Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 27th July

Syllabus: Inequality and Trigonometric Ratios & Identities

Sub: Mathematics **CT-04 JEE Main Solution** Prof. Chetan Sir

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51. Solve $\sqrt{x-1} < 4$

(1) $x < 15$ (2) $x > 5$ (3) $1 < x < 17$ (4) $1 \leq x < 17$

Solution:

For the expression $\sqrt{x-1}$ to be defined, we must have:
$x - 1 \geq 0 \implies x \geq 1. \quad \cdots (i)$

Now, we solve the inequality $\sqrt{x-1} < 4$.
Since both sides are non-negative, we can square both sides without changing the inequality sign.
$(\sqrt{x-1})^2 < 4^2$.
$x - 1 < 16$.
$x < 17. \quad \cdots (ii)$

The final solution must satisfy both conditions (i) and (ii).
Taking the intersection of $x \geq 1$ and $x < 17$, we get:
$1 \leq x < 17$.

The correct option is **(4)**.

52. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$ is

(1) 1/16 (2) $-1/16$ (3) 1 (4) 0

Solution:

Let $P = \cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}$.

Let $\theta = \dfrac{\pi}{15}$. The product is of the form $\cos\theta\cos(2\theta)\cos(4\theta)\cos(8\theta)$.

This matches the identity with $n = 4$.

$P = \dfrac{\sin(2^4\theta)}{2^4\sin\theta} = \dfrac{\sin(16\theta)}{16\sin\theta}$.

Substitute $\theta = \pi/15$:

$P = \dfrac{\sin(16\pi/15)}{16\sin(\pi/15)}$.

Using $\sin(\pi+\alpha) = -\sin\alpha$:

$\sin\left(\dfrac{16\pi}{15}\right) = \sin\left(\pi + \dfrac{\pi}{15}\right) = -\sin\left(\dfrac{\pi}{15}\right)$.

$P = \dfrac{-\sin(\pi/15)}{16\sin(\pi/15)} = -\dfrac{1}{16}$.

The correct option is **(2)**.

53. Solve $\sqrt{2x+1} \geq 3$

(1) $x \geq 4$ (2) $x \leq 4$ (3) $x = 4$ (4) $x \geq 3$

Solution:

For the expression $\sqrt{2x+1}$ to be defined, we must have:

$2x+1 \geq 0 \implies 2x \geq -1 \implies x \geq -1/2. \quad \cdots (i)$

Now, we solve the inequality $\sqrt{2x+1} \geq 3$.

Since both sides are non-negative, we can square both sides.

$2x + 1 \geq 9$.

$2x \geq 8$.

$x \geq 4. \quad \cdots (ii)$

The final solution must satisfy both conditions (i) and (ii).

The intersection of $x \geq -1/2$ and $x \geq 4$ is $x \geq 4$.

The correct option is **(1)**.

54. The maximum value of the function $y = \frac{1}{3-2\sin^2 x}$ is

(1) 1/3 (2) 1 (3) 1/5 (4) 3

Solution:

Let the denominator be $D(x) = 3 - 2\sin^2 x$.
We know the range of $\sin x$ is $[-1, 1]$.
Therefore, the range of $\sin^2 x$ is $[0, 1]$.
$0 \le \sin^2 x \le 1$.

Multiply by -2 (this reverses the inequality signs):
$0 \ge -2\sin^2 x \ge -2$.
Add 3 to all parts:
$3 + 0 \ge 3 - 2\sin^2 x \ge 3 - 2$.
$3 \ge D(x) \ge 1$.
The range of the denominator is $[1, 3]$.

The function is $y = \dfrac{1}{D(x)}$.
The maximum value of y occurs when D(x) is minimum.
$y_{max} = \dfrac{1}{D_{min}} = \dfrac{1}{1} = 1$.

The correct option is **(2)**.

55. Solve $\sqrt{x^2 - 4x + 4} \le x - 1$

(1) $x \ge 2$ (2) $x \le 2$ (3) $x = 2$ (4) $x > 2$

Solution:

$\sqrt{x^2 - 4x + 4} \le x - 1$.
$\sqrt{(x-2)^2} \le x - 1$.
$|x - 2| \le x - 1$.

Since the LHS is always non-negative, the RHS must also be non-negative.
$x - 1 \ge 0 \implies x \ge 1. \quad \cdots (i)$

For $x \ge 1,$ both sides are non-negative, so we can square them:
$|x-2|^2 \le (x-1)^2$.
$(x-2)^2 \le (x-1)^2$.
$x^2 - 4x + 4 \le x^2 - 2x + 1$.
$-4x + 4 \le -2x + 1$.
$3 \le 2x$.
$x \ge 3/2. \quad \cdots (ii)$

The correct option is **(1)**.

