Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

JEE Repeater CT-05: 27 July 2025

Topic: Sequence and Series & Trigonometry

Sub: Mathematics **JEE Main CT-05** Prof. Chetan Sir

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51. Let $a_1, a_2, \ldots$ be a GP of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its 3^{rd} and 5^{th} terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(1) $2\sqrt{2}$ (2) 2 (3) $3\sqrt{3}$ (4) 3

Solution:

Let the GP have first term 'a' and common ratio 'r'. Since it is increasing and positive, $a > 0, r > 1$.

Given: $a_6 + a_8 = ar^5 + ar^7 = ar^5(1 + r^2) = 2 \quad \cdots(1)$.

Given: $a_3 \cdot a_5 = (ar^2)(ar^4) = a^2r^6 = \dfrac{1}{9}$.

From the second condition, $(ar^3)^2 = \dfrac{1}{9} \implies a_4 = ar^3 = \dfrac{1}{3} \quad \cdots(2)$.

From (2), $a = \dfrac{1}{3r^3}$. Substitute this into (1):

$$\left(\frac{1}{3r^3}\right) r^5(1 + r^2) = 2 \implies \frac{r^2(1 + r^2)}{3} = 2 \implies r^4 + r^2 = 6.$$

$r^4 + r^2 - 6 = 0 \implies (r^2 + 3)(r^2 - 2) = 0$.

Since r^2 must be positive, $r^2 = 2 \implies r = \sqrt{2}$ (as r greater than 1).

$$a = \frac{1}{3(\sqrt{2})^3} = \frac{1}{6\sqrt{2}}.$$

Now, evaluate the required expression:
$E = 6(a_2 + a_4)(a_4 + a_6) = 6(ar + ar^3)(ar^3 + ar^5) = 6[ar(1 + r^2)][ar^3(1 + r^2)]$.
$E = 6a^2r^4(1 + r^2)^2 = 6a^2(r^2)^2(1 + r^2)^2$.
$$E = 6\left(\frac{1}{6\sqrt{2}}\right)^2 (2)^2(1+2)^2 = 6\left(\frac{1}{72}\right)(4)(9) = \frac{6 \times 36}{72} = 3.$$

The correct option is **(4)**.

52. If $S_n = 4 + 11 + 21 + 34 + 50 + ...$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to

(1) 227 (2) 226 (3) 220 (4) 223

Solution:

Let the nth term be T_n. The differences between terms are:
$11 - 4 = 7, \quad 21 - 11 = 10, \quad 34 - 21 = 13, \quad 50 - 34 = 16, \ldots$
The first differences are in an AP with first term 7 and common difference 3.
So, T_n is a quadratic of the form $an^2 + bn + c$.
$$T_n = T_1 + \sum_{k=1}^{n-1}(7 + (k-1)3) = 4 + \frac{n-1}{2}[14 + (n-2)3] = 4 + \frac{n-1}{2}[3n + 8].$$
$$2T_n = 8 + (n-1)(3n+8) = 8 + 3n^2 + 5n - 8 = 3n^2 + 5n \implies T_n = \frac{3n^2 + 5n}{2}.$$

$$S_{29} - S_9 = \sum_{n=10}^{29} T_n = \sum_{n=10}^{29} \frac{3n^2 + 5n}{2} = \frac{3}{2}\sum_{10}^{29} n^2 + \frac{5}{2}\sum_{10}^{29} n.$$
$$\sum_{10}^{29} n^2 = \sum_{1}^{29} n^2 - \sum_{1}^{9} n^2 = \frac{29(30)(59)}{6} - \frac{9(10)(19)}{6} = 8555 - 285 = 8270.$$
$$\sum_{10}^{29} n = \sum_{1}^{29} n - \sum_{1}^{9} n = \frac{29(30)}{2} - \frac{9(10)}{2} = 435 - 45 = 390.$$
$$S_{29} - S_9 = \frac{3}{2}(8270) + \frac{5}{2}(390) = 3(4135) + 5(195) = 12405 + 975 = 13380.$$
$$\frac{1}{60}(S_{29} - S_9) = \frac{13380}{60} = \frac{1338}{6} = 223.$$

The correct option is **(4)**.

