Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 27th July (Star Batch)

Syllabus: Inequality and Trigonometric Ratios & Identities

Sub: Mathematics **CT-04 JEE Main Solution** Prof. Chetan Sir

Index of Questions

51. Solution of the inequality $\sqrt{\frac{-2025}{-x^2+2x-10}} \geq 0$

(1) ϕ (2) $(-\infty, 0]$ (3) $[0, \infty)$ (4) $(-\infty, \infty)$

Solution:

The square root of an expression is defined only if the expression inside the root is non-negative.

Let's analyze the expression inside the root: $\dfrac{-2025}{-x^2+2x-10}$.

Numerator is -2025, which is negative.

Denominator is $-x^2+2x-10 = -(x^2-2x+10) = -((x-1)^2+9)$.

Since $(x-1)^2 \geq 0$, it follows that $(x-1)^2+9 \geq 9$.

Therefore, the denominator $-((x-1)^2+9)$ is always negative (specifically, ≤ -9).

The expression inside the root is $\dfrac{\text{negative}}{\text{negative}}$, which is always positive.

A positive number is always greater than or equal to 0.

Thus, the inequality holds true for all real numbers.

$x \in (-\infty, \infty)$.

The correct option is **(4)**.

52. If $\theta \in (\frac{\pi}{2}, \frac{3\pi}{2})$ then the value of $\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2(\frac{\pi}{4} - \frac{\theta}{2})$ is

(1) $-2\cot\theta$ (2) $2\cot\theta$ (3) $2\cos\theta$ (4) $2\sin\theta$

Solution:

Let's simplify the first term:

$$\sqrt{4\cos^4\theta + \sin^2 2\theta}$$
$$= \sqrt{4\cos^4\theta + (2\sin\theta\cos\theta)^2}$$
$$= \sqrt{4\cos^4\theta + 4\sin^2\theta\cos^2\theta}$$
$$= \sqrt{4\cos^2\theta(\cos^2\theta + \sin^2\theta)} = \sqrt{4\cos^2\theta} = 2|\cos\theta|.$$

Given $\theta \in (\pi/2, 3\pi/2)$, which covers Q2 and Q3. In both quadrants, $\cos\theta$ is negative. Therefore, $2|\cos\theta| = -2\cos\theta$.

Now, simplify the second term: $4\cot\theta\cos^2(\frac{\pi}{4} - \frac{\theta}{2})$.

Using $2\cos^2 A = 1 + \cos 2A$:

$$= 4\cot\theta \cdot \frac{1}{2}\left(1 + \cos\left(2(\frac{\pi}{4} - \frac{\theta}{2})\right)\right)$$
$$= 2\cot\theta(1 + \cos(\frac{\pi}{2} - \theta)).$$
$$= 2\cot\theta(1 + \sin\theta)$$
$$= 2\frac{\cos\theta}{\sin\theta}(1 + \sin\theta) = 2\cos\theta\left(\frac{1}{\sin\theta} + 1\right) = 2\cos\theta\csc\theta + 2\cos\theta.$$

Combine the simplified terms:

$$\text{Expression} = -2\cos\theta + (2\cos\theta\csc\theta + 2\cos\theta) = 2\cos\theta\csc\theta = 2\cos\theta\frac{1}{\sin\theta} = 2\cot\theta.$$

The correct option is **(2)**.

53. Solution of the inequality $\sqrt{x+2} > \sqrt{x-2}$ is

(1) $(2, \infty)$ (2) $[2, \infty)$ (3) $(-\infty, \infty)$ (4) $(-\infty, 2]$

Solution:

First, we must determine the domain for which both square roots are defined.

For $\sqrt{x+2} : x + 2 \geq 0 \implies x \geq -2.$

For $\sqrt{x-2} : x - 2 \geq 0 \implies x \geq 2.$

The intersection of these two domains is $x \geq 2$.

For $x \geq 2$, both sides of the inequality are defined and non-negative.

We can square both sides without changing the inequality sign.

$(\sqrt{x+2})^2 > (\sqrt{x-2})^2.$

$x + 2 > x - 2.$
$2 > -2.$
This statement is always true.
Therefore, the inequality is true for all x in the established domain.
The solution is $x \geq 2,$ or $[2, \infty).$

The correct option is **(2)**.

