



Topic: Sequence and Series

Sub: Mathematics

JEE Main PYQ (2019-2025)

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Arithmetic Progression (A.P.)

Finding Terms, Common Difference, and Basic Properties

1. The interior angles of a polygon with n sides, are in an A.P. with common difference 6° . If the largest interior angle of the polygon is 219° then n is equal to: **[JEE Main 2025]**
2. The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$ is: **[JEE Main 2024]**
(1) -118 (2) -110 (3) -115 (4) -100
3. In an A.P., the sixth term $a_6 = 2$. If $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P., is equal to: **[JEE Main 2024]**
(1) $\frac{8}{5}$ (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$
4. Let a_1, a_2, \dots, a_n be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}+\sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}+\sqrt{a_{18}}}}\right)$ is equal to: **[JEE Main 2023]**
5. Let a_1, a_2, a_3, \dots be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to: **[JEE Main 2023]**
(1) 24 (2) $\frac{33}{4}$ (3) $\frac{381}{4}$ (4) 9
6. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is: **[JEE Main 2022]**
7. The common difference of the A.P. b_1, b_2, \dots, b_m is 2 more than the common difference of A.P. a_1, a_2, \dots, a_n . If $a_{40} = -159, a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to: **[JEE Main 2020]**
(a) 81 (b) -127 (c) -81 (d) 127
8. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is: **[JEE Main 2019]**
(a) 4 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

Sum of n Terms of an A.P. (S_n)

9. Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is: **[JEE Main 2025]**
(1) 90 (2) 84 (3) 122 (4) 108
10. Let $a_1, a_2, \dots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + \dots + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \dots + a_{2024}$ is equal to: **[JEE Main 2025]**

11. Let T_r be the r^{th} term of an A.P. If for some m , $n \in \mathbb{N}$, $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$ and $\sum_{r=1}^{25} T_r = 13$, then $5m \sum_{r=m}^{2m} T_r$ is equal to: [JEE Main 2025]
- (1) 98 (2) 126 (3) 142 (4) 112
12. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to: [JEE Main 2025]
- (1) 525 (2) 510 (3) 515 (4) 505
13. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to: [JEE Main 2025]
- (1) -1080 (2) -1020 (3) -1200 (4) -120
14. Suppose that the number of terms in an A.P. is $2k$, $k \in \mathbb{N}$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to: [JEE Main 2025]
- (1) 6 (2) 5 (3) 8 (4) 4
15. Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$ then a_n is equal to: [JEE Main 2025]
- (A) 65 (B) 56 (C) 70 (D) 64
16. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is: [JEE Main 2025]
- (A) 6 (B) 4 (C) 8 (D) 10
17. Let a_1, a_2, a_3, \dots be an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$, $a_1 \neq 0$. If $\sum_{k=1}^n a_k = 0$, then n is: [JEE Main 2025]
- (A) 18 (B) 17 (C) 11 (D) 10
18. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to: [JEE Main 2024]
- (1) 800 (2) 890 (3) 790 (4) 690
19. Let S_n denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is: [JEE Main 2024]
- (1) 395 (2) 390 (3) 405 (4) 410
20. Let $s_1, s_2, s_3, \dots, s_{10}$ respectively be the sum of 12 terms of 10 A.P.s whose first terms are $1, 2, 3, \dots, 10$ and the common differences are $1, 3, 5, \dots, 19$ respectively. Then $\sum_{i=1}^{10} s_i$ is equal to: [JEE Main 2023]
- (1) 7220 (2) 7360 (3) 7260 (4) 7380
21. Let $a_1 = 8, a_2, a_3, \dots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is: [JEE Main 2023]
22. Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms to the sum of first nine terms of the progression is $5 : 17$ and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to: [JEE Main 2022]
- (A) 290 (B) 380 (C) 460 (D) 510
23. If $\{a_i\}_{i=1}^n$ where n is an even integer is an arithmetic progression with common difference 1, and $\sum_{i=1}^n a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to: [JEE Main 2022]
- (A) 48 (B) 96 (C) 92 (D) 104

24. Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to: [JEE Main 2021]
- (1) 1000 (2) 7000 (3) 5000 (4) 3000
25. Let S_n denote the sum of first n -terms of an arithmetic progression. If $S_{10} = 530, S_5 = 140$, then $S_{20} - S_6$ is equal to: [JEE Main 2021]
- (1) 1862 (2) 1842 (3) 1852 (4) 1872
26. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is: [JEE Main 2021]
- (1) 6 (2) 4 (3) 2 (4) 8
27. Let a_1, a_2, \dots, a_{10} be an AP with common difference -3 and b_1, b_2, \dots, b_{10} be a GP with common ratio 2 . Let $c_k = a_k + b_k, k = 1, 2, \dots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to: [JEE Main 2021]
28. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is: [JEE Main 2020]
- (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{7}$
29. In the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ upto n^{th} term is 488 and then n^{th} term is negative, then : [JEE Main 2020]
- (a) $n = 60$ (b) n^{th} term is -4
(c) $n = 41$ (d) n^{th} term is $-4\frac{2}{5}$
30. If the sum of first 11 terms of an A.P., a_1, a_2, a_3, \dots is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \dots, a_{23}$ is ka_1 , where k is equal to: [JEE Main 2020]
- (a) $-\frac{121}{10}$ (b) $\frac{121}{10}$ (c) $\frac{72}{5}$ (d) $-\frac{72}{5}$
31. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is: [JEE Main 2020]
- (a) 50 (b) $50\frac{1}{4}$ (c) 100 (d) $100\frac{1}{2}$
32. Five numbers are in A.P., whose sum is 25 and product is 2520 . If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: [JEE Main 2020]
- (a) 27 (b) 7 (c) $\frac{21}{2}$ (d) 16
33. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to: [JEE Main 2019]
- (a) -260 (b) -410 (c) -320 (d) -380
34. If a_1, a_2, a_3, \dots are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is: [JEE Main 2019]
- (a) 200 (b) 280 (c) 120 (d) 150
35. If a_1, a_2, \dots, a_n are in A.P. and $a_1 + a_4 + a_7 + \dots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to: [JEE Main 2019]
- (a) 98 (b) 76 (c) 38 (d) 64
36. If the sum and product of the first three terms in an A.P. are 33 and 1155 , respectively, then a value of its 11^{th} term is: [JEE Main 2019]
- (a) -35 (b) 25 (c) -36 (d) -25
37. Let a_1, a_2, \dots, a_{30} be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to: [JEE Main 2019]
- (a) 52 (b) 57 (c) 47 (d) 42

