Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 20th July

Syllabus: Sets & Relation and Trigonometric Ratios & Identities

Sub: Mathematics **CT-01 MHT CET Solution** Prof. Chetan Sir

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101. $\sin 12^\circ \sin 48^\circ \sin 54^\circ =$

(1) $\frac{1}{16}$ (2) $\frac{1}{32}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

Solution:

Let the expression be E.

$E = \sin 12^\circ \sin 48^\circ \sin 54^\circ$.

We can write this as $\sin 12^\circ \sin(60^\circ - 12^\circ) \sin 54^\circ$.

To use the identity $\sin\theta \sin(60-\theta)\sin(60+\theta) = \dfrac{1}{4}\sin 3\theta$, we need $\sin(60+12)^\circ = \sin 72^\circ$.

$$E = \frac{\sin 12^\circ \sin 48^\circ \sin 72^\circ}{\sin 72^\circ} \sin 54^\circ.$$

The product in the numerator becomes $\dfrac{1}{4}\sin(3 \times 12^\circ) = \dfrac{1}{4}\sin 36^\circ$.

$$E = \frac{\frac{1}{4}\sin 36^\circ}{\sin 72^\circ} \sin 54^\circ.$$

$$E = \frac{\frac{1}{4}\sin 36^\circ}{2\sin 36^\circ \cos 36^\circ} \cos 36^\circ \quad (\text{since } \sin 54^\circ = \cos 36^\circ).$$

$$E = \frac{1}{8}.$$

The correct option is **(3)**.

102. The set 1,4,9,16,25,... can be written as

(1) $\{x : x = n^2, n \in N\}$ (2) $\{x : x = n^2, n \in I\}$ (3) $\{x : x = n, n \in N\}$ (4) none of these

Solution:

The given set is $\{1, 4, 9, 16, 25, \ldots\}$.
We can observe that the elements of the set are squares of natural numbers.
$1 = 1^2$
$4 = 2^2$
$9 = 3^2$
$16 = 4^2$
$\ldots$
So, the general element is $x = n^2$, where n is a natural number ($n \in N$).
This corresponds to the set-builder form $\{x : x = n^2, n \in N\}$.

The correct option is **(1)**.

103. $\tan 75^\circ - \cot 75^\circ =$

(1) $2\sqrt{3}$ (2) $2 + \sqrt{3}$ (3) $2 - \sqrt{3}$ (4) $-2\sqrt{3}$

Solution:

$$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}.$$

$$\cot 75^\circ = \frac{1}{\tan 75^\circ} = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}.$$

The expression is $\tan 75^\circ - \cot 75^\circ = (2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$.
Alternatively,

$$\tan\theta - \cot\theta = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} = \frac{-\cos 2\theta}{\frac{1}{2}\sin 2\theta} = -2\cot 2\theta.$$

$$\tan 75^\circ - \cot 75^\circ = -2\cot(2 \times 75^\circ) = -2\cot 150^\circ = -2(-\cot 30^\circ) = 2\cot 30^\circ = 2\sqrt{3}.$$

The correct option is **(1)**.

104. If A and B are two sets such that A has 12 elements, B has 17 elements and $A \cup B$ has 21 elements, then number of elements in $A \cap B$ are

(1) 6 (2) 4 (3) 8 (4) None of these

Solution:

We are given the number of elements in the sets:
$n(A) = 12$

$n(B) = 17$
$n(A \cup B) = 21$

We use the Principle of Inclusion-Exclusion for two sets:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
$21 = 12 + 17 - n(A \cap B)$.
$21 = 29 - n(A \cap B)$.
$n(A \cap B) = 29 - 21 = 8$.

The correct option is **(3)**.

105. At $x = \frac{5\pi}{6}$, the value of $2\sin 3x + 3\cos 3x$ is

(1) 0 (2) 1 (3) -1 (4) 2

Solution:

We need to evaluate the expression at $x = \dfrac{5\pi}{6}$.
First, find the angle $3x$:
$3x = 3 \times \dfrac{5\pi}{6} = \dfrac{5\pi}{2}$.

Now evaluate the trigonometric functions at this angle:
$\sin(3x) = \sin\left(\frac{5\pi}{2}\right) = \sin\left(2\pi + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1.$
$\cos(3x) = \cos\left(\frac{5\pi}{2}\right) = \cos\left(2\pi + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0.$

Substitute these values into the expression:
$2\sin(3x) + 3\cos(3x) = 2(1) + 3(0) = 2.$

The correct option is **(4)**.

106. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, then number of elements $S \cup T$ has

(1) 42 (2) 50 (3) 48 (4) none of these

Solution:

We are given the number of elements in the sets:
$n(S) = 21$
$n(T) = 32$
$n(S \cap T) = 11$

We use the Principle of Inclusion-Exclusion for two sets:
$n(S \cup T) = n(S) + n(T) - n(S \cap T)$.
$n(S \cup T) = 21 + 32 - 11$.

$$n(S \cup T) = 53 - 11 = 42.$$

The correct option is **(1)**.

107. $8 \sin \frac{x}{8} \cos \frac{x}{8} \cos \frac{x}{4} \cos \frac{x}{2}$

(1) 8 sin x (2) sin x (3) cos x (4) 8 cos x

Solution:

Let the expression be E.

$E = 8 \sin \frac{x}{8} \cos \frac{x}{8} \cos \frac{x}{4} \cos \frac{x}{2}$.

We use the double angle identity $2 \sin \theta \cos \theta = \sin(2\theta)$ repeatedly.

$E = 4 \cdot \left(2 \sin \frac{x}{8} \cos \frac{x}{8}\right) \cos \frac{x}{4} \cos \frac{x}{2}$.

$E = 4 \sin \left(2 \cdot \frac{x}{8}\right) \cos \frac{x}{4} \cos \frac{x}{2} = 4 \sin \frac{x}{4} \cos \frac{x}{4} \cos \frac{x}{2}$.

$E = 2 \cdot \left(2 \sin \frac{x}{4} \cos \frac{x}{4}\right) \cos \frac{x}{2}$.

$E = 2 \sin \left(2 \cdot \frac{x}{4}\right) \cos \frac{x}{2} = 2 \sin \frac{x}{2} \cos \frac{x}{2}$.

$E = \sin \left(2 \cdot \frac{x}{2}\right) = \sin x$.

The correct option is **(2)**.

108. If set $A = \{x : 4 \leq x \leq 7, x \in Z\}$ then sum of all elements of set A is

(1) 22 (2) 23 (3) 20 (4) 21

Solution:

The set A is defined as $\{x : 4 \leq x \leq 7, x \in Z\}$.