54. $\cot 12^\circ \cot 102^\circ + \cot 102^\circ \cot 66^\circ + \cot 66^\circ \cot 12^\circ$ is

(1) -2 (2) 1 (3) -1 (4) 2

Solution:

Let the expression be E.
$E = \cot 12^\circ \cot 102^\circ + \cot 102^\circ \cot 66^\circ + \cot 66^\circ \cot 12^\circ.$

$\cot 102^\circ = \cot(90^\circ + 12^\circ) = -\tan 12^\circ.$

$E = \cot 12^\circ(-\tan 12^\circ) + (-\tan 12^\circ)\cot 66^\circ + \cot 66^\circ \cot 12^\circ.$

Since $\cot\theta \cdot \tan\theta = 1,$ the first term is -1. Now, we factor out $\cot 66^\circ$ from the other two terms:
$E = -1 - \tan 12^\circ \cot 66^\circ + \cot 66^\circ \cot 12^\circ.$
$E = -1 + \cot 66^\circ(\cot 12^\circ - \tan 12^\circ).$

Next, we use the identity $\cot\theta - \tan\theta = 2\cot(2\theta)$:
$\cot 12^\circ - \tan 12^\circ = 2\cot(2 \times 12^\circ) = 2\cot 24^\circ.$

Substitute this result back into the expression for E:
$E = -1 + \cot 66^\circ(2\cot 24^\circ).$

$\cot 24^\circ = \tan(90^\circ - 24^\circ) = \tan 66^\circ.$

$E = -1 + 2\cot 66^\circ \tan 66^\circ.$
$E = -1 + 2(1) = 1.$

The correct option is **(2)**.

55. Solution of the inequality $\sqrt{5 - x^2} > x - 1$

(1) $[-\sqrt{5}, \sqrt{5}]$ (2) $[-\sqrt{5}, 2]$ (3) $[-\sqrt{5}, 2)$ (4) None.

Solution:

First, the domain: $5 - x^2 \geq 0 \implies x^2 \leq 5 \implies -\sqrt{5} \leq x \leq \sqrt{5}.$

Case 1: $x - 1 < 0 \implies x < 1$.
The RHS is negative, while the LHS is non-negative. The inequality is always true in this case.
The solution for this case is the intersection of the domain and $x < 1$.
$[-\sqrt{5}, \sqrt{5}] \cap (-\infty, 1) = [-\sqrt{5}, 1)$.

Case 2: $x - 1 \geq 0 \implies x \geq 1$.
Both sides are non-negative, so we can square them.
$5 - x^2 > (x-1)^2 \implies 5 - x^2 > x^2 - 2x + 1$.
$0 > 2x^2 - 2x - 4 \implies 0 > x^2 - x - 2$.
$x^2 - x - 2 < 0 \implies (x-2)(x+1) < 0$.
This is true for $-1 < x < 2$.
The solution for this case is the intersection of the domain, the case condition, and the result.
$[-\sqrt{5}, \sqrt{5}] \cap [1, \infty) \cap (-1, 2) = [1, 2)$.

The final solution is the union of the results from both cases.
$[-\sqrt{5}, 1) \cup [1, 2) = [-\sqrt{5}, 2)$.

The correct option is **(3)**.

56. The value of $16 \sin(\frac{\pi}{18}) \sin(\frac{5\pi}{18}) \sin(\frac{7\pi}{18})$ is equal to

(1) 1 (2) 2 (3) 4 (4) 8

Solution:

Let the expression be E.

$$E = 16 \sin\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right).$$

First, we convert the angles from radians to degrees for clarity:

$$\frac{\pi}{18} = 10^\circ, \quad \frac{5\pi}{18} = 50^\circ, \quad \frac{7\pi}{18} = 70^\circ.$$

$E = 16 \sin(10^\circ) \sin(50^\circ) \sin(70^\circ)$.

We use the product identity $\sin\theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4}\sin(3\theta)$.

