Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 14th July

Syllabus: Trigonometric Ratios & Identities (till multiple angle)

Topic: Trigonometry

Sub: Mathematics

CT-03 Solution

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Regular Batch Questions

1. The value of $8\cos(10°)\cos(50°)\cos(70°)$ is:

(A) 1
(B) $\sqrt{3}$
(C) $2\sqrt{3}$
(D) $3/2$

Solution:
We use the identity $\cos A\cos(60° - A)\cos(60° + A) = \frac{1}{4}\cos 3A$. Let $A = 10°$. Then $60° - A = 50°$ and $60° + A = 70°$.

$$\begin{aligned}\text{Expression} &= 8[\cos 10° \cos(60° - 10°)\cos(60° + 10°)] \\ &= 8\left[\frac{1}{4}\cos(3 \times 10°)\right] \\ &= 2\cos(30°) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}.\end{aligned}$$

The correct option is **(B)**.

2. The value of the expression $\operatorname{cosec} 10° - \sqrt{3}\sec 10°$ is equal to:

(A) 2
(B) 4
(C) 1
(D) 0

Solution:
Convert to sin and cos.

$$\text{LHS} = \frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10° \cos 10°}$$

Multiply and divide the numerator by 2.

$$= \frac{2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{1}{2}(2\sin 10^\circ \cos 10^\circ)}$$
$$= \frac{2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\frac{1}{2}\sin 20^\circ}$$
$$= \frac{2\sin(30^\circ - 10^\circ)}{\frac{1}{2}\sin 20^\circ} = \frac{2\sin 20^\circ}{\frac{1}{2}\sin 20^\circ} = 4.$$

The correct option is **(B)**.

3. The value of $(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10})$ is:

(A) $1/8$ (B) $-1/8$
(C) $1/16$ (D) $-1/16$

Solution:
Use the identity $\cos(\pi - x) = -\cos x$. $\cos(9\pi/10) = \cos(\pi - \pi/10) = -\cos(\pi/10)$. $\cos(7\pi/10) = \cos(\pi - 3\pi/10) = -\cos(3\pi/10)$. Substitute these into the expression:

$$\begin{aligned}\text{LHS} &= (1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1-\cos\frac{3\pi}{10})(1-\cos\frac{\pi}{10}) \\ &= \left[(1+\cos\frac{\pi}{10})(1-\cos\frac{\pi}{10})\right]\left[(1+\cos\frac{3\pi}{10})(1-\cos\frac{3\pi}{10})\right] \\ &= (1-\cos^2\frac{\pi}{10})(1-\cos^2\frac{3\pi}{10}) = \sin^2\frac{\pi}{10}\sin^2\frac{3\pi}{10}\end{aligned}$$

Convert to degrees: $\sin^2 18^\circ \sin^2 54^\circ$.

$$= \sin^2 18^\circ \cos^2 36^\circ \qquad \text{[Using } \sin 54^\circ = \cos 36^\circ\text{]}$$
$$= \left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2 = \left[\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)\right]^2$$
$$= \left[\frac{5-1}{16}\right]^2 = \left(\frac{4}{16}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

The correct option is **(C)**.

4. If $\cos x = \frac{4}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, then the value of $\tan\frac{x}{2}$ is:

(A) $1/3$ (B) $-1/3$
(C) 3 (D) -3

Solution:
The given range for x is Quadrant IV. If $\frac{3\pi}{2} < x < 2\pi$, then dividing by 2 gives the range for $x/2$: $\frac{3\pi}{4} < \frac{x}{2} < \pi$. This means the angle $x/2$ lies in Quadrant II, where $\tan(x/2)$ is negative. We use the half-angle identity for tangent:

$$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} = \frac{1-4/5}{1+4/5} = \frac{1/5}{9/5} = \frac{1}{9}.$$

Taking the square root:

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1}{9}} = \pm\frac{1}{3}.$$

Since $x/2$ is in Quadrant II, we choose the negative value. $\tan\frac{x}{2} = -1/3$.
The correct option is **(B)**.