Relationship between Terms and Sums (T_n vs S_n)

38. Let $\langle a_n \rangle$ be a sequence such that $a_1 + a_2 + \dots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $\sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \dots p_m$, where p_1, p_2, \dots, p_m are the first m prime numbers, then m is equal to: [JEE Main 2023]
(1) 5 (2) 8 (3) 6 (4) 7
39. Let a_1, a_2, a_3, \dots be an A. P. If $\frac{a_1+a_2+\dots+a_{10}}{a_1+a_2+\dots+a_p} = \frac{100}{p^2}, p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to: [JEE Main 2021]
(1) $\frac{19}{21}$ (2) $\frac{100}{121}$ (3) $\frac{21}{19}$ (4) $\frac{121}{100}$
40. Let a_1, a_2, \dots, a_n be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \dots + a_n$. If $a_1 = 1, a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to: [JEE Main 2020]
(a) (2490,249) (b) (2480,249) (c) (2480,248) (d) (2490,248)
41. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, \dots be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to: [JEE Main 2019]
(a) (50, 50 + 46A) (b) (50, 50 + 45A)
(c) (A, 50 + 45A) (d) (A, 50 + 46A)

Problems involving two or more A.P.s / Common Terms

42. Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is: [JEE Main 2025]
(A) 3814 (B) 4003 (C) 4027 (D) 3761
43. Let 3, 7, 11, 15, ..., 403 and 2, 5, 8, 11, ..., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to: [JEE Main 2024]
44. The number of common terms in the progressions 4, 9, 14, 19, ..., up to 25th term and 3, 6, 9, 12, ..., up to 37th term is: [JEE Main 2024]
(1) 9 (2) 5 (3) 7 (4) 8
45. The 8th common term of the series $S_1 = 3 + 7 + 11 + 15 + 19 + \dots$ and $S_2 = 1 + 6 + 11 + 16 + 21 + \dots$ is: [JEE Main 2023]
46. The sum of the common terms of the following three arithmetic progressions. 3, 7, 11, 15, ..., 399, 2, 5, 8, 11, ..., 359 and 2, 7, 12, 17, ..., 197, is equal to: [JEE Main 2023]
47. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to: [JEE Main 2022]
48. Let a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are A.P. and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$ then $a_4 b_4$ is equal to: [JEE Main 2022]
(A) $\frac{35}{27}$ (B) 1 (C) $\frac{27}{28}$ (D) $\frac{28}{27}$
49. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to: [JEE Main 2021]
50. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is [JEE Main 2020]

Geometric Progression (G.P.)

Finding Terms, Common Ratio, and Basic Properties

51. Let a_1, a_2, a_3, \dots be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to: [JEE Main 2025]
(1) 628 (2) 812 (3) 526 (4) 784
52. Let a_1, a_2, a_3, \dots be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to: [JEE Main 2025]
(A) 128 (B) 129 (C) 131 (D) 130
53. Let a and b be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b , is equal to p^{th} term of another GP, whose first term is a and fifth term is b . Then p is equal to: [JEE Main 2024]
(1) 20 (2) 25 (3) 21 (4) 24
54. In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the $4^{th}, 6^{th}$ and 8^{th} terms is equal to: [JEE Main 2024]
(1) 96 (2) 91 (3) 84 (4) 78
55. The 4^{th} term of a GP is 500 and its common ratio is $\frac{1}{m}, m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is: [JEE Main 2023]
56. Let a_1, a_2, a_3, \dots be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to: [JEE Main 2023]
(1) 3 (2) $3\sqrt{3}$ (3) 2 (4) $2\sqrt{2}$
57. Let a_1, a_2, a_3, \dots be a G.P. of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$ is equal to: [JEE Main 2023]
58. Let A_1, A_2, A_3, \dots be an increasing geometric progression of positive real numbers. If $A_1 A_3 A_5 A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to: [JEE Main 2022]
(A) 33 (B) 37 (C) 43 (D) 47
59. In an increasing, geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} terms is equal to: [JEE Main 2021]
(1) 35 (2) 30 (3) 26 (4) 32
60. Let a_1, a_2, \dots, a_{10} be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals: [JEE Main 2019]
(a) 5^4 (b) $4(5^2)$ (c) 5^3 (d) $2(5^2)$
61. If a, b and c be three distinct real numbers in G.P. and $a+b+c = xb$, then x cannot be: [JEE Main 2019]
(a) -2 (b) -3 (c) 4 (d) 2