53. Let a_n be the nth term of the series $5 + 8 + 14 + 23 + 35 + 50 + ...$ and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(1) 11280 (2) 11290 (3) 11310 (4) 11260

Solution:

Let $T_n = a_n$. The differences between terms are:
$8 - 5 = 3, \quad 14 - 8 = 6, \quad 23 - 14 = 9, \quad 35 - 23 = 12, \ldots$
The first differences are in AP: 3, 6, 9, ... with first term 3 and common difference 3.

$$a_n = a_1 + \sum_{k=1}^{n-1}(3k) = 5 + 3\frac{(n-1)n}{2}.$$
$$a_{40} = 5 + 3\frac{39 \times 40}{2} = 5 + 3(39 \times 20) = 5 + 2340 = 2345.$$

$$S_{30} = \sum_{n=1}^{30} a_n = \sum_{n=1}^{30}\left(5 + \frac{3}{2}(n^2 - n)\right) = \sum 5 + \frac{3}{2}\sum n^2 - \frac{3}{2}\sum n.$$
$$= 5(30) + \frac{3}{2}\left(\frac{30(31)(61)}{6}\right) - \frac{3}{2}\left(\frac{30(31)}{2}\right).$$
$$= 150 + \frac{3}{2}(5 \cdot 31 \cdot 61) - \frac{3}{2}(15 \cdot 31) = 150 + \frac{3}{2}(9455) - \frac{3}{2}(465).$$
$$= 150 + \frac{3}{2}(8990) = 150 + 3(4495) = 150 + 13485 = 13635.$$

$$S_{30} - a_{40} = 13635 - 2345 = 11290.$$

The correct option is **(2)**.

54. If gcd $(m, n) = 1$ and $1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ then $m^2 - n^2$ is equal to

(1) 220 (2) 200 (3) 240 (4) 180

Solution:

Let S be the sum. Group the terms in pairs:
$S = (1^2 - 2^2) + (3^2 - 4^2) + \cdots + (2021^2 - 2022^2) + 2023^2.$
Using $a^2 - b^2 = (a-b)(a+b)$:
$S = (-1)(1+2) + (-1)(3+4) + \cdots + (-1)(2021 + 2022) + 2023^2.$
$S = -(1 + 2 + 3 + 4 + \cdots + 2022) + 2023^2.$
$S = -\frac{2022 \times 2023}{2} + 2023^2 = -1011 \times 2023 + 2023^2.$
$S = 2023(-1011 + 2023) = 2023 \times 1012.$

We are given $S = 1012m^2n.$
$2023 \times 1012 = 1012m^2n \implies m^2n = 2023.$
Prime factorization of 2023: $2023 = 7 \times 17^2 = 7 \times 289.$
$m^2n = 7 \times 17^2.$
Since gcd(m,n)=1, by comparison, we must have $m = 17$ and $n = 7$.

We need to find $m^2 - n^2$.
$m^2 - n^2 = 17^2 - 7^2 = 289 - 49 = 240.$

The correct option is **(3)**.

55. The sum of 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...$ is

(1) $\frac{58}{111}$ (2) $\frac{56}{111}$ (3) $\frac{55}{111}$ (4) $\frac{59}{111}$

Solution:

The nth term is $T_n = \dfrac{n}{1+n^2+n^4}$.

The denominator can be factored: $n^4+n^2+1 = (n^2+1)^2 - n^2 = (n^2-n+1)(n^2+n+1)$.

$T_n = \dfrac{n}{(n^2-n+1)(n^2+n+1)}$.

Using partial fractions, we note that $(n^2+n+1)-(n^2-n+1) = 2n$.

$T_n = \dfrac{1}{2}\dfrac{2n}{(n^2-n+1)(n^2+n+1)} = \dfrac{1}{2}\left[\dfrac{1}{n^2-n+1} - \dfrac{1}{n^2+n+1}\right]$.

Let $f(n) = \dfrac{1}{n^2-n+1}$. Then $f(n+1) = \dfrac{1}{(n+1)^2-(n+1)+1} = \dfrac{1}{n^2+n+1}$.

So, $T_n = \dfrac{1}{2}[f(n)-f(n+1)]$.

The sum of 10 terms is a telescoping series:

$S_{10} = \sum_{n=1}^{10} T_n = \dfrac{1}{2}[f(1)-f(11)]$.

$f(1) = \dfrac{1}{1-1+1} = 1$.