5. If $\cot\theta = \frac{a}{b}$, then $a\cos 2\theta + b\sin 2\theta$ is equal to:

(A) b
(B) 1
(C) a
(D) 0

Solution:
Given $\cot\theta = a/b$, we can say $\tan\theta = b/a$. We use the t-formulas for $\cos 2\theta$ and $\sin 2\theta$.

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-(b/a)^2}{1+(b/a)^2} = \frac{a^2-b^2}{a^2+b^2}.$$
$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2(b/a)}{1+(b/a)^2} = \frac{2ab}{a^2+b^2}.$$

Substitute these into the expression:

$$\begin{aligned} a\cos 2\theta + b\sin 2\theta &= a\left(\frac{a^2-b^2}{a^2+b^2}\right) + b\left(\frac{2ab}{a^2+b^2}\right) \\ &= \frac{a^3-ab^2+2ab^2}{a^2+b^2} = \frac{a^3+ab^2}{a^2+b^2} = \frac{a(a^2+b^2)}{a^2+b^2} = a. \end{aligned}$$

The correct option is **(C)**.

6. If $\cos\theta = -\frac{5}{13}$ and $\sin\phi = \frac{3}{5}$, where θ lies in the second quadrant and ϕ lies in the first quadrant, then $\sin(\theta+\phi) =$

(A) $\frac{33}{65}$
(B) $\frac{56}{65}$
(C) $-\frac{33}{65}$
(D) $-\frac{63}{65}$

Solution:
We need to find the other trigonometric ratios based on the given quadrants. For θ in Q2, $\sin\theta$ is positive.

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(-5/13)^2} = \sqrt{1-25/169} = \sqrt{144/169} = 12/13.$$

For ϕ in Q1, $\cos\phi$ is positive.

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-(3/5)^2} = \sqrt{1-9/25} = \sqrt{16/25} = 4/5.$$

Now use the sum formula for sine.

$$\begin{aligned} \sin(\theta+\phi) &= \sin\theta\cos\phi + \cos\theta\sin\phi \\ &= \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) + \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right) \\ &= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}. \end{aligned}$$

The correct option is **(A)**.

7. The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is:

(A) 0 (B) 2/3
(C) 1 (D) 3/2

Solution:
We simplify the expression inside the parenthesis.

$$\begin{aligned}\text{Expression} &= \sin 70° \left(\frac{\cos 10°}{\sin 10°} \frac{\cos 70°}{\sin 70°} - 1 \right) \\ &= \sin 70° \left(\frac{\cos 10° \cos 70° - \sin 10° \sin 70°}{\sin 10° \sin 70°} \right) \\ &= \sin 70° \left(\frac{\cos(10° + 70°)}{\sin 10° \sin 70°} \right) = \frac{\cos 80°}{\sin 10°}\end{aligned}$$

Using the co-function identity $\cos 80° = \sin(90° - 80°) = \sin 10°$.

$$= \frac{\sin 10°}{\sin 10°} = 1.$$

The correct option is **(C)**.

8. $\sin\left(\frac{3\pi}{4} - x\right) + \cos\left(\frac{\pi}{4} + x\right) =$

(A) $\sqrt{2}\cos x$ (B) $\sqrt{2}\sin x$
(C) $-\sqrt{2}\cos x$ (D) 0

Solution:
We expand both terms using sum/difference formulas.

$$\begin{aligned}\sin\left(\frac{3\pi}{4} - x\right) &= \sin\frac{3\pi}{4}\cos x - \cos\frac{3\pi}{4}\sin x \\ &= \left(\frac{1}{\sqrt{2}}\right)\cos x - \left(-\frac{1}{\sqrt{2}}\right)\sin x = \frac{1}{\sqrt{2}}(\cos x + \sin x).\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{\pi}{4} + x\right) &= \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x \\ &= \left(\frac{1}{\sqrt{2}}\right)\cos x - \left(\frac{1}{\sqrt{2}}\right)\sin x = \frac{1}{\sqrt{2}}(\cos x - \sin x).\end{aligned}$$

Adding the two results:

$$\begin{aligned}\text{LHS} &= \frac{1}{\sqrt{2}}(\cos x + \sin x) + \frac{1}{\sqrt{2}}(\cos x - \sin x) \\ &= \frac{1}{\sqrt{2}}(2\cos x) = \sqrt{2}\cos x.\end{aligned}$$

The correct option is **(A)**.

9. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°}$ is:

(A) 1 (B) 2
(C) 1/2 (D) 0

Solution:
We can write $\tan 70^\circ = \tan(50^\circ + 20^\circ)$ and expand it.

$$\begin{aligned} \tan 70^\circ &= \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ} \\ \tan 70^\circ(1 - \tan 50^\circ \tan 20^\circ) &= \tan 50^\circ + \tan 20^\circ \\ \tan 70^\circ - \tan 70^\circ \tan 50^\circ \tan 20^\circ &= \tan 50^\circ + \tan 20^\circ \end{aligned}$$

Since $\tan 70^\circ = \cot 20^\circ$, we have $\tan 70^\circ \tan 20^\circ = 1$.

$$\begin{aligned} \tan 70^\circ - \tan 50^\circ &= \tan 50^\circ + \tan 20^\circ \\ \tan 70^\circ - \tan 20^\circ &= 2\tan 50^\circ \end{aligned}$$

Dividing by $\tan 50^\circ$:

$$\frac{\tan 70^\circ - \tan 20^\circ}{\tan 50^\circ} = 2.$$

The correct option is **(B)**.

10. The value of $\frac{\sin 300^\circ \tan 330^\circ \sec 420^\circ}{\tan 135^\circ \sin 210^\circ \sec 315^\circ}$ is:

(A) -1 (B) 1
(C) $\sqrt{2}$ (D) $\sqrt{3}$

Solution:
We evaluate each trigonometric function using allied angles.

- Numerator: $\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$, $\tan 330^\circ = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$, $\sec 420^\circ = \sec 60^\circ = 2$.
- Denominator: $\tan 135^\circ = -1$, $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$, $\sec 315^\circ = \sec 45^\circ = \sqrt{2}$.

$$\text{Expression} = \frac{(-\frac{\sqrt{3}}{2})(-\frac{1}{\sqrt{3}})(2)}{(-1)(-\frac{1}{2})(\sqrt{2})} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \boldsymbol{\sqrt{2}}.$$

The correct option is **(C)**.

11. The expression $\frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)}$ is equal to:

(A) $\tan\theta$ (B) $\cot\theta$
(C) $\tan\phi$ (D) $\cot\phi$

Solution:
We group terms in the numerator and denominator and apply sum-to-product formulas.

$$\begin{aligned} \text{LHS} &= \frac{(\sin(\theta+\phi) + \sin(\theta-\phi)) - 2\sin\theta}{(\cos(\theta+\phi) + \cos(\theta-\phi)) - 2\cos\theta} \\ &= \frac{2\sin\theta\cos\phi - 2\sin\theta}{2\cos\theta\cos\phi - 2\cos\theta} \\ &= \frac{2\sin\theta(\cos\phi - 1)}{2\cos\theta(\cos\phi - 1)} = \frac{\sin\theta}{\cos\theta} = \tan\theta. \end{aligned}$$

The correct option is **(A)**.

12. The value of $\cos^3\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)+\sin^3\left(\frac{\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$ is:

(A) $1/2$ (B) $1/4$

(C) $\frac{\sqrt{3}}{4}$ (D) 1

Solution:

Let $A=\pi/12=15^\circ$. Then $5\pi/12=75^\circ$. We use the co-function identities: $\cos 75^\circ=\sin 15^\circ$ and $\sin 75^\circ=\cos 15^\circ$. The expression becomes:

$$\begin{aligned}
\text{LHS} &= \cos^3(15^\circ)\sin(15^\circ)+\sin^3(15^\circ)\cos(15^\circ)\\
&= \sin(15^\circ)\cos(15^\circ)[\cos^2(15^\circ)+\sin^2(15^\circ)]\\
&= \sin(15^\circ)\cos(15^\circ)\cdot(1)\\
&= \frac{1}{2}(2\sin 15^\circ\cos 15^\circ) && [\text{Using } \sin 2A=2\sin A\cos A]\\
&= \frac{1}{2}\sin(2\times 15^\circ)=\frac{1}{2}\sin(30^\circ)=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}.
\end{aligned}$$

The correct option is **(B)**.