Applying the identity, the product of the sine terms is:

$$\sin(10^\circ) \sin(50^\circ) \sin(70^\circ) = \frac{1}{4}\sin(3 \times 10^\circ) = \frac{1}{4}\sin(30^\circ).$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}.$$

Now, substitute this value back into the expression for E:

$$E = 16 \times \frac{1}{8} = 2.$$

The correct option is **(2)**.

57. Solution of the inequality $\sqrt{3x - x^2} < 4 - x$

(1) (-3, 3) (2) [-3,3] (3) (0,3) (4) [0,3]

Solution:

Domain: $3x - x^2 \geq 0 \implies x(3 - x) \geq 0 \implies 0 \leq x \leq 3$.
The RHS is $4 - x$. For the domain $[0, 3], 4 - x$ is always positive.
Since both sides are non-negative in the domain, we can square them:
$3x - x^2 < (4 - x)^2 \implies 3x - x^2 < 16 - 8x + x^2$.
$0 < 2x^2 - 11x + 16$.
Check the discriminant of $2x^2 - 11x + 16 : D = b^2 - 4ac = (-11)^2 - 4(2)(16) = 121 - 128 = -7$.
Since the leading coefficient (2) is positive and the discriminant is negative, the quadratic $2x^2 - 11x + 1($
The inequality $0 < 2x^2 - 11x + 16$ is true for all real x.
The final solution is the intersection of the domain and all real x, which is just the domain.
Solution is $[0, 3]$.

The correct option is **(4)**.

58. The value of $5 \cos \theta + 3 \cos(\theta + \frac{\pi}{3}) + 3$ lies between

(1) -2 and 5 (2) -1 and 8 (3) -3 and 6 (4) -4 and 10

Solution:

Let $E = 5 \cos \theta + 3 \cos(\theta + \dfrac{\pi}{3}) + 3$.

Expand $\cos(\theta + \pi/3) = \cos \theta \cos(\pi/3) - \sin \theta \sin(\pi/3) = \dfrac{1}{2} \cos \theta - \dfrac{\sqrt{3}}{2} \sin \theta$.

$E = 5 \cos \theta + 3(\dfrac{1}{2} \cos \theta - \dfrac{\sqrt{3}}{2} \sin \theta) + 3$.

$E = 5 \cos \theta + \dfrac{3}{2} \cos \theta - \dfrac{3\sqrt{3}}{2} \sin \theta + 3 = \dfrac{13}{2} \cos \theta - \dfrac{3\sqrt{3}}{2} \sin \theta + 3$.

The range of $a \cos \theta + b \sin \theta$ is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$.

Here, $a = 13/2, b = -3\sqrt{3}/2$.

$\sqrt{a^2 + b^2} = \sqrt{(\dfrac{13}{2})^2 + (-\dfrac{3\sqrt{3}}{2})^2} = \sqrt{\dfrac{169}{4} + \dfrac{27}{4}} = \sqrt{\dfrac{196}{4}} = \sqrt{49} = 7$.

The range of $\dfrac{13}{2} \cos \theta - \dfrac{3\sqrt{3}}{2} \sin \theta$ is $[-7, 7]$.

The range of E is $[-7 + 3, 7 + 3] = [-4, 10]$.

The correct option is **(4)**.

59. Solution of the inequality $\sqrt{2 - x} > x$

(1) $(-\infty, 1)$ (2) $(-\infty, 1]$ (3) $[1, \infty)$ (4) $[-1, \infty)$

Solution:

Domain: $2 - x \geq 0 \implies x \leq 2$.
Case 1: $x < 0$. LHS is non-negative, RHS is negative. Inequality holds.
Solution 1 is the intersection of the domain and the case: $(-\infty, 0)$.
Case 2: $x \geq 0$. Both sides are non-negative. We can square.
$2 - x > x^2 \implies x^2 + x - 2 < 0 \implies (x+2)(x-1) < 0$.
This holds for $-2 < x < 1$.
Solution 2 is the intersection of domain, case, and result: $[0, 2] \cap [0, \infty) \cap (-2, 1) = [0, 1)$.
Final solution is the union of both cases: $(-\infty, 0) \cup [0, 1) = (-\infty, 1)$.