$f(11) = \dfrac{1}{11^2-11+1} = \dfrac{1}{121-11+1} = \dfrac{1}{111}$.

$S_{10} = \dfrac{1}{2}\left(1-\dfrac{1}{111}\right) = \dfrac{1}{2}\left(\dfrac{110}{111}\right) = \dfrac{55}{111}$.

The correct option is **(3)**.

56. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively then the sum of common ratios of all such GPs is

(1) 7 (2) 14 (3) 3 (4) 9/2

Solution:

Let the four positive consecutive terms of the G.P. be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

From the product of the terms:

$\left(\frac{a}{r^3}\right)\left(\frac{a}{r}\right)(ar)(ar^3) = 1296$.

$a^4 = 1296 = 6^4$.

Since the terms are positive, $a = 6$.

From the sum of the terms:

$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$.

$a\left(\frac{1}{r^3} + \frac{1}{r} + r + r^3\right) = 126$.

$6\left((r^3+\frac{1}{r^3})+(r+\frac{1}{r})\right)=126.$
$(r^3+\frac{1}{r^3})+(r+\frac{1}{r})=21.$

Let $x=r+\frac{1}{r}$. Then $r^3+\frac{1}{r^3}=\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=x^3-3x.$
Substituting this into the equation:
$(x^3-3x)+x=21.$
$x^3-2x-21=0.$

By inspection, we can see that x=3 is a root, since $(3)^3-2(3)-21=27-6-21=0.$
Factoring the cubic polynomial gives $(x-3)(x^2+3x+7)=0.$
The quadratic factor x^2+3x+7 has a discriminant $D=3^2-4(1)(7)=9-28=-19<0,$ so it has no

Therefore, the only real solution is $x=3,$ which means $r+\frac{1}{r}=3.$
$r^2-3r+1=0.$
Solving for r gives two possible values for the common ratio:
$r=\frac{3+\sqrt{5}}{2}$ and $r=\frac{3-\sqrt{5}}{2}.$

The question asks for the sum of common ratios of all such possible GPs.
$\text{Sum}=\left(\frac{3+\sqrt{5}}{2}\right)+\left(\frac{3-\sqrt{5}}{2}\right)=\frac{6}{2}=3.$
The correct option is **(3)**.

57. The sum $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$ is equal to

(1) $\frac{7}{87}$ (2) $\frac{7}{29}$ (3) $\frac{14}{87}$ (4) $\frac{21}{29}$

Solution:

Let $T_n=\frac{3}{(4n-1)(4n+3)}$. We use partial fractions.
$T_n=A\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right)=A\frac{4n+3-(4n-1)}{(4n-1)(4n+3)}=A\frac{4}{(4n-1)(4n+3)}.$
So $4A=3 \implies A=3/4.$
$T_n=\frac{3}{4}\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right).$
The sum is a telescoping series:
$$S_{21}=\frac{3}{4}\sum_{n=1}^{21}\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right).$$
$$=\frac{3}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+...+\left(\frac{1}{4(21)-1}-\frac{1}{4(21)+3}\right)\right].$$
$$=\frac{3}{4}\left[\frac{1}{3}-\frac{1}{87}\right]=\frac{3}{4}\left[\frac{29-1}{87}\right]=\frac{3}{4}\frac{28}{87}=\frac{3\cdot 7}{87}=\frac{7}{29}.$$

The correct option is **(2)**.

58. If 'n' arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1: 7 and $a + n = 33$, then the value of n is

(1) 21 (2) 22 (3) 23 (4) 24

Solution:

Let the n means be $A_1, A_2, \ldots, A_n$.
The sequence $a, A_1, \ldots, A_n, 100$ is an AP with $n + 2$ terms.
$A_1 = a + d$ and $A_n = a + nd = 100 - d$.
Given $\frac{A_1}{A_n} = \frac{a+d}{100-d} = \frac{1}{7} \implies 7a + 7d = 100 - d \implies 7a + 8d = 100.$
The last term is $100 = a + (n+1)d \implies d = \frac{100-a}{n+1}$.
$7a + 8\frac{100-a}{n+1} = 100.$
$7a(n+1) + 800 - 8a = 100(n+1) \implies 7an + 7a + 800 - 8a = 100n + 100.$
$7an - a + 700 = 100n.$
Given $a + n = 33 \implies a = 33 - n.$
$7(33-n)n - (33-n) + 700 = 100n.$
$231n - 7n^2 - 33 + n + 700 = 100n.$
$-7n^2 + 232n + 667 = 100n \implies 7n^2 - 132n - 667 = 0.$
Check options. Let n=23: $7(23^2) - 132(23) - 667 = 7(529) - 3036 - 667 = 3703 - 3036 - 667 = 3703 -$
So n=23 is the solution.