Sum of n Terms of a G.P. (S_n)

62. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is: [JEE Main 2025]
(A) 757 (B) 755 (C) 750 (D) 760
63. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to: [JEE Main 2024]
(1) 7 (2) 4 (3) 5 (4) 6

64. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to: **[JEE Main 2023]**
- (1) 241 (2) 231 (3) 210 (4) 220
65. If $a_1 (> 0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to: **[JEE Main 2022]**

66. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α , then 2α is: **[JEE Main 2021]**

67. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to: **[JEE Main 2020]**
- (a) $3^{11} - 2^{12}$ (b) 3^{11}
(c) $\frac{3^{11}}{2} + 2^{10}$ (d) $2 \cdot 3^{11}$

68. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is: **[JEE Main 2020]**
- (a) $\frac{1}{26}(3^{49} - 1)$ (b) $\frac{1}{26}(3^{50} - 1)$
(c) $\frac{2}{13}(3^{50} - 1)$ (d) $\frac{1}{13}(3^{50} - 1)$

69. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in: **[JEE Main 2020]**
- (a) $(-\infty, -9] \cup [3, \infty)$ (b) $[-3, 8)$
(c) $(-\infty, -3] \cup [9, \infty)$ (d) $(-\infty, -9]$

70. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to: **[JEE Main 2020]**
- (a) 300 (b) 225 (c) 175 (d) 150

71. Let a_1, a_2, a_3, \dots be a G. P. such that $a_1 < 0, a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to: **[JEE Main 2020]**
- (a) -513 (b) -171 (c) 171 (d) $\frac{511}{3}$

Sum of Infinite G.P.

72. Let a, ar, ar^2, \dots be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then $a + 18r$ is equal to: **[JEE Main 2024]**
- (1) 46 (2) 38 (3) 31 (4) 27

73. Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$, then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to: **[JEE Main 2023]**

74. Let the sum of an infinite G.P., whose first term is a and the common ratio is r , be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10ar$, n th term is a_n and the common difference is $10ar^2$, is equal to: **[JEE Main 2022]**
- (A) $21a_{11}$ (B) $22a_{11}$ (C) $15a_{16}$ (D) $14a_{16}$

75. Let a_1, a_2, a_3, \dots be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to: **[JEE Main 2022]**

76. If the sum of an infinite GP a, ar, ar^2, ar^3, \dots is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, \dots is: [JEE Main 2021]
- (1) $\frac{5}{2}$ (2) $\frac{1}{2}$ (3) $\frac{25}{2}$ (4) $\frac{9}{2}$
77. The value of $(0.16)^{\log_{2.5}(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty)}$ is equal to: [JEE Main 2020]
78. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \dots$ to ∞ is equal to: [JEE Main 2020]
- (a) $2^{\frac{1}{2}}$ (b) $2^{\frac{1}{4}}$ (c) 1 (d) 2
79. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is: [JEE Main 2019]
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{2}{9}$

Harmonic Progression (H.P.)

80. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then: [JEE Main 2022]
- (A) x, y, z are in A.P. (B) x, y, z are in G.P.
 (C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. (D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Relationship between Progressions (A.P., G.P., H.P.)

Terms of one progression forming another

81. Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4 , then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1 x_2 x_3 x_4)$ is: [JEE Main 2025]
- (A) 18 (B) 216 (C) 36 (D) 72
82. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$ then $p - q$ is equal to: [JEE Main 2025]
- (A) 540 (B) 450 (C) 600 (D) 630
83. Let 3, a, b, c be in A.P. and 3, $a - 1, b + 1, c + 9$ be in G.P. Then, the arithmetic mean of a, b and c is: [JEE Main 2024]
- (1) -4 (2) -1 (3) 13 (4) 11
84. If each term of a geometric progression a_1, a_2, a_3, \dots with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \dots + a_n$, then $S_{20} - S_{18}$ is equal to: [JEE Main 2024]
- (1) 2^{15} (2) -2^{18} (3) 2^{18} (4) -2^{15}
85. Let $2^{nd}, 8^{th}$ and 44^{th} terms of a non-constant A.P. be respectively the $1^{st}, 2^{nd}$ and 3^{rd} terms of a G.P. If the first term of A.P. is 1, then the sum of its first 20 terms is equal to: [JEE Main 2024]
- (1) 980 (2) 960 (3) 990 (4) 970
86. Let the first three terms 2, p and q, with $q \neq 2$ of a G.P. be respectively the $7^{th}, 8^{th}$ and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to: [JEE Main 2024]
- (1) 163 (2) 151 (3) 177 (4) 169
87. Let three real numbers a, b, c be in arithmetic progression and $a + 1, b, c + 3$ be in geometric progression. If $a > 10$ and the arithmetic mean of a, b and c is 8, then the cube of the geometric mean of a, b and c is: [JEE Main 2024]
- (1) 128 (2) 316 (3) 120 (4) 312

