Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Exercise 2: MHT CET PYQ

Topic: Trigonometric Ratios and Identities

Sub: Mathematics **Exercise 2: Solution** Prof. Chetan Sir

Index of Questions

1. The value of $\cos 20^\circ + 2\sin^2 55^\circ - \sqrt{2}\sin 65^\circ$ is: **[MHT CET: 2024]**

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

Solution:

Concept Used:

- Double Angle Identity: $2\sin^2 A = 1 - \cos 2A$.
- Sum-to-Product Identity: $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.
- Co-function Identity: $\sin(90^\circ - \theta) = \cos\theta$.

Hint:

- Convert $2\sin^2 55^\circ$ to an expression involving $\cos 110^\circ$.
- Group the cosine terms and apply the sum-to-product formula.
- Simplify the resulting expression.

The given expression is $\cos 20^\circ + 2\sin^2 55^\circ - \sqrt{2}\sin 65^\circ$.
First, using the identity $2\sin^2 A = 1 - \cos 2A$, we have:
$2\sin^2 55^\circ = 1 - \cos(2 \times 55^\circ) = 1 - \cos 110^\circ$.

Substitute this into the expression:
$= \cos 20^\circ + (1 - \cos 110^\circ) - \sqrt{2}\sin 65^\circ$
$= 1 + (\cos 20^\circ - \cos 110^\circ) - \sqrt{2}\sin 65^\circ$.

Now, apply the sum-to-product formula $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$:
$$\cos 20^\circ - \cos 110^\circ = -2\sin\left(\frac{20^\circ + 110^\circ}{2}\right)\sin\left(\frac{20^\circ - 110^\circ}{2}\right).$$
$= -2\sin(65^\circ)\sin(-45^\circ) = -2\sin(65^\circ)(-\sin 45^\circ)$.
$$= 2\sin(65^\circ)\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}\sin 65^\circ.$$

Substitute this back into the main expression:
$= 1 + \sqrt{2}\sin 65^\circ - \sqrt{2}\sin 65^\circ$.
$= 1$.

The correct option is **(B)**.

2. The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)\ldots(\cos\alpha_n)$ under the constraints $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1)\cdot(\cot\alpha_2)\ldots(\cot\alpha_n) = 1$ is: **[MHT CET: 2024]**

(A) $\frac{1}{2^{(n/2)}}$ (B) $\frac{1}{2^n}$ (C) 2^n (D) $2^{\frac{n}{2}}$

Solution:

Concept Used:

- Trigonometric identity: $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$.
- Double angle identity: $\sin(2\alpha) = 2\sin\alpha\cos\alpha$.
- The maximum value of $\sin\theta$ is 1 for $\theta \in [0, \pi]$.

Hint:

- Let the required product be y. Square the expression and use the given constraint to form an equation for y^2.
- Analyze the range of the trigonometric functions involved to find the maximum possible value of y^2.

We are given that,

$$(\cot\alpha_1)\cdot(\cot\alpha_2)\cdots(\cot\alpha_n) = 1$$

$$\implies \frac{\cos\alpha_1}{\sin\alpha_1}\cdot\frac{\cos\alpha_2}{\sin\alpha_2}\cdots\frac{\cos\alpha_n}{\sin\alpha_n} = 1$$

$$\implies (\cos\alpha_1)\cdot(\cos\alpha_2)\cdots(\cos\alpha_n) = (\sin\alpha_1)\cdot(\sin\alpha_2)\cdots(\sin\alpha_n) \quad \cdots(i)$$

Let $y = (\cos\alpha_1)\cdot(\cos\alpha_2)\cdots(\cos\alpha_n)$ (to be maximized).

Squaring both sides, we get

$$y^2 = (\cos^2\alpha_1)\cdot(\cos^2\alpha_2)\cdots(\cos^2\alpha_n)$$

$$= (\cos\alpha_1\cdot\sin\alpha_1)\cdot(\cos\alpha_2\cdot\sin\alpha_2)\cdots(\cos\alpha_n\cdot\sin\alpha_n) \quad \text{[using (i)]}$$

$$= \left(\frac{\sin 2\alpha_1}{2}\right)\cdot\left(\frac{\sin 2\alpha_2}{2}\right)\cdots\left(\frac{\sin 2\alpha_n}{2}\right)$$

$$= \frac{1}{2^n}[\sin 2\alpha_1\cdot\sin 2\alpha_2\cdots\sin 2\alpha_n]$$

As $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \dfrac{\pi}{2}$

$$\therefore 0 \le 2\alpha_1, 2\alpha_2, \ldots, 2\alpha_n \le \pi$$

$$\implies 0 \le \sin 2\alpha_1, \sin 2\alpha_2, \ldots, \sin 2\alpha_n \le 1$$

$$\therefore y^2 \le \frac{1}{2^n}\cdot 1$$

$$\implies y \le \frac{1}{\sqrt{2^n}} = \frac{1}{2^{n/2}}$$

$\therefore$ Maximum value of y is $\dfrac{1}{2^{n/2}}$.

The correct option is **(A)**.

3. If $A + B = 225°$, then $\frac{\cot A}{1+\cot A} \cdot \frac{\cot B}{1+\cot B}$, if it exists, is equal to: **[MHT CET: 2024]**

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

Solution:

Concept Used:

- Tangent addition formula: $\tan(A + B) = \frac{\tan A+\tan B}{1-\tan A \tan B}$.
- Identity: $\cot x = 1/\tan x$.

Hint:

- Use the condition $A + B = 225°$ to find a relationship between $\tan A$ and $\tan B$.
- Rewrite the given expression in terms of $\tan A$ and $\tan B$ and simplify.

$$\begin{aligned}
&\text{Given } A + B = 225°.\\
&\tan(A + B) = \tan(225°) = \tan(180° + 45°) = \tan(45°) = 1.\\
&\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1.\\
&\tan A + \tan B = 1 - \tan A \tan B.\\
&\tan A + \tan B + \tan A \tan B = 1.\\
&\text{Adding 1 to both sides:}\\
&1 + \tan A + \tan B + \tan A \tan B = 2.\\
&(1 + \tan A)(1 + \tan B) = 2.\\
\\
&\text{Now, consider the given expression:}\\
&\frac{\cot A}{1 + \cot A} \cdot \frac{\cot B}{1 + \cot B} = \frac{\frac{1}{\tan A}}{1 + \frac{1}{\tan A}} \cdot \frac{\frac{1}{\tan B}}{1 + \frac{1}{\tan B}}.\\
&= \left(\frac{\frac{1}{\tan A}}{\frac{\tan A+1}{\tan A}}\right) \cdot \left(\frac{\frac{1}{\tan B}}{\frac{\tan B+1}{\tan B}}\right).\\
&= \frac{1}{1 + \tan A} \cdot \frac{1}{1 + \tan B} = \frac{1}{(1 + \tan A)(1 + \tan B)}.\\
\\
&\text{Substitute the result from our first step:}\\
&= \frac{1}{2}.
\end{aligned}$$

The correct option is **(D)**.

4. The value of $\cos(18° - A)\cos(18° + A) - \cos(72° - A)\cos(72° + A)$ is equal to: **[MHT CET: 2024]**

(A) $\cos 54°$ (B) $\cos 36°$ (C) $\sin 54°$ (D) $\sin 36°$

Solution:

Concept Used:

- Product-to-Sum/Difference Identity: $\cos(X - Y)\cos(X + Y) = \cos^2 X - \sin^2 Y$.
- Co-function Identity: $\cos(90° - \theta) = \sin\theta$.
- Double Angle Identity: $\cos^2\theta - \sin^2\theta = \cos(2\theta)$.

Hint:

- Apply the identity $\cos(X - Y)\cos(X + Y) = \cos^2 X - \sin^2 Y$ to both parts of the expression.
- Use co-function identities to simplify the terms and combine them.

$$\begin{aligned}
&\text{The expression is } \cos(18° - A)\cos(18° + A) - \cos(72° - A)\cos(72° + A).\\
&\text{Using the identity } \cos(X - Y)\cos(X + Y) = \cos^2 X - \sin^2 Y:\\
&\text{First part: } \cos^2 18° - \sin^2 A.\\
&\text{Second part: } \cos^2 72° - \sin^2 A.\\
\\
&\text{The full expression becomes:}\\
&(\cos^2 18° - \sin^2 A) - (\cos^2 72° - \sin^2 A)\\
&= \cos^2 18° - \sin^2 A - \cos^2 72° + \sin^2 A\\
&= \cos^2 18° - \cos^2 72°.\\
\\
&\text{Using the co-function identity } \cos 72° = \cos(90° - 18°) = \sin 18°.\\
&\text{The expression becomes } \cos^2 18° - \sin^2 18°.\\
\\
&\text{Using the double angle identity } \cos(2\theta) = \cos^2\theta - \sin^2\theta:\\
&= \cos(2 \times 18°) = \cos 36°.\\
\\
&\text{Finally, using the co-function identity again: } \cos 36° = \cos(90° - 54°) = \sin 54°.
\end{aligned}$$

The correct option is **(C)**.

5. The value of $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$ is: **[MHT CET: 2024]**

(A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{16}$ (D) $-\frac{1}{16}$

Solution:

Concept Used:

- Identity: $\cos(\pi-\theta) = -\cos\theta$.
- Pythagorean Identity: $1-\cos^2\theta = \sin^2\theta$.
- Double Angle Identity: $2\sin A\cos A = \sin 2A$.

Hint:

- Use the identity $\cos(\pi-\theta) = -\cos\theta$ to simplify the terms with angles $5\pi/8$ and $7\pi/8$.
- Group the terms to form a difference of squares.
- Use the identity $\sin^2\theta\sin^2(\pi/2-\theta) = \sin^2\theta\cos^2\theta$ and simplify.

The expression is $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$.