The correct option is **(3)**.

59. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$ Then 4S is equal to

(1) $(\frac{7}{3})^3$ (2) $\frac{7^3}{3^2}$ (3) $(\frac{7}{3})^2$ (4) $\frac{7^2}{3^3}$

Solution:

Let S be the given sum.

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots \quad \cdots (1)$$

This is a series where the numerators do not form a simple AP. We apply the S-rS method.
Multiply equation (1) by the common ratio $r = 1/7$:

$$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots \quad \cdots (2)$$

Subtract equation (2) from (1):

$$S - \frac{S}{7} = \frac{6S}{7} = 2 + \left(\frac{6-2}{7}\right) + \left(\frac{12-6}{7^2}\right) + \left(\frac{20-12}{7^3}\right) + \ldots$$

$$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \dots$$

Let this new series be S'. $S' = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \dots \quad \cdots(3)$

S' is an Arithmetico-Geometric Progression (AGP). We apply the method again.

Multiply equation (3) by $r = 1/7$:

$$\frac{S'}{7} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \dots \quad \cdots(4)$$

Subtract equation (4) from (3):

$$S' - \frac{S'}{7} = \frac{6S'}{7} = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \dots$$

$$\frac{6S'}{7} = 2 + 2\left(\frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots\right).$$

The series in the parenthesis is an infinite GP with first term $a' = \frac{1}{7}$ and ratio $r' = \frac{1}{7}$.

Sum of GP $= \frac{a'}{1 - r'} = \frac{1/7}{1 - 1/7} = \frac{1/7}{6/7} = \frac{1}{6}$.

Substitute this back to find S':

$$\frac{6S'}{7} = 2 + 2\left(\frac{1}{6}\right) = 2 + \frac{1}{3} = \frac{7}{3}.$$

$$S' = \frac{7}{3} \times \frac{7}{6} = \frac{49}{18}.$$

Now substitute S' back into the equation for S, where $\frac{6S}{7} = S'$:

$$\frac{6S}{7} = \frac{49}{18}.$$

$$S = \frac{49}{18} \times \frac{7}{6} = \frac{343}{108}.$$

The question asks for the value of 4S.

$$4S = 4 \times \frac{343}{108} = \frac{343}{27}.$$

Since $7^3 = 343$ and $3^3 = 27,$ this is $\frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$.

The correct option is **(1)**.

60. $15(\tan 2\theta + \sin 2\theta) + 8 = 0$ if:

(1) $\tan\theta = 1/2$ (2) $\sin\theta = 1/4$ (3) $\tan\theta = 2$ (4) $\cos\theta = 1/5$

Solution:

Let $t = \tan\theta$. Use half-angle formulas for 2θ.

$15\left(\frac{2t}{1-t^2}+\frac{2t}{1+t^2}\right)+8=0.$

$15\cdot 2t\left(\frac{1}{1-t^2}+\frac{1}{1+t^2}\right)+8=0.$

$30t\left(\frac{1+t^2+1-t^2}{(1-t^2)(1+t^2)}\right)+8=0.$

$30t\left(\frac{2}{1-t^4}\right)+8=0 \implies \frac{60t}{1-t^4}=-8.$

$60t=-8+8t^4 \implies 8t^4-60t-8=0 \implies 2t^4-15t-2=0.$

Check options. Let $t=1/2$. $2(1/16)-15(1/2)-2=1/8-15/2-2\neq 0.$

Let $t=2$. $2(16)-15(2)-2=32-30-2=0.$

So $t=\tan\theta=2$ is a solution.

The correct option is **(3)**.

61. The value of $(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10})$ is

(1) 1/8 (2) -1/8 (3) 1/16 (4) -1/16

Solution:

Let E be the expression. Use $\cos(\pi-\theta)=-\cos\theta$.