This means A contains all integers from 4 to 7, inclusive.

Writing A in roster form:

$A = \{4, 5, 6, 7\}$.

The sum of all elements in set A is:

$\text{Sum} = 4 + 5 + 6 + 7 = 22$.

The correct option is **(1)**.

109. $2 \sin A \cos^3 A - 2 \sin^3 A \cos A =$

(1) sin 4A (2) $\frac{1}{2} sin\ 4A$ (3) $\frac{1}{4} sin\ 4A$ (4) $\frac{1}{8} sin\ 4A$

Solution:

Let the expression be E.

$E = 2 \sin A \cos^3 A - 2 \sin^3 A \cos A$.

Factor out the common term $2 \sin A \cos A$:

$E = 2\sin A\cos A(\cos^2 A - \sin^2 A)$.

We use the double angle identities:
$2\sin A\cos A = \sin(2A)$.
$\cos^2 A - \sin^2 A = \cos(2A)$.

Substitute these back into the expression:
$E = \sin(2A)\cos(2A)$.

Apply the sine double angle identity again:
$E = \frac{1}{2}(2\sin(2A)\cos(2A)) = \frac{1}{2}\sin(2\cdot 2A) = \frac{1}{2}\sin(4A)$.

The correct option is **(2)**.

110. Let A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A\cup B$?

(1) 3 (2) 6 (3) 9 (4) 18

Solution:

We are given:
$n(A) = 3$
$n(B) = 6$

The formula for the number of elements in the union of two sets is:
$n(A\cup B) = n(A) + n(B) - n(A\cap B)$.
$n(A\cup B) = 3 + 6 - n(A\cap B) = 9 - n(A\cap B)$.

To find the minimum value of $n(A\cup B)$, we must maximize the value of $n(A\cap B)$.
The number of elements in the intersection, $n(A\cap B)$,
cannot be more than the number of elements in the smaller set.
So, the maximum possible value for $n(A\cap B)$ is $\min(n(A), n(B)) = \min(3, 6) = 3$.
This occurs when A is a subset of B ($A\subset B$).

The minimum value of $n(A\cup B)$ is therefore:
$n(A\cup B)_{min} = 9 - n(A\cap B)_{max} = 9 - 3 = 6$.

The correct option is **(2)**.

111. The value of tan $(7.5°)$ is equal to

(1) $\sqrt{6}+\sqrt{3}+\sqrt{2}-2$ (2) $\sqrt{6}-\sqrt{3}+\sqrt{2}-2$
(3) $\sqrt{6}-\sqrt{3}+\sqrt{2}+2$ (4) $\sqrt{6}-\sqrt{3}-\sqrt{2}-2$

Solution:

We want to find the value of $\tan(7.5°)$.

We use the identity $\tan(\theta/2) = \operatorname{cosec}\theta - \cot\theta$. Let $\theta = 15°$.
$\tan(7.5°) = \operatorname{cosec} 15° - \cot 15°$.

First, find the values for $15°$:

$\sin 15° = \sin(45° - 30°) = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

$\cos 15° = \cos(45° - 30°) = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

$$\operatorname{cosec} 15° = \frac{1}{\sin 15°} = \frac{2\sqrt{2}}{\sqrt{3}-1} = \frac{2\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{2\sqrt{6}+2\sqrt{2}}{2} = \sqrt{6}+\sqrt{2}.$$

$$\cot 15° = \frac{\cos 15°}{\sin 15°} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{2} = 2+\sqrt{3}.$$

$$\tan(7.5°) = (\sqrt{6}+\sqrt{2}) - (2+\sqrt{3}) = \sqrt{6}+\sqrt{2}-\sqrt{3}-2.$$

The correct option is **(2)**.

112. The value of $\frac{\sin 55° - \cos 55°}{\sin 10°}$ is :

(1) $\frac{1}{\sqrt{2}}$ (2) 2 (3) 1 (4) $\sqrt{2}$

Solution:

Let the expression be E.

$E = \frac{\sin 55° - \cos 55°}{\sin 10°}$.

Using the co-function identity $\cos\theta = \sin(90° - \theta)$:

$\cos 55° = \sin(90° - 55°) = \sin 35°$.

$E = \frac{\sin 55° - \sin 35°}{\sin 10°}$.

Apply the sum-to-product formula $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$.

$$E = \frac{2\cos\left(\frac{55°+35°}{2}\right)\sin\left(\frac{55°-35°}{2}\right)}{\sin 10°}.$$

$$E = \frac{2\cos(45°)\sin(10°)}{\sin 10°}.$$

$$E = 2\cos(45°) = 2\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}.$$

The correct option is **(4)**.

113. Of the members of three athletic teams in a school, 21 are in the basket-ball team, 26 in hockey team and 29 in the football team. 14 play hockey and basket ball, 15 play hockey and foot ball, 12 play foot-ball and basket-ball and 8 play all the games. The numbers of members there in all is

(1) 43 (2) 37 (3) 54 (4) 60

Solution:

Let B, H, and F be the sets of members in the Basketball, Hockey, and Football teams, respectively.
We are given:
$n(B) = 21, \quad n(H) = 26, \quad n(F) = 29.$
$n(H \cap B) = 14.$
$n(H \cap F) = 15.$
$n(F \cap B) = 12.$
$n(B \cap H \cap F) = 8.$

We need to find the total number of members, which is $n(B \cup H \cup F)$.
Using the Principle of Inclusion-Exclusion for three sets:
$n(B \cup H \cup F) = n(B) + n(H) + n(F) - [n(B \cap H) + n(H \cap F) + n(F \cap B)] + n(B \cap H \cap F).$
$n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8.$
$n(B \cup H \cup F) = 76 - 41 + 8.$
$n(B \cup H \cup F) = 35 + 8 = 43.$

The correct option is **(1)**.

114. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 180^\circ =$

(1) 0 (2) 1 (3) -1 (4) 2

Solution:

Let the sum be S.
$S = \cos 1^\circ + \cos 2^\circ + \cdots + \cos 89^\circ + \cos 90^\circ + \cos 91^\circ + \cdots + \cos 179^\circ + \cos 180^\circ.$
We use the identity $\cos(180^\circ - \theta) = -\cos\theta.$
$\cos 179^\circ = \cos(180^\circ - 1^\circ) = -\cos 1^\circ.$
$\cos 178^\circ = \cos(180^\circ - 2^\circ) = -\cos 2^\circ.$
$\cdots$
$\cos 91^\circ = \cos(180^\circ - 89^\circ) = -\cos 89^\circ.$

We can pair the terms:
$S = (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) + \cdots + (\cos 89^\circ + \cos 91^\circ) + \cos 90^\circ + \cos 180^\circ.$
$S = (\cos 1^\circ - \cos 1^\circ) + (\cos 2^\circ - \cos 2^\circ) + \cdots + (\cos 89^\circ - \cos 89^\circ) + \cos 90^\circ + \cos 180^\circ.$
$S = 0 + 0 + \cdots + 0 + \cos 90^\circ + \cos 180^\circ.$
$S = 0 + 0 + (-1) = -1.$

The correct option is **(3)**.