56. The value of $\tan 67\frac{1}{2}^\circ + \cot 67\frac{1}{2}^\circ$ is

(1) $\sqrt{2}$ (2) $3\sqrt{2}$ (3) $2\sqrt{2}$ (4) $2-\sqrt{2}$

Solution:

Assuming the question is to find $\tan 67.5^\circ + \cot 67.5^\circ$.

We use the identity $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\frac{1}{2}\sin 2\theta} = 2\operatorname{cosec} 2\theta$.

Let $\theta = 67.5^\circ$. Then $2\theta = 135^\circ$.

The expression becomes $2\operatorname{cosec}(135^\circ)$.

$\operatorname{cosec}(135^\circ) = \operatorname{cosec}(180^\circ - 45^\circ) = \operatorname{cosec}(45^\circ) = \sqrt{2}$.

So, the value is $2\sqrt{2}$.

The correct option is **(3)**.

57. Solve $3 > \sqrt{x^2-1}$

(1) $-2 < x < 2$ (2) $x < -2$ or $x > 2$ (3) $x \in (-\sqrt{10}, \sqrt{10})$ (4) $(-\sqrt{10}, -1] \cup [1, \sqrt{10})$

Solution:

The inequality is $\sqrt{x^2-1} < 3$.

Domain: The expression under the square root must be non-negative.

$x^2 - 1 \geq 0 \implies (x-1)(x+1) \geq 0$.

This holds for $x \in (-\infty, -1] \cup [1, \infty)$. $\cdots(i)$

Since both sides of $\sqrt{x^2-1} < 3$ are non-negative, we can square them:

$x^2 - 1 < 9$.

$x^2 - 10 < 0$.

$(x - \sqrt{10})(x + \sqrt{10}) < 0$.

This holds for $x \in (-\sqrt{10}, \sqrt{10})$. $\cdots(ii)$

The final solution is the intersection of (i) and (ii).

$(-\infty, -1] \cup [1, \infty) \cap (-\sqrt{10}, \sqrt{10})$.

Since $\sqrt{10} \approx 3.16$, the intersection is $(-\sqrt{10}, -1] \cup [1, \sqrt{10})$.

The correct option is **(4)**.

58. Which one of the following is possible?

(1) $\sin\ \theta = \frac{a^2+b^2}{a^2-b^2}, (a \neq b)$ (2) $sec\ \theta = \frac{4}{5}$ (3) $tan\ \theta = 45$ (4) $cos\ \theta = \frac{7}{3}$

Solution:

We check each option against the standard ranges.

(1) Range of $\sin\theta$ is $[-1,1]$. The expression $\dfrac{a^2+b^2}{a^2-b^2}$ has magnitude $\left|\dfrac{1+(b/a)^2}{1-(b/a)^2}\right| > 1$. Impossible.

(2) Range of $\sec\theta$ is $(-\infty,-1]\cup[1,\infty)$. The value $4/5 = 0.8,$ which is not in the range. Impossible.

(3) Range of $\tan\theta$ is $(-\infty,\infty)$. The value 45 is in this range. Possible.

(4) Range of $\cos\theta$ is $[-1,1]$. The value $7/3 \approx 2.33,$ which is not in the range. Impossible.

The correct option is **(3)**.

59. Solve $\frac{1}{\sqrt{x^2-15x+56}}$

(1) $(-\infty, 15)$ (2) $(-\infty,-56)$ (3) $(-\infty,7)\cup[8,\infty)$ (4) $(-\infty,7)\cup(8,\infty)$

Solution:

The question asks for the domain of the expression (the values of x for which it is defined).
For the square root to be defined, we need $x^2-15x+56 \geq 0$.
For the fraction to be defined, the denominator cannot be zero, so $\sqrt{x^2-15x+56} \neq 0$.
Combining these, we need $x^2-15x+56 > 0$.

Factor the quadratic:
$(x-7)(x-8) > 0$.

The roots are 7 and 8. The quadratic is positive outside of its roots.
$x < 7$ or $x > 8$.
The solution set is $(-\infty,7)\cup(8,\infty)$.

The correct option is **(4)**.