$\cos(9\pi/10)=\cos(\pi-\pi/10)=-\cos(\pi/10).$

$\cos(7\pi/10)=\cos(\pi-3\pi/10)=-\cos(3\pi/10).$

$E=(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1-\cos\frac{3\pi}{10})(1-\cos\frac{\pi}{10}).$

$E=(1-\cos^2\frac{\pi}{10})(1-\cos^2\frac{3\pi}{10})=\sin^2\frac{\pi}{10}\sin^2\frac{3\pi}{10}.$

This is $\sin^2 18^\circ\sin^2 54^\circ=\sin^2 18^\circ\cos^2 36^\circ$.

$=\left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2=\left(\frac{(\sqrt{5}-1)(\sqrt{5}+1)}{16}\right)^2.$

$=\left(\frac{5-1}{16}\right)^2=(\frac{4}{16})^2=(\frac{1}{4})^2=\frac{1}{16}.$

The correct option is **(3)**.

62. The value of $\cos 12^\circ+\cos 84^\circ+\cos 156^\circ+\cos 132^\circ$ is

(1) 1/8 (2) -1/2 (3) 1 (4) 1/2

Solution:

Let S be the sum. Group the terms:

$S=(\cos 12^\circ+\cos 132^\circ)+(\cos 84^\circ+\cos 156^\circ).$

Use $\cos C+\cos D=2\cos\frac{C+D}{2}\cos\frac{C-D}{2}.$

$S=2\cos\frac{144}{2}\cos\frac{-120}{2}+2\cos\frac{240}{2}\cos\frac{-72}{2}.$

$S=2\cos 72^\circ\cos 60^\circ+2\cos 120^\circ\cos 36^\circ.$

$$S = 2(\sin 18^\circ)(\frac{1}{2}) + 2(-\frac{1}{2})(\cos 36^\circ).$$
$$S = \sin 18^\circ - \cos 36^\circ = \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} = \frac{-2}{4} = -\frac{1}{2}.$$

The correct option is **(2)**.

63. The value of $\cos^3(\frac{\pi}{8})\cos(\frac{3\pi}{8}) + \sin^3(\frac{\pi}{8})\sin(\frac{3\pi}{8}) =$

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Solution:

Let E be the expression. Use $\cos(3\pi/8) = \cos(\pi/2 - \pi/8) = \sin(\pi/8)$.
And $\sin(3\pi/8) = \sin(\pi/2 - \pi/8) = \cos(\pi/8)$.

$E = \cos^3(\pi/8)\sin(\pi/8) + \sin^3(\pi/8)\cos(\pi/8)$.
$E = \sin(\pi/8)\cos(\pi/8)[\cos^2(\pi/8) + \sin^2(\pi/8)]$.

$$E = \sin(\pi/8)\cos(\pi/8) = \frac{1}{2}\sin(2\cdot\pi/8) = \frac{1}{2}\sin(\pi/4) = \frac{1}{2\sqrt{2}}.$$

The correct option is **(1)**.

64. If $\cos x = \frac{4}{5}$ and $\frac{3\pi}{2} < x < 2\pi$ then the value of $\tan\frac{x}{2}$ is

(1) 1/3 (2) -1/3 (3) 3 (4) -3

Solution:

Given $x \in (3\pi/2, 2\pi)$ (Quadrant IV).
Then $x/2 \in (3\pi/4, \pi)$ (Quadrant II). In Q2, $\tan(x/2)$ is negative.

$$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x}.$$
$$\tan^2\frac{x}{2} = \frac{1-4/5}{1+4/5} = \frac{1/5}{9/5} = \frac{1}{9}.$$
$$\tan\frac{x}{2} = \pm\frac{1}{3}.$$

Since x/2 is in Q2, we choose the negative value. $\tan(x/2) = -1/3$.

The correct option is **(2)**.

65. If $\cos\theta = -\frac{5}{13}$ and $\sin\phi = \frac{3}{5}$ where θ lies in the second quadrant and ϕ lies in the first quadrant, then $\sin(\theta+\phi) =$

(1) 33/65 (2) 56/65 (3) $-33/65$ (4) $-63/65$

Solution:

$\theta \in Q2 \implies \sin\theta > 0. \quad \sin\theta = \sqrt{1-(-5/13)^2} = \sqrt{1-25/169} = 12/13.$
$\phi \in Q1 \implies \cos\phi > 0. \quad \cos\phi = \sqrt{1-(3/5)^2} = \sqrt{1-9/25} = 4/5.$
$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi.$
$= (\frac{12}{13})(\frac{4}{5}) + (-\frac{5}{13})(\frac{3}{5}) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}.$

The correct option is **(1)**.