115. If set $A = \{x : x \text{ is a positive integer and } x^2 < 40)$ then its roster form is

(1) 1,2,3,4,5,6,7 (2) 1,2,3,4,5 (3) 1,2,3,4,5,6 (4) 1,2,3

Solution:

We are given the set A in set-builder form: $A = \{x : x \text{ is a positive integer and } x^2 < 40\}.$

We need to find all positive integers whose square is less than 40.
$1^2 = 1 < 40.$
$2^2 = 4 < 40.$
$3^2 = 9 < 40.$
$4^2 = 16 < 40.$
$5^2 = 25 < 40.$
$6^2 = 36 < 40.$
$7^2 = 49 \not< 40.$
The positive integers satisfying the condition are 1, 2, 3, 4, 5, and 6.
So, the roster form of set A is $\{1, 2, 3, 4, 5, 6\}$.

The correct option is **(3)**.

116. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

(1) 2 (2) 3 (3) 4 (4) 6

Solution:

Let $E = \dfrac{1}{D}$, where $D = \sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$.
To maximize E, we must minimize the denominator D.
We convert D to an expression in terms of 2θ :

$$D = \left(\frac{1-\cos 2\theta}{2}\right) + 3\left(\frac{\sin 2\theta}{2}\right) + 5\left(\frac{1+\cos 2\theta}{2}\right).$$

$$D = \frac{1-\cos 2\theta + 3\sin 2\theta + 5 + 5\cos 2\theta}{2} = \frac{6 + 4\cos 2\theta + 3\sin 2\theta}{2}.$$

$$D = 3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta.$$

The minimum value of an expression $a\cos x + b\sin x$ is $-\sqrt{a^2+b^2}$.
For $2\cos 2\theta + \dfrac{3}{2}\sin 2\theta$, the minimum value is $-\sqrt{2^2 + (\frac{3}{2})^2} = -\sqrt{4+\frac{9}{4}} = -\sqrt{\frac{25}{4}} = -\frac{5}{2}$.

$$D_{min} = 3 + \left(-\frac{5}{2}\right) = 3 - \frac{5}{2} = \frac{1}{2}.$$

$$E_{max} = \frac{1}{D_{min}} = \frac{1}{1/2} = 2.$$

The correct option is **(1)**.

117. If set $A = \{x : 2x - 5 < 4, x \in Z\}$ then its roster form is

(1) 0,1,2,3,4,.....
(2) $\{-2, -1, 0, 1, 2, 3, 4, ...\}$
(3) $\{......, -2, -1, 0, 1, 2, 3, 4\}$
(4) $\{1, 2, 3, 4, ...\}$

Solution:

We need to solve the inequality for integers x.

$2x - 5 < 4.$
$2x < 9.$
$x < 4.5.$

The set A contains all integers x such that x is less than 4.5.
The integers are ..., -3, -2, -1, 0, 1, 2, 3, 4.
In roster form, this is written as $A = \{..., -2, -1, 0, 1, 2, 3, 4\}$.

The correct option is **(3)**.

118. $\cos^2 48° - \sin^2 12° =$

(1) $\frac{-\sqrt{5}+1}{8}$ (2) $\frac{\sqrt{5}-1}{8}$ (3) $\frac{\sqrt{5}+1}{8}$ (4) $\frac{-\sqrt{5}-1}{8}$

Solution:

We use the identity $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$.
Here, $A = 48°, B = 12°$.
$\cos^2 48° - \sin^2 12° = \cos(48° + 12°)\cos(48° - 12°)$.
$= \cos(60°)\cos(36°)$.

We know the standard values:
$\cos 60° = \frac{1}{2}$.
$\cos 36° = \frac{\sqrt{5}+1}{4}$.

The expression equals $\frac{1}{2} \times \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{8}$.

The correct option is **(3)**.

119. If A and B are subsets of a set U, then

(1) $n(A) \geq n(U)$ and $n(B) \geq n(U)$ (2) $n(B) < n(A)$
(3) $n(B) = n(A)$ (4) $n(U) \geq n(A)$ and $n(U) \geq n(B)$

Solution:

By definition, if A is a subset of a universal set U (denoted $A \subseteq U$),
then every element of A is also an element of U.
This implies that the number of elements in A cannot be greater than the number of elements in U.
Therefore, $n(A) \leq n(U)$, which is equivalent to $n(U) \geq n(A)$.

Similarly, if B is a subset of U, then $n(B) \leq n(U)$, or $n(U) \geq n(B)$.

Options (1), (2), and (3) are not necessarily true.
For example, A could have more elements than B, or they could be equal.
Option (4) correctly states the fundamental property of subsets.

The correct option is **(4)**.

120. $sin^2(10°) + sin^2(20°) + sin^2(30°) + ... + sin^2(70°) + sin^2(80°) + sin^2(90) =$

(1) $\frac{8}{2}$ (2) $\frac{9}{2}$ (3) $\frac{7}{2}$ (4) 5

Solution:

Let the sum be S.

$S = \sin^2 10° + \sin^2 20° + \sin^2 30° + \sin^2 40° + \sin^2 50° + \sin^2 60° + \sin^2 70° + \sin^2 80° + \sin^2 90°$.

We use the identity $\sin\theta = \cos(90° - \theta)$, so $\sin^2\theta = \cos^2(90° - \theta)$.

$\sin^2 80° = \cos^2 10°$.

$\sin^2 70° = \cos^2 20°$.

$\sin^2 60° = \cos^2 30°$.

$\sin^2 50° = \cos^2 40°$.

Let's pair the terms:

$S = (\sin^2 10° + \sin^2 80°) + (\sin^2 20° + \sin^2 70°) + (\sin^2 30° + \sin^2 60°) + (\sin^2 40° + \sin^2 50°) + \sin^2 90°$.

$S = (\sin^2 10° + \cos^2 10°) + (\sin^2 20° + \cos^2 20°) + (\sin^2 30° + \cos^2 30°) + (\sin^2 40° + \cos^2 40°) + \sin^2 90°$.

Using the identity $\sin^2\theta + \cos^2\theta = 1$:

$S = 1 + 1 + 1 + 1 + \sin^2 90°$.

