JEEM-CT-03 Solution

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Topic: Quadratic Equation

Sub: Mathematics **JEEM-CT-03** Prof. Chetan Sir

SECTION-A

51. The integer 'k' for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R is:

(1) 3
(2) 2
(3) 4
(4) 0

Solution:
A quadratic expression $Ax^2 + Bx + C$ is positive for all real x if two conditions are met:
1. The parabola opens upwards, which means the leading coefficient $A > 0$.
2. The parabola does not touch or cross the x-axis, which means it has no real roots, so the discriminant $D < 0$.

In the given inequality, $A = 1$, which is positive. So, we only need to satisfy the discriminant condition.

$$D = B^2 - 4AC < 0$$
$$[-2(3k-1)]^2 - 4(1)(8k^2-7) < 0$$
$$4(3k-1)^2 - 4(8k^2-7) < 0$$
$$(9k^2 - 6k + 1) - (8k^2 - 7) < 0$$
$$k^2 - 6k + 8 < 0$$
$$(k-2)(k-4) < 0$$

This inequality holds for $2 < k < 4$.
The only integer value of 'k' in this interval is 3.
The correct option is **(1)**.

52. The set of all real values of l for which the quadratic equation $(l^2+1)x^2 - 4lx + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is:

(1) $(-3, -1)$
(2) $(0, 2)$
(3) (1,3]
(4) $(2, 4]$

Solution:
For a quadratic equation to have exactly one root in an open interval (k_1, k_2), the values of the function at the endpoints must have opposite signs.
Let $f(x) = (l^2+1)x^2 - 4lx + 2$.
The condition is $f(0) \cdot f(1) < 0$.
First, evaluate the function at the endpoints.

$$f(0) = (l^2+1)(0)^2 - 4l(0) + 2 = 2.$$
$$f(1) = (l^2+1)(1)^2 - 4l(1) + 2 = l^2 + 1 - 4l + 2 = l^2 - 4l + 3.$$

Now, apply the condition.

$$(2)(l^2 - 4l + 3) < 0$$
$$l^2 - 4l + 3 < 0$$
$$(l - 1)(l - 3) < 0$$

This inequality holds for $1 < l < 3$. The set is $(1, 3)$.
The closest matching option is (3). Note that if $l = 3$, $f(1) = 0$, so one root is exactly 1, which is not *in* the open interval $(0, 1)$.
The correct option is **(3)**.

53. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1,2) and its other root lies in the interval (2, 3), is:

(1) 2
(2) 0
(3) 1
(4) 3

Solution:
Let $f(x) = 2x^2 - 8x + k$. This is an upward-opening parabola ($A = 2 > 0$).
For the roots to be separated by the numbers 1, 2, and 3 as described, the following conditions must be met:

- $f(1) > 0$
- $f(2) < 0$
- $f(3) > 0$

Let's evaluate each condition.

$$f(1) = 2(1)^2 - 8(1) + k > 0 \implies 2 - 8 + k > 0 \implies k > 6.$$
$$f(2) = 2(2)^2 - 8(2) + k < 0 \implies 8 - 16 + k < 0 \implies k < 8.$$
$$f(3) = 2(3)^2 - 8(3) + k > 0 \implies 18 - 24 + k > 0 \implies k > 6.$$

We need to find the intersection of these conditions: $k > 6$ and $k < 8$. This gives the interval $6 < k < 8$. The only integer value of k in this interval is 7. There is only one such integral value.
The correct option is **(3)**.

54. The set of real values of x satisfying $log_{1/2}(x^2 - 6x + 12) \geq -2$ is:

(1) $(-\infty, 2]$
(2) $[2, 4]$
(3) $[4, +\infty)$
(4) None of these

Solution:
First, the argument of the logarithm must be positive: $x^2 - 6x + 12 > 0$. The discriminant of this quadratic is $D = (-6)^2 - 4(1)(12) = 36 - 48 = -12 < 0$. Since the leading coefficient is positive, the quadratic is always positive. This condition holds for all real x.

Now, we solve the main inequality. Since the base of the logarithm is 1/2 (which is between 0 and 1), we must reverse the inequality sign when removing the log.

$$x^2 - 6x + 12 \leq (1/2)^{-2}$$
$$x^2 - 6x + 12 \leq 4$$
$$x^2 - 6x + 8 \leq 0$$
$$(x - 2)(x - 4) \leq 0$$

This inequality holds for $2 \leq x \leq 4$. The solution set is $[2, 4]$.
The correct option is **(2)**.

55. If $(log_{10}x)^2 - 4|log_{10}x| + 3 = 0$, then product of roots of the equation is

(1) 3 (2) 10^4
(3) 10^8 (4) 1

Solution:
Let $y = |log_{10}x|$. Since $|a|^2 = a^2$, we have $y^2 = (log_{10}x)^2$. The equation becomes a quadratic in y.

$$y^2 - 4y + 3 = 0$$
$$(y-1)(y-3) = 0$$

This gives two possible values for y: $y = 1$ or $y = 3$.