We use $\cos(\pi-\theta) = -\cos\theta$.

$$\cos\frac{5\pi}{8} = \cos(\pi-\frac{3\pi}{8}) = -\cos\frac{3\pi}{8}.$$

$$\cos\frac{7\pi}{8} = \cos(\pi-\frac{\pi}{8}) = -\cos\frac{\pi}{8}.$$

Substitute these back into the expression:

$$(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1-\cos\frac{3\pi}{8})(1-\cos\frac{\pi}{8}).$$

$$= \left[(1+\cos\frac{\pi}{8})(1-\cos\frac{\pi}{8})\right]\left[(1+\cos\frac{3\pi}{8})(1-\cos\frac{3\pi}{8})\right].$$

$$= (1-\cos^2\frac{\pi}{8})(1-\cos^2\frac{3\pi}{8}).$$

$$= (\sin^2\frac{\pi}{8})(\sin^2\frac{3\pi}{8}).$$

Now use $\sin\frac{3\pi}{8} = \sin(\frac{\pi}{2}-\frac{\pi}{8}) = \cos\frac{\pi}{8}$.

The expression becomes $(\sin^2\frac{\pi}{8})(\cos^2\frac{\pi}{8}) = (\sin\frac{\pi}{8}\cos\frac{\pi}{8})^2$.

$$= \left(\frac{1}{2}\sin(2\cdot\frac{\pi}{8})\right)^2 = \left(\frac{1}{2}\sin\frac{\pi}{4}\right)^2.$$

$$= \left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{8}.$$

The correct option is **(A)**.

6. If angle θ in $[0, 2\pi]$ satisfies both the equations $\cot\theta = \sqrt{3}$ and $\sqrt{3}\sec\theta + 2 = 0$ then θ is: **[MHT CET: 2024]**

(A) $\frac{\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

Solution:

Concept Used:

- Solving basic trigonometric equations.
- Identifying the quadrants where trigonometric functions are positive or negative.

Hint:

- Solve each equation separately to find the possible values of θ in the interval $[0, 2\pi]$.
- Find the common solution that satisfies both equations.

First equation: $\cot\theta = \sqrt{3}$.

This means $\tan\theta = \dfrac{1}{\sqrt{3}}$.

Tangent is positive in Quadrant I and Quadrant III.

The principal value is $\theta = \dfrac{\pi}{6}$.

The solutions in $[0, 2\pi]$ are $\theta = \dfrac{\pi}{6}$ and $\theta = \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$.

Second equation: $\sqrt{3}\sec\theta + 2 = 0$.

$\sec\theta = -\dfrac{2}{\sqrt{3}} \implies \cos\theta = -\dfrac{\sqrt{3}}{2}$.

Cosine is negative in Quadrant II and Quadrant III.

The principal value related angle is $\dfrac{\pi}{6}$.

The solutions in $[0, 2\pi]$ are $\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$ and $\theta = \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$.

Comparing the solution sets for both equations:

Set 1: $\{\dfrac{\pi}{6}, \dfrac{7\pi}{6}\}$

Set 2: $\{\dfrac{5\pi}{6}, \dfrac{7\pi}{6}\}$

The common solution is $\theta = \dfrac{7\pi}{6}$.

The correct option is **(B)**.

7. $\cos^3(\frac{\pi}{8})\cos(\frac{3\pi}{8}) + \sin^3(\frac{\pi}{8})\sin(\frac{3\pi}{8}) =$ **[MHT CET: 2024]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

Solution:

Concept Used:

- Co-function identities: $\cos(\frac{\pi}{2}-\theta) = \sin\theta$ and $\sin(\frac{\pi}{2}-\theta) = \cos\theta$.
- Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$.
- Double angle identity: $\sin(2\theta) = 2\sin\theta\cos\theta$.

Hint:

- Use co-function identities to express $\cos(3\pi/8)$ and $\sin(3\pi/8)$ in terms of $\pi/8$.
- Factor the resulting expression and simplify using trigonometric identities.

The expression is $\cos^3(\dfrac{\pi}{8})\cos(\dfrac{3\pi}{8}) + \sin^3(\dfrac{\pi}{8})\sin(\dfrac{3\pi}{8})$.

We use the identities:

$$\cos(\frac{3\pi}{8}) = \cos(\frac{4\pi}{8} - \frac{\pi}{8}) = \cos(\frac{\pi}{2} - \frac{\pi}{8}) = \sin(\frac{\pi}{8}).$$

$$\sin(\frac{3\pi}{8}) = \sin(\frac{\pi}{2} - \frac{\pi}{8}) = \cos(\frac{\pi}{8}).$$

Substitute these into the expression:

$$\cos^3(\frac{\pi}{8})\sin(\frac{\pi}{8}) + \sin^3(\frac{\pi}{8})\cos(\frac{\pi}{8}).$$

$$= \sin(\frac{\pi}{8})\cos(\frac{\pi}{8})\left[\cos^2(\frac{\pi}{8}) + \sin^2(\frac{\pi}{8})\right].$$

$$= \sin(\frac{\pi}{8})\cos(\frac{\pi}{8})\cdot(1).$$

Using the double angle identity $2\sin\theta\cos\theta = \sin(2\theta)$:

$$= \frac{1}{2}\left(2\sin(\frac{\pi}{8})\cos(\frac{\pi}{8})\right) = \frac{1}{2}\sin(2\cdot\frac{\pi}{8}).$$

$$= \frac{1}{2}\sin(\frac{\pi}{4}) = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

The correct option is **(A)**.

8. The value of the expression $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$ is equal to: **[MHT CET: 2024]**

(A) 2 (B) $\frac{2\sin 20°}{\sin 40°}$ (C) 4 (D) $4\frac{\sin 20°}{\sin 40°}$

Solution:

Concept Used:

- Reciprocal identities: $\text{cosec}\,\theta = 1/\sin\theta, \sec\theta = 1/\cos\theta$.
- Sine difference identity: $\sin(A - B) = \sin A \cos B - \cos A \sin B$.
- Double angle identity: $\sin(2\theta) = 2\sin\theta\cos\theta$.

Hint:

- Convert the expression into terms of sine and cosine and find a common denominator.
- Multiply and divide the numerator by 2 to use the form $R\sin(A - B)$.
- Use the double angle identity on the denominator to simplify the final expression.

The expression is $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$.

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}.$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}.$$

For the numerator, multiply and divide by 2:

$$2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right) = 2(\sin 60° \cos 20° - \cos 60° \sin 20°).$$

$$= 2\sin(60° - 20°) = 2\sin 40°.$$

For the denominator, use the double angle identity:

$$\sin 20° \cos 20° = \frac{1}{2}(2\sin 20° \cos 20°) = \frac{1}{2}\sin 40°.$$

The expression becomes $\dfrac{2\sin 40°}{\frac{1}{2}\sin 40°} = 4$.

The correct option is **(C)**.

9. If $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ then $\cos\frac{x}{2} =$ **[MHT CET: 2024]**

(A) $\frac{-2}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{\sqrt{10}}$ (D) $\frac{-1}{\sqrt{10}}$

Solution:

Concept Used:

- Half-angle identity: $2\cos^2(\frac{x}{2}) = 1 + \cos x$.
- Determining the sign of a trigonometric function based on the quadrant.

Hint:

- From the given quadrant for x, determine the quadrant for $x/2$ to find the sign of $\cos(x/2)$.
- Find the value of $\cos x$ from $\tan x$.
- Use the half-angle identity to find $\cos^2(x/2)$ and then take the square root with the correct sign.

Given $\pi < x < \dfrac{3\pi}{2}$, which is Quadrant III.

Dividing by 2, we get $\dfrac{\pi}{2} < \dfrac{x}{2} < \dfrac{3\pi}{4}$. This is Quadrant II.

In Quadrant II, $\cos(\dfrac{x}{2})$ is negative.

Given $\tan x = 3/4$. Since x is in Q3, both sin x and cos x are negative.

We can form a right triangle with opposite=3, adjacent=4, hypotenuse=5.

$$\cos x = -\frac{\text{adjacent}}{\text{hypotenuse}} = -\frac{4}{5}.$$

Using the half-angle identity:

$$2\cos^2(\frac{x}{2}) = 1 + \cos x = 1 - \frac{4}{5} = \frac{1}{5}.$$

$$\cos^2(\frac{x}{2}) = \frac{1}{10}.$$

$$\cos(\frac{x}{2}) = \pm\frac{1}{\sqrt{10}}.$$

Since $x/2$ is in Quadrant II, we choose the negative value.

$$\cos(\frac{x}{2}) = -\frac{1}{\sqrt{10}}.$$

The correct option is **(D)**.

10. $\cos^2 48° - \sin^2 12° =$ if $\sin 18° = \frac{\sqrt{5}-1}{4}$ **[MHT CET: 2023]**

(A) $\frac{-\sqrt{5}+1}{8}$ (B) $\frac{\sqrt{5}-1}{8}$ (C) $\frac{\sqrt{5}+1}{8}$ (D) $\frac{-\sqrt{5}-1}{8}$

Solution:

Concept Used:

- Product-to-Sum/Difference Identity: $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$.
- Standard trigonometric values, specifically $\cos 60°$ and $\cos 36°$.

Hint:

- Apply the identity $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$.
- Substitute the known values of the resulting cosine terms.

$$\begin{aligned}
&\text{The expression is } \cos^2 48° - \sin^2 12°.\\
&\text{Using the identity } \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B):\\
&= \cos(48° + 12°)\cos(48° - 12°).\\
&= \cos(60°)\cos(36°).\\
&\\
&\text{We know the standard values:}\\
&\cos 60° = \frac{1}{2}.\\
&\cos 36° = \frac{\sqrt{5}+1}{4}.\\
&\\
&\text{Substitute these values:}\\
&= \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}.\\
&\text{(Note: The value of } \sin 18° \text{ is provided but not needed if } \cos 36° \text{ is known.)}
\end{aligned}$$

The correct option is **(C)**.

11. If $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, then $\cos^2 48^\circ - \sin^2 12^\circ$ has the value: **[MHT CET: 2023]**

(A) $\frac{-\sqrt{5}+1}{8}$ (B) $\frac{\sqrt{5}-1}{8}$ (C) $\frac{\sqrt{5}+1}{8}$ (D) $\frac{-1-\sqrt{5}}{8}$

Solution:

Concept Used:

- Identity: $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$.
- Double angle identity: $\cos(2\theta) = 1 - 2\sin^2\theta$.