The correct option is **(1)**.

60. $cos\frac{\pi}{19} + cos\frac{3\pi}{19} + \cdots + cos\frac{17\pi}{19} =$

(1) 1 (2) $-1/2$ (3) $1/2$ (4) 0

Solution:

$$\text{First term } a = \frac{\pi}{19}, \text{ common difference } d = \frac{2\pi}{19}, \text{ number of terms } n = 9.$$
$$\text{Sum} = \frac{\sin(nd/2)}{\sin(d/2)} \cos\left(\frac{\text{first} + \text{last}}{2}\right) = \frac{\sin(9\pi/19)}{\sin(\pi/19)} \cos\left(\frac{9\pi}{19}\right).$$
$$= \frac{\frac{1}{2}\sin(18\pi/19)}{\sin(\pi/19)} = \frac{\frac{1}{2}\sin(\pi/19)}{\sin(\pi/19)} = \frac{1}{2}.$$

The correct option is **(3)**.

61. Solution of the inequality $\sqrt{x^2 - 20} \leq x - 5$

(1) $(-\infty, \infty)$ (2) $[0, \infty)$ (3) ϕ (4) $(-\infty, 0]$

Solution:

Domain: $x^2 - 20 \geq 0 \implies x^2 \geq 20 \implies x \in (-\infty, -2\sqrt{5}] \cup [2\sqrt{5}, \infty)$.
Condition for RHS: Since LHS is non-negative, we must have $x - 5 \geq 0 \implies x \geq 5$.
The intersection of the domain and this condition is $x \geq 5$ (since $2\sqrt{5} \approx 4.47$).
For $x \geq 5$, both sides are non-negative, so we can square them:
$x^2 - 20 \leq (x-5)^2 \implies x^2 - 20 \leq x^2 - 10x + 25$.
$-20 \leq -10x + 25 \implies 10x \leq 45 \implies x \leq 4.5$.
We need to find the intersection of $x \geq 5$ and $x \leq 4.5$.
This intersection is empty. There is no solution.

The correct option is **(3)**.

62. *cos* 12° · *cos* 24° · *cos* 36° · *cos* 48° · *cos* 72° · *cos* 84° is

(1) 1/32 (2) 1/16 (3) 1/64 (4) 1/128

Solution:

Let the product be P.
$P = \cos 12° \cdot \cos 24° \cdot \cos 36° \cdot \cos 48° \cdot \cos 72° \cdot \cos 84°$.

We will use the identity $\cos\theta\cos(60° - \theta)\cos(60° + \theta) = \frac{1}{4}\cos(3\theta)$.

Let's group the terms into triplets that match this identity's form:
$P = (\cos 12° \cdot \cos 48° \cdot \cos 72°) \cdot (\cos 24° \cdot \cos 36° \cdot \cos 84°)$.

For the first triplet, let $\theta = 12°$:
$\cos 12° \cdot \cos(60° - 12°) \cdot \cos(60° + 12°) = \cos 12° \cos 48° \cos 72°$.
Applying the identity, this product is $\frac{1}{4}\cos(3 \times 12°) = \frac{1}{4}\cos 36°$.

For the second triplet, let $\theta = 24°$:
$\cos 24° \cdot \cos(60° - 24°) \cdot \cos(60° + 24°) = \cos 24° \cos 36° \cos 84°$.
Applying the identity, this product is $\frac{1}{4}\cos(3 \times 24°) = \frac{1}{4}\cos 72°$.

Now, substitute these results back into the expression for P:
$P = \left(\frac{1}{4}\cos 36°\right) \cdot \left(\frac{1}{4}\cos 72°\right) = \frac{1}{16}\cos 36° \cos 72°$.

Using the standard values $\cos 36° = \frac{\sqrt{5}+1}{4}$ and $\cos 72° = \sin 18° = \frac{\sqrt{5}-1}{4}$:
$P = \frac{1}{16}\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)$.
$P = \frac{1}{16}\left(\frac{5-1}{16}\right) = \frac{1}{16} \cdot \frac{4}{16} = \frac{1}{64}$.

The correct option is **(3)**.