60. If $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ then $\cos\frac{x}{2}$ is equal to

(1) $\frac{2}{5}$ (2) $\frac{-2}{5}$ (3) $\frac{1}{\sqrt{10}}$ (4) $\frac{-1}{\sqrt{10}}$

Solution:

Given the quadrant for x: $\pi < x < \dfrac{3\pi}{2}$ (Quadrant III).

For x/2, we divide the inequality by 2: $\dfrac{\pi}{2} < \dfrac{x}{2} < \dfrac{3\pi}{4}$.

This means x/2 is in Quadrant II. In Q2, cosine is negative.

Given $\tan x = 3/4$. In Q3, both sin x and cos x are negative.

From the 3-4-5 triangle, we get $\cos x = -4/5$.

Now use the half-angle identity:

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} = \frac{1+(-4/5)}{2} = \frac{1/5}{2} = \frac{1}{10}.$$

$$\cos\left(\frac{x}{2}\right) = \pm\frac{1}{\sqrt{10}}.$$

Since x/2 is in Quadrant II, we choose the negative value.

$$\cos\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{10}}.$$

The correct option is **(4)**.

61. Solve $\sqrt{9-x^2} > 0$

(1) (0,3) (2) [-3,3] (3) (-3, 3) (4) None

Solution:

For the square root to be defined, we need $9 - x^2 \geq 0$.

For the inequality $\sqrt{9-x^2} > 0$, the value inside the root must be strictly positive.

$9 - x^2 > 0.$

$9 > x^2 \implies x^2 < 9.$

$-3 < x < 3.$

The solution set is $(-3, 3)$.

The correct option is **(3)**.

62. The value of $\cos(-1500°)$ is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 1

Solution:

The given expression is $\cos(-1500°)$.

First, we use the identity $\cos(-\theta) = \cos(\theta)$:

$\cos(-1500°) = \cos(1500°).$

$1500° = 4 \times 360° + 60°.$

This means the angle corresponds to 4 full rotations plus an additional $60°$.

$\cos(1500°) = \cos(4 \times 360° + 60°) = \cos(60°).$

The value of $\cos(60°)$ is well-known:

$$\cos(60°) = \frac{1}{2}.$$

The correct option is **(2)**.

63. Solve $x + |x| = 0$

(1) $x \in R$ (2) $[0, \infty)$ (3) $(-\infty, 0]$ (4) Φ

Solution:

We consider two cases based on the definition of absolute value.
Case 1: $x \geq 0$.
In this case, $|x| = x$.
The equation becomes $x + x = 0 \implies 2x = 0 \implies x = 0$.
This solution is consistent with our assumption $x \geq 0$.

Case 2: $x < 0$.
In this case, $|x| = -x$.
The equation becomes $x + (-x) = 0 \implies 0 = 0$.
This is always true. So, all values in the assumed domain $x < 0$ are solutions.

Combining the solutions from both cases, we have $\{x|x < 0\} \cup \{0\}$.
This is the set $(-\infty, 0]$.

The correct option is **(3)**.

64. $cos\frac{\pi}{19} + cos\frac{3\pi}{19} + cos\frac{5\pi}{19} + ... + cos\frac{17\pi}{19} =$

(1) 1 (2) $-1/2$ (3) $1/2$ (4) 0

Solution:

First term $a = \frac{\pi}{19}$, common difference $d = \frac{2\pi}{19}$, number of terms $n = 9$.

$$\text{Sum} = \frac{\sin(nd/2)}{\sin(d/2)} \cos\left(\frac{\text{first + last}}{2}\right) = \frac{\sin(9\pi/19)}{\sin(\pi/19)} \cos\left(\frac{9\pi}{19}\right).$$

$$= \frac{\frac{1}{2}\sin(18\pi/19)}{\sin(\pi/19)} = \frac{\frac{1}{2}\sin(\pi/19)}{\sin(\pi/19)} = \frac{1}{2}.$$

The correct option is **(3)**.

65. Solve $|x| = 15$

(1) ± 16 (2) ± 15 (3) ± 17 (4) ± 14

Solution:

The equation is $|x| = 15$.

By definition of absolute value, this means the distance of x from 0 is 15.
This is true for two values:
$x = 15$ or $x = -15$.
This can be written as $x = \pm 15$.

The correct option is **(2)**.

66. The range of the function $f(x) = 4\sin^2 x - 4\sin x + 5$ is

(1) [4,5] (2) [5,13] (3) [1,13] (4) [4,13]

Solution:

The function is $f(x) = 4\sin^2 x - 4\sin x + 5$.
First, we complete the square for the expression in terms of $\sin x$:
$f(x) = (4\sin^2 x - 4\sin x + 1) + 4$.
$f(x) = (2\sin x - 1)^2 + 4$.

