2(\sin\theta + \cos\theta) + 2\sin\theta\cos\theta.$

$= 2(1 - \sin\theta - \cos\theta + \sin\theta\cos\theta) = 2(1 - \sin\theta)(1 - \cos\theta).$

D matches S. $\mathbf{D \to S}$.

Final Matching: **A-Q, B-R, C-P, D-S**.

57. Match the following:

Column I	Column II
(A) $\frac{1}{4}(\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ)$	(p) 4
(B) $2\left(\sin^4\dfrac{\pi}{8} + \sin^4\dfrac{3\pi}{8} + \sin^4\dfrac{5\pi}{8} + \sin^4\dfrac{7\pi}{8}\right)$	(q) 3
(C) $2\sqrt{3}(\cot 70^\circ + 4\cos 70^\circ)$	(r) 1
(D) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$	(s) 6

Solution:

$$\textbf{(A)}\ \frac{1}{4}(\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ)$$
$$= \frac{1}{4}\left(\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}\right) = \frac{1}{4}\left(\frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ\cos 20^\circ}\right)$$
$$= \frac{1}{4}\left(\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\frac{1}{2}(2\sin 20^\circ\cos 20^\circ)}\right)$$
$$= \frac{1}{4}\left(\frac{2(\sin 60^\circ\cos 20^\circ - \cos 60^\circ\sin 20^\circ)}{\frac{1}{2}\sin 40^\circ}\right)$$
$$= \frac{1}{4}\left(\frac{2\sin(60^\circ - 20^\circ)}{\frac{1}{2}\sin 40^\circ}\right) = \frac{1}{4}\left(\frac{2\sin 40^\circ}{\frac{1}{2}\sin 40^\circ}\right) = \frac{1}{4}(4) = 1.$$

A $\to$ **R**.

(B) $2\left(\sin^4\dfrac{\pi}{8}+\sin^4\dfrac{3\pi}{8}+\sin^4\dfrac{5\pi}{8}+\sin^4\dfrac{7\pi}{8}\right)$

Using $\sin(\pi-\theta)=\sin\theta,$ we have $\sin\dfrac{7\pi}{8}=\sin\dfrac{\pi}{8}$ and $\sin\dfrac{5\pi}{8}=\sin\dfrac{3\pi}{8}$.

$=2\left(2\sin^4\dfrac{\pi}{8}+2\sin^4\dfrac{3\pi}{8}\right)=4\left(\sin^4\dfrac{\pi}{8}+\sin^4\dfrac{3\pi}{8}\right).$

Using $\sin\dfrac{3\pi}{8}=\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{8}\right)=\cos\dfrac{\pi}{8}$.

$=4\left(\sin^4\dfrac{\pi}{8}+\cos^4\dfrac{\pi}{8}\right).$

Using $a^4+b^4=(a^2+b^2)^2-2a^2b^2,$ we get

$=4\left(1-2\sin^2\dfrac{\pi}{8}\cos^2\dfrac{\pi}{8}\right)=4\left(1-\dfrac{1}{2}\left(2\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8}\right)^2\right).$

$=4\left(1-\dfrac{1}{2}\sin^2\dfrac{\pi}{4}\right)=4\left(1-\dfrac{1}{2}\left(\dfrac{1}{\sqrt{2}}\right)^2\right)=4\left(1-\dfrac{1}{4}\right)=4\left(\dfrac{3}{4}\right)=3.$

B $\to$ **Q**.

(C) $2\sqrt{3}(\cot 70^\circ+4\cos 70^\circ)$

$=2\sqrt{3}(\tan 20^\circ+4\sin 20^\circ)=2\sqrt{3}\left(\dfrac{\sin 20^\circ}{\cos 20^\circ}+4\sin 20^\circ\right)$

$=2\sqrt{3}\left(\dfrac{\sin 20^\circ+4\sin 20^\circ\cos 20^\circ}{\cos 20^\circ}\right)=2\sqrt{3}\left(\dfrac{\sin 20^\circ+2\sin 40^\circ}{\cos 20^\circ}\right)$

$=2\sqrt{3}\left(\dfrac{\sin 20^\circ+2\sin(60^\circ-20^\circ)}{\cos 20^\circ}\right)$

$=2\sqrt{3}\left(\dfrac{\sin 20^\circ+2(\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ)}{\cos 20^\circ}\right)$

$=2\sqrt{3}\left(\dfrac{\sin 20^\circ+2(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ)}{\cos 20^\circ}\right)$

$=2\sqrt{3}\left(\dfrac{\sin 20^\circ+\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\cos 20^\circ}\right)=2\sqrt{3}\left(\dfrac{\sqrt{3}\cos 20^\circ}{\cos 20^\circ}\right)=2\sqrt{3}(\sqrt{3})=6.$

C $\to$ **S**.