Star Batch Questions

1. $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$ is equal to:

(A) $(1/\sqrt{2})\sin A$ (B) $(1/\sqrt{2})\cos A$
(C) $\sqrt{2}\sin A$ (D) $\sqrt{2}\cos A$

Solution:
We use the identity $\sin^2 X-\sin^2 Y=\sin(X+Y)\sin(X-Y)$.
Let $X=\frac{\pi}{8}+\frac{A}{2}$ and $Y=\frac{\pi}{8}-\frac{A}{2}$.

$$X+Y=\left(\frac{\pi}{8}+\frac{A}{2}\right)+\left(\frac{\pi}{8}-\frac{A}{2}\right)=\frac{2\pi}{8}=\frac{\pi}{4}.$$
$$X-Y=\left(\frac{\pi}{8}+\frac{A}{2}\right)-\left(\frac{\pi}{8}-\frac{A}{2}\right)=A.$$

Substituting these into the identity:

$$\begin{aligned}\text{Expression}&=\sin(X+Y)\sin(X-Y)\\&=\sin\left(\frac{\pi}{4}\right)\sin(A)\\&=\frac{1}{\sqrt{2}}\sin A.\end{aligned}$$

The correct option is **(A)**.

2. $15[\tan 2\theta+\sin 2\theta]+8=0$ if:

(A) $\tan\theta=1/2$ (B) $\sin\theta=1/4$
(C) $\tan\theta=2$ (D) $\cos\theta=1/5$

Solution:
Let $t=\tan\theta$. We use the t-formulas for $\tan 2\theta$ and $\sin 2\theta$.

$$\tan 2\theta=\frac{2t}{1-t^2}\quad\text{and}\quad\sin 2\theta=\frac{2t}{1+t^2}.$$

Substitute these into the given equation:

$$\begin{aligned}15\left[\frac{2t}{1-t^2}+\frac{2t}{1+t^2}\right]+8&=0\\15\cdot 2t\left[\frac{1}{1-t^2}+\frac{1}{1+t^2}\right]+8&=0\\30t\left[\frac{(1+t^2)+(1-t^2)}{(1-t^2)(1+t^2)}\right]+8&=0\\30t\left[\frac{2}{1-t^4}\right]+8&=0\\\frac{60t}{1-t^4}&=-8\\60t&=-8(1-t^4)=8t^4-8\\8t^4-60t-8&=0\\2t^4-15t-2&=0.\end{aligned}$$

Now we check the given options. If $\tan\theta = 2$, then $t = 2$.

$$2(2)^4 - 15(2) - 2 = 2(16) - 30 - 2 = 32 - 32 = 0.$$

Since $t = 2$ satisfies the equation, $\tan\theta = 2$ is a valid solution.
The correct option is **(C)**.

3. What is $\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$ equal to ?

(A) 0 (B) 1
(C) 2 (D) 3

Solution:
We evaluate each term using the properties of trigonometric functions for integer multiples of π.

- $\sin(n\pi) = 0$ for any integer n. So, $\sin(3\pi) = 0$.
- $\cos(n\pi) = (-1)^n$ for any integer n. So, $\cos(4\pi) = (-1)^4 = 1$.
- $\tan(n\pi) = 0$ for any integer n. So, $\tan(5\pi) = 0$.

Now substitute these values into the expression:

$$\begin{aligned}\text{Expression} &= (\sin(3\pi))^2 + (\cos(4\pi))^2 + (\tan(5\pi))^2 \\ &= (0)^2 + (1)^2 + (0)^2 = 0 + 1 + 0 = 1.\end{aligned}$$

The correct option is **(B)**.