To find the range, we start with the fundamental range of the sine function:
$-1 \leq \sin x \leq 1$.

Now, we build our expression from this inequality, step by step.
Multiply all parts by 2:
$-2 \leq 2\sin x \leq 2$.

Subtract 1 from all parts:
$-2 - 1 \leq 2\sin x - 1 \leq 2 - 1$.
$-3 \leq 2\sin x - 1 \leq 1$.

Square the expression. Since the interval $[-3, 1]$ contains zero,
the minimum value of the square will be 0.
The maximum value will be the larger of $(-3)^2$ and 1^2, which is 9.
$0 \leq (2\sin x - 1)^2 \leq 9$.

Finally, add 4 to all parts to get the full expression for f(x):
$0 + 4 \leq (2\sin x - 1)^2 + 4 \leq 9 + 4$.
$4 \leq f(x) \leq 13$.

Therefore, the range of the function is $[4, 13]$.

The correct option is **(4)**.

67. Solve $|x + 3| = 8$

(1) 5 and 6 (2) 5 and 10 (3) 5 and -10 (4) 5 and -11

Solution:

The equation $|x+3| = 8$ implies two possibilities:
Case 1: $x + 3 = 8 \implies x = 8 - 3 = 5$.
Case 2: $x + 3 = -8 \implies x = -8 - 3 = -11$.
The solutions are 5 and -11.

The correct option is **(4)**.

68. Solve $\sqrt{x^2 - 5x + 6} \geq 0$

(1) $(-\infty, 3) \cup (2, \infty)$ (2) $(-\infty, 3)$ (3) $(2, \infty)$ (4) $(-\infty, 2] \cup [3, \infty)$

Solution:

The square root of any real number, if it exists, is always non-negative (greater than or equal to 0).
So, the inequality $\sqrt{x^2 - 5x + 6} \geq 0$ is true for all values of x for which the expression is defined.
The expression is defined when the term inside the square root is non-negative:
$x^2 - 5x + 6 \geq 0$.
Factor the quadratic:
$(x-2)(x-3) \geq 0$.
The roots are 2 and 3. The quadratic is non-negative at or outside of its roots.
The solution is $x \leq 2$ or $x \geq 3$.
In interval notation, this is $(-\infty, 2] \cup [3, \infty)$.

The correct option is **(4)**.

69. $(4\cos^2 9° - 3)(4\cos^2 27° - 3) =$

(1) $\sin 9°$ (2) $\cos 9°$ (3) $\tan 9°$ (4) $\cot 9°$

Solution:

We use the triple angle identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

This implies $\dfrac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3$, for $\cos\theta \neq 0$.

Let the expression be E.

$$E = \left(4\cos^2 9° - 3\right)\left(4\cos^2 27° - 3\right).$$

$$E = \left(\frac{\cos(3 \times 9°)}{\cos 9°}\right)\left(\frac{\cos(3 \times 27°)}{\cos 27°}\right).$$

$$E = \frac{\cos 27°}{\cos 9°} \cdot \frac{\cos 81°}{\cos 27°}.$$

$$E = \frac{\cos 81°}{\cos 9°}.$$

Using $\cos 81° = \cos(90° - 9°) = \sin 9°$.

$$E = \frac{\sin 9°}{\cos 9°} = \tan 9°.$$

The correct option is **(3)**.

70. Solution of the inequality $\sqrt{x^2 - 2x} > 1 - x$

(1) $[2, \infty)$ (2) $(-\infty, -2]$ (3) $(2, \infty)$ (4) $(-\infty, -2)$

Solution:

We are solving the inequality $\sqrt{x^2 - 2x} > 1 - x$.

First, we determine the domain where the square root is defined:
$x^2 - 2x \geq 0 \implies x(x-2) \geq 0$.
This holds for $x \in (-\infty, 0] \cup [2, \infty)$. (Domain)

We now consider two cases based on the sign of the right-hand side.

Case 1: $1 - x < 0 \implies x > 1$.
In this case, the LHS is non-negative and the RHS is negative.
The inequality (non-negative) ¿ (negative) is always true.
The solution for this case is the intersection of its condition ($x > 1$) and the domain.
Solution 1 = $((-\infty, 0] \cup [2, \infty)) \cap (1, \infty) = [2, \infty)$.