88. If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P., then $a : b : c$ is equal to: **[JEE Main 2024]**
- (1) $9 : 6 : 4$ (2) $16 : 4 : 1$ (3) $25 : 10 : 4$ (4) $6 : 3 : 2$
89. Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$, is -444, then abc is equal to: **[JEE Main 2023]**
- (1) 343 (2) 216 (3) $\frac{343}{8}$ (4) $\frac{125}{8}$
90. For the two positive numbers a, b , if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}, 10$ and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to: **[JEE Main 2023]**
91. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to: **[JEE Main 2023]**
92. For three positive integers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to: **[JEE Main 2023]**
- (1) 2 (2) 6 (3) 12 (4) -6
93. Three numbers are in an increasing geometric progression with common ratio r . If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d . If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to: **[JEE Main 2021]**
- (1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$ (3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$
94. Let $\frac{1}{16}, a$ and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where $a, b > 0$. Then $72(a + b)$ is equal to: **[JEE Main 2021]**
95. If $3^{2\sin 2\alpha - 1}, 14$ and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an A.P. for some α , then the sixth term of this A.P. is: **[JEE Main 2020]**
- (a) 66 (b) 81 (c) 65 (d) 78
96. Let a, b and c be in G.P. with common ratio r , where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a, 7b$ and $15c$ are the first three terms of an A.P., then the 4th term of this A.P. is: **[JEE Main 2019]**
- (a) $\frac{2}{3}a$ (b) $5a$ (c) $\frac{7}{3}a$ (d) a
97. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is: **[JEE Main 2019]**
- (a) 36 (b) 32 (c) 24 (d) 28
98. Let a, b and c be the $7^{th}, 11^{th}$ and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to: **[JEE Main 2019]**
- (a) 2 (b) $\frac{1}{2}$ (c) $\frac{7}{13}$ (d) 4

Progressions involving roots of equations

99. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to: **[JEE Main 2025]**

100. For $p, q \in R$, consider the real valued function $f(x) = (x - p)^2 - q$, $x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is: [JEE Main 2022]
101. If the arithmetic mean and geometric mean of the p th and q th terms of the sequence $-16, 8, -4, 2, \dots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to: [JEE Main 2021]

Insertion of Means (A.M. and G.M.)

102. Let A_1 and A_2 be two arithmetic means and G_1, G_2 and G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_2^2$ is equal to: [JEE Main 2023]
- (1) $(A_1 + A_2)^2 G_1 G_3$ (2) $2(A_1 + A_2) G_1 G_3$
 (3) $(A_1 + A_2) G_1^2 G_3^2$ (4) $2(A_1 + A_2) G_1^2 G_3^2$
103. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is: [JEE Main 2022]
- (A) 21 (B) 22 (C) 23 (D) 24
104. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to: [JEE Main 2020]

AM-GM-HM Inequality and Other Inequalities

105. For $x \geq 0$ the least value of K , for which $4^{1+x} + 4^{1-x}, \frac{K}{2}, 16^x + 16^{-x}$ are three consecutive terms of an A.P., is equal to: [JEE Main 2024]
- (1) 8 (2) 4 (3) 10 (4) 16
106. Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750β , then the value of β is: [JEE Main 2023]
- (1) 90 (2) 110 (3) 55 (4) 108
107. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to: [JEE Main 2021]
- (1) $a + \frac{1}{a}$ (2) $a + 1$ (3) $2a$ (4) $2\sqrt{a}$
108. Let $f: R \rightarrow R$ be such that for all $x \in R$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is: [JEE Main 2020]
- (a) 2 (b) 3 (c) 0 (d) 4

Special Series and Summations

Arithmetico-Geometric Progression (A.G.P.)

109. If $7 = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \frac{1}{7^3}(5 + 3\alpha) + \dots \infty$, then the value of α is: [JEE Main 2025]
- (1) $\frac{6}{7}$ (2) 6 (3) $\frac{1}{7}$ (4) 1
110. If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \dots \infty$, then the value of p is: [JEE Main 2024]
111. For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots$ is 10 , then the value of k is: [JEE Main 2023]
112. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is: [JEE Main 2023]

113. Suppose $a_1, a_2 = 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to: [JEE Main 2023]
114. The sum $1 + 2 \cdot 3 + 3 \cdot 3^2 + \dots + 10 \cdot 3^9$ is equal to: [JEE Main 2022]
- (A) $\frac{2 \cdot 3^{12} + 10}{4}$ (B) $\frac{19 \cdot 3^{10} + 1}{4}$
 (C) $5 \cdot 3^{10} - 2$ (D) $\frac{9 \cdot 3^{10} + 1}{2}$
115. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \dots$, then $4S$ is equal to: [JEE Main 2022]
- (A) $(\frac{7}{3})^2$ (B) $\frac{7^3}{3^2}$ (C) $(\frac{7}{3})^3$ (D) $4 \cdot (\frac{7}{3})^3$
116. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$ is equal to: [JEE Main 2021]
- (1) $\frac{9}{4}$ (2) $\frac{15}{4}$ (3) $\frac{13}{4}$ (4) $\frac{11}{4}$
117. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$, then $160S$ is equal to: [JEE Main 2021]

118. Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + \dots$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to: [JEE Main 2020]
- (a) -5 (b) 1 (c) -3 (d) 3
119. The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to: [JEE Main 2019]
- (a) $2 - \frac{3}{2^{17}}$ (b) $1 - \frac{11}{2^{20}}$ (c) $2 - \frac{11}{2^{19}}$ (d) $2 - \frac{21}{2^{20}}$

Summation using Method of Differences (V_n Method)