4. What is $\frac{1-\tan 2^\circ \cot 62^\circ}{\tan 152^\circ - \cot 88^\circ}$ equal to?

(A) $\sqrt{3}$ (B) $-\sqrt{3}$
(C) $\sqrt{2}-1$ (D) $1-\sqrt{2}$

Solution:
We use co-function and allied angle identities to simplify the expression.

- $\cot 62^\circ = \cot(90^\circ - 28^\circ) = \tan 28^\circ$.
- $\tan 152^\circ = \tan(180^\circ - 28^\circ) = -\tan 28^\circ$.
- $\cot 88^\circ = \cot(90^\circ - 2^\circ) = \tan 2^\circ$.

Substitute these into the expression:

$$\begin{aligned}\text{Expression} &= \frac{1 - \tan 2^\circ \tan 28^\circ}{-\tan 28^\circ - \tan 2^\circ} \\ &= -\left(\frac{1 - \tan 2^\circ \tan 28^\circ}{\tan 28^\circ + \tan 2^\circ}\right)\end{aligned}$$

This is the reciprocal of the $\tan(A + B)$ formula.

$$\begin{aligned}&= -\frac{1}{\frac{\tan 28^\circ + \tan 2^\circ}{1 - \tan 2^\circ \tan 28^\circ}} = -\frac{1}{\tan(28^\circ + 2^\circ)} \\ &= -\frac{1}{\tan 30^\circ} = -\cot 30^\circ = -\sqrt{3}.\end{aligned}$$

The correct option is **(B)**.

5. If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to:

(A) 5 (B) 3
(C) 4 (D) none of these

Solution:
Let $3\sin\theta + 5\cos\theta = 5 \quad \cdots(1)$.
Let $5\sin\theta - 3\cos\theta = x \quad \cdots(2)$.
We square both equations and add them.

$$\begin{aligned}(3\sin\theta+5\cos\theta)^2+(5\sin\theta-3\cos\theta)^2 &= 5^2+x^2\\ (9\sin^2\theta+25\cos^2\theta+30\sin\theta\cos\theta)+(25\sin^2\theta+9\cos^2\theta-30\sin\theta\cos\theta) &= 25+x^2\\ 9(\sin^2\theta+\cos^2\theta)+25(\cos^2\theta+\sin^2\theta) &= 25+x^2\\ 9(1)+25(1) &= 25+x^2\\ 34 &= 25+x^2\\ x^2 &= 9 \implies x=\pm 3.\end{aligned}$$

One possible value is 3.
The correct option is **(B)**.

6. If $\sin A + \sin B = a$ and $\cos A + \cos B = b$, then $\cos(A+B)$ is:

(A) $\frac{a^2+b^2}{b^2-a^2}$ (B) $\frac{2ab}{a^2+b^2}$
(C) $\frac{b^2-a^2}{a^2+b^2}$ (D) $\frac{a^2-b^2}{a^2+b^2}$

Solution:
We square and add the two given equations.

$$\begin{aligned}a^2+b^2 &= (\sin A+\sin B)^2+(\cos A+\cos B)^2\\ &= (\sin^2 A+\sin^2 B+2\sin A\sin B)+(\cos^2 A+\cos^2 B+2\cos A\cos B)\\ &= (\sin^2 A+\cos^2 A)+(\sin^2 B+\cos^2 B)+2(\cos A\cos B+\sin A\sin B)\\ &= 1+1+2\cos(A-B)=2+2\cos(A-B). \quad \cdots(i)\end{aligned}$$

Now, we find b^2-a^2.

$$\begin{aligned}b^2-a^2 &= (\cos A+\cos B)^2-(\sin A+\sin B)^2\\ &= (\cos^2 A+\cos^2 B+2\cos A\cos B)-(\sin^2 A+\sin^2 B+2\sin A\sin B)\\ &= (\cos^2 A-\sin^2 A)+(\cos^2 B-\sin^2 B)+2(\cos A\cos B-\sin A\sin B)\\ &= \cos 2A+\cos 2B+2\cos(A+B)\\ &= 2\cos(A+B)\cos(A-B)+2\cos(A+B)\\ &= 2\cos(A+B)[\cos(A-B)+1].\end{aligned}$$