$S = 4 + (1)^2 = 5$.

The correct option is **(4)**.

121. Solve: $\frac{x^2+x-6}{x^2-5x+6} < 0$

(1) $x \in (2, 3) \cup (-3, -2)$ (2) $x \in (-3, 2) \cup (2, 3)$

(3) $x \in (-3, 2) \cup (3, \infty)$ (4) $x \in (-\infty, -3) \cup (2, 3)$

Solution:

We need to solve the rational inequality.

$$\frac{x^2 + x - 6}{x^2 - 5x + 6} < 0.$$

Factorize the numerator and the denominator:

Numerator: $x^2 + x - 6 = (x + 3)(x - 2)$.

Denominator: $x^2 - 5x + 6 = (x - 2)(x - 3)$.

The inequality becomes $\frac{(x + 3)(x - 2)}{(x - 2)(x - 3)} < 0$.

We can cancel the term $(x - 2)$,

but we must note that $x \neq 2$ because it makes the original denominator zero.

$$\frac{x + 3}{x - 3} < 0, \quad \text{with the condition } x \neq 2.$$

To solve $\frac{x + 3}{x - 3} < 0$, we find the critical points $x = -3$ and $x = 3$.

The expression is negative between the roots.
So, the solution is $x \in (-3, 3)$.

Now, we must apply the condition $x \neq 2$.
We exclude the point x=2 from the interval $(-3, 3)$.
The final solution is $x \in (-3, 2) \cup (2, 3)$.

The correct option is **(2)**.

122. $x^2(x-1)^2(x+2) < 0$

(1) $x \in (-\infty, -2)$
(2) $x \in (-\infty, -2) \cup (1, \infty)$
(3) $x \in (-2, 0) \cup (1, \infty)$
(4) $x \in (-2, 0) \cup (0, 1)$

Solution:

We need to solve the inequality $x^2(x-1)^2(x+2) < 0$.
The terms x^2 and $(x-1)^2$ are always non-negative (greater than or equal to 0).
For the product to be strictly negative, these terms cannot be zero, so $x \neq 0$ and $x \neq 1$.
Since $x^2 > 0$ and $(x-1)^2 > 0$ for $x \neq 0, 1$,
we can divide the inequality by them without changing the sign.
The inequality simplifies to:
$x + 2 < 0, \quad \text{with conditions } x \neq 0, x \neq 1.$
$x < -2.$

The solution set is $(-\infty, -2)$.
This interval does not contain 0 or 1, so the conditions are met.

The correct option is **(1)**.

123. If A lies in the third quadrant and $3\tan A - 4 = 0$, then $5\sin 2A + 3\sin A + 4\cos A =$

(1) 0
(2) $\frac{-24}{5}$
(3) $\frac{24}{5}$
(4) $\frac{48}{5}$

Solution:

Given $3\tan A - 4 = 0 \implies \tan A = 4/3$.
A is in the third quadrant, where sin A and cos A are both negative.
From $\tan A = 4/3$, we can form a right triangle with opposite=4, adjacent=3, and hypotenuse=5.
$\sin A = -4/5$.
$\cos A = -3/5$.

Now, calculate $\sin 2A$:
$\sin 2A = 2\sin A\cos A = 2(-4/5)(-3/5) = 24/25$.

Evaluate the expression:
$5\sin 2A + 3\sin A + 4\cos A = 5(24/25) + 3(-4/5) + 4(-3/5)$.

$$= \frac{24}{5} - \frac{12}{5} - \frac{12}{5}.$$
$$= \frac{24 - 12 - 12}{5} = \frac{0}{5} = 0.$$

The correct option is **(1)**.

124. Solve: $\frac{x^2-4}{x^2-1} < 0$

(1) $x \in (-2, -1) \cup (1, 2)$
(2) $x \in (-\infty, -2) \cup (2, \infty)$
(3) $x \in (-\infty, -2) \cup (-1, 1) \cup (2, \infty)$
(4) $x \in (-2, -1) \cup (1, 2) \cup (-1, 1)$

Solution:

We need to solve the inequality $\dfrac{x^2 - 4}{x^2 - 1} < 0.$

Factorize the numerator and denominator:

$$\frac{(x-2)(x+2)}{(x-1)(x+1)} < 0.$$

The critical points are $-2, -1, 1, 2.$

We use the wavy curve method to analyze the sign of the expression.

For $x > 2 : \dfrac{(+)(+)}{(+)(+)} = (+).$

For $1 < x < 2 : \dfrac{(-)(+)}{(+)(+)} = (-).$

For $-1 < x < 1 : \dfrac{(-)(+)}{(-)(+)} = (+).$

For $-2 < x < -1 : \dfrac{(-)(+)}{(-)(-)} = (-).$

For $x < -2 : \dfrac{(-)(-)}{(-)(-)} = (+).$

We need the intervals where the expression is negative.

The solution set is $x \in (-2, -1) \cup (1, 2).$

The correct option is **(1)**.

125. The value of $\sin \frac{31\pi}{3}$ is

(1) $\frac{\sqrt{3}}{2}$
(2) $\frac{1}{\sqrt{2}}$
(3) $\frac{-\sqrt{3}}{2}$
(4) $\frac{-1}{\sqrt{2}}$

Solution:

We need to evaluate $\sin\left(\dfrac{31\pi}{3}\right).$

First, simplify the angle by removing full rotations of $2\pi.$

$$\frac{31\pi}{3} = \frac{30\pi + \pi}{3} = 10\pi + \frac{\pi}{3}.$$

$10\pi = 5 \times 2\pi,$ which represents 5 full rotations.

Using the periodicity of the sine function, $\sin(\theta + 2k\pi) = \sin\theta$ for any integer k.

$$\sin\left(\frac{31\pi}{3}\right) = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right).$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

The correct option is **(1)**.

126. Value of x not satisfying the inequality $4x - 5 < 3x + 1$ is?

(1) $x = -1$ (2) $x = 0$ (3) $x = 2$ (4) $x = 10$

Solution:

First, let's solve the inequality to find the values of x that *do* satisfy it.

$4x - 5 < 3x + 1.$

$4x - 3x < 1 + 5.$

$x < 6.$

The inequality is satisfied for any real number less than 6.

The question asks for a value that does *not* satisfy this condition.

Let's check the options:

$(1)x = -1 : -1 < 6.(Satisfies)$

$(2)x = 0 : 0 < 6.(Satisfies)$

$(3)x = 2 : 2 < 6.(Satisfies)$

$(4)x = 10 : 10 \not< 6.(Doesnotsatisfy)$

The correct option is **(4)**.