Hint:

- This is the same question as the previous one. The solution is identical. The value of $\sin 18^\circ$ can be used to derive the value of $\cos 36^\circ$.

$$\text{The expression is } \cos^2 48^\circ - \sin^2 12^\circ.$$
$$= \cos(48^\circ + 12^\circ)\cos(48^\circ - 12^\circ) = \cos(60^\circ)\cos(36^\circ).$$
$$= \frac{1}{2}\cos(36^\circ).$$

$$\text{We can derive } \cos 36^\circ \text{ from } \sin 18^\circ.$$
$$\cos(36^\circ) = \cos(2 \times 18^\circ) = 1 - 2\sin^2(18^\circ).$$
$$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2 = 1 - 2\left(\frac{5+1-2\sqrt{5}}{16}\right).$$
$$= 1 - \frac{6-2\sqrt{5}}{8} = \frac{8-(6-2\sqrt{5})}{8} = \frac{2+2\sqrt{5}}{8} = \frac{1+\sqrt{5}}{4}.$$

Substitute this value back:

$$\frac{1}{2}\cos(36^\circ) = \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}.$$

The correct option is **(C)**.

12. If $p = \tan 20°$, then value of $\frac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°}$ in terms of p is: **[MHT CET: 2022]**

(A) $\frac{1+p^2}{2p^2}$ (B) $\frac{1+p^2}{2p}$ (C) $\frac{1-p^2}{2p}$ (D) $\frac{1-p^2}{2p^2}$

Solution:

Concept Used:

- Tangent difference formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.
- Co-function identity: $\tan(90° - \theta) = \cot \theta$.
- Double angle identity for cotangent: $\cot(2\theta) = \frac{1 - \tan^2 \theta}{2 \tan \theta}$.

Hint:

- Recognize the expression as the formula for $\tan(A - B)$.
- Simplify the angle and use co-function and double angle identities to express the result in terms of $p = \tan 20°$.

The expression is of the form $\dfrac{\tan A - \tan B}{1 + \tan A \tan B} = \tan(A - B)$.
Here, $A = 160°$ and $B = 110°$.
The value is $\tan(160° - 110°) = \tan(50°)$.

We need to express $\tan 50°$ in terms of $p = \tan 20°$.
$\tan 50° = \tan(90° - 40°) = \cot 40°$.
$\cot 40° = \cot(2 \times 20°)$.

Using the double angle identity for cotangent:
$$\cot(2\theta) = \frac{1}{\tan(2\theta)} = \frac{1 - \tan^2 \theta}{2 \tan \theta}.$$
Let $\theta = 20°$.
$$\cot(40°) = \frac{1 - \tan^2 20°}{2 \tan 20°} = \frac{1 - p^2}{2p}.$$

The correct option is **(C)**.

13. $\frac{\sin^2(-160^\circ)}{\sin^2 70^\circ} + \frac{\sin(180^\circ - \theta)}{\sin\theta} =$ **[MHT CET: 2022]**

(A) $\sec^2(20^\circ)$ (B) $\cot^2(20^\circ)$ (C) $\tan^2(20^\circ)$ (D) $\text{cosec}^2(20^\circ)$

Solution:

Concept Used:

- Basic trigonometric identities: $\sin(-\theta) = -\sin\theta$, $\sin(180^\circ - \theta) = \sin\theta$, $\sin(90^\circ - \theta) = \cos\theta$.
- Pythagorean identity: $1 + \tan^2\theta = \sec^2\theta$.

Hint:

- Simplify each part of the expression using reduction and co-function formulas.

Consider the first term: $\dfrac{\sin^2(-160^\circ)}{\sin^2 70^\circ}$.

$\sin(-160^\circ) = -\sin(160^\circ) = -\sin(180^\circ - 20^\circ) = -\sin 20^\circ$.

$\sin^2(-160^\circ) = (-\sin 20^\circ)^2 = \sin^2 20^\circ$.

$\sin 70^\circ = \sin(90^\circ - 20^\circ) = \cos 20^\circ$.

$\sin^2 70^\circ = \cos^2 20^\circ$.

So, the first term is $\dfrac{\sin^2 20^\circ}{\cos^2 20^\circ} = \tan^2 20^\circ$.

Consider the second term: $\dfrac{\sin(180^\circ - \theta)}{\sin\theta}$.

$\sin(180^\circ - \theta) = \sin\theta$.

So, the second term is $\dfrac{\sin\theta}{\sin\theta} = 1$.

The entire expression is $\tan^2 20^\circ + 1$.

Using the Pythagorean identity $1 + \tan^2 A = \sec^2 A$,

The value is $\sec^2 20^\circ$.

The correct option is **(A)**.

14. The value of $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$ is: **[MHT CET: 2022]**

(A) $2\sin^2(\frac{\alpha-\beta}{2})$ (B) $2\cos^2(\frac{\alpha-\beta}{2})$ (C) $4\cos^2(\frac{\alpha-\beta}{2})$ (D) $4\sin^2(\frac{\alpha-\beta}{2})$

Solution:

Concept Used:

- Expansion of squares and Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.
- Cosine difference identity: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.
- Half-angle identity: $1 + \cos\theta = 2\cos^2(\theta/2)$.

Hint:

- Expand both squared terms.
- Group terms using Pythagorean identities and the cosine difference formula.
- Use the half-angle identity to get the final form.

$$
\begin{aligned}
&(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 \\
&= (\cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta) + (\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta). \\
&= (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta). \\
&= 1 + 1 + 2\cos(\alpha - \beta). \\
&= 2 + 2\cos(\alpha - \beta) = 2(1 + \cos(\alpha - \beta)). \\
\\
&\text{Using the half-angle identity } 1 + \cos\theta = 2\cos^2(\theta/2): \\
&= 2\left(2\cos^2\left(\frac{\alpha-\beta}{2}\right)\right). \\
&= 4\cos^2\left(\frac{\alpha-\beta}{2}\right).
\end{aligned}
$$

The correct option is **(C)**.

15. If $\tan\theta = \frac{a}{b}$ then $b\cos 2\theta + a\sin 2\theta =$ **[MHT CET: 2022]**

(A) b (B) a (C) 0 (D) 1

Solution:

Concept Used:

- Double angle identities in terms of tangent: $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.

Hint:

- Substitute the half-angle formulas for $\cos 2\theta$ and $\sin 2\theta$ into the expression.
- Replace $\tan\theta$ with a/b and simplify.

$$\begin{aligned}
&\text{The expression is } b\cos 2\theta + a\sin 2\theta.\\
&\text{Substitute the double angle identities in terms of } \tan\theta:\\
&= b\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + a\left(\frac{2\tan\theta}{1+\tan^2\theta}\right).\\
&\text{Substitute } \tan\theta = a/b:\\
&= b\left(\frac{1-(a/b)^2}{1+(a/b)^2}\right) + a\left(\frac{2(a/b)}{1+(a/b)^2}\right).\\
&= b\left(\frac{(b^2-a^2)/b^2}{(b^2+a^2)/b^2}\right) + a\left(\frac{2a/b}{(b^2+a^2)/b^2}\right).\\
&= b\left(\frac{b^2-a^2}{a^2+b^2}\right) + a\left(\frac{2ab}{a^2+b^2}\right).\\
&= \frac{b(b^2-a^2)+a(2ab)}{a^2+b^2}.\\
&= \frac{b^3-a^2b+2a^2b}{a^2+b^2} = \frac{b^3+a^2b}{a^2+b^2}.\\
&= \frac{b(b^2+a^2)}{a^2+b^2} = b.
\end{aligned}$$

The correct option is **(A)**.

16. If $\cot\alpha = \frac{1}{2}$ and $\sec\beta = -\frac{5}{3}$ where $\alpha \in (\pi, \frac{3\pi}{2})$ and $\beta \in (\frac{\pi}{2}, \pi)$, then $\tan(\alpha+\beta)$ has the value: **[MHT CET: 2022]**

(A) $\frac{3}{11}$ (B) $\frac{22}{9}$ (C) $\frac{9}{11}$ (D) $\frac{2}{11}$

Solution:

Concept Used:

- Determining the sign of trigonometric functions based on the quadrant.
- Tangent addition formula: $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$.

Hint:

- Find the values of $\tan\alpha$ and $\tan\beta$ using the given information and quadrant rules.
- Substitute these values into the tangent addition formula.

$$\begin{aligned}
&\text{For angle } \alpha : \alpha \in (\pi, 3\pi/2) \text{ is Quadrant III.}\\
&\text{In Q3, tangent and cotangent are positive.}\\
&\text{Given } \cot\alpha = 1/2 \implies \tan\alpha = 2.\\
\\
&\text{For angle } \beta : \beta \in (\pi/2, \pi) \text{ is Quadrant II.}\\
&\text{In Q2, secant and tangent are negative.}\\
&\text{Given } \sec\beta = -5/3.\\
&\text{We use the identity } \tan^2\beta = \sec^2\beta - 1.\\
&\tan^2\beta = (-5/3)^2 - 1 = 25/9 - 1 = 16/9.\\
&\tan\beta = \pm 4/3. \text{ Since } \beta \text{ is in Q2, } \tan\beta = -4/3.\\
\\
&\text{Now, calculate } \tan(\alpha+\beta):\\
&\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{2+(-4/3)}{1-(2)(-4/3)}.\\
&= \frac{2/3}{1+8/3} = \frac{2/3}{11/3} = \frac{2}{11}.
\end{aligned}$$

The correct option is **(D)**.

17. $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A =$ **[MHT CET: 2021]**

(A) $\tan 2A$ (B) $\cot A$ (C) $\tan A$ (D) $\cot 2A$

Solution:

Concept Used:

- Double angle identity for tangent: $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$, which implies $\cot(2\theta) = \frac{1-\tan^2\theta}{2\tan\theta}$.

Hint:

- Start from the right side of the expression.
- Repeatedly combine terms by expressing the cotangent term using the double angle identity for tangent.

The expression is $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$.