- Case 1: $|log_{10}x| = 1 \implies log_{10}x = 1$ or $log_{10}x = -1$. This gives roots $x_1 = 10^1 = 10$ and $x_2 = 10^{-1}$.
- Case 2: $|log_{10}x| = 3 \implies log_{10}x = 3$ or $log_{10}x = -3$. This gives roots $x_3 = 10^3$ and $x_4 = 10^{-3}$.

The four roots of the equation are $10, 10^{-1}, 10^3, 10^{-3}$. The product of the roots is:

$$\text{Product} = 10^1 \times 10^{-1} \times 10^3 \times 10^{-3} = 10^{1-1+3-3} = 10^0 = 1.$$

The correct option is **(4)**.

56. The solution of the system of equations $log_3x + log_3y = 2 + log_32$ and $log_{27}(x+y) = \frac{2}{3}$ is

(1) {6,3} (2) {4,6}
(3) {6,12} (4) {10,6}

Solution:
Simplify the first equation using logarithm properties.

$$\log_3(xy) = \log_3(3^2) + \log_3(2)$$
$$\log_3(xy) = \log_3(9 \times 2) = \log_3(18)$$
$$xy = 18 \quad \cdots (i)$$

Simplify the second equation.

$$x + y = 27^{2/3} = (3^3)^{2/3} = 3^2 = 9 \quad \cdots (ii)$$

We need to find two numbers whose sum is 9 and product is 18. These are the roots of the quadratic equation $t^2 - (\text{sum})t + (\text{product}) = 0$.

$$t^2 - 9t + 18 = 0$$
$$(t-3)(t-6) = 0$$

The solutions are $t = 3$ and $t = 6$. So, the pair $\{x, y\}$ is $\{3, 6\}$.
The correct option is **(1)**.

57. For what values of k, the equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots?

(1) $2, -\frac{10}{9}$
(2) $2 \ \& \ -\frac{10}{9}$
(3) $2, -\frac{10}{9}$
(4) $4 \ \& \ -\frac{10}{9}$

Solution:
For an equation to have equal roots, its discriminant (D) must be zero.

$$\begin{aligned} D = B^2 - 4AC &= 0 \\ [-2(1+3k)]^2 - 4(1)(7(3+2k)) &= 0 \\ 4(1+3k)^2 - 28(3+2k) &= 0 \\ (1+6k+9k^2) - 7(3+2k) &= 0 \qquad \text{[Dividing by 4]} \\ 9k^2 + 6k + 1 - 21 - 14k &= 0 \\ 9k^2 - 8k - 20 &= 0 \end{aligned}$$

We solve this quadratic for k by factoring.

$$\begin{aligned} 9k^2 - 18k + 10k - 20 &= 0 \\ 9k(k-2) + 10(k-2) &= 0 \\ (9k+10)(k-2) &= 0 \end{aligned}$$

The solutions are $k = 2$ and $k = -10/9$.
The correct option is **(2)**.

58. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is

(1) $11 + \sqrt{3}$
(2) $9 + \sqrt{3}$
(3) $9 - \sqrt{3}$
(4) $11 - \sqrt{3}$

Solution:
First, rewrite the equation as $|x^2 - 8x + 15| = 2x - 7$. For a solution to exist, the RHS must be non-negative: $2x - 7 \geq 0 \implies x \geq 3.5$. The expression inside the absolute value is $x^2 - 8x + 15 = (x-3)(x-5)$. We consider two cases.

- **Case 1:** $x^2 - 8x + 15 \geq 0$ (i.e., $x \leq 3$ or $x \geq 5$). The equation becomes $x^2 - 8x + 15 = 2x - 7 \implies x^2 - 10x + 22 = 0$. The roots are $x = \frac{10 \pm \sqrt{100-88}}{2} = 5 \pm \sqrt{3}$. Check against conditions: $x = 5 + \sqrt{3} \approx 6.73$ (valid, since $x \geq 5$). $x = 5 - \sqrt{3} \approx 3.27$ (not valid, since it's not ≤ 3 or ≥ 5).
- **Case 2:** $x^2 - 8x + 15 < 0$ (i.e., $3 < x < 5$). The equation becomes $-(x^2 - 8x + 15) = 2x - 7 \implies x^2 - 6x + 8 = 0$. Factoring gives $(x-2)(x-4) = 0$. Roots are $x = 2, x = 4$. Check against conditions: $x = 2$ is not in $(3,5)$. $x = 4$ is in $(3,5)$ and satisfies $x \geq 3.5$. (Valid).