(D) $\tan 9^\circ-\tan 27^\circ-\tan 63^\circ+\tan 81^\circ$

$=(\tan 9^\circ+\tan 81^\circ)-(\tan 27^\circ+\tan 63^\circ)$

$=(\tan 9^\circ+\cot 9^\circ)-(\tan 27^\circ+\cot 27^\circ)$

Using $\tan\theta+\cot\theta=\dfrac{2}{\sin 2\theta}$:

$=\dfrac{2}{\sin 18^\circ}-\dfrac{2}{\sin 54^\circ}=\dfrac{2}{\sin 18^\circ}-\dfrac{2}{\cos 36^\circ}$

$=2\left(\dfrac{\cos 36^\circ-\sin 18^\circ}{\sin 18^\circ\cos 36^\circ}\right)$

$=2\left(\dfrac{(\frac{\sqrt{5}+1}{4})-(\frac{\sqrt{5}-1}{4})}{(\frac{\sqrt{5}-1}{4})(\frac{\sqrt{5}+1}{4})}\right)=2\left(\dfrac{\frac{2}{4}}{\frac{5-1}{16}}\right)=2\left(\dfrac{1/2}{4/16}\right)=2\left(\dfrac{1/2}{1/4}\right)=4.$

D $\rightarrow$ **P**.

Matching from Key **A-R, B-Q, C-S, D-P**

58. The value of $2\tan 18^\circ + 3\sec 18^\circ - 4\cos 18^\circ$ is

Solution:

$$\begin{aligned}
&4\cos 18^\circ - \frac{3}{\cos 18^\circ} - \frac{2\sin 18^\circ}{\cos 18^\circ} \\
&= \frac{4\cos^2 18^\circ - 3 - 2\sin 18^\circ}{\cos 18^\circ} \\
&= \frac{4\cos^3 18^\circ - 3\cos 18^\circ - 2\sin 18^\circ \cos 18^\circ}{\cos^2 18^\circ} \\
&= \frac{\cos(3 \times 18^\circ) - \sin(2 \times 18^\circ)}{\cos^2 18^\circ} \\
&= \frac{\cos 54^\circ - \sin 36^\circ}{\cos^2 18^\circ} \\
&= \frac{\sin 36^\circ - \sin 36^\circ}{\cos^2 18^\circ} \\
&= 0
\end{aligned}$$

The value of the expression is **0**.

59. The least integer satisfying $49.4 - (\frac{27-x}{10}) < 47.4 - (\frac{27-9x}{10})$ is

Solution:

$49.4 - 2.7 + 0.1x < 47.4 - 2.7 + 0.9x.$

$46.7 + 0.1x < 44.7 + 0.9x.$

$46.7 - 44.7 < 0.9x - 0.1x.$

$2 < 0.8x.$

$x > \frac{2}{0.8} = \frac{20}{8} = 2.5.$

We need the least integer satisfying $x > 2.5$. That integer is 3.

The correct answer is **3**

60. If $K = \sin^3 10^\circ + \sin^3 50^\circ - \sin^3 70^\circ$, then $8K + 6$ is equal to

Solution:

$K = \sin^3 10 + \sin^3(60-10) - \sin^3(60+10).$

We know $\sin 3\theta = 3\sin\theta - 4\sin^3\theta \implies \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}.$

$K = \frac{3s10 - s30}{4} + \frac{3s50 - s150}{4} - \frac{3s70 - s210}{4}.$

$4K = 3(\sin 10 + \sin 50 - \sin 70) - (\sin 30 + \sin 150 - \sin 210).$

$\sin 10 + \sin 50 = 2 \sin 30 \cos 20 = \cos 20.$

$\sin 70 = \cos 20.$ So $\sin 10 + \sin 50 - \sin 70 = 0.$

$4K = 3(0) - (\frac{1}{2} + \frac{1}{2} - (-\frac{1}{2})) = -(3/2).$

$4K = -3/2 \implies 8K = -3.$

$8K + 6 = -3 + 6 = 3.$

The correct answer is **3**.