First, let's simplify the last two terms: $4\tan 4A + 8\cot 8A$.

$$= 4\tan 4A + 8\left(\frac{1-\tan^2 4A}{2\tan 4A}\right)$$

$$= 4\tan 4A + 4\left(\frac{1-\tan^2 4A}{\tan 4A}\right)$$

$$= \frac{4\tan^2 4A + 4(1-\tan^2 4A)}{\tan 4A}$$

$$= \frac{4\tan^2 4A + 4 - 4\tan^2 4A}{\tan 4A} = \frac{4}{\tan 4A} = 4\cot 4A.$$

Now the expression becomes: $\tan A + 2\tan 2A + 4\cot 4A$.

Let's simplify the last two terms: $2\tan 2A + 4\cot 4A$.

$$= 2\tan 2A + 4\left(\frac{1-\tan^2 2A}{2\tan 2A}\right)$$

$$= 2\tan 2A + 2\left(\frac{1-\tan^2 2A}{\tan 2A}\right)$$

$$= \frac{2\tan^2 2A + 2(1-\tan^2 2A)}{\tan 2A} = \frac{2}{\tan 2A} = 2\cot 2A.$$

The expression becomes: $\tan A + 2\cot 2A$.

$$= \tan A + 2\left(\frac{1-\tan^2 A}{2\tan A}\right)$$

$$= \tan A + \frac{1-\tan^2 A}{\tan A} = \frac{\tan^2 A + 1 - \tan^2 A}{\tan A} = \frac{1}{\tan A} = \cot A.$$

The correct option is **(B)**.

18. $\tan 3A \cdot \tan 2A \cdot \tan A =$ **[MHT CET: 2021]**

(A) $\tan 3A + \tan 2A - \tan A$
(B) $\tan 3A - \tan 2A - \tan A$
(C) $\tan 3A + \tan 2A + \tan A$
(D) $\tan 3A - \tan 2A + \tan A$

Solution:

Concept Used:

- Tangent addition formula: $\tan(X + Y) = \frac{\tan X + \tan Y}{1 - \tan X \tan Y}$.

Hint:

- Write $3A$ as $2A + A$.
- Apply the tangent addition formula to $\tan(2A + A)$.
- Rearrange the resulting equation to isolate the product $\tan 3A \tan 2A \tan A$.

We start with the identity for $\tan(3A)$.

$$\tan(3A) = \tan(2A + A).$$

$$\tan(3A) = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}.$$

Multiply both sides by the denominator:

$$\tan(3A)(1 - \tan 2A \tan A) = \tan 2A + \tan A.$$

$$\tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A.$$

Rearrange the terms to solve for the product:

$$\tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A.$$

The correct option is **(B)**.

19. If $\theta+\phi=\alpha$ and $\tan\theta = k\tan\phi$ (where $K>1$), then the value of $\sin(\theta-\phi)$ is: **[MHT CET: 2021]**

(A) $k\tan\phi$ (B) $\sin\alpha$ (C) $(\frac{k-1}{k+1})\sin\alpha$ (D) $k\cos\phi$

Solution:

Concept Used:

- Componendo and Dividendo rule.
- Sine addition and subtraction formulas.

Hint:

- Rewrite the condition $\tan\theta = k\tan\phi$ as $\frac{\tan\theta}{\tan\phi} = \frac{k}{1}$.
- Apply Componendo and Dividendo.
- Convert the tangent terms to sine and cosine and simplify to get the desired expression.

$$\text{Given } \tan\theta = k\tan\phi \implies \frac{\tan\theta}{\tan\phi} = \frac{k}{1}.$$

Applying Componendo and Dividendo:

$$\frac{\tan\theta - \tan\phi}{\tan\theta + \tan\phi} = \frac{k-1}{k+1}.$$

Now, express the left side in terms of sine and cosine:

$$\frac{\frac{\sin\theta}{\cos\theta} - \frac{\sin\phi}{\cos\phi}}{\frac{\sin\theta}{\cos\theta} + \frac{\sin\phi}{\cos\phi}} = \frac{\sin\theta\cos\phi - \cos\theta\sin\phi}{\sin\theta\cos\phi + \cos\theta\sin\phi}.$$

$$= \frac{\sin(\theta-\phi)}{\sin(\theta+\phi)}.$$

So, we have the relation:

$$\frac{\sin(\theta-\phi)}{\sin(\theta+\phi)} = \frac{k-1}{k+1}.$$

Given $\theta+\phi=\alpha$, so $\sin(\theta+\phi)=\sin\alpha$.

$$\frac{\sin(\theta-\phi)}{\sin\alpha} = \frac{k-1}{k+1}.$$

$$\sin(\theta-\phi) = \left(\frac{k-1}{k+1}\right)\sin\alpha.$$

The correct option is **(C)**.

20. $\frac{\sin A+\sin 7A+\sin 13A}{\cos A+\cos 7A+\cos 13A} =$ **[MHT CET: 2020]**

(A) $\cot 7A$ (B) $\tan 6A$ (C) $\tan 7A$ (D) $\cot 6A$

Solution:

Concept Used:

- Sum-to-Product formulas:
- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

Hint:

- Rearrange the terms in the numerator and denominator to group the first and last terms.
- Apply the sum-to-product formulas to these groups.
- Factor out the common term and simplify the fraction.

The expression is $\frac{(\sin 13A + \sin A) + \sin 7A}{(\cos 13A + \cos A) + \cos 7A}$.

Numerator:

$(\sin 13A + \sin A) + \sin 7A = 2\sin\left(\frac{13A + A}{2}\right)\cos\left(\frac{13A - A}{2}\right) + \sin 7A.$

$= 2\sin(7A)\cos(6A) + \sin 7A.$

$= \sin 7A(2\cos 6A + 1).$

Denominator:

$(\cos 13A + \cos A) + \cos 7A = 2\cos\left(\frac{13A + A}{2}\right)\cos\left(\frac{13A - A}{2}\right) + \cos 7A.$

$= 2\cos(7A)\cos(6A) + \cos 7A.$

$= \cos 7A(2\cos 6A + 1).$

The fraction becomes:

$\frac{\sin 7A(2\cos 6A + 1)}{\cos 7A(2\cos 6A + 1)}.$

$= \frac{\sin 7A}{\cos 7A} = \tan 7A.$

The correct option is **(C)**.

21. The value of $\sin^2(\frac{\pi}{8}) =$ **[MHT CET: 2020]**

(A) $\frac{\sqrt{2}+1}{2\sqrt{2}}$ (B) $\frac{\sqrt{5}+1}{2\sqrt{2}}$ (C) $\frac{\sqrt{5}-1}{2\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{2\sqrt{2}}$

Solution:

Concept Used:

- The half-angle identity for cosine: $\cos(2\theta) = 1 - 2\sin^2\theta$.

Hint:

- Rearrange the half-angle identity to solve for $\sin^2\theta$, which is $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$.
- Apply this formula with $\theta = \pi/8$.

$$\text{We use the identity } \cos(2\theta) = 1 - 2\sin^2\theta.$$
$$\text{Rearranging gives } 2\sin^2\theta = 1 - \cos(2\theta) \implies \sin^2\theta = \frac{1-\cos(2\theta)}{2}.$$
$$\text{Let } \theta = \frac{\pi}{8}. \text{ Then } 2\theta = \frac{\pi}{4}.$$
$$\sin^2\left(\frac{\pi}{8}\right) = \frac{1-\cos(\frac{\pi}{4})}{2}.$$
$$\text{We know that } \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}.$$
$$\sin^2\left(\frac{\pi}{8}\right) = \frac{1-\frac{1}{\sqrt{2}}}{2} = \frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}.$$

The correct option is **(D)**.

22. If $\sin x + \operatorname{cosec} x = 3$, then value of $\sin^4 x + \operatorname{cosec}^4 x$ is: **[MHT CET: 2020]**

(A) 7 (B) 47 (C) 14 (D) 49

Solution:

Concept Used:

- Algebraic identity: $(a+b)^2 = a^2 + b^2 + 2ab$.
- The reciprocal relationship $\operatorname{cosec} x = 1/\sin x$.

Hint:

- Let $y = \sin x$. The given equation is $y + \frac{1}{y} = 3$.
- Square the equation to find the value of $y^2 + \frac{1}{y^2}$.
- Square the result again to find the value of $y^4 + \frac{1}{y^4}$.

$$\text{Let } y = \sin x. \text{ Then } \operatorname{cosec} x = \frac{1}{y}.$$

$$\text{Given: } y + \frac{1}{y} = 3.$$

Square both sides:

$$\left(y + \frac{1}{y}\right)^2 = 3^2.$$

$$y^2 + 2(y)\left(\frac{1}{y}\right) + \frac{1}{y^2} = 9.$$

$$y^2 + 2 + \frac{1}{y^2} = 9 \implies y^2 + \frac{1}{y^2} = 7.$$

Square both sides again:

$$\left(y^2 + \frac{1}{y^2}\right)^2 = 7^2.$$

$$y^4 + 2(y^2)\left(\frac{1}{y^2}\right) + \frac{1}{y^4} = 49.$$

$$y^4 + 2 + \frac{1}{y^4} = 49 \implies y^4 + \frac{1}{y^4} = 47.$$

$$\text{Therefore, } \sin^4 x + \operatorname{cosec}^4 x = 47.$$

The correct option is **(B)**.

23. If $\sin\theta = \sin 15^\circ + \sin 45^\circ$, where $0^\circ < \theta < 180^\circ$ then $\theta =$ **[MHT CET: 2020]**

(A) 75° (B) 150° (C) 45° (D) 60°

Solution:

Concept Used:

- Sum-to-Product formula: $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$.
- Co-function identity: $\cos\theta = \sin(90^\circ - \theta)$.

Hint:

- Apply the sum-to-product formula to the right-hand side of the equation.
- Use co-function identities to express the result in terms of sine.
- Find the general solution for θ and select the value within the given interval.

$\sin\theta = \sin 15^\circ + \sin 45^\circ$.

Using the sum-to-product formula:

$$\sin 15^\circ + \sin 45^\circ = 2\sin\left(\frac{15^\circ + 45^\circ}{2}\right)\cos\left(\frac{45^\circ - 15^\circ}{2}\right).$$

$$= 2\sin(30^\circ)\cos(15^\circ).$$

$$= 2\left(\frac{1}{2}\right)\cos(15^\circ) = \cos(15^\circ).$$

So, $\sin\theta = \cos 15^\circ$.