120. For positive integers n , if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^n \left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is: [JEE Main 2025]
- (1) 540 (2) 675 (3) 1350 (4) 135
121. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots$ upto n terms. If the sum of the first six terms of an A.P. with first term $-p$ and common difference p is $\sqrt{2026S_{2025}}$, then the absolute difference between 20^{th} and 15^{th} terms of the A.P. is: [JEE Main 2025]
- (1) 20 (2) 90 (3) 45 (4) 25
122. If the sum of the first 10 terms of the series $\frac{4 \cdot 1}{1+4 \cdot 1^4} + \frac{4 \cdot 2}{1+4 \cdot 2^4} + \frac{4 \cdot 3}{1+4 \cdot 3^4} + \dots$ is $\frac{m}{n}$, where $gcd(m, n) = 1$, then $m + n$ is equal to: [JEE Main 2025]
123. If the sum of the first 20 terms of the series $\frac{4 \cdot 1}{4+3 \cdot 1^2+1^4} + \frac{4 \cdot 2}{4+3 \cdot 2^2+2^4} + \frac{4 \cdot 3}{4+3 \cdot 3^2+3^4} + \dots$ is $\frac{m}{n}$, where m and n are coprime, then $m + n$ is equal to: [JEE Main 2025]
- (A) 423 (B) 421 (C) 422 (D) 420
124. If $\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \dots + \frac{1}{\sqrt{99+\sqrt{100}}} = m$ and $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{99 \cdot 100} = n$, then the point (m, n) lies on the line: [JEE Main 2024]
- (1) $11(x - 1) - 100(y - 2) = 0$ (2) $11x - 100y = 0$
 (3) $11(x - 2) - 100(y - 1) = 0$ (4) $11(x - 1) - 100y = 0$
125. The sum of the series $\frac{1}{1-3 \cdot 1^2+1^4} + \frac{2}{1-3 \cdot 2^2+2^4} + \frac{3}{1-3 \cdot 3^2+3^4} + \dots$ up to 10 terms is: [JEE Main 2024]
- (1) $\frac{45}{109}$ (2) $-\frac{45}{109}$ (3) $\frac{55}{109}$ (4) $-\frac{55}{109}$
126. If the sum of the series $\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)}$ is equal to 5, then $50d$ is equal to: [JEE Main 2024]
- (1) 10 (2) 5 (3) 15 (4) 20

127. If the sum of the series $(\frac{1}{2} - \frac{1}{3}) + (\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}) + (\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}) + \dots$ is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to: [JEE Main 2023]
128. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \dots + a_{25}$ is equal to: [JEE Main 2023]
 (1) $\frac{51}{144}$ (2) $\frac{49}{138}$ (3) $\frac{50}{141}$ (4) $\frac{52}{147}$
129. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ is: [JEE Main 2023]
 (1) $\frac{59}{111}$ (2) $\frac{55}{111}$ (3) $\frac{56}{111}$ (4) $\frac{58}{111}$
130. Let a_1, a_2, \dots, a_n be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then $\lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} (\frac{1}{\sqrt{a_1 + \sqrt{a_2}}} + \frac{1}{\sqrt{a_2 + \sqrt{a_3}}} + \dots + \frac{1}{\sqrt{a_{n-1} + \sqrt{a_n}}})$ is: [JEE Main 2023]
 (1) $\frac{1}{\sqrt{d}}$ (2) \sqrt{d} (3) 1 (4) 2
131. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to: [JEE Main 2022]
 (A) $\frac{7}{87}$ (B) $\frac{7}{29}$ (C) $\frac{14}{87}$ (D) $\frac{21}{89}$
132. If $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$, where m and n are co-prime, then $m + n$ is equal to: [JEE Main 2022]
133. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \dots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is: [JEE Main 2022]
 (A) 198 (B) 202 (C) 212 (D) 218
134. If $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots + \frac{1}{100 \times 101 \times 102} = \frac{k}{101}$, then $34k$ is equal to: [JEE Main 2022]
135. If the sum of the first ten terms of the series $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$ is $\frac{m}{n}$, where m and n are co-prime numbers, then $m + n$ is equal to: [JEE Main 2022]
136. The sum $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1}$ is equal to: [JEE Main 2021]
 (1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$
137. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{K=0}^{20} \frac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100(\frac{A_{14}+A_{15}}{A_{13}})^2$ is equal to: [JEE Main 2021]
138. The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots$ is: [JEE Main 2021]
 (1) 1 (2) $\frac{120}{121}$ (3) $\frac{99}{100}$ (4) $\frac{143}{144}$
139. Let a_1, a_2, \dots, a_{21} be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to: [JEE Main 2021]
 (1) 57 (2) 72 (3) 48 (4) 36
140. The sum of the series $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \dots + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is: [JEE Main 2021]
 (1) $1 + \frac{2^{101}}{4^{101}-1}$ (2) $1 + \frac{2^{100}}{4^{101}-1}$
 (3) $1 - \frac{2^{100}}{4^{100}-1}$ (4) $1 - \frac{2^{101}}{4^{101}-1}$

Summation involving $\Sigma n, \Sigma n^2, \Sigma n^3$

141. The sum of the series $1 + 3 + 5^2 + 7 + 9^2 + \dots$ upto 40 terms is equal to: [JEE Main 2025]
 (A) 40870 (B) 41880 (C) 43890 (D) 33980