Case 2: $1 - x \geq 0 \implies x \leq 1$.
In this case, both sides are non-negative, so we can square them:
$(\sqrt{x^2 - 2x})^2 > (1 - x)^2$.
$x^2 - 2x > 1 - 2x + x^2$.
$0 > 1$.
This is a false statement, so there are no solutions in this case.

The overall solution is the union of the solutions from all cases.
Final Solution = $[2, \infty) \cup \emptyset = [2, \infty)$.

The correct option is **(1)**.

SECTION-B

71. If $\sin\ \theta = -\frac{24}{25}$ and θ is in the 4th quadrant, then $7\tan\ \theta + 100\cos\ \theta$ is equal to

Solution:

Given $\sin\theta = -24/25$ and θ is in Q4.

In Q4, $\cos\theta > 0$ and $\tan\theta < 0$.
From the Pythagorean identity:

$\cos^2\theta = 1 - \sin^2\theta = 1 - (-24/25)^2 = 1 - 576/625 = \dfrac{625 - 576}{625} = \dfrac{49}{625}$.

$\cos\theta = \sqrt{49/625} = 7/25$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-24/25}{7/25} = -24/7$.

Now, evaluate the expression:
$7\tan\theta + 100\cos\theta = 7(-24/7) + 100(7/25)$.
$= -24 + 4 \times 7 = -24 + 28 = 4$.

The answer is **4**.

72. Number of integral solutions of the inequality $-5 > \sqrt{x^2 - 5x + 6}$

Solution:

The given inequality is $\sqrt{x^2 - 5x + 6} < -5$.
The expression on the left-hand side, $\sqrt{x^2 - 5x + 6}$, represents the principal square root.
By definition, the principal square root of a real number is always non-negative.
So, LHS ≥ 0.
The right-hand side is -5, which is a negative number.
The inequality is of the form (non-negative number) ¡ (negative number), which is impossible.
There are no real values of x that can satisfy this inequality.
Therefore, the number of integral solutions is 0.

The answer is **0**.

73. $cos\frac{\pi}{12} + cos\frac{5\pi}{12} + cos\frac{7\pi}{12} + cos\frac{11\pi}{12}$ is

Solution:

Let S be the sum.
$S = \cos\dfrac{\pi}{12} + \cos\dfrac{5\pi}{12} + \cos\dfrac{7\pi}{12} + \cos\dfrac{11\pi}{12}$.
We use the identity $\cos(\pi - \theta) = -\cos\theta$.
$\cos\dfrac{11\pi}{12} = \cos\left(\pi - \dfrac{\pi}{12}\right) = -\cos\dfrac{\pi}{12}$.
$\cos\dfrac{7\pi}{12} = \cos\left(\pi - \dfrac{5\pi}{12}\right) = -\cos\dfrac{5\pi}{12}$.

Substitute these into the sum:

$S = \cos\dfrac{\pi}{12} + \cos\dfrac{5\pi}{12} - \cos\dfrac{5\pi}{12} - \cos\dfrac{\pi}{12}$.

$$S = 0.$$

The answer is **0**.

74. Number of integral solutions of the inequality $x^2 + 3|x| + 2 = 0$

Solution:

The given equation is $x^2 + 3|x| + 2 = 0$.
We know that for any real number x, $x^2 \geq 0$ and $|x| \geq 0$.
Therefore, the terms on the left-hand side satisfy:
$x^2 \geq 0$.
$3|x| \geq 0$.
$2 > 0$.
The sum of these terms is $x^2 + 3|x| + 2 \geq 0 + 0 + 2 = 2$.
The expression on the left-hand side is always greater than or equal to 2.
It can never be equal to 0.
Therefore, there are no real solutions, and thus no integral solutions.

The answer is **0**.

75. The maximum value of $12 \sin x - 5 \cos x + 3$ is

Solution:

Let the function be $f(x) = 12 \sin x - 5 \cos x + 3$.
First, we find the range of the part $12 \sin x - 5 \cos x$.
This is of the form $a \sin x + b \cos x$ with $a = 12, b = -5$.
The range of this part is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$.
$\sqrt{a^2 + b^2} = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$.
So, the range of $12 \sin x - 5 \cos x$ is $[-13, 13]$.

To find the range of the full function, we add 3.
$-13 + 3 \leq 12 \sin x - 5 \cos x + 3 \leq 13 + 3$.
$-10 \leq f(x) \leq 16$.
The maximum value of the function is 16.

The answer is **16**.