Using the co-function identity, $\cos 15^\circ = \sin(90^\circ - 15^\circ) = \sin 75^\circ$.

$\sin\theta = \sin 75^\circ$.

From the given options, $\theta = 75^\circ$ is a valid solution.

The correct option is **(A)**.

24. If A, B, C, D are the angles of a cyclic quadrilateral taken in order, then $\cos A + \cos B + \cos C + \cos D =$ **[MHT CET: 2020]**

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0

Solution:

Concept Used:

- A key property of a cyclic quadrilateral is that its opposite angles are supplementary (add up to $180°$ or π radians).
- The identity $\cos(180° - \theta) = -\cos\theta$.

Hint:

- Use the cyclic quadrilateral property to relate angles A and C, and angles B and D.
- Substitute these relationships into the given expression.

$$\begin{aligned}
&\text{For a cyclic quadrilateral, opposite angles are supplementary.}\\
&A + C = 180° \implies C = 180° - A.\\
&B + D = 180° \implies D = 180° - B.\\
&\\
&\text{Now, consider the expression } \cos A + \cos B + \cos C + \cos D.\\
&\text{Substitute for C and D:}\\
&= \cos A + \cos B + \cos(180° - A) + \cos(180° - B).\\
&\text{Using the identity } \cos(180° - \theta) = -\cos\theta :\\
&= \cos A + \cos B - \cos A - \cos B.\\
&= 0.
\end{aligned}$$

The correct option is **(D)**.

25. If $\tan\theta + \cot\theta = 4$ then $\tan^4\theta + \cot^4\theta =$ **[MHT CET: 2020]**

(A) 194 (B) 110 (C) 80 (D) 191

Solution:

Concept Used:

- Algebraic identity: $(x+y)^2 = x^2 + y^2 + 2xy$.

Hint:

- Let $y = \tan\theta$. The given equation is $y + \frac{1}{y} = 4$.
- Square the equation to find $y^2 + \frac{1}{y^2}$.
- Square the result again to find $y^4 + \frac{1}{y^4}$.

$$\text{Given } \tan\theta + \cot\theta = 4.$$
$$\text{Let } y = \tan\theta, \text{ so } \cot\theta = 1/y.$$
$$y + \frac{1}{y} = 4.$$

$$\text{Squaring both sides:}$$
$$\left(y + \frac{1}{y}\right)^2 = 4^2.$$
$$y^2 + 2 + \frac{1}{y^2} = 16.$$
$$y^2 + \frac{1}{y^2} = 14.$$

$$\text{Squaring both sides again:}$$
$$\left(y^2 + \frac{1}{y^2}\right)^2 = 14^2.$$
$$y^4 + 2 + \frac{1}{y^4} = 196.$$
$$y^4 + \frac{1}{y^4} = 194.$$

$$\text{Therefore, } \tan^4\theta + \cot^4\theta = 194.$$

The correct option is **(A)**.

26. If A and B are supplementary angles, then $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} =$ **[MHT CET: 2020]**

(A) 1 (B) $\frac{1}{3}$ (C) 0 (D) $\frac{1}{2}$

Solution:

Concept Used:

- Supplementary angles add up to 180° or π radians.
- Co-function identity: $\sin(90° - \theta) = \cos\theta$.
- Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$.

Hint:

- Use the supplementary condition $A + B = 180°$ to express B in terms of A.
- Substitute this into the expression and simplify using trigonometric identities.

Given A and B are supplementary, so $A + B = 180°$.

$B = 180° - A$.

$$\frac{B}{2} = \frac{180° - A}{2} = 90° - \frac{A}{2}.$$

The expression is $\sin^2 \dfrac{A}{2} + \sin^2 \dfrac{B}{2}$.

Substitute for $B/2$:

$$= \sin^2 \frac{A}{2} + \sin^2\left(90° - \frac{A}{2}\right).$$

Using the identity $\sin(90° - \theta) = \cos\theta$:

$$= \sin^2 \frac{A}{2} + \cos^2 \frac{A}{2}.$$

Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$:

$$= 1.$$

The correct option is **(A)**.

27. $\frac{1-\sin\theta+\cos\theta}{1-\sin\theta-\cos\theta} =$ **[MHT CET: 2020]**

(A) $\cot\frac{\theta}{2}$ (B) $-\cot\frac{\theta}{2}$ (C) $\tan\frac{\theta}{2}$ (D) $-\tan\frac{\theta}{2}$

Solution:

Concept Used:

- Half-angle identities:
- $1+\cos\theta = 2\cos^2(\theta/2)$
- $1-\cos\theta = 2\sin^2(\theta/2)$
- $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$

Hint:

- Group the '1' and 'cos' terms in the numerator and denominator.
- Apply the half-angle identities and factor the resulting expressions.

$$\text{The expression is } \frac{(1+\cos\theta)-\sin\theta}{(1-\cos\theta)-\sin\theta}.$$

$$\text{Numerator: } (1+\cos\theta)-\sin\theta = 2\cos^2(\frac{\theta}{2}) - 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2}).$$
$$= 2\cos(\frac{\theta}{2})\left[\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right].$$

$$\text{Denominator: } (1-\cos\theta)-\sin\theta = 2\sin^2(\frac{\theta}{2}) - 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2}).$$
$$= 2\sin(\frac{\theta}{2})\left[\sin(\frac{\theta}{2}) - \cos(\frac{\theta}{2})\right].$$

$$\text{The fraction becomes: } \frac{2\cos(\frac{\theta}{2})\left[\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right]}{2\sin(\frac{\theta}{2})\left[\sin(\frac{\theta}{2}) - \cos(\frac{\theta}{2})\right]}.$$
$$= \frac{\cos(\frac{\theta}{2})}{\sin(\frac{\theta}{2})} \cdot \frac{\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})}{-\left(\cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2})\right)}.$$
$$= \cot(\frac{\theta}{2}) \cdot (-1) = -\cot(\frac{\theta}{2}).$$

The correct option is **(B)**.

28. $\operatorname{cosec} 2\theta - \cot 2\theta =$ **[MHT CET: 2020]**

(A) $\tan\theta$ (B) $\sin 2\theta$ (C) $\cos\theta$ (D) $\tan 2\theta$

Solution:

Concept Used:

- Reciprocal identities: $\operatorname{cosec} x = 1/\sin x$, $\cot x = \cos x/\sin x$.
- Double angle identities: $1 - \cos(2\theta) = 2\sin^2\theta$ and $\sin(2\theta) = 2\sin\theta\cos\theta$.

Hint:

- Convert the expression to sine and cosine terms and simplify.

$$\operatorname{cosec} 2\theta - \cot 2\theta = \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}.$$

$$= \frac{1 - \cos 2\theta}{\sin 2\theta}.$$

Using double angle identities:

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}.$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

The correct option is **(A)**.

29. If A and B are two angles such that A, $B \in (0, \pi)$ and they are not supplementary angles such that $\sin A - \sin B = 0$ then: **[MHT CET: 2020]**

(A) $A - B = \frac{\pi}{3}$ (B) $A - B = \frac{\pi}{2}$ (C) $A = B$ (D) $A \neq B$

Solution:

Concept Used:

- General solution for the equation $\sin A = \sin B$.

Hint:

- The condition $\sin A = \sin B$ implies either $A = B$ or $A + B = \pi$.
- Use the given constraints on A and B to determine which case is valid.

Given $\sin A - \sin B = 0 \implies \sin A = \sin B$.

The question states that A and B are not supplementary angles.
Therefore, Case 2 is rejected, and we are left with the only possibility from Case 1.
$A = B$.

The correct option is **(C)**.

30. $\cos(36^\circ - A)\cos(36^\circ + A) + \cos(54^\circ + A)\cos(54^\circ - A) =$ **[MHT CET: 2020]**

(A) $\cos 2A$ (B) $\sin 2A$ (C) $\cos A$ (D) $\sin A$

Solution:

Concept Used:

- Identity: $\cos(X - Y)\cos(X + Y) = \cos^2 X - \sin^2 Y$.
- Co-function and Pythagorean identities.
- Double angle identity for cosine.

Hint:

- Apply the identity $\cos(X - Y)\cos(X + Y)$ to both parts of the expression.
- Use the identity $\cos 54^\circ = \sin 36^\circ$ to simplify.

The expression can be written as:

$$\left[\cos^2(36^\circ) - \sin^2(A)\right] + \left[\cos^2(54^\circ) - \sin^2(A)\right].$$

$$= \cos^2 36^\circ + \cos^2 54^\circ - 2\sin^2 A.$$

Using the co-function identity $\cos 54^\circ = \cos(90^\circ - 36^\circ) = \sin 36^\circ$.

$$= \cos^2 36^\circ + \sin^2 36^\circ - 2\sin^2 A.$$

Using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$:

$$= 1 - 2\sin^2 A.$$

Using the double angle identity $\cos(2A) = 1 - 2\sin^2 A$:

$$= \cos 2A.$$

The correct option is **(A)**.

31. If $\sin\theta = \frac{-12}{13}$, $\cos\phi = \frac{-4}{5}$ and θ, ϕ lie in the third quadrant, then $\tan(\theta - \phi) =$ **[MHT CET: 2020]**

(A) $\frac{-33}{56}$ (B) $\frac{-56}{33}$ (C) $\frac{56}{33}$ (D) $\frac{33}{56}$

Solution:

Concept Used:

- Determining the signs and values of trigonometric functions in each quadrant.
- Tangent difference formula: $\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$.

Hint:

- Find $\cos\theta$ from $\sin\theta$ using $\sin^2\theta + \cos^2\theta = 1$, noting that cosine is negative in Q3.
- Find $\sin\phi$ from $\cos\phi$, noting that sine is negative in Q3.
- Calculate $\tan\theta$ and $\tan\phi$ and substitute into the difference formula.