142. The sum $1 + 3 + 11 + 25 + 45 + 71 + \dots$ upto 20 terms, is equal to: [JEE Main 2025]
 (A) 7240 (B) 8124 (C) 7130 (D) 6982
143. Let $\alpha = \sum_{n=1}^{10} n\left(\frac{n^2}{2} + \frac{3n}{2} - 1\right)^2$ and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to: [JEE Main 2024]
144. The value of $\frac{1 \times 2^2 + 2 \times 3^2 + \dots + 100 \times (101)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + 100^2 \times 101}$ is: [JEE Main 2024]
 (1) $\frac{32}{31}$ (2) $\frac{31}{30}$ (3) $\frac{306}{305}$ (4) $\frac{305}{301}$
145. The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \dots$ is: [JEE Main 2023]
 (1) 3520 (2) 3450 (3) 3250 (4) 3420
146. Let a_n be n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \dots$ and $S_n = \sum_{k=1}^n a_k$. Then $S_{30} - a_{40}$ is equal to: [JEE Main 2023]
 (1) 11310 (2) 11260 (3) 11290 (4) 11280
147. If $S_n = 4 + 11 + 21 + 34 + 50 + \dots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to: [JEE Main 2023]
 (1) 223 (2) 226 (3) 220 (4) 227
148. The sum to 20 terms of the series $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - \dots$ is equal to: [JEE Main 2023]
149. If $\frac{1^3+2^3+3^3+\dots+n \text{ terms}}{1 \cdot 3+2 \cdot 5+3 \cdot 7+\dots+n \text{ terms}} = \frac{9}{5}$, then the value of n is: [JEE Main 2023]
150. The sum of the series $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \dots - 10 \cdot 21^2$ is: [JEE Main 2023]
151. The sum of the infinite series $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - \dots + 15 \cdot 29^2$ is: [JEE Main 2023]
152. Consider two G.Ps. $2, 2^2, 2^3, \dots$ and $4, 4^2, 4^3, \dots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^n k(n - k)$ is equal to: [JEE Main 2022]
 (A) 560 (B) 1540 (C) 1330 (D) 2600
153. The series of positive multiples of 3 is divided into sets: $3, 6, 9, 12, 15, 18, 21, 24, 27, \dots$. Then the sum of the elements in the 11^{th} set is equal to: [JEE Main 2022]
154. The sum of the series $\frac{2^3-1^3}{1 \times 7} + \frac{4^3-3^3+2^3-1^3}{2 \times 11} + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3 \times 15} + \dots + \frac{30^3-29^3+\dots+2^3-1^3}{15 \times 63}$ is equal to: [JEE Main 2022]
155. Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$. Then $A + B$ is equal to: [JEE Main 2022]
156. The sum, $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$ is equal to: [JEE Main 2020]
157. The sum $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$ is: [JEE Main 2020]
158. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots$ is $(102)m$, then m is equal to: [JEE Main 2020]
 (a) 20 (b) 25 (c) 5 (d) 10

159. The sum $\frac{3 \times 1^3}{1^2} + \frac{5 \times (1^3 + 2^3)}{1^2 + 2^2} + \frac{7 \times (1^3 + 2^3 + 3^3)}{1^2 + 2^2 + 3^2} + \dots$ upto 10^{th} term, is: [JEE Main 2019]
 (a) 680 (b) 600 (c) 660 (d) 620
160. The sum $1 + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + \dots + \frac{1^3 + 2^3 + \dots + 15^3}{1 + 2 + \dots + 15} - \frac{1}{2}(1 + 2 + \dots + 15)$ is equal to: [JEE Main 2019]
 (a) 620 (b) 1240 (c) 1860 (d) 660
161. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \dots$ upto 11^{th} term is: [JEE Main 2019]
 (a) 915 (b) 946 (c) 945 (d) 916
162. Let $S_k = \frac{1+2+3+\dots+k}{k}$. If $S_1^2 + S_2^2 + \dots + S_{10}^2 = \frac{5}{12}A$. Then A is equal to: [JEE Main 2019]
 (a) 283 (b) 301 (c) 303 (d) 156
163. If the sum of the first 15 terms of the series $(\frac{3}{4})^3 + (1\frac{1}{2})^3 + (2\frac{1}{4})^3 + 3^3 + (3\frac{3}{4})^3 + \dots$ is equal to $225k$, then k is equal to: [JEE Main 2019]
 (a) 108 (b) 27 (c) 54 (d) 9
164. The sum of the following series $1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+\dots+5^2)}{11} + \dots$ up to 15 terms, is: [JEE Main 2019]
 (a) 7520 (b) 7510 (c) 7830 (d) 7820
165. The sum of the series $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots +$ upto n terms is: [JEE Main 2023]

Summation involving Factorials and Exponentials

166. The value of $\lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!} \right)$ is: [JEE Main 2025]
 (1) $\frac{4}{3}$ (2) 2 (3) $\frac{7}{3}$ (4) $\frac{5}{3}$
167. The sum $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \dots$ upto ∞ terms, is equal to: [JEE Main 2025]
 (A) $3e$ (B) $2e$ (C) $4e$ (D) $6e$
168. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Then $a^2 - b + c$ is equal to: [JEE Main 2023]
169. $\sum_{r=1}^{20} (r^2 + 1)(r!)$ is equal to: [JEE Main 2022]
 (A) $22! - 21!$ (B) $22! - 2(21!)$
 (C) $21! - 2(20!)$ (D) $21! - 20!$
170. If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to: [JEE Main 2021]
171. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to: [JEE Main 2021]
 (1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$ (2) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
 (3) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$ (4) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$
172. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \dots + (n-1) \cdot 1, n \geq 4$. The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to: [JEE Main 2021]
 (1) $\frac{e-1}{3}$ (2) $\frac{e-2}{6}$ (3) $\frac{e}{3}$ (4) $\frac{e}{6}$