From (i), $a^2+b^2-2 = 2\cos(A-B) \implies \cos(A-B) = \frac{a^2+b^2-2}{2}$. Substitute this into the expression for b^2-a^2:

$$\begin{aligned}b^2-a^2 &= 2\cos(A+B)\left[\frac{a^2+b^2-2}{2}+1\right]\\ &= 2\cos(A+B)\left[\frac{a^2+b^2}{2}\right] = \cos(A+B)(a^2+b^2).\end{aligned}$$

Therefore, $\cos(A+B) = \frac{b^2-a^2}{a^2+b^2}$.
The correct option is **(C)**.

7. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is:

(A) $1/8$ (B) $-1/2$

(C) 1 (D) $1/2$

Solution:
We group the terms and use the sum-to-product formula $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$.

$$\begin{aligned}
\text{LHS} &= (\cos 12° + \cos 132°) + (\cos 84° + \cos 156°) \\
&= 2\cos\left(\frac{144}{2}\right)\cos\left(\frac{-120}{2}\right) + 2\cos\left(\frac{240}{2}\right)\cos\left(\frac{-72}{2}\right) \\
&= 2\cos 72° \cos(-60°) + 2\cos 120° \cos(-36°) \\
&= 2\cos 72° \cos 60° + 2\cos 120° \cos 36° \\
&= 2\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4}\right) \\
&= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}-1-\sqrt{5}-1}{4} = \frac{-2}{4} = -1/2.
\end{aligned}$$

The correct option is **(B)**.

8. If $\sin A, \cos A$ and $\tan A$ are in G.P., then $\cot^6 A - \cot^2 A =$ ______.

Solution:
If three terms are in G.P., the square of the middle term is equal to the product of the other two.

$$\begin{aligned}
(\cos A)^2 &= (\sin A)(\tan A) \\
\cos^2 A &= \sin A\left(\frac{\sin A}{\cos A}\right) \\
\cos^3 A &= \sin^2 A
\end{aligned}$$

We need to find the value of $\cot^6 A - \cot^2 A$. Let's use the relation we just found. Divide the relation $\cos^3 A = \sin^2 A$ by $\sin^3 A$:

$$\begin{aligned}
\frac{\cos^3 A}{\sin^3 A} &= \frac{\sin^2 A}{\sin^3 A} \\
\cot^3 A &= \frac{1}{\sin A} = \operatorname{cosec} A
\end{aligned}$$

Now, square both sides:

$$\begin{aligned}
(\cot^3 A)^2 &= (\operatorname{cosec} A)^2 \\
\cot^6 A &= \operatorname{cosec}^2 A
\end{aligned}$$

Using the Pythagorean identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$:

$$\cot^6 A = 1 + \cot^2 A$$

Rearranging the terms gives the desired expression:

$$\cot^6 A - \cot^2 A = 1.$$

The answer is **1**.

9. The value of $\cos^3\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)=$:

(A) $\frac{1}{2\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{3}}{2}$

Solution:
Let $A=\pi/8$. The expression is $\cos^3 A\cos 3A+\sin^3 A\sin 3A$. We use the triple angle identities: $\cos 3A = 4\cos^3 A - 3\cos A \implies \cos^3 A = \frac{\cos 3A + 3\cos A}{4}$. $\sin 3A = 3\sin A - 4\sin^3 A \implies \sin^3 A = \frac{3\sin A - \sin 3A}{4}$. Substitute these into the expression:

$$\begin{aligned}\text{LHS} &= \left(\frac{\cos 3A + 3\cos A}{4}\right)\cos 3A + \left(\frac{3\sin A - \sin 3A}{4}\right)\sin 3A\\ &= \frac{1}{4}[\cos^2 3A + 3\cos A\cos 3A + 3\sin A\sin 3A - \sin^2 3A]\\ &= \frac{1}{4}[(\cos^2 3A - \sin^2 3A) + 3(\cos 3A\cos A + \sin 3A\sin A)]\\ &= \frac{1}{4}[\cos(2\cdot 3A) + 3\cos(3A - A)]\\ &= \frac{1}{4}[\cos 6A + 3\cos 2A]\end{aligned}$$