For angle θ in Quadrant III:

$$\sin\theta = -12/13.$$

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-(-12/13)^2} = -\sqrt{1-144/169} = -\sqrt{25/169} = -5/13.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-12/13}{-5/13} = \frac{12}{5}.$$

For angle ϕ in Quadrant III:

$$\cos\phi = -4/5.$$

$$\sin\phi = -\sqrt{1-\cos^2\phi} = -\sqrt{1-(-4/5)^2} = -\sqrt{1-16/25} = -\sqrt{9/25} = -3/5.$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{-3/5}{-4/5} = \frac{3}{4}.$$

Now, find $\tan(\theta - \phi)$:

$$\tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi} = \frac{12/5 - 3/4}{1+(12/5)(3/4)}.$$

$$= \frac{\frac{48-15}{20}}{1+\frac{36}{20}} = \frac{33/20}{56/20} = \frac{33}{56}.$$

The correct option is **(D)**.

32. If $a = \sin 175° + \cos 175°$ then: **[MHT CET: 2020]**

(A) $a > 0$ (B) $a = 0$ (C) $a < 0$ (D) $a = 1$

Solution:

Concept Used:

- Signs of trigonometric functions in the second quadrant.
- Comparing values of sine and cosine for angles less than 45°.

Hint:

- Use reduction formulas to express the terms using an acute angle (e.g., 5°).
- Compare the magnitudes of $\sin 5°$ and $\cos 5°$ to determine the sign of their difference.

$a = \sin 175° + \cos 175°$.
The angle 175° is in Quadrant II.
In Q2, sine is positive and cosine is negative.

$\sin 175° = \sin(180° - 5°) = \sin 5°$.
$\cos 175° = \cos(180° - 5°) = -\cos 5°$.

So, $a = \sin 5° - \cos 5°$.

For any acute angle $\theta \in (0°, 45°)$, we know that $\cos\theta > \sin\theta$.
Since 5° is in this range, $\cos 5° > \sin 5°$.
Therefore, $\sin 5° - \cos 5°$ is a negative value.
So, $a < 0$.

The correct option is **(C)**.

33. $\sqrt{2+\sqrt{2+2\cos 4\theta}} =$ **[MHT CET: 2020]**

(A) $2\cos\theta$ (B) $\frac{\cos\theta}{2}$ (C) $\frac{\cos\theta}{\sqrt{2}}$ (D) $\sqrt{2}\cdot\cos\theta$

Solution:

Concept Used:

- The half-angle identity: $1+\cos(2A) = 2\cos^2 A$.

Hint:

- Work from the inside out.
- First, simplify $\sqrt{2+2\cos 4\theta}$ by factoring out 2 and using the half-angle identity.
- Repeat the process for the outer square root. Assume the angles are such that the cosine terms are positive.

$$\begin{aligned}
&\text{Start with the innermost term: } 2+2\cos 4\theta = 2(1+\cos 4\theta).\\
&\text{Using } 1+\cos(2A) = 2\cos^2 A, \text{ with } A = 2\theta:\\
&2(1+\cos 4\theta) = 2(2\cos^2 2\theta) = 4\cos^2 2\theta.\\
&\text{So, } \sqrt{2+2\cos 4\theta} = \sqrt{4\cos^2 2\theta} = 2|\cos 2\theta|.\\
&\text{Assuming } \cos 2\theta \geq 0, \text{ this is } 2\cos 2\theta.\\
\\
&\text{The expression becomes } \sqrt{2+2\cos 2\theta}.\\
&= \sqrt{2(1+\cos 2\theta)}.\\
&\text{Using the identity again with } A = \theta:\\
&= \sqrt{2(2\cos^2\theta)} = \sqrt{4\cos^2\theta} = 2|\cos\theta|.\\
\\
&\text{Assuming } \cos\theta \geq 0, \text{ the final answer is } 2\cos\theta.
\end{aligned}$$

The correct option is **(A)**.

34. If $A + B + C = 180°$, then the value of $\tan(\frac{A}{2})\tan(\frac{B}{2}) + \tan(\frac{B}{2})\tan(\frac{C}{2}) + \tan(\frac{C}{2})\tan(\frac{A}{2})$ is: **[MHT CET: 2020]**

(A) 1 (B) -2 (C) -1 (D) 2

Solution:

Concept Used:

- Properties of angles in a triangle.
- Tangent addition formula and co-function identities.

Hint:

- From $A + B + C = 180°$, we get $\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$.
- Apply the tangent function to both sides and use the formula for $\tan(X + Y)$.

Given $A + B + C = 180°$.

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°.$$

$$\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}.$$

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(90° - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right).$$

Using the tangent addition formula:

$$\frac{\tan(\frac{A}{2}) + \tan(\frac{B}{2})}{1 - \tan(\frac{A}{2})\tan(\frac{B}{2})} = \frac{1}{\tan(\frac{C}{2})}.$$

$$\tan(\frac{C}{2})\left[\tan(\frac{A}{2}) + \tan(\frac{B}{2})\right] = 1 - \tan(\frac{A}{2})\tan(\frac{B}{2}).$$

$$\tan(\frac{A}{2})\tan(\frac{C}{2}) + \tan(\frac{B}{2})\tan(\frac{C}{2}) = 1 - \tan(\frac{A}{2})\tan(\frac{B}{2}).$$

Rearranging the terms gives the required expression:

$$\tan(\frac{A}{2})\tan(\frac{B}{2}) + \tan(\frac{B}{2})\tan(\frac{C}{2}) + \tan(\frac{C}{2})\tan(\frac{A}{2}) = 1.$$

The correct option is **(A)**.

35. If A, B, C are angles of $\triangle ABC$, then $\tan 2A + \tan 2B + \tan 2C =$ **[MHT CET: 2020]**

(A) $\tan 2A \tan 3B \tan 2C$

(B) $\tan 2A \tan 2B \tan 2C$

(C) $\tan A \tan B \tan C$

(D) $\tan 3A \tan 2B \tan 2C$

Solution:

Concept Used:

- Property of angles in a triangle: $A + B + C = \pi$.
- Tangent addition formula for two angles.

Hint:

- Since $A + B + C = \pi$, we have $2A + 2B + 2C = 2\pi$.
- Let $X = 2A, Y = 2B, Z = 2C$. We have $X + Y + Z = 2\pi$.
- Use $\tan(X + Y) = \tan(2\pi - Z)$ to derive the identity.

Since A, B, C are angles of a triangle, $A + B + C = \pi$.

$\implies 2A + 2B + 2C = 2\pi$.

$2A + 2B = 2\pi - 2C$.

$\tan(2A + 2B) = \tan(2\pi - 2C)$.

Using the tangent addition formula on the LHS and periodicity on the RHS:

$$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = \tan(-2C) = -\tan 2C.$$

$\tan 2A + \tan 2B = -\tan 2C(1 - \tan 2A \tan 2B)$.

$\tan 2A + \tan 2B = -\tan 2C + \tan 2A \tan 2B \tan 2C$.

$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$.

The correct option is **(B)**.

36. $\sin(690^\circ) \times \sec(240^\circ) =$ **[MHT CET: 2020]**

(A) 1 (B) -1 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

Solution:

Concept Used:

- Trigonometric reduction formulas for large angles.
- $\sin(n \cdot 360^\circ - \theta) = \sin(-\theta) = -\sin\theta$.
- $\sec(180^\circ + \theta) = -\sec\theta$.

Hint:

- Reduce each angle to a corresponding acute angle in the first quadrant.

$$\sin(690^\circ) = \sin(2 \times 360^\circ - 30^\circ) = \sin(-30^\circ) = -\sin(30^\circ) = -\frac{1}{2}.$$
$$\sec(240^\circ) = \sec(180^\circ + 60^\circ) = -\sec(60^\circ) = -2.$$

$$\text{The product is } \left(-\frac{1}{2}\right) \times (-2) = 1.$$

The correct option is **(A)**.

37. If $x = 3\sin\theta, y = 3\cos\theta\cos\phi, z = 3\cos\theta\sin\phi$, then $x^2 + y^2 + z^2 =$ **[MHT CET: 2020]**

(A) 18 (B) 27 (C) 9 (D) 3

Solution:

Concept Used:

- The Pythagorean identity $\sin^2 A + \cos^2 A = 1$.

Hint:

- Calculate x^2, y^2, z^2 individually.
- For the sum $y^2 + z^2$, factor out the common term $9\cos^2\theta$ and use the Pythagorean identity.
- Add x^2 to the result and use the Pythagorean identity again.

$$x^2 = (3\sin\theta)^2 = 9\sin^2\theta.$$
$$y^2 = (3\cos\theta\cos\phi)^2 = 9\cos^2\theta\cos^2\phi.$$
$$z^2 = (3\cos\theta\sin\phi)^2 = 9\cos^2\theta\sin^2\phi.$$

$$y^2 + z^2 = 9\cos^2\theta\cos^2\phi + 9\cos^2\theta\sin^2\phi$$
$$= 9\cos^2\theta(\cos^2\phi + \sin^2\phi).$$
$$= 9\cos^2\theta(1) = 9\cos^2\theta.$$

$$x^2 + y^2 + z^2 = 9\sin^2\theta + 9\cos^2\theta.$$
$$= 9(\sin^2\theta + \cos^2\theta) = 9(1) = 9.$$

The correct option is **(C)**.

38. $\sin(\frac{\pi}{3}+x)-\cos(\frac{\pi}{6}+x)=$ **[MHT CET: 2020]**

(A) $\cos x$ (B) $\sin x$ (C) $-\cos x$ (D) $\sin x$

Solution:

Concept Used:

- Sum and difference identities for sine and cosine.
- $\sin(A+B)=\sin A\cos B+\cos A\sin B$.
- $\cos(A+B)=\cos A\cos B-\sin A\sin B$.

Hint:

- Expand both terms using the sum identities and simplify.

$$\text{First term: } \sin(\frac{\pi}{3}+x)=\sin(\frac{\pi}{3})\cos x+\cos(\frac{\pi}{3})\sin x=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x.$$

$$\text{Second term: } \cos(\frac{\pi}{6}+x)=\cos(\frac{\pi}{6})\cos x-\sin(\frac{\pi}{6})\sin x=\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x.$$

The expression is the difference between these two:

$$\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)-\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$$

$$=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x$$

$$=\sin x.$$

The correct option is **(D)**.

