Trigonometry

Lecture notes

World of Trigonometry

JEE Main

2025 → 6 Jan (10) + April (10) = 20 papers

2024 → 7

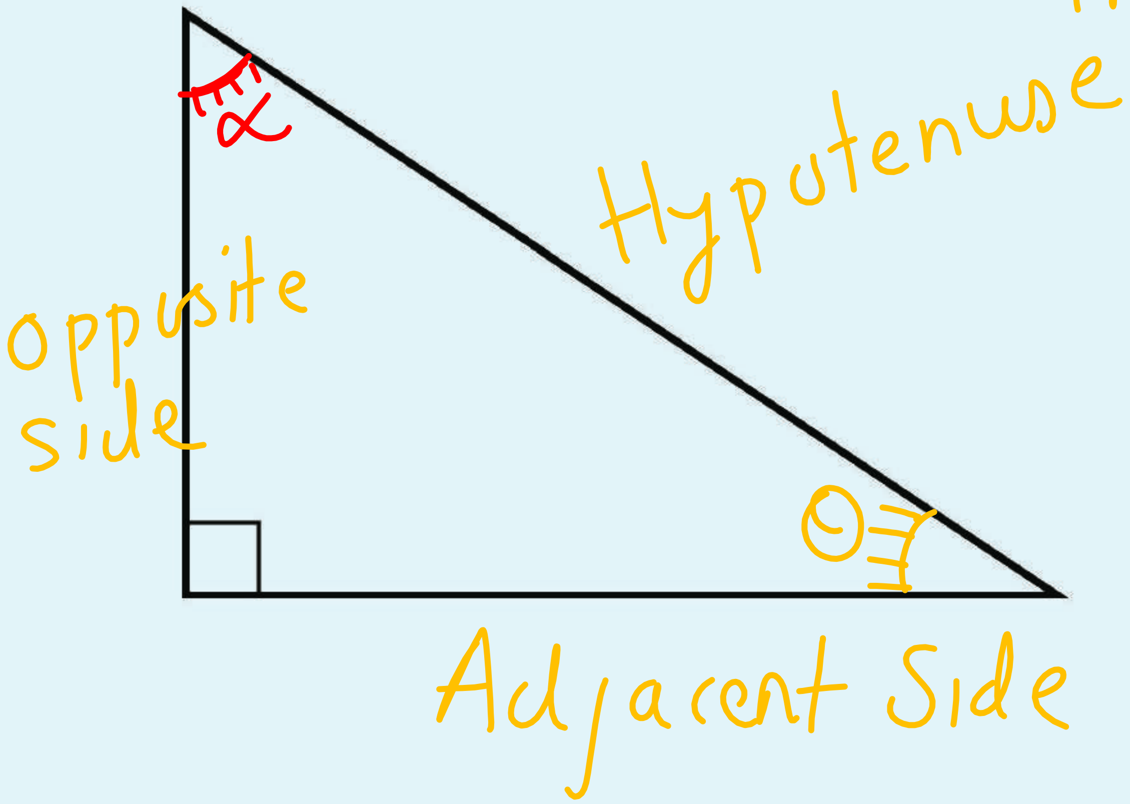
2023 → 6

2022 → 7

1. Trigonometric Ratios and Identities (Compound Angle) → 45-50 formula + Problem Solving Tech
2. Trigonometric Equation
3. Solution of Triangle
4. Inverse Trigonometric Function (ITF) (12th Class)

Definition of T-Ratios

Trigonometry
Tri → Three
gono → Side
metry → Measurement (metron)