63. Solution of the inequality $\sqrt{\frac{x-2}{x-5}} > -4$

(1) $(-\infty, 2] \cup [5, \infty)$ (2) $(-\infty, 2) \cup (5, \infty)$ (3) $(-\infty, 2] \cup (5, \infty)$ (4) None of these

Solution:

The LHS, being a square root, is always non-negative where it is defined.
The RHS is -4, a negative number.
The inequality (non-negative) ¿ (negative) is always true.
So, the solution is simply the domain of the expression.

For the square root to be defined, $\frac{x-2}{x-5} \geq 0$.
The critical points are x=2 and x=5. The expression is positive outside the roots.
$x \leq 2$ or $x > 5$. (x cannot be 5 as it makes the denominator zero).
The solution is $(-\infty, 2] \cup (5, \infty)$.

The correct option is **(3)**.

64. The value of $\sin 36° \sin 72° \sin 108° \sin 144°$ is

(1) 1/4 (2) 1/16 (3) 3/4 (4) 5/16

Solution:

Let P be the expression. First, we use the identity $\sin(180° - \theta) = \sin\theta$.
$\sin 144° = \sin(180° - 36°) = \sin 36°$.
$\sin 108° = \sin(180° - 72°) = \sin 72°$.

The expression becomes:
$P = \sin 36° \sin 72° \sin 72° \sin 36°$
$= (\sin 36° \sin 72°)^2$.
$= \left(\frac{2\sin 36° \sin 72°}{2}\right)^2$
$= \left(\frac{1}{2}(\cos(72° - 36°) - \cos(72° + 36°))\right)^2$
$= \left(\frac{1}{2}(\cos 36° - \cos 108°)\right)^2$
$= \left(\frac{1}{2}(\cos 36° - (-\sin 18°))\right)^2$
$= \left(\frac{\cos 36° + \sin 18°}{2}\right)^2$

Substitute the standard values $\cos 36° = \frac{\sqrt{5}+1}{4}$ and $\sin 18° = \frac{\sqrt{5}-1}{4}$:
$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4} + \frac{\sqrt{5}-1}{4}\right) = \frac{1}{2}\left(\frac{2\sqrt{5}}{4}\right) = \frac{\sqrt{5}}{4}$.

Finally, we square this result to find P:
$P = \left(\frac{\sqrt{5}}{4}\right)^2 = \frac{5}{16}$.

The correct option is **(4)**.

65. Solution of the inequality $\sqrt{x^2 - 5x + 4} < 2$

(1) $(-\infty, 1] \cup [4, \infty)$ (2) $[0, 1] \cup [4, 5)$ (3) $(0, 1] \cup [4, 5)$ (4) $(0, 1] \cup [4, 5]$

Solution:

Domain: $x^2 - 5x + 4 \ge 0 \implies (x-1)(x-4) \ge 0 \implies x \in (-\infty, 1] \cup [4, \infty)$.
Since both sides are non-negative, we can square them:
$x^2 - 5x + 4 < 4 \implies x^2 - 5x < 0 \implies x(x-5) < 0$.
This holds for $0 < x < 5$.
The final solution is the intersection of the domain and this result:
$((-\infty, 1] \cup [4, \infty)) \cap (0, 5) = (0, 1] \cup [4, 5)$.

The correct option is **(3)**.

66. The maximum and minimum values of $\cos^6\theta + \sin^6\theta$ are m and n respectively then the value of 3m+4n is (Typo in original question)

(1) 2 (2) 1 (3) 4 (4) 3

Solution:

$$E = \cos^6\theta + \sin^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - \frac{3}{4}\sin^2(2\theta).$$

The range of $\sin^2(2\theta)$ is $[0, 1]$.
Max value (m) occurs when $\sin^2(2\theta)$ is min (0): $m = 1 - 0 = 1$.
Min value (n) occurs when $\sin^2(2\theta)$ is max (1): $n = 1 - 3/4 = 1/4$.
The expression to find is likely $3m + 4n$.
$3(1) + 4(1/4) = 3 + 1 = 4$.

The correct option is **(3)**.