Summation involving Log

- 173.** If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots$, then the value of e^{1+y} at $x = \frac{1}{2}$ is: [JEE Main 2021]
(1) $\frac{1}{2}e^2$ (2) $2e$ (3) $\frac{1}{2}\sqrt{e}$ (4) $2e^2$
- 174.** If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to: [JEE Main 2021]
(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$ (2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$
(3) $\frac{1-x}{1+x} + \log_e(1-x)$ (4) $\frac{1+x}{1-x} + \log_e(1-x)$
- 175.** If $\log_3 2, \log_3(2^x - 5), \log_3(2^x - \frac{7}{2})$ are in an arithmetic progression, then the value of x is equal to: [JEE Main 2021]
- 176.** If $\tan(\frac{\pi}{9}), x, \tan(\frac{7\pi}{18})$ are in arithmetic progression and $\tan(\frac{\pi}{9}), y, \tan(\frac{5\pi}{18})$ are also in arithmetic progression, then $|x - 2y|$ is equal to: [JEE Main 2021]
(1) 4 (2) 3 (3) 0 (4) 1
- 177.** Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \dots$ up to n -terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to: [JEE Main 2021]
- 178.** If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \dots$, where $x > 0$ is 504, then x is equal to: [JEE Main 2021]
(1) 243 (2) 9 (3) 7 (4) 81
- 179.** If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to: [JEE Main 2020]
(a) 7^2 (b) $7^{1/2}$ (c) e^2 (d) $7^{46/21}$

Miscellaneous Problems

Geometric Applications of Progressions

- 180.** Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is the sum of areas of all the triangles formed in this process, then : [JEE Main 2024]
(1) $P^2 = 6\sqrt{3}Q$ (2) $P^2 = 36\sqrt{3}Q$
(3) $P = 36\sqrt{3}Q^2$ (4) $P^2 = 72\sqrt{3}Q$
- 181.** If three successive terms of a G.P. with common ratio $r (r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r , then $3[r] + [-r]$ is equal to: [JEE Main 2024]
- 182.** Let A_1, A_2, A_3, \dots be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1} . If the length of the side of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is: [JEE Main 2021]
- 183.** Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is: [JEE Main 2019]
(a) 157 (b) 262 (c) 225 (d) 190

Recurrence Relations and Sequences

184. Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n, n = 0, 1, 2, 3, \dots$. Then $\sum_{k=1}^{100} a_k$ is equal to: **[JEE Main 2025]**
- (1) $3a_{99} - 100$ (2) $3a_{100} - 100$ (3) $3a_{99} + 100$ (4) $3a_{100} + 100$
185. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r, r = 2, 3, \dots, n$. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)(4 \cdot 6^n - 5 \cdot 3^n + 1)$, then n is equal to: **[JEE Main 2024]**
186. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}$, for $n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^8 \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to: **[JEE Main 2023]**
187. Let $a_1 = b_1 = 1, a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to: **[JEE Main 2022]**
188. Consider the sequence a_1, a_2, a_3, \dots such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $n = 1, 2, 3, \dots$. If $(\frac{a_1 + \frac{1}{a_2}}{a_3})(\frac{a_2 + \frac{1}{a_3}}{a_4}) \dots (\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}) = 2^\alpha ({}^{61}C_{31})$, then α is equal to: **[JEE Main 2022]**
- (A) -30 (B) -31 (C) -60 (D) -61
189. Let $\{a_n\}_{n=0}^\infty$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0$. Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to: **[JEE Main 2022]**
- (A) 483 (B) 528 (C) 575 (D) 624
190. Let $\{a_n\}_{n=0}^\infty$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^\infty \frac{a_n}{7^n}$ is equal to: **[JEE Main 2022]**
- (A) $\frac{6}{343}$ (B) $\frac{7}{216}$ (C) $\frac{8}{343}$ (D) $\frac{49}{216}$
191. Let $\{a_n\}_{n=1}^\infty$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^\infty \frac{a_n}{2^{3n}}$ is equal to: **[JEE Main 2021]**

Other Miscellaneous Problems

192. If $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90}$,
 $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \infty = \alpha$, and
 $\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots \infty = \beta$, then $\frac{\alpha}{\beta}$ is equal to: **[JEE Main 2025]**
- (A) 23 (B) 14 (C) 18 (D) 15
193. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to: **[JEE Main 2024]**
- (1) 150 (2) 180 (3) 160 (4) 125
194. If $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \dots + 60(1+x)^{60}, x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in \mathbb{N}$ then $(a+b)$ equal to: **[JEE Main 2024]**
195. If the set $R = \{(a, b) : a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^m (1 - i^{n!}) = x + iy$, where $i = \sqrt{-1}$, then the value of $m + x + y$ is: **[JEE Main 2024]**
- (1) 12 (2) 4 (3) 8 (4) 5