The valid roots are $5 + \sqrt{3}$ and 4. The sum of all roots is $4 + (5 + \sqrt{3}) = 9 + \sqrt{3}$.
The correct option is **(2)**.

59. If the equation $2x^2 + 3x + 5\lambda = 0$ and $x^2 + 2x + 3\lambda = 0$ have a common root then λ is equal to

(1) 0
(2) -1
(3) 0, -1
(4) 2, -1

Solution:

Let the common root be α.

$$2\alpha^2 + 3\alpha + 5\lambda = 0 \quad \cdots (1)$$
$$\alpha^2 + 2\alpha + 3\lambda = 0 \quad \cdots (2)$$

To eliminate the α^2 term, multiply equation (2) by 2 and subtract from equation (1).

$$(2\alpha^2 + 3\alpha + 5\lambda) - 2(\alpha^2 + 2\alpha + 3\lambda) = 0$$
$$2\alpha^2 + 3\alpha + 5\lambda - 2\alpha^2 - 4\alpha - 6\lambda = 0$$
$$-\alpha - \lambda = 0 \implies \alpha = -\lambda.$$

Now substitute $\alpha = -\lambda$ back into the simpler equation (2).

$$(-\lambda)^2 + 2(-\lambda) + 3\lambda = 0$$
$$\lambda^2 - 2\lambda + 3\lambda = 0$$
$$\lambda^2 + \lambda = 0$$
$$\lambda(\lambda + 1) = 0$$

The possible values for λ are $\lambda = 0$ or $\lambda = -1$.
The correct option is **(3)**.

60. The value of a for which $2x^2 - 2(2a+1)x + a(a+1) = 0$ may have one root less than a and other root greater than a.

(1) $-1 < a < 0$
(2) $a > 0$ or $a < -1$
(3) $a \geq 0$
(4) $-\frac{1}{2} < a < 0$

Solution:
Let $f(x) = 2x^2 - 2(2a+1)x + a(a+1)$. The condition that 'a' lies between the roots of the quadratic equation means that $A \cdot f(a) < 0$. Here, the leading coefficient is $A = 2$, which is positive. So the condition simplifies to $f(a) < 0$.

$$f(a) = 2a^2 - 2(2a+1)a + a(a+1) < 0$$
$$= 2a^2 - (4a^2 + 2a) + a^2 + a < 0$$
$$= 2a^2 - 4a^2 - 2a + a^2 + a < 0$$
$$= -a^2 - a < 0$$
$$= a^2 + a > 0 \qquad \text{[Multiplying by -1 reverses the inequality]}$$
$$= a(a+1) > 0$$

This inequality holds when $a > 0$ or $a < -1$.
The correct option is **(2)**.

61. If roots of equation $x^2 - 5x + 16 = 0$ are α, β and roots of equation $x^2 + px + q = 0$ are $\alpha^2 + \beta^2$ and $\frac{\alpha\beta}{2}$ then

(1) $p = 1$ and $q = -56$
(2) $p = -1$ and $q = -56$
(3) $p = 1$ and $q = 56$
(4) $p = -1$ and $q = 56$

Solution:
From the first equation, $x^2 - 5x + 16 = 0$:

- Sum of roots: $\alpha + \beta = 5$.

- Product of roots: $\alpha\beta = 16$.

The roots of the second equation are $r_1 = \alpha^2 + \beta^2$ and $r_2 = \alpha\beta/2$. We find the values of these new roots.

$$r_1 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (5)^2 - 2(16) = 25 - 32 = -7.$$
$$r_2 = \frac{\alpha\beta}{2} = \frac{16}{2} = 8.$$

For the second equation, $x^2 + px + q = 0$, we find p and q using Vieta's formulas.

$$\text{Sum of new roots: } r_1 + r_2 = -7 + 8 = 1.$$
$$\text{Product of new roots: } r_1 r_2 = (-7)(8) = -56.$$

From the new equation: Sum is $-p$ and Product is q.

$$-p = 1 \implies p = -1.$$
$$q = -56.$$

So, $p = -1$ and $q = -56$.
The correct option is **(2)**.

62. The least value of a, for which roots of the equation $x^2 - 2x - log_4 a = 0$ are real, is

(1) $\frac{1}{16}$
(2) $\frac{1}{4}$
(3) 4
(4) 20

Solution:
For the roots of the equation to be real, the discriminant D must be non-negative ($D \geq 0$).

$$D = (-2)^2 - 4(1)(-\log_4 a) \geq 0$$
$$4 + 4\log_4 a \geq 0$$
$$1 + \log_4 a \geq 0$$
$$\log_4 a \geq -1$$

To remove the logarithm, we exponentiate both sides with base 4.

$$a \geq 4^{-1} \implies a \geq \frac{1}{16}.$$

Also, for $\log_4 a$ to be defined, we must have $a > 0$. The condition $a \geq 1/16$ satisfies this. The least value of a is $1/16$.
The correct option is **(1)**.