127. If $\sin\alpha = \frac{1}{\sqrt{5}}$ and $\sin\beta = 3/5$ then $\beta - \alpha$ lies in the interval

(1) $(0, \frac{\pi}{4})$ (2) $(\frac{\pi}{2}, \frac{3\pi}{4})$ (3) $[0, \pi]$ (4) $[\pi, \frac{5\pi}{4}]$

Solution:

Assuming α and β are acute angles (in Quadrant I), since their sines are positive.

$$\sin\alpha = \frac{1}{\sqrt{5}} \implies \cos\alpha = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}}.$$

$$\sin\beta = \frac{3}{5} \implies \cos\beta = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}.$$

We evaluate $\sin(\beta - \alpha)$ and $\cos(\beta - \alpha)$ to determine the quadrant of $\beta - \alpha$.

$$\sin(\beta - \alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha = \left(\frac{3}{5}\right)\left(\frac{2}{\sqrt{5}}\right) - \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{5}}\right) = \frac{6-4}{5\sqrt{5}} = \frac{2}{5\sqrt{5}} > 0.$$

$$\cos(\beta - \alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha = \left(\frac{4}{5}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{\sqrt{5}}\right) = \frac{8+3}{5\sqrt{5}} = \frac{11}{5\sqrt{5}} > 0.$$

Since both sine and cosine of $(\beta - \alpha)$ are positive, the angle lies in Quadrant I, i.e., $(0, \pi/2)$.

To check if it's in $(0, \pi/4)$, we find $\tan(\beta - \alpha)$.

$$\tan(\beta - \alpha) = \frac{\sin(\beta - \alpha)}{\cos(\beta - \alpha)} = \frac{2/(5\sqrt{5})}{11/(5\sqrt{5})} = \frac{2}{11}.$$

Since $\tan(\beta - \alpha) = \frac{2}{11} < 1$, and $\tan(\pi/4) = 1$, we have $\beta - \alpha < \frac{\pi}{4}$.

Therefore, $\beta - \alpha \in (0, \pi/4)$.

The correct option is **(1)**.

128. Number of subsets of set of letter of word 'MONOTONE'

(1) 8 (2) 256 (3) 64 (4) 32

Solution:

First, we find the set of distinct letters in the word 'MONOTONE'.
The letters are M, O, N, T, O, N, E.
The distinct letters are M, O, N, T, E.
Let S be the set of these letters: $S = \{M, O, N, T, E\}$.

The number of elements in the set S is $n(S) = 5$.
The number of subsets of a set with n elements is given by the formula 2^n.
Number of subsets $= 2^5 = 32$.

The correct option is **(4)**.

129. $4\sin(\frac{\pi}{3} + \theta) sin(\frac{\pi}{3} - \theta) =$

(1) $1 + \cos 2\theta$ (2) $1 - \cos 2\theta$ (3) $2\cos 2\theta - 1$ (4) $1 + 2\cos 2\theta$

Solution:

We use the product-to-sum identity $2\sin A \sin B = \cos(A - B) - \cos(A + B)$.

Let $A = \frac{\pi}{3} + \theta$ and $B = \frac{\pi}{3} - \theta$.

$A - B = (\frac{\pi}{3} + \theta) - (\frac{\pi}{3} - \theta) = 2\theta$.

$A + B = (\frac{\pi}{3} + \theta) + (\frac{\pi}{3} - \theta) = \frac{2\pi}{3}$.

The expression is $2 \cdot \left[2\sin\left(\frac{\pi}{3} + \theta\right)\sin\left(\frac{\pi}{3} - \theta\right)\right]$.

$= 2\left[\cos(2\theta) - \cos\left(\frac{2\pi}{3}\right)\right]$.

We know that $\cos\left(\frac{2\pi}{3}\right) = \cos(120^\circ) = -\frac{1}{2}$.

$= 2\left[\cos(2\theta) - \left(-\frac{1}{2}\right)\right] = 2\left(\cos 2\theta + \frac{1}{2}\right)$.

$= 2\cos 2\theta + 1$.

The correct option is **(4)**.

130. If set A has n elements, then total number of subsets of A is

(1) n (2) n^2 (3) 2^n (4) 2n

Solution:

Let A be a set with n elements, i.e., $n(A) = n$.
The total number of subsets of a set is the number of elements in its power set, P(A).
For a set with n elements, the number of possible subsets is given by the formula 2^n.
This includes the empty set and the set A itself.

The correct option is **(3)**.

131. $\frac{\sin 3A - \cos(\frac{\pi}{2} - A)}{\cos A + \cos(\pi + 3A)} =$

(1) tan A (2) cot A (3) tan 2A (4) cot 2A

Solution:

We simplify the expression using reduction formulas first.

$\cos\left(\frac{\pi}{2} - A\right) = \sin A.$

$\cos(\pi + 3A) = -\cos 3A.$

The expression becomes $\frac{\sin 3A - \sin A}{\cos A - \cos 3A}$.

Now we apply the sum-to-product formulas:

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right).$

$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right).$

Numerator: $\sin 3A - \sin A = 2\cos\left(\frac{3A+A}{2}\right)\sin\left(\frac{3A-A}{2}\right) = 2\cos(2A)\sin(A).$

Denominator:

$\cos A - \cos 3A = -2\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)$

$= -2\sin(2A)\sin(-A) = 2\sin(2A)\sin(A).$

The expression is $\frac{2\cos(2A)\sin(A)}{2\sin(2A)\sin(A)}$.

Assuming $\sin A \neq 0$, we can cancel the term.

$= \frac{\cos(2A)}{\sin(2A)} = \cot(2A).$

The correct option is **(4)**.

132. Solve: $\frac{(x+2)^2}{(x-1)^2} < 1$

(1) $x \in (-\infty, -1) \cup (0, 2)$
(2) $x \in (-2, 0) \cup (2, \infty)$
(3) $x \in (-\infty, -2) \cup (0, 1) \cup (1, 2)$
(4) $x \in (-\infty, -1/2)$

Solution:

$\dfrac{(x+2)^2}{(x-1)^2} < 1.$

First, note that $x \neq 1$.

$\dfrac{(x+2)^2}{(x-1)^2} - 1 < 0.$

$\dfrac{(x+2)^2 - (x-1)^2}{(x-1)^2} < 0.$

The denominator $(x-1)^2$ is always positive for $x \neq 1$.

So, the sign of the fraction is determined by the sign of the numerator.

$(x+2)^2 - (x-1)^2 < 0.$

Using $a^2 - b^2 = (a-b)(a+b)$:

$[(x+2) - (x-1)][(x+2) + (x-1)] < 0.$

$(x+2-x+1)(x+2+x-1) < 0.$

$(3)(2x+1) < 0.$

$2x+1 < 0 \implies 2x < -1 \implies x < -1/2.$

The condition $x \neq 1$ is already satisfied by $x < -1/2$.