207. If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then $\frac{A}{B}$ is equal to: [JEE Main 2022]
 (A) $\frac{11}{9}$ (B) 1 (C) $-\frac{11}{9}$ (D) $-\frac{11}{3}$
208. Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50} (S_n + \frac{2}{n+1} - n - 1)$ is equal to: [JEE Main 2022]
209. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \dots$ is equal to: [JEE Main 2022]
 (A) $\frac{425}{216}$ (B) $\frac{429}{216}$ (C) $\frac{288}{125}$ (D) $\frac{280}{125}$
210. Let α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$, then the slope of the line passing through (α, β) and origin is: [JEE Main 2021]
 (1) 540 (2) 550 (3) 530 (4) 510
211. If the value of $(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \text{ upto } \infty)^{\log_{0.25}(\frac{1}{3} + \frac{1}{3^2} + \dots)}$ is l , then l^2 is equal to: [JEE Main 2021]
212. If $[x]$ be the greatest integer less than or equal to x , then $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right]$ is equal to: [JEE Main 2021]
 (1) 0 (2) 4 (3) -2 (4) 2
213. The sum of the series $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots$ upto n terms is: [JEE Main 2023]
214. If $\tan(\frac{\pi}{9}), x, \tan(\frac{7\pi}{18})$ are in arithmetic progression and $\tan(\frac{\pi}{9}), y, \tan(\frac{5\pi}{18})$ are also in arithmetic progression, then $|x - 2y|$ is equal to: [JEE Main 2021]
 (1) 4 (2) 3 (3) 0 (4) 1
215. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + \dots$ up to n -terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to: [JEE Main 2021]
216. The sum of the infinite series $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - \dots + 15 \cdot 29^2$ is: [JEE Main 2023]
217. The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to: [JEE Main 2023]
218. The sum of the series $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \dots + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is: [JEE Main 2021]
 (1) $1 + \frac{2^{101}}{4^{101}-1}$ (2) $1 + \frac{2^{100}}{4^{101}-1}$
 (3) $1 - \frac{2^{100}}{4^{100}-1}$ (4) $1 - \frac{2^{101}}{4^{101}-1}$
219. Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$, is -444, then abc is equal to: [JEE Main 2023]
 (1) 343 (2) 216 (3) $\frac{343}{8}$ (4) $\frac{125}{8}$

Answer Key

1 (20)	2 (C)	3 (D)	4 (8)	5 (A)	6 (53)	7 (C)	8 (C)
9 (A)	10 (11132)	11 (B)	12 (C)	13 (A)	14 (B)	15 (D)	16 (B)
17 (C)	18 (C)	19 (A)	20 (C)	21 (754)	22 (B)	23 (B)	24 (D)
25 (A)	26 (A)	27 (2021)	28 (A)	29 (B)	30 (D)	31 (D)	32 (D)
33 (C)	34 (A)	35 (B)	36 (D)	37 (A)	38 (C)	39 (C)	40 (D)
41 (D)	42 (D)	43 (6699)	44 (C)	45 (151)	46 (321)	47 (2223)	48 (D)
49 (3)	50 (14)	51 (D)	52 (B)	53 (C)	54 (B)	55 (12)	56 (A)
57 (60)	58 (C)	59 (A)	60 (A)	61 (D)	62 (A)	63 (D)	64 (B)
65 (40)	66 (3)	67 (B)	68 (B)	69 (C)	70 (D)	71 (B)	72 (C)
73 (9)	74 (A)	75 (16)	76 (B)	77 (4)	78 (A)	79 (B)	80 (C)
81 (B)	82 (A)	83 (D)	84 (D)	85 (D)	86 (A)	87 (C)	88 (A)
89 (B)	90 (3)	91 (150)	92 (A)	93 (B)	94 (14)	95 (A)	96 (D)
97 (D)	98 (D)	99 (474)	100 (50)	101 (10)	102 (A)	103 (C)	104 (39)
105 (C)	106 (A)	107 (D)	108 (B)	109 (B)	110 (9)	111 (2)	112 (2175)
113 (16)	114 (B)	115 (C)	116 (C)	117 (305)	118 (C)	119 (C)	120 (B)
121 (D)	122 (441)	123 (B)	124 (B)	125 (D)	126 (B)	127 (7)	128 (C)
129 (B)	130 (C)	131 (B)	132 (166)	133 (C)	134 (286)	135 (276)	136 (B)
137 (9)	138 (B)	139 (B)	140 (D)	141 (B)	142 (D)	143 (353)	144 (D)
145 (A)	146 (C)	147 (A)	148 (1310)	149 (5)	150 (M)	151 (6952)	152 (C)
153 (6993)	154 (120)	155 (1100)	156 (504)	157 (1540)	158 (A)	159 (C)	160 (A)
161 (B)	162 (C)	163 (B)	164 (D)	165 (M)	166 (D)	167 (B)	168 (26)
169 (B)	170 (160)	171 (C)	172 (A)	173 (A)	174 (A)	175 (3)	176 (C)
177 (16)	178 (D)	179 (A)	180 (B)	181 (1)	182 (9)	183 (D)	184 (B)
185 (6)	186 (461)	187 (27560)	188 (C)	189 (B)	190 (B)	191 (7)	192 (D)
193 (C)	194 (3660)	195 (A)	196 (1505)	197 (103)	198 (1011)	199 (76)	200 (A)
201 (400)	202 (C)	203 (M)	204 (12)	205 (D)	206 (98)	207 (C)	208 (41651)
209 (C)	210 (B)	211 (3)	212 (B)	213 (M)	214 (C)	215 (16)	216 (6952)
217 (9525)	218 (D)	219 (B)					