67. Solution of the equality $x^2 + 3|x| + 2 = 0$

(1) 1,2 (2) -1,-2 (3) $\pm 1, \pm 2$ (4) Φ

Solution:

Let $y = |x|$. Then $x^2 = |x|^2 = y^2$. We know $y \ge 0$.
The equation becomes $y^2 + 3y + 2 = 0$.
$(y+1)(y+2) = 0$.
$y = -1$ or $y = -2$.
This means $|x| = -1$ or $|x| = -2$.
The absolute value of a real number cannot be negative. Therefore, there is no real solution.

The correct option is **(4)**.

68. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$ is

(1) $\frac{1}{16}$ (2) $-\frac{1}{16}$ (3) 1 (4) 0

Solution:

Let $P = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$.

Let $\theta = \frac{\pi}{15}$. The product is $\cos\theta\cos(2\theta)\cos(4\theta)\cos(8\theta)$.

$P = \frac{\sin(2^4\theta)}{2^4\sin\theta} = \frac{\sin(16\pi/15)}{16\sin(\pi/15)} = \frac{-\sin(\pi/15)}{16\sin(\pi/15)} = -\frac{1}{16}$.

The correct option is **(2)**.

69. Solution of the equality $|x-1|+|x-2|+|x+3|=0$

(1) ϕ (2) 1, 2, 3 (3) 2,3 (4) None of these

Solution:

The absolute value of any real number is non-negative.

$|x-1| \geq 0$.

$|x-2| \geq 0$.

$|x+3| \geq 0$.

The sum of three non-negative numbers can be zero only if each term is individually zero.

$|x-1| = 0 \implies x = 1$.

$|x-2| = 0 \implies x = 2$.

$|x+3| = 0 \implies x = -3$.

There is no value of x that can satisfy all three conditions simultaneously.

Therefore, there is no solution. The solution set is the empty set, ϕ.

The correct option is **(1)**.

70. The value of $8\sin(\frac{\pi}{14})\sin(\frac{3\pi}{14})\sin(\frac{5\pi}{14}) =$

(1) 4 (2) 2 (3) 8 (4) 1

Solution:

Let the expression be E.

$E = 8\sin\left(\frac{\pi}{14}\right)\sin\left(\frac{3\pi}{14}\right)\sin\left(\frac{5\pi}{14}\right)$.

First, we convert the sine terms to cosine using the identity $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$:

$\sin\left(\frac{5\pi}{14}\right) = \cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right) = \cos\left(\frac{2\pi}{14}\right) = \cos\left(\frac{\pi}{7}\right)$.

$\sin\left(\frac{3\pi}{14}\right) = \cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right) = \cos\left(\frac{4\pi}{14}\right) = \cos\left(\frac{2\pi}{7}\right)$.

$\sin\left(\frac{\pi}{14}\right) = \cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right) = \cos\left(\frac{6\pi}{14}\right) = \cos\left(\frac{3\pi}{7}\right)$.

Substituting these back into the expression for E:

$$E = 8\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right).$$

Next, we use the identity $\cos\theta = -\cos(\pi - \theta)$

$$\cos\left(\frac{3\pi}{7}\right) = -\cos\left(\pi - \frac{3\pi}{7}\right) = -\cos\left(\frac{4\pi}{7}\right).$$

Substitute this into the expression for E:

$$E = 8\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\left[-\cos\left(\frac{4\pi}{7}\right)\right].$$

$$E = -8\left[\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\right].$$

The product in the bracket is of the form $\cos\theta\cos(2\theta)\cos(4\theta)$, which equals $\frac{\sin(8\theta)}{8\sin\theta}$.

Let $\theta = \pi/7$:

$$\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right) = \frac{\sin(8\pi/7)}{8\sin(\pi/7)} = \frac{\sin(\pi + \pi/7)}{8\sin(\pi/7)} = \frac{-\sin(\pi/7)}{8\sin(\pi/7)} = -\frac{1}{8}.$$

Finally, substitute this value back into the expression for E:

$$E = -8\left(-\frac{1}{8}\right) = 1.$$

The correct option is **(4)**.

SECTION-B

71. Sum of the solution of the equality is k then $|k|$? $|x| - |x-1| + |x+1| = 6$

Solution:

We solve by considering intervals based on the critical points $-1, 0, 1$.