63. The minimum and maximum values of the expression $\frac{x^2}{x^2+x+1}$, $x \in R$ are

(1) 0, 3/4
(2) $0, 4/3$
(3) 0, 1
(4) 0, 3

Solution:
Let $y = \frac{x^2}{x^2+x+1}$. We find the range of this expression.

$$y(x^2 + x + 1) = x^2$$
$$yx^2 + yx + y = x^2$$
$$(y-1)x^2 + yx + y = 0$$

Since x is real, the discriminant of this quadratic in x must be non-negative.

$$\begin{aligned} D = (y)^2 - 4(y-1)(y) &\geq 0 \\ y^2 - 4y^2 + 4y &\geq 0 \\ -3y^2 + 4y &\geq 0 \\ 3y^2 - 4y &\leq 0 \qquad \text{[Multiplying by -1 reverses the inequality]} \\ y(3y-4) &\leq 0 \end{aligned}$$

This inequality holds for $0 \leq y \leq 4/3$. The minimum value is 0 and the maximum value is 4/3.
The correct option is **(2)**.

64. The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$ is

(1) 1 (2) 2
(3) 3 (4) 4

Solution:
We consider cases based on the absolute value expressions. The critical points are $x = 0$ and $x = -2$.

- **Case 1:** $x \geq 0$. The equation becomes $x^2 - 5(x+2) + 6 = 0 \implies x^2 - 5x - 4 = 0$. Roots are $x = \frac{5 \pm \sqrt{25 - 4(1)(-4)}}{2} = \frac{5 \pm \sqrt{41}}{2}$. $x_1 = \frac{5+\sqrt{41}}{2} \approx 6.7 > 0$ (Valid). $x_2 = \frac{5-\sqrt{41}}{2} \approx -0.7 < 0$ (Not valid for this case).
- **Case 2:** $-2 \leq x < 0$. The equation becomes $-x^2 - 5(x+2) + 6 = 0 \implies -x^2 - 5x - 4 = 0 \implies x^2 + 5x + 4 = 0$. Factoring gives $(x+1)(x+4) = 0$. Roots are $x = -1, x = -4$. Only $x = -1$ is in the interval $[-2, 0)$. (Valid).
- **Case 3:** $x < -2$. The equation becomes $-x^2 - 5(-x-2) + 6 = 0 \implies -x^2 + 5x + 16 = 0 \implies x^2 - 5x - 16 = 0$. Roots are $x = \frac{5 \pm \sqrt{25 - 4(1)(-16)}}{2} = \frac{5 \pm \sqrt{89}}{2}$. $x_3 = \frac{5+\sqrt{89}}{2} > 0$ (Not valid). $x_4 = \frac{5-\sqrt{89}}{2} \approx \frac{5-9.43}{2} \approx -2.21$ (Valid, since it is less than -2).

The real roots are $\frac{5+\sqrt{41}}{2}$, -1, and $\frac{5-\sqrt{89}}{2}$. There are 3 real roots.
The correct option is **(3)**.

65. The least integral value of k for which the equation $x^2 - 2(k+2)x + 12 + k^2 = 0$ has two distinct real roots is

(1) 3 (2) 6
(3) 9 (4) 12

Solution:
For two distinct real roots, the discriminant D must be strictly positive ($D > 0$).

$$\begin{aligned} D = [-2(k+2)]^2 - 4(1)(12+k^2) > 0 \\ 4(k+2)^2 - 4(12+k^2) > 0 \\ (k^2 + 4k + 4) - (12 + k^2) > 0 \\ 4k - 8 > 0 \\ 4k > 8 \implies k > 2. \end{aligned}$$

The question asks for the least integral value of k. The smallest integer greater than 2 is 3.
The correct option is **(1)**.

66. Let S be the set positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0$, $\forall x \in R$. Then, the number of elements in S is:

(1) 2
(2) 0
(3) 1
(4) 5

Solution:
Let's analyze the denominator: $D(x) = x^2 - 8x + 32$. Its discriminant is $D_{den} = (-8)^2 - 4(1)(32) = 64 - 128 = -64 < 0$. Since the leading coefficient is positive and the discriminant is negative, the denominator is always positive for all real x. For the entire fraction to be negative, the numerator must be negative for all real x. Let $N(x) = ax^2 + 2(a+1)x + 9a + 4$. For $N(x) < 0$ for all x, two conditions must be met:

(a) The leading coefficient must be negative: $a < 0$.

(b) The discriminant must be negative: $D_{num} < 0$.

The question asks for positive integral values of a. The condition $a < 0$ means there are no positive values of a for which the numerator is always negative. Therefore, the set S is empty. The number of elements is 0.
The correct option is **(2)**.