Now substitute $A=\pi/8$.

$$\begin{aligned}&= \frac{1}{4}\left[\cos\left(\frac{6\pi}{8}\right) + 3\cos\left(\frac{2\pi}{8}\right)\right] = \frac{1}{4}\left[\cos\left(\frac{3\pi}{4}\right) + 3\cos\left(\frac{\pi}{4}\right)\right]\\ &= \frac{1}{4}\left[-\frac{1}{\sqrt{2}} + 3\left(\frac{1}{\sqrt{2}}\right)\right] = \frac{1}{4}\left[\frac{2}{\sqrt{2}}\right] = \frac{1}{4}(\sqrt{2}) = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}.\end{aligned}$$

The correct option is **(A)**.

10. The value of $8\cos(10°)\cos(50°)\cos(70°)$ is:

(A) 1
(B) $\sqrt{3}$
(C) $2\sqrt{3}$
(D) $3/2$

Solution:
We use the identity $\cos A\cos(60° - A)\cos(60° + A) = \frac{1}{4}\cos 3A$. Let $A = 10°$. Then $60° - A = 50°$ and $60° + A = 70°$.

$$\begin{aligned}\text{Expression} &= 8[\cos 10°\cos(60° - 10°)\cos(60° + 10°)]\\ &= 8\left[\frac{1}{4}\cos(3\times 10°)\right]\\ &= 2\cos(30°) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}.\end{aligned}$$

The correct option is **(B)**.

11. The value of the expression $\operatorname{cosec} 10° - \sqrt{3}\sec 10°$ is equal to:

(A) 2
(B) 4
(C) 1
(D) 0

Solution:
Convert to sin and cos.

$$\text{LHS} = \frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°\cos 10°}$$

Multiply and divide the numerator by 2.

$$= \frac{2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{1}{2}(2\sin 10^\circ \cos 10^\circ)}$$
$$= \frac{2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\frac{1}{2}\sin 20^\circ}$$
$$= \frac{2\sin(30^\circ - 10^\circ)}{\frac{1}{2}\sin 20^\circ} = \frac{2\sin 20^\circ}{\frac{1}{2}\sin 20^\circ} = 4.$$

The correct option is **(B)**.

12. The value of $(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10})$ is:

(A) $1/8$ (B) $-1/8$
(C) $1/16$ (D) $-1/16$

Solution:
Use the identity $\cos(\pi - x) = -\cos x$. $\cos(9\pi/10) = \cos(\pi - \pi/10) = -\cos(\pi/10)$. $\cos(7\pi/10) = \cos(\pi - 3\pi/10) = -\cos(3\pi/10)$. Substitute these into the expression:

$$\text{LHS} = (1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1-\cos\frac{3\pi}{10})(1-\cos\frac{\pi}{10})$$
$$= \left[(1+\cos\frac{\pi}{10})(1-\cos\frac{\pi}{10})\right]\left[(1+\cos\frac{3\pi}{10})(1-\cos\frac{3\pi}{10})\right]$$
$$= (1-\cos^2\frac{\pi}{10})(1-\cos^2\frac{3\pi}{10}) = \sin^2\frac{\pi}{10}\sin^2\frac{3\pi}{10}$$

Convert to degrees: $\sin^2 18^\circ \sin^2 54^\circ$.

$$= \sin^2 18^\circ \cos^2 36^\circ \quad [\text{Using } \sin 54^\circ = \cos 36^\circ]$$
$$= \left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2 = \left[\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)\right]^2$$
$$= \left[\frac{5-1}{16}\right]^2 = \left(\frac{4}{16}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

The correct option is **(C)**.