Case 1: $x \geq 1$. $x - (x-1) + (x+1) = 6 \implies x + 2 = 6 \implies x = 4.$ (Valid)

Case 2: $0 \leq x < 1$. $x - (-(x-1)) + (x+1) = 6 \implies x + x - 1 + x + 1 = 6 \implies 3x = 6 \implies x = 2.$ (

Case 3: $-1 \leq x < 0$. $-x - (-(x-1)) + (x+1) = 6 \implies -x + x - 1 + x + 1 = 6 \implies x = 6.$ (Not i

Case 4: $x < -1$. $-x - (-(x-1)) - (x+1) = 6 \implies -x + x - 1 - x - 1 = 6 \implies -x - 2 = 6 \implies$

The solutions are 4 and -8. The sum is $k = 4 + (-8) = -4$.

$|k| = |-4| = 4$.

The answer is **4**.

72. $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ is equal to

Solution:

$$\text{We know } \tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$
$$\text{Let } \theta = 10^\circ, t = \tan 10^\circ.$$
$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{3t - t^3}{1 - 3t^2}.$$
$$(1 - 3t^2)^2 = (\sqrt{3}(3t - t^3))^2 \implies 1 - 6t^2 + 9t^4 = 3(9t^2 - 6t^4 + t^6).$$
$$1 - 6t^2 + 9t^4 = 27t^2 - 18t^4 + 3t^6.$$
$$3t^6 - 27t^4 + 33t^2 - 1 = 0.$$
$$3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1.$$

The answer is **1**.

73. Sum of the Solution of the equality $|x^2 - 4x + 3| + 3 - x = 0$

Solution:

$|x^2 - 4x + 3| = x - 3.$
LHS is non-negative, so RHS must be non-negative: $x - 3 \geq 0 \implies x \geq 3.$
For $x \geq 3,$ the expression $x^2 - 4x + 3 = (x - 1)(x - 3)$ is non-negative.
So, $|x^2 - 4x + 3| = x^2 - 4x + 3.$
The equation becomes $x^2 - 4x + 3 = x - 3 \implies x^2 - 5x + 6 = 0 \implies (x - 2)(x - 3) = 0.$
The solutions are $x = 2$ and $x = 3.$
We must check these against the condition $x \geq 3$. Only $x = 3$ is a valid solution.
The sum of solutions is just 3.

The answer is **3**.

74. If $k = \cos\frac{\pi}{7}\cos\frac{\pi}{5}cos\frac{2\pi}{7}cos\frac{2\pi}{5}cos\frac{4\pi}{7} =$ is equal to then $32k + 4$

Solution:

$$k = \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cdot\cos\frac{\pi}{5}\cos\frac{2\pi}{5}.$$
$$\text{Part 1: } \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = \frac{\sin(8\pi/7)}{8\sin(\pi/7)} = \frac{-\sin(\pi/7)}{8\sin(\pi/7)} = -1/8.$$
$$\text{Part 2: } \cos\frac{\pi}{5}\cos\frac{2\pi}{5} = \cos 36^\circ\cos 72^\circ = \frac{\sqrt{5}+1}{4}\frac{\sqrt{5}-1}{4} = \frac{4}{16} = 1/4.$$
$$k = (-1/8)(1/4) = -1/32.$$
$$32k + 4 = 32(-1/32) + 4 = -1 + 4 = 3.$$

The answer is **3**.

75. Number of solution of the equality $|x| + x^3 = 0$

Solution:

Case 1: $x \geq 0$. The equation is $x + x^3 = 0 \implies x(1 + x^2) = 0$.
Since $1 + x^2$ is always positive, the only solution is $x = 0$. This is in the case's domain.
Case 2: $x < 0$. The equation is $-x + x^3 = 0 \implies x(x^2 - 1) = 0 \implies x(x-1)(x+1) = 0$.
The potential solutions are $x = 0, x = 1, x = -1$.
Only $x = -1$ is in the case's domain ($x < 0$).
The solutions are $\{0, -1\}$. There are 2 solutions.

The answer is **2**.