The solution is $x \in (-\infty, -1/2)$.

The correct option is **(4)**.

133. If $\sin\alpha = \frac{-3}{5}$ where $\pi < \alpha < \frac{3\pi}{2}$, then $\cos(\frac{\alpha}{2}) =$

(1) $\frac{3}{\sqrt{10}}$
(2) $\frac{1}{\sqrt{10}}$
(3) $\frac{-1}{\sqrt{10}}$
(4) $\frac{-3}{\sqrt{10}}$

Solution:

Given the quadrant for $\alpha : \pi < \alpha < \dfrac{3\pi}{2}$.

Dividing by 2 to find the quadrant for $\alpha/2 : \dfrac{\pi}{2} < \dfrac{\alpha}{2} < \dfrac{3\pi}{4}$.

This range is in Quadrant II, where $\cos(\alpha/2)$ is negative.

We use the half-angle identity $\cos^2\left(\dfrac{\alpha}{2}\right) = \dfrac{1+\cos\alpha}{2}$.

First, find $\cos\alpha$. Since α is in Q3, $\cos\alpha$ is negative.

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - (-3/5)^2 = 1 - 9/25 = 16/25.$

$\cos\alpha = -\dfrac{4}{5}.$

$\cos^2\left(\dfrac{\alpha}{2}\right) = \dfrac{1+(-4/5)}{2} = \dfrac{1/5}{2} = \dfrac{1}{10}.$

$\cos\left(\dfrac{\alpha}{2}\right) = \pm\dfrac{1}{\sqrt{10}}.$

Since $\alpha/2$ is in Q2, we choose the negative value.

$$\cos\left(\frac{\alpha}{2}\right) = -\frac{1}{\sqrt{10}}.$$

The correct option is **(3)**.

134. Solve: $\frac{(x-1)^2}{x^2-4x+3} > 0$

(1) $x < 1$
(2) $x \in (-\infty, 1) \cup (3, \infty)$
(3) $x \in (1, 3)$
(4) $x \in (-\infty, 1) \cup (1, 3) \cup (3, \infty)$

Solution:

$$\frac{(x-1)^2}{x^2-4x+3} > 0.$$

Factor the denominator: $x^2 - 4x + 3 = (x-1)(x-3)$.

$$\frac{(x-1)^2}{(x-1)(x-3)} > 0.$$

For the expression to be defined, $x \neq 1$ and $x \neq 3$.

For $x \neq 1$, we can cancel a factor of $(x-1)$:

$$\frac{x-1}{x-3} > 0.$$

The critical points are $x = 1$ and $x = 3$.

Using the wavy curve method, the expression is positive outside the roots.

$x \in (-\infty, 1) \cup (3, \infty)$.

The original restrictions $x \neq 1$ and $x \neq 3$ are already handled by the open intervals.

The correct option is **(2)**.

135. $cos\frac{\pi}{7} - cos\frac{2\pi}{7} + cos\frac{3\pi}{7} - cos\frac{4\pi}{7} + cos\frac{5\pi}{7} - cos\frac{6\pi}{7} =$

(1) 0
(2) 3/2
(3) 3/4
(4) 1

Solution:

Let the sum be S.

$$S = \cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} - \cos\frac{6\pi}{7}.$$

First, we group terms using the identity $\cos(\pi - \theta) = -\cos\theta$.

$$\cos\frac{6\pi}{7} = -\cos\frac{\pi}{7}, \quad \cos\frac{5\pi}{7} = -\cos\frac{2\pi}{7}, \quad \cos\frac{4\pi}{7} = -\cos\frac{3\pi}{7}.$$

$$S = \left(\cos\frac{\pi}{7} - \cos\frac{6\pi}{7}\right) - \left(\cos\frac{2\pi}{7} - \cos\frac{5\pi}{7}\right) + \left(\cos\frac{3\pi}{7} - \cos\frac{4\pi}{7}\right).$$

$$S = \left(\cos\frac{\pi}{7} - (-\cos\frac{\pi}{7})\right) - \left(\cos\frac{2\pi}{7} - (-\cos\frac{2\pi}{7})\right) + \left(\cos\frac{3\pi}{7} - (-\cos\frac{3\pi}{7})\right).$$

$$S = 2\cos\frac{\pi}{7} - 2\cos\frac{2\pi}{7} + 2\cos\frac{3\pi}{7}.$$

$S = 2\left(\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7}\right).$

Multiply and divide the expression inside the parenthesis by $2\sin\frac{\pi}{7}$:

$S = \frac{2}{2\sin\frac{\pi}{7}}\left[2\sin\frac{\pi}{7}\cos\frac{\pi}{7} - 2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{3\pi}{7}\right].$

Using the product-to-sum formulas $2\sin A\cos A = \sin 2A$ and $2\sin A\cos B = \sin(A+B) + \sin(A-B)$:
The numerator becomes:

$= \sin\frac{2\pi}{7} - \left[\sin\left(\frac{3\pi}{7}\right) + \sin\left(-\frac{\pi}{7}\right)\right] + \left[\sin\left(\frac{4\pi}{7}\right) + \sin\left(-\frac{2\pi}{7}\right)\right]$

$= \sin\frac{2\pi}{7} - \sin\frac{3\pi}{7} + \sin\frac{\pi}{7} + \sin\frac{4\pi}{7} - \sin\frac{2\pi}{7}.$

Cancelling terms and using $\sin\frac{4\pi}{7} = \sin\left(\pi - \frac{3\pi}{7}\right) = \sin\frac{3\pi}{7}$:

$= \sin\frac{\pi}{7} - \sin\frac{3\pi}{7} + \sin\frac{3\pi}{7} = \sin\frac{\pi}{7}.$

Substitute this back into the expression for S:

$S = \frac{2}{2\sin\frac{\pi}{7}}\left(\sin\frac{\pi}{7}\right) = \frac{\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}} = 1.$

The correct option is **(4)**.

136. Solve: $(x^2-4)(x+1)^2 \leq 0$

(1) $-2 \leq x \leq 0$
(2) $0 \leq x \leq 2$
(3) $-2 \leq x < -1 \cup -1 < x \leq 2$
(4) $-2 \leq x \leq 2$

Solution:

$(x^2-4)(x+1)^2 \leq 0.$
$(x-2)(x+2)(x+1)^2 \leq 0.$

The term $(x+1)^2$ is always non-negative.
Case 1: $(x+1)^2 > 0,$ which means $x \neq -1.$
In this case, we can divide by $(x+1)^2$ without changing the inequality sign.
$(x-2)(x+2) \leq 0.$
The roots are -2 and 2. The expression is non-positive between the roots.
$-2 \leq x \leq 2.$
Combining with the condition $x \neq -1,$ the solution for this case is $[-2,-1) \cup (-1,2]$.