66. The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$ is

(1) 0 (2) 2/3 (3) 1 (4) 3/2

Solution:

$$\begin{aligned} E &= (\sin 70^\circ)(\frac{\cos 10^\circ \cos 70^\circ}{\sin 10^\circ \sin 70^\circ} - 1) \\ &= \sin 70^\circ \frac{\cos 10^\circ \cos 70^\circ - \sin 10^\circ \sin 70^\circ}{\sin 10^\circ \sin 70^\circ}. \\ &= \frac{\cos(10^\circ + 70^\circ)}{\sin 10^\circ} \\ &= \frac{\cos 80^\circ}{\sin 10^\circ} \\ &= \frac{\sin 10^\circ}{\sin 10^\circ} \\ &= 1. \end{aligned}$$

The correct option is **(3)**.

67. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by 10.5. Then the number of terms is: (Corrected question)

(1) 4 (2) 10 (3) 6 (4) 8

Solution:

Let the number of terms be $2n$.
Let the AP be $a, a+d, \ldots, a+(2n-1)d$.

$S_{odd} = a_1 + a_3 + \cdots + a_{2n-1} = n/2(2a + (n-1)2d) = n(a + (n-1)d) = 24.$

$S_{even} = a_2 + a_4 + \cdots + a_{2n} = n/2(2(a+d) + (n-1)2d) = n(a + d + (n-1)d) = n(a + nd) = 30.$

$S_{even} - S_{odd} = n(a+nd) - n(a+nd-d) = nd = 6.$
Last term - First term: $a_{2n} - a_1 = (a + (2n-1)d) - a = (2n-1)d = 10.5 = 21/2.$

From $nd = 6$ and $(2n-1)d = 10.5$, divide them: $\frac{nd}{(2n-1)d} = \frac{6}{10.5} = \frac{12}{21} = \frac{4}{7}.$

$$\frac{n}{2n-1} = \frac{4}{7} \implies 7n = 8n - 4 \implies n = 4.$$

The total number of terms is $2n = 8$.

The correct option is **(4)**.

68. $1 + 3 + 5^2 + 7 + 9^2 + ...$ upto 40 terms is equal to

Solution:

Let S be the sum of the series. The series can be split into two separate series based on the term positio
Series 1 (Odd positions): $1^2 + 5^2 + 9^2 + \dots$ (20 terms)
Series 2 (Even positions): $3 + 7 + 11 + \dots$ (20 terms)
Calculate the sum of Series 1 (S1):
The terms are squares of numbers in an AP: 1, 5, 9, ...
The nth term of this AP is $a_n = 1 + (n-1)4 = 4n - 3$.
$S_1 = \sum_{n=1}^{20}(4n-3)^2 = \sum_{n=1}^{20}(16n^2 - 24n + 9)$.
$= 16\sum n^2 - 24\sum n + \sum 9$.
$= 16\left(\frac{20(21)(41)}{6}\right) - 24\left(\frac{20(21)}{2}\right) + 9(20)$.
$= 16(2870) - 24(210) + 180$.
$= 45920 - 5040 + 180 = 41060$.

Calculate the sum of Series 2 (S2):
This is an AP with first term $a = 3,$ common difference $d = 4,$ and $n = 20$ terms.
$S_2 = \frac{n}{2}[2a + (n-1)d] = \frac{20}{2}[2(3) + (19)4]$.
$= 10[6 + 76] = 10(82) = 820$.

The total sum is S = S1 + S2:
$S = 41060 + 820 = 41880$.

69. Let X_1, X_2, X_3, X_4 be in geometric progression. If 2,7,9,5 are subtracted respectively from them, then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(X_1X_2X_3X_4)$ is:

(1) 72 (2) 18 (3) 36 (4) 216

Solution:

Let the terms of the Geometric Progression (GP) be a, ar, ar^2, ar^3.

After subtracting the given numbers, the new terms are in an Arithmetic Progression (AP): $a-2, \quad ar-7, \quad ar^2-9, \quad ar^3-5$.