39. If $\tan\theta + \sin\theta = a$ and $\tan\theta - \sin\theta = b$ then the values of $\cot\theta$ and $\operatorname{cosec}\theta$ are respectively: **[MHT CET: 2020]**

(A) $\frac{1}{a+b}, \frac{1}{a-b}$ (B) $\frac{2}{a+b}, \frac{2}{a-b}$ (C) $\frac{2}{a-b}, \frac{2}{a+b}$ (D) $\frac{1}{a-b}, \frac{1}{a+b}$

Solution:

Concept Used:

- Solving a system of linear equations.
- Reciprocal identities for trigonometric functions.

Hint:

- Add the two given equations to find an expression for $\tan\theta$.
- Subtract the second equation from the first to find an expression for $\sin\theta$.
- Take the reciprocals to find $\cot\theta$ and $\operatorname{cosec}\theta$.

Given the two equations:

$$\tan\theta + \sin\theta = a \quad \cdots(1)$$
$$\tan\theta - \sin\theta = b \quad \cdots(2)$$

Add (1) and (2):

$$2\tan\theta = a + b \implies \tan\theta = \frac{a+b}{2}.$$

$$\text{Therefore, } \cot\theta = \frac{1}{\tan\theta} = \frac{2}{a+b}.$$

Subtract (2) from (1):

$$2\sin\theta = a - b \implies \sin\theta = \frac{a-b}{2}.$$

$$\text{Therefore, } \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{2}{a-b}.$$

$$\text{The values are } \cot\theta = \frac{2}{a+b} \text{ and } \operatorname{cosec}\theta = \frac{2}{a-b}.$$

The correct option is **(B)**.

40. $\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} =$ **[MHT CET: 2020]**

(A) -2 (B) 0 (C) -1 (D) 1

Solution:

Concept Used:

- The identity $\frac{1-\tan A}{1+\tan A} = \tan(45^\circ - A)$.
- The identity $\tan(180^\circ - A) = -\tan A$.

Hint:

- For the first term, divide the numerator and denominator by $\cos 12^\circ$ to express it in terms of $\tan 12^\circ$.
- The second term is simply $\tan 147^\circ$.
- Use reduction formulas to show that the two terms cancel each other out.

Consider the first term:

$$\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} = \frac{1 - \frac{\sin 12^\circ}{\cos 12^\circ}}{1 + \frac{\sin 12^\circ}{\cos 12^\circ}} = \frac{1 - \tan 12^\circ}{1 + \tan 12^\circ}.$$

Using the identity $\tan(45^\circ - A) = \dfrac{\tan 45^\circ - \tan A}{1 + \tan 45^\circ \tan A} = \dfrac{1 - \tan A}{1 + \tan A}$.

The first term is equal to $\tan(45^\circ - 12^\circ) = \tan 33^\circ$.

Consider the second term:

$$\frac{\sin 147^\circ}{\cos 147^\circ} = \tan 147^\circ.$$

$$\tan 147^\circ = \tan(180^\circ - 33^\circ) = -\tan 33^\circ.$$

The expression becomes:

$$\tan 33^\circ + (-\tan 33^\circ) = 0.$$

The correct option is **(B)**.

41. $\cos x \cdot \cos 7x - \cos 5x \cdot \cos 13x =$ **[MHT CET: 2020]**

(A) $2\cos^2 6x \cdot \cos 12x$ (B) $2\sin^2 6x \cdot \cos 6x$ (C) $2\sin 6x \cdot \sin 12x$ (D) $2\sin 6x \cdot \cos 12x$

Solution:

Concept Used:

- Product-to-Sum Identity: $2\cos A \cos B = \cos(A+B) + \cos(A-B)$.
- Sum-to-Product Identity: $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.

Hint:

- Apply the product-to-sum formula to both terms in the expression.
- Simplify the resulting expression and then use a sum-to-product formula to get the final form.

The expression is $\cos x \cos 7x - \cos 5x \cos 13x$.

$$= \frac{1}{2}(2\cos 7x \cos x) - \frac{1}{2}(2\cos 13x \cos 5x).$$
$$= \frac{1}{2}[\cos(7x + x) + \cos(7x - x)] - \frac{1}{2}[\cos(13x + 5x) + \cos(13x - 5x)].$$
$$= \frac{1}{2}[\cos 8x + \cos 6x] - \frac{1}{2}[\cos 18x + \cos 8x].$$
$$= \frac{1}{2}(\cos 8x + \cos 6x - \cos 18x - \cos 8x).$$
$$= \frac{1}{2}(\cos 6x - \cos 18x).$$

Using the identity $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$:

$$= \frac{1}{2}\left[-2\sin\left(\frac{6x+18x}{2}\right)\sin\left(\frac{6x-18x}{2}\right)\right].$$
$$= -\sin(12x)\sin(-6x)$$
$$= \sin(12x)\sin(6x).$$
$$= [2\sin(6x)\cos(6x)]\sin(6x)$$
$$= 2\sin^2(6x)\cos(6x)$$

The correct option is **(B)**.

42. If $\cos x + \cos y = -\cos\alpha$, $\sin x + \sin y = -\sin\alpha$, then $\cot(\frac{x+y}{2}) =$ **[MHT CET: 2020]**

(A) $\sin\alpha$ (B) $\cot\alpha$ (C) $\tan\alpha$ (D) $\cos\alpha$

Solution:

Concept Used:

- Sum-to-Product formulas:
- $\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$
- $\sin x + \sin y = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$

Hint:

- Apply the sum-to-product formulas to both given equations.
- Divide the first resulting equation by the second one to find the value of $\cot(\frac{x+y}{2})$.

$$\text{Given } \cos x + \cos y = -\cos\alpha.$$
$$\implies 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\cos\alpha \quad \cdots(1).$$

$$\text{Given } \sin x + \sin y = -\sin\alpha.$$
$$\implies 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\sin\alpha \quad \cdots(2).$$

Divide equation (1) by equation (2):
$$\frac{2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})}{2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})} = \frac{-\cos\alpha}{-\sin\alpha}.$$
$$\frac{\cos(\frac{x+y}{2})}{\sin(\frac{x+y}{2})} = \frac{\cos\alpha}{\sin\alpha}.$$
$$\cot\left(\frac{x+y}{2}\right) = \cot\alpha.$$

The correct option is **(B)**.

43. If $\tan\theta = \frac{1}{3}$ then $\cos 2\theta =$ **[MHT CET: 2020]**

(A) $\frac{1}{4}$ (B) $\frac{1}{10}$ (C) $\frac{1}{5}$ (D) $\frac{4}{5}$

Solution:

Concept Used:

- Double angle identity for cosine in terms of tangent: $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$.

Hint:

- Directly substitute the given value of $\tan\theta$ into the formula for $\cos 2\theta$.

$$\text{We use the identity } \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}.$$

$$\text{Given } \tan\theta = \frac{1}{3}.$$

$$\cos 2\theta = \frac{1-(1/3)^2}{1+(1/3)^2} = \frac{1-1/9}{1+1/9}.$$

$$= \frac{8/9}{10/9} = \frac{8}{10} = \frac{4}{5}.$$

The correct option is **(D)**.

44. If $\sec\theta = \frac{13}{12}$ lies in 4^{th} quadrant, then $\tan\theta \times \operatorname{cosec}\theta \times \sin\theta \times \cos\theta =$ **[MHT CET: 2020]**

(A) $-\frac{5}{13}$ (B) $\frac{144}{169}$ (C) $\frac{25}{169}$ (D) $\frac{5}{13}$

Solution:

Concept Used:

- Basic trigonometric identities and definitions.
- Signs of trigonometric functions in the 4th quadrant.

Hint:

- Simplify the expression first. Notice that some terms cancel out.
- Determine the value of the remaining function using the given information about $\sec\theta$ and the quadrant.

Let's simplify the expression first:

$$\tan\theta \times \operatorname{cosec}\theta \times \sin\theta \times \cos\theta$$
$$= \left(\frac{\sin\theta}{\cos\theta}\right) \times \left(\frac{1}{\sin\theta}\right) \times \sin\theta \times \cos\theta.$$
$$= \sin\theta.$$

We are given $\sec\theta = \dfrac{13}{12}$ and θ is in the 4th quadrant.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{12}{13}.$$

In the 4th quadrant, sine is negative.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{12}{13}\right)^2}.$$
$$= -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{169-144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}.$$

The correct option is **(A)**.

45. $\sec 2\theta - \tan 2\theta =$ **[MHT CET: 2020]**

(A) $\tan(\frac{\pi}{4} - \theta)$ (B) $\tan 2\theta$ (C) $\cot 2\theta$ (D) $\cot(\frac{\pi}{4} - \theta)$

Solution:

Concept Used:

- Double angle and Pythagorean identities.
- Tangent difference formula.

Hint:

- Express the function in terms of sine and cosine.
- Use the identities $1 = \cos^2\theta + \sin^2\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$, and $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

$$\sec 2\theta - \tan 2\theta = \frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta} = \frac{1 - \sin 2\theta}{\cos 2\theta}.$$

Numerator: $1 - \sin 2\theta = (\cos^2\theta + \sin^2\theta) - 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)^2$.

Denominator: $\cos 2\theta = \cos^2\theta - \sin^2\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$.

The expression becomes $\dfrac{(\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$.

Divide numerator and denominator by $\cos\theta$:

$$= \frac{1 - \tan\theta}{1 + \tan\theta}.$$

This is the formula for $\tan(45^\circ - \theta)$ or $\tan(\frac{\pi}{4} - \theta)$.

The correct option is **(A)**.

46. If $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$ then $A + B =$ **[MHT CET: 2020]**

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Solution:

Concept Used:

- The tangent addition formula: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

Hint:

- Apply the formula for $\tan(A+B)$ using the given values.
- The result will be a standard value, from which $A+B$ can be determined.

We use the tangent addition formula:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Substitute the given values:

$$\tan(A+B) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \cdot \frac{1}{11}} = \frac{\frac{55+6}{66}}{1 - \frac{5}{66}}.$$

$$= \frac{\frac{61}{66}}{\frac{66-5}{66}} = \frac{61/66}{61/66} = 1.$$

Since $\tan(A+B) = 1$, one possible value is $A + B = \dfrac{\pi}{4}$.