Case 2: $(x+1)^2 = 0,$ which means $x = -1.$
If $x = -1,$ the inequality becomes $((-1)^2-4)((-1)+1)^2 \leq 0 \implies (-3)(0) \leq 0 \implies 0 \leq 0.$
This is true, so $x = -1$ is also a solution.

The complete solution set is the union of the results from both cases:
$([-2,-1)\cup(-1,2])\cup\{-1\}=[-2,2]$.

The correct option is **(4)**.

137. If $tan\ A=\frac{1}{2}, tan\ B=\frac{1}{3}$, then $\cos 2A=$

(1) sin B (2) sin 2B (3) sin 3B (4) $-sin\ 2B$

Solution:

First, find $\cos 2A$ using $\tan A = 1/2$.

$$\cos 2A=\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-(1/2)^2}{1+(1/2)^2}=\frac{1-1/4}{1+1/4}=\frac{3/4}{5/4}=\frac{3}{5}.$$

Now, find $\sin 2B$ using $\tan B = 1/3$.

$$\sin 2B=\frac{2\tan B}{1+\tan^2 B}=\frac{2(1/3)}{1+(1/3)^2}=\frac{2/3}{1+1/9}=\frac{2/3}{10/9}=\frac{2}{3}\times\frac{9}{10}=\frac{3}{5}.$$

Comparing the results, we see that $\cos 2A=\sin 2B$.

The correct option is **(2)**.

138. If $3(x+2)\geq 2x+5$, then

(1) $x\leq -1$ (2) $x\geq -1$ (3) $x\leq 1$ (4) $x\geq 1$

Solution:

We solve the linear inequality for x.

$3(x+2)\geq 2x+5.$

$3x+6\geq 2x+5.$

$3x-2x\geq 5-6.$

$x\geq -1.$

The correct option is **(2)**.

139. $\sin(\frac{\pi}{10})\sin(\frac{3\pi}{10})=$

(1) $\frac{1}{2}$ (2) $\frac{-1}{2}$ (3) $\frac{1}{4}$ (4) 1

Solution:

The angles are $\pi/10=18^\circ$ and $3\pi/10=54^\circ$.

The expression is $\sin 18^\circ \sin 54^\circ$.

Using $\sin 54^\circ=\sin(90^\circ-36^\circ)=\cos 36^\circ$.

$=\sin 18^\circ\cos 36^\circ$.

We know the standard values:

$$\sin 18^\circ=\frac{\sqrt{5}-1}{4}.$$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}.$$

$$\text{The product is } \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right) = \frac{(\sqrt{5})^2 - 1^2}{16} = \frac{5-1}{16} = \frac{4}{16} = \frac{1}{4}.$$

The correct option is **(3)**.

140. Solve: $\frac{x-1}{x^2+2x+1} \geq 0$

(1) $x \in [1, \infty)$
(2) $x = 1$
(3) $x \in (-\infty, -1) \cup [1, \infty)$
(4) $x \leq 1$

Solution:

$$\frac{x-1}{x^2+2x+1} \geq 0.$$
$$\frac{x-1}{(x+1)^2} \geq 0.$$

The denominator $(x+1)^2$ is always positive, except when $x = -1$, where it is zero.
The expression is undefined at $x = -1$.

The solution is $x \geq 1$. This interval does not include -1, so no further restrictions are needed.
The solution set is $[1, \infty)$.

The correct option is **(1)**.

141. $\frac{\cot^2 15^\circ - 1}{\cot^2 15^\circ + 1} =$

(1) $\frac{1}{2}$
(2) $\frac{\sqrt{3}}{2}$
(3) $\frac{3\sqrt{3}}{4}$
(4) $\sqrt{3}$

Solution:

Let $\theta = 15^\circ$. The expression is $\dfrac{\cot^2\theta - 1}{\cot^2\theta + 1}$.

Convert to sine and cosine:

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{\frac{\cos^2\theta}{\sin^2\theta} + 1} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}.$$
$$= \frac{\cos(2\theta)}{1} = \cos(2\theta).$$

Substitute $\theta = 15^\circ$:

$$= \cos(2 \times 15^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}.$$

The correct option is **(2)**.

142. Solve: $(x+1)(x-2)^2(x+3) \leq 0$

(1) $x \in [-3,-1] \cup [2,\infty]$ (2) $x \in [-3,-1]$ (3) $x \in (-3,-1]$ (4) $x \in [-3,-1] \cup \{2\}$

Solution:

$(x+1)(x-2)^2(x+3) \leq 0.$
The term $(x-2)^2$ is always non-negative.

Case 1: $(x-2)^2 > 0,$ i.e., $x \neq 2.$
We can divide by $(x-2)^2$:
$(x+1)(x+3) \leq 0.$
The roots are -3 and -1. The expression is non-positive between the roots.
$-3 \leq x \leq -1.$ This interval does not contain 2.

Case 2: $(x-2)^2 = 0,$ i.e., $x = 2.$
Check if x=2 is a solution: $(2+1)(2-2)^2(2+3) \leq 0 \implies (3)(0)(5) \leq 0 \implies 0 \leq 0.$
This is true, so x=2 is also a solution.

The complete solution is the union of the two cases:
$[-3,-1] \cup \{2\}.$

The correct option is **(4)**.

143. $\frac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°} =$

(1) $tan\ 55°$ (2) $cot\ 55°$ (3) $-tan\ 35°$ (4) $-cot\ 35°$

Solution:

Divide the numerator and denominator by $\cos 10°.$

$$\frac{1 + \frac{\sin 10°}{\cos 10°}}{1 - \frac{\sin 10°}{\cos 10°}} = \frac{1 + \tan 10°}{1 - \tan 10°}.$$

We use the identity $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}.$
Let $A = 45°, B = 10°.$ We know $\tan 45° = 1.$

$$\frac{\tan 45° + \tan 10°}{1 - \tan 45° \tan 10°} = \tan(45° + 10°) = \tan 55°.$$

The correct option is **(1)**.