For an AP, the difference between consecutive terms is constant. So, $2b = a + c$.
Using the first three terms:
$2(ar-7) = (a-2) + (ar^2-9)$.
$2ar - 14 = a + ar^2 - 11$.
$a(r^2 - 2r + 1) = -3 \implies a(r-1)^2 = -3 \quad \cdots(1)$.

Using the last three terms:
$2(ar^2-9) = (ar-7) + (ar^3-5)$.
$2ar^2 - 18 = ar + ar^3 - 12$.
$ar(r^2 - 2r + 1) = -6 \implies ar(r-1)^2 = -6 \quad \cdots(2)$.

Divide equation (2) by equation (1):
$\frac{ar(r-1)^2}{a(r-1)^2} = \frac{-6}{-3}$.
$r = 2$.

Substitute r=2 back into equation (1):
$a(2-1)^2 = -3 \implies a(1)^2 = -3 \implies a = -3$.

The terms of the GP are:
$X_1 = a = -3$.
$X_2 = ar = -3(2) = -6$.
$X_3 = ar^2 = -3(4) = -12$.
$X_4 = ar^3 = -3(8) = -24$.

The product of these terms is:
$X_1X_2X_3X_4 = (-3)(-6)(-12)(-24) = (18)(288) = 5184$.

The required value is:
$\frac{1}{24}(X_1X_2X_3X_4) = \frac{1}{24}(5184) = 216$.

The correct option is **(4)**.

70. If $\frac{1}{1^4} + \frac{1}{2^4} + ... = \frac{\pi^4}{90}$, $\frac{1}{1^4} + \frac{1}{3^4} + ... = \alpha$, $\frac{1}{2^4} + \frac{1}{4^4} + ... = \beta$ then $\frac{\alpha}{\beta}$ is equal to

(1) 23 (2) 18 (3) 15 (4) 14

Solution:

$$\text{Let } S = \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

$$S = (\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots) + (\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots).$$
$$S = \alpha + \beta.$$
$$\beta = \frac{1}{(2 \cdot 1)^4} + \frac{1}{(2 \cdot 2)^4} + \frac{1}{(2 \cdot 3)^4} + \dots = \frac{1}{2^4} \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{S}{16}.$$
$$\alpha = S - \beta = S - \frac{S}{16} = \frac{15S}{16}.$$
$$\frac{\alpha}{\beta} = \frac{15S/16}{S/16} = 15.$$

The correct option is **(3)**.

SECTION-B

71. The value of the expression $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$ is equal to

Solution:

$$\begin{aligned} E &= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} \\ &= \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cos 10^\circ}. \\ &= \frac{2(\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ)}{\frac{1}{2} \sin 20^\circ} \\ &= \frac{2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\frac{1}{2} \sin 20^\circ}. \\ &= \frac{2 \sin(30^\circ - 10^\circ)}{\frac{1}{2} \sin 20^\circ} \\ &= \frac{2 \sin 20^\circ}{\frac{1}{2} \sin 20^\circ} \\ &= 4. \end{aligned}$$

The answer is **4**.

72. The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - 7^2 ...$ is 118k, then find k

Solution:

The series has 20 terms, which we can group into 10 pairs.
The general form for the k-th pair is: $2 \cdot (2k)^2 - (2k+1)^2$.
The sum to 20 terms is the sum of the first 10 such pairs:

$$S_{20} = \sum_{k=1}^{10} [2 \cdot (2k)^2 - (2k+1)^2].$$

First, simplify the general term:
$T_k = 2(4k^2) - (4k^2 + 4k + 1)$
$T_k = 8k^2 - 4k^2 - 4k - 1 = 4k^2 - 4k - 1.$

Now, we sum this simplified expression from k=1 to 10:

$$S_{20} = \sum_{k=1}^{10}(4k^2 - 4k - 1).$$

$$S_{20} = 4\sum_{k=1}^{10} k^2 - 4\sum_{k=1}^{10} k - \sum_{k=1}^{10} 1.$$

Using the standard summation formulas:

$$\sum_{k=1}^{10} k^2 = \frac{10(10+1)(2 \cdot 10 + 1)}{6} = \frac{10 \cdot 11 \cdot 21}{6} = 385.$$

$$\sum_{k=1}^{10} k = \frac{10(10+1)}{2} = \frac{10 \cdot 11}{2} = 55.$$

$$\sum_{k=1}^{10} 1 = 10.$$

Substitute these values back into the sum:
$S_{20} = 4(385) - 4(55) - 10.$
$S_{20} = 1540 - 220 - 10 = 1310.$