As $\tan A$ and $\tan B$ are positive, we can assume A and B are acute angles, so their sum is also acute.

The correct option is **(D)**.

47. The maximum value of the function $y = e^{5+\sqrt{3}\sin x+\cos x}$ is: **[MHT CET: 2020]**

(A) e^7 (B) e^2 (C) e^5 (D) e^8

Solution:

Concept Used:

- The maximum value of the expression $a\sin x + b\cos x$ is $\sqrt{a^2+b^2}$.
- The exponential function e^u is an increasing function, so it is maximized when its exponent u is maximized.

Hint:

- Find the maximum value of the exponent, $5 + \sqrt{3}\sin x + \cos x$.
- The maximum value of the function will be e raised to this maximum exponent.

The function is $y = e^{5+\sqrt{3}\sin x+\cos x}$.
To maximize y, we must maximize the exponent $E(x) = 5 + \sqrt{3}\sin x + \cos x$.

Consider the part $\sqrt{3}\sin x + 1\cos x$.
This is of the form $a\sin x + b\cos x$, where $a = \sqrt{3}$ and $b = 1$.
The maximum value of this part is $\sqrt{a^2+b^2} = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{3+1} = \sqrt{4} = 2$.

The maximum value of the exponent E(x) is $5 + 2 = 7$.
Therefore, the maximum value of the function y is e^7.

The correct option is **(A)**.

48. If $\theta = \frac{17\pi}{3}$ then $(\tan\theta - \cot\theta)$ is: **[MHT CET: 2019]**

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{-1}{2\sqrt{3}}$ (C) $\frac{2}{\sqrt{3}}$ (D) $-\frac{2}{\sqrt{3}}$

Solution:

Concept Used:

- Periodicity of trigonometric functions: $\tan(\theta + n\pi) = \tan\theta$.
- Values of trigonometric functions for standard angles.

Hint:

- Reduce the angle $\frac{17\pi}{3}$ to an equivalent angle in the range $[0, 2\pi)$.
- Find the values of $\tan\theta$ and $\cot\theta$ and compute their difference.

$$\theta = \frac{17\pi}{3} = \frac{18\pi - \pi}{3} = 6\pi - \frac{\pi}{3}.$$
$$\tan\theta = \tan(6\pi - \frac{\pi}{3}) = \tan(-\frac{\pi}{3}) = -\tan(\frac{\pi}{3}) = -\sqrt{3}.$$
$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{\sqrt{3}}.$$

$$\tan\theta - \cot\theta = -\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right) = -\sqrt{3} + \frac{1}{\sqrt{3}}.$$
$$= \frac{-3+1}{\sqrt{3}} = -\frac{2}{\sqrt{3}}.$$

The correct option is **(D)**.

49. In $\triangle ABC$, if $\tan A + \tan B + \tan C = 6$ and $\tan A \cdot \tan B = 2$ then $\tan C =$ **[MHT CET: 2019]**

(A) 3 (B) 4 (C) 1 (D) 2

Solution:

Concept Used:

- For any triangle ABC, $A + B + C = \pi$. This leads to the identity $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Hint:

- Use the identity for the sum of tangents in a triangle.
- Substitute the given values into the identity to solve for $\tan C$.

$$\begin{aligned}
&\text{In any } \triangle ABC, \text{ we have the identity } \tan A + \tan B + \tan C = \tan A \tan B \tan C. \\
&\text{We are given } \tan A + \tan B + \tan C = 6. \\
&\text{Therefore, } \tan A \tan B \tan C = 6. \\
\\
&\text{We are also given } \tan A \tan B = 2. \\
&\text{Substitute this into the previous equation:} \\
&(2) \tan C = 6. \\
&\tan C = \frac{6}{2} = 3.
\end{aligned}$$

The correct option is **(A)**.

50. $\frac{1-2(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ} =$ **[MHT CET: 2019]**

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{2}$

Solution:

Concept Used:

- Value of $\cos 60^\circ$.
- Co-function identity $\cos(90^\circ - \theta) = \sin\theta$.

Hint:

- Substitute the value of $\cos 60^\circ$ and simplify the numerator.
- Use the co-function identity to relate $\cos 80^\circ$ and $\sin 10^\circ$.

$$\text{The expression is } \frac{1-2(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ}.$$
$$\text{Since } \cos 60^\circ = 1/2:$$
$$= \frac{1-2(\frac{1}{2} - \cos 80^\circ)}{2\sin 10^\circ}.$$
$$= \frac{1-1+2\cos 80^\circ}{2\sin 10^\circ} = \frac{2\cos 80^\circ}{2\sin 10^\circ} = \frac{\cos 80^\circ}{\sin 10^\circ}.$$

$$\text{Using the co-function identity } \cos 80^\circ = \cos(90^\circ - 10^\circ) = \sin 10^\circ.$$
$$= \frac{\sin 10^\circ}{\sin 10^\circ} = 1.$$

The correct option is **(B)**.

51. If A, B, C are the angles of $\triangle ABC$ then $\cot A \cot B + \cot B \cot C + \cot C \cot A =$ **[MHT CET: 2018]**

(A) 0 (B) 1 (C) 2 (D) -1

Solution:

Concept Used:

- For angles of a triangle, $A + B + C = \pi$.
- The identity for $\tan(A + B)$.

Hint:

- Start with $A + B = \pi - C$. Apply the tangent function to both sides.
- Use the $\tan(A + B)$ formula and rearrange the terms.
- Divide the resulting identity by $\tan A \tan B \tan C$ to get the desired result in terms of cotangent.

Since A, B, C are angles of a triangle, $A + B + C = \pi \implies A + B = \pi - C$.

$\tan(A + B) = \tan(\pi - C) = -\tan C$.

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C.$$

$\tan A + \tan B = -\tan C(1 - \tan A \tan B)$.

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$.

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Assuming none of the tangents are zero, divide the entire equation by $\tan A \tan B \tan C$:

$$\frac{1}{\tan B \tan C} + \frac{1}{\tan A \tan C} + \frac{1}{\tan A \tan B} = 1.$$

$\cot B \cot C + \cot A \cot C + \cot A \cot B = 1$.

The correct option is **(B)**.

52. If $2\sin(\theta + \frac{\pi}{3}) = \cos(\theta - \frac{\pi}{6})$, then $\tan\theta =$ **[MHT CET: 2018]**

(A) $\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$

Solution:

Concept Used:

- Sum and difference formulas for sine and cosine.

Hint:

- Expand both sides of the equation using the appropriate sum/difference formulas.
- Group the terms with $\sin\theta$ and $\cos\theta$ and simplify to find $\tan\theta$.

$$\text{Given } 2\sin(\theta + \frac{\pi}{3}) = \cos(\theta - \frac{\pi}{6}).$$
$$2[\sin\theta\cos(\pi/3) + \cos\theta\sin(\pi/3)] = \cos\theta\cos(\pi/6) + \sin\theta\sin(\pi/6).$$
$$2[\sin\theta \cdot \frac{1}{2} + \cos\theta \cdot \frac{\sqrt{3}}{2}] = \cos\theta \cdot \frac{\sqrt{3}}{2} + \sin\theta \cdot \frac{1}{2}.$$
$$\sin\theta + \sqrt{3}\cos\theta = \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta.$$

$$\text{Group terms with } \sin\theta \text{ and } \cos\theta:$$
$$\sin\theta - \frac{1}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta - \sqrt{3}\cos\theta.$$
$$\frac{1}{2}\sin\theta = -\frac{\sqrt{3}}{2}\cos\theta.$$
$$\sin\theta = -\sqrt{3}\cos\theta.$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\sqrt{3}.$$

The correct option is **(D)**.

53. $\sin(\frac{\pi}{3}+x)-\cos(\frac{\pi}{6}+x)=$ **[MHT CET: 2020]**

(A) $\cos x$ (B) $\sin x$ (C) $-\cos x$ (D) $\sin x$

Solution:

Concept Used:

- Sum identities for sine and cosine.

Hint:

- Expand both terms using their respective sum identities and simplify.

$$\text{First term: } \sin(\frac{\pi}{3}+x)=\sin(\frac{\pi}{3})\cos x+\cos(\frac{\pi}{3})\sin x=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x.$$

$$\text{Second term: } \cos(\frac{\pi}{6}+x)=\cos(\frac{\pi}{6})\cos x-\sin(\frac{\pi}{6})\sin x=\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x.$$

The expression is the difference:

$$\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)-\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$$

$$=\frac{1}{2}\sin x+\frac{1}{2}\sin x=\sin x.$$

The correct option is **(D)**.

54. If $\tan\theta + \sin\theta = a$ and $\tan\theta - \sin\theta = b$ then the values of $\cot\theta$ and $\operatorname{cosec}\theta$ are respectively: **[MHT CET: 2020]**

(A) $\frac{1}{a+b}, \frac{1}{a-b}$ (B) $\frac{2}{a+b}, \frac{2}{a-b}$ (C) $\frac{2}{a-b}, \frac{2}{a+b}$ (D) $\frac{1}{a-b}, \frac{1}{a+b}$

Solution:

Concept Used:

- Solving a system of linear equations.
- Reciprocal identities: $\cot\theta = 1/\tan\theta$, $\operatorname{cosec}\theta = 1/\sin\theta$.

Hint:

- Add the two equations to solve for $\tan\theta$ in terms of a and b.
- Subtract the second equation from the first to solve for $\sin\theta$.
- Find the reciprocals to get the required values.

Given equations:

$$\tan\theta + \sin\theta = a \quad \cdots(1)$$
$$\tan\theta - \sin\theta = b \quad \cdots(2)$$

Adding (1) and (2):

$$2\tan\theta = a + b \implies \tan\theta = \frac{a+b}{2}.$$
$$\implies \cot\theta = \frac{2}{a+b}.$$

Subtracting (2) from (1):

$$2\sin\theta = a - b \implies \sin\theta = \frac{a-b}{2}.$$
$$\implies \operatorname{cosec}\theta = \frac{2}{a-b}.$$

The values are $\frac{2}{a+b}$ and $\frac{2}{a-b}$ respectively.

The correct option is **(B)**.