144. Find the set of x for which $x^3 - 4x < 0$

(1) $x \in (-\infty,-2) \cup (0,2)$ (2) $x \in (-2,0) \cup (2,\infty)$
(3) $x \in (-\infty,-2) \cup (2,\infty)$ (4) $x \in (-2,2)$

Solution:

$x^3 - 4x < 0.$
Factor the expression:
$x(x^2 - 4) < 0.$

$x(x-2)(x+2) < 0.$

The critical points are $-2, 0, 2.$
Using the wavy curve method:
For $x > 2 : (+)(+)(+) = (+).$
For $0 < x < 2 : (+)(-)(+) = (-).$
For $-2 < x < 0 : (-)(-)(+) = (+).$
For $x < -2 : (-)(-)(-) = (-).$

We need the intervals where the expression is negative.
$x \in (-\infty, -2) \cup (0, 2).$

The correct option is **(1)**.

145. $\frac{1}{4}(\sqrt{3}\cos 23^\circ - \sin 23^\circ) =$

(1) $cos\ 43^\circ$ (2) $cos\ 7^\circ$ (3) $\frac{1}{4}cos\ 53^\circ$ (4) $\frac{1}{2}cos\ 53^\circ$

Solution:

Let $E = \dfrac{1}{4}(\sqrt{3}\cos 23^\circ - \sin 23^\circ).$
The expression inside the parenthesis is of the form $a\cos\theta + b\sin\theta.$
$E = \dfrac{1}{4} \cdot 2\left(\dfrac{\sqrt{3}}{2}\cos 23^\circ - \dfrac{1}{2}\sin 23^\circ\right).$
$E = \dfrac{1}{2}\left(\sin 60^\circ \cos 23^\circ - \cos 60^\circ \sin 23^\circ\right).$
Using the identity $\sin(A-B) = \sin A\cos B - \cos A\sin B:$
$E = \dfrac{1}{2}\sin(60^\circ - 23^\circ) = \dfrac{1}{2}\sin(37^\circ).$

We can also write it as:
$E = \dfrac{1}{2}\left(\cos 30^\circ \cos 23^\circ - \sin 30^\circ \sin 23^\circ\right).$
Using the identity $\cos(A+B) = \cos A\cos B - \sin A\sin B:$
$E = \dfrac{1}{2}\cos(30^\circ + 23^\circ) = \dfrac{1}{2}\cos(53^\circ).$

The correct option is **(4)**.

146. If $5x + 1 > 3x - 9$ then

(1) $x < -5$ (2) $x > -5$ (3) $x > 5$ (4) $x < 5$

Solution:

We solve the linear inequality for x.
$5x + 1 > 3x - 9.$
$5x - 3x > -9 - 1.$
$2x > -10.$

$x > -5.$

The correct option is **(2)**.

147. The range of the function $y = -3\sin(\frac{x}{2}) - \frac{1}{3}$ is

(1) $[-3, 3]$ (2) $[-\frac{8}{3}, \frac{10}{3}]$ (3) $[-\frac{10}{3}, \frac{8}{3}]$ (4) [-1,1]

Solution:

We start with the basic range of the sine function.

$$-1 \leq \sin\left(\frac{x}{2}\right) \leq 1.$$

Multiply by -3. This reverses the inequality signs.

$$(-1) \times (-3) \geq -3\sin\left(\frac{x}{2}\right) \geq 1 \times (-3).$$

$$3 \geq -3\sin\left(\frac{x}{2}\right) \geq -3.$$

Subtract 1/3 from all parts.

$$3 - \frac{1}{3} \geq -3\sin\left(\frac{x}{2}\right) - \frac{1}{3} \geq -3 - \frac{1}{3}.$$

$$\frac{8}{3} \geq y \geq -\frac{10}{3}.$$

The range is $\left[-\frac{10}{3}, \frac{8}{3}\right]$.

The correct option is **(3)**.

148. Solve: $x^3 - 3x^2 + 2x > 0$

(1) $x < 0$ or $0 < x < 1$ (2) $0 < x < 1$ or $x > 2$ (3) $x < 1$ or $x > 2$ (4) $x > 0$ or $x > 2$

Solution:

$x^3 - 3x^2 + 2x > 0.$

Factor the expression:

$x(x^2 - 3x + 2) > 0.$

$x(x - 1)(x - 2) > 0.$

The critical points are $0, 1, 2$.

Using the wavy curve method:

For $x > 2 : (+)(+)(+) = (+).$

For $1 < x < 2 : (+)(+)(-) = (-).$

For $0 < x < 1 : (+)(-)(-) = (+).$

For $x < 0 : (-)(-)(-) = (-).$

We need the intervals where the expression is positive.
$x \in (0, 1) \cup (2, \infty)$.
This means $0 < x < 1$ or $x > 2$.

The correct option is **(2)**.

149. The range of the function $y = \frac{1}{\sqrt{3}\cos(2x) - \sin(2x) + 3}$ is

(1) $[\frac{1}{5}, 1]$ (2) [1,5] (3) $[\frac{1}{4}, 1]$ (4) $[-\frac{1}{5}, \frac{1}{2}]$

Solution:

To find the range of y, we first find the range of the denominator, D.
$D = \sqrt{3}\cos(2x) - \sin(2x) + 3$.
Let's find the range of $\sqrt{3}\cos(2x) - \sin(2x)$.
This is of the form $a\cos\theta + b\sin\theta$ with $a = \sqrt{3}, b = -1$.
The range is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$.
Range $= [-\sqrt{(\sqrt{3})^2 + (-1)^2}, \sqrt{(\sqrt{3})^2 + (-1)^2}] = [-\sqrt{3 + 1}, \sqrt{3 + 1}] = [-2, 2]$.

Now, find the range of the entire denominator D:
$D_{min} = (\text{min value}) + 3 = -2 + 3 = 1$.
$D_{max} = (\text{max value}) + 3 = 2 + 3 = 5$.
The range of D is $[1, 5]$.

The range of $y = \frac{1}{D}$ is $\left[\frac{1}{D_{max}}, \frac{1}{D_{min}}\right]$.
Range of y is $[\frac{1}{5}, 1]$.

The correct option is **(1)**.

150. Solve: $x^4 - 16 < 0$

(1) $x \in (-2, 2)$ (2) $x \in (-4, 4)$
(3) $x \in (-\sqrt{16}, 0)$ (4) $x \in (-\infty, -2) \cup (2, \infty)$

Solution:

$x^4 - 16 < 0$.
Factor the expression as a difference of squares:
$(x^2 - 4)(x^2 + 4) < 0$.

The term $(x^2 + 4)$ is always positive for any real number x.
Since it's a positive factor, we can divide both sides of the inequality by it.
$x^2 - 4 < 0$.

Factor again:

$(x-2)(x+2) < 0.$

The roots are -2 and 2. The expression is negative between the roots.
$-2 < x < 2.$
The solution set is $x \in (-2, 2)$.

The correct option is **(1)**.