We are given that the sum is 118k:
$118k = 1310.$

$$k = \frac{1310}{118}.$$

73. Suppose $a_1, a_2, 2, a_4, a_5$ be in an arithmetico-geometric progression. If the common ratio of the corresponding GP is 2 and the sum of all 5 terms is 49/2 then a_4 is equal to

Solution:

Let the Arithmetico-Geometric Progression (AGP) have an AP component with first term 'a' and comm
and a GP component with common ratio $r = 2$.
The terms of the AGP are given by $a_k = (a + (k-1)d)r^{k-1}$.

We are given that the third term is 2:
$a_3 = (a + (3-1)d)r^{3-1} = (a + 2d) \cdot 2^2 = 4(a + 2d).$

$$4(a + 2d) = 2 \implies a + 2d = \frac{1}{2} \quad \cdots (1).$$

The sum of the five terms is given as 49/2:

$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = \frac{49}{2}$.

$S_5 = a(2^0) + (a+d)2^1 + (a+2d)2^2 + (a+3d)2^3 + (a+4d)2^4 = \frac{49}{2}$.

$a + 2a + 2d + 4a + 8d + 8a + 24d + 16a + 64d = \frac{49}{2}$.

Group the terms with 'a' and 'd':

$a(1+2+4+8+16) + d(2+8+24+64) = \frac{49}{2}$.

$31a + 98d = \frac{49}{2} \implies 62a + 196d = 49 \quad \cdots (2)$.

Now we solve the system of linear equations (1) and (2).

From (1), $a = \frac{1}{2} - 2d$.

Substitute 'a' into (2):

$62\left(\frac{1}{2} - 2d\right) + 196d = 49$.

$31 - 124d + 196d = 49$.

$72d = 18 \implies d = \frac{18}{72} = \frac{1}{4}$.

$a = \frac{1}{2} - 2\left(\frac{1}{4}\right) = \frac{1}{2} - \frac{1}{2} = 0$.

We need to find the fourth term, a_4 :

$a_4 = (a+3d)r^3 = \left(0 + 3\left(\frac{1}{4}\right)\right) \cdot 2^3$.

$a_4 = \left(\frac{3}{4}\right) \cdot 8 = 6$.

74. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to

Solution:

For AMs: $3, A_1, \ldots, A_m, 243. \quad d = \frac{243-3}{m+1} = \frac{240}{m+1}$.

$A_4 = 3 + 4d = 3 + 4(\frac{240}{m+1})$.

For GMs: $3, G_1, G_2, G_3, 243. \quad 243 = 3r^4 \implies r^4 = 81 \implies r = 3$.

$G_2 = 3r^2 = 3(3^2) = 27$.

Given $A_4 = G_2$:

$3 + \frac{960}{m+1} = 27 \implies \frac{960}{m+1} = 24 \implies m+1 = \frac{960}{24} = 40$.

$$m = 39.$$

The answer is **39**.

75. If sin A, cos A and tan A are in G.P., then $cot^6 A - cot^2 A =$

Solution:

If three terms are in G.P., the square of the middle term is equal to the product of the other two.

$$(\cos A)^2 = (\sin A)(\tan A)$$
$$\cos^2 A = \sin A \left(\frac{\sin A}{\cos A}\right)$$
$$\cos^3 A = \sin^2 A$$

We need to find the value of $\cot^6 A - \cot^2 A$. Let's use the relation we just found. Divide the relation $\cos^3 A = \sin^2 A$ by $\sin^3 A$:

$$\frac{\cos^3 A}{\sin^3 A} = \frac{\sin^2 A}{\sin^3 A}$$
$$\cot^3 A = \frac{1}{\sin A} = \text{cosec } A$$

Now, square both sides:

$$(\cot^3 A)^2 = (\text{cosec } A)^2$$
$$\cot^6 A = \text{cosec}^2 A$$

Using the Pythagorean identity $\text{cosec}^2 A = 1 + \cot^2 A$:

$$\cot^6 A = 1 + \cot^2 A$$

Rearranging the terms gives the desired expression:

$$\cot^6 A - \cot^2 A = 1.$$

The answer is **1**.