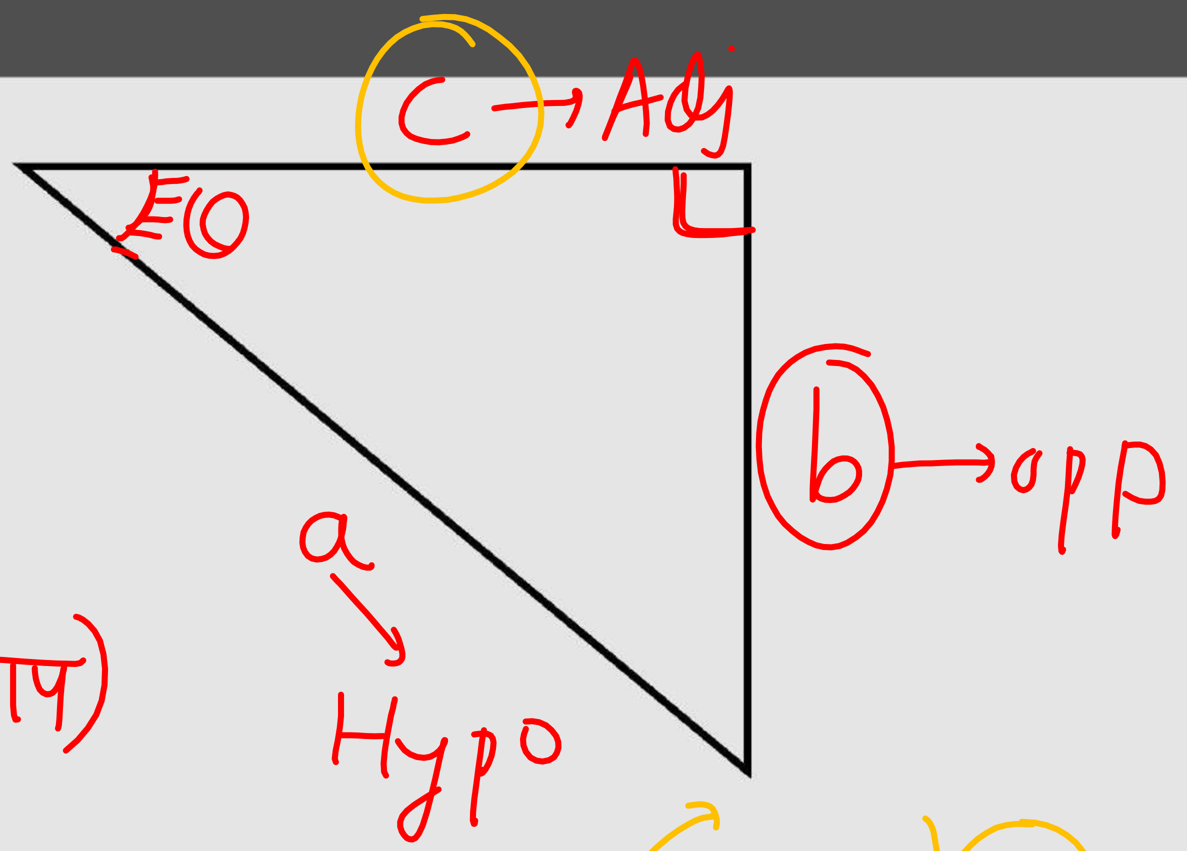
In the right-angled triangle,

The trigonometric ratios are defined as:

- $\sin\theta = \dfrac{\text{Opposite Side}}{\text{Hypotenuse}}$ $\qquad \operatorname{cosec}\theta = \dfrac{\text{Hypo}}{\text{Opp}}$
- $\cos\theta = \dfrac{\text{Adjacent Side}}{\text{Hypotenuse}}$ $\qquad \sec\theta = \dfrac{\text{Hypo}}{\text{Adj}}$
- $\tan\theta = \dfrac{\text{Opposites side}}{\text{Adjacent Side}}$ $\qquad \cot\theta = \dfrac{\text{Adj}}{\text{Opp}}$

Example: Finding Trigonometric Ratios

Example 1



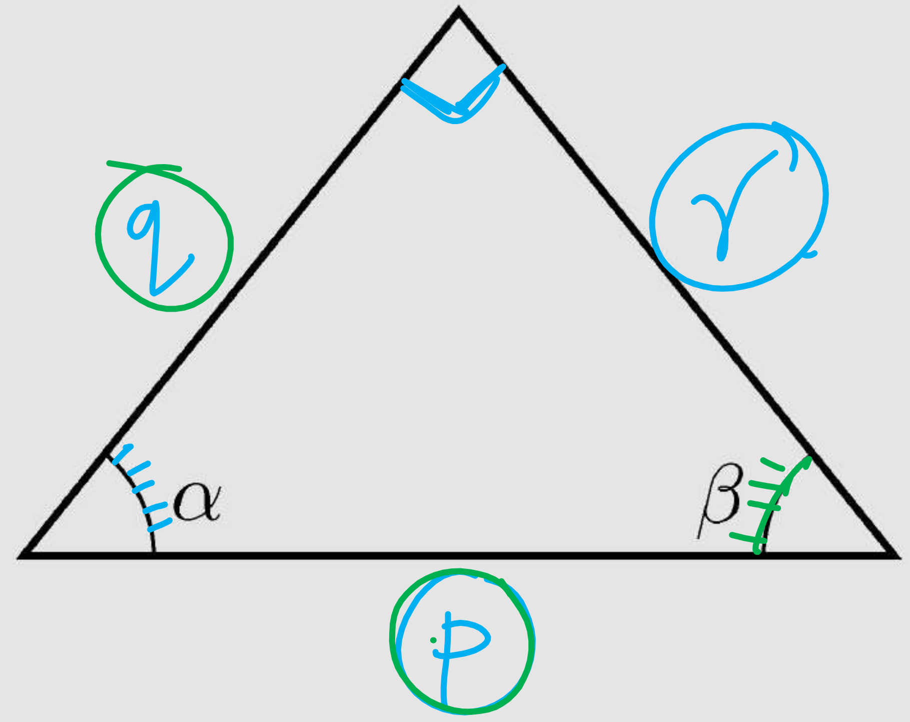
Sine (पाप)

Find:

- $\sin\theta = \frac{b}{a}$
- $\cos\theta =$
- $\tan\theta = \frac{b}{c}$

$\cos\theta = \frac{c}{a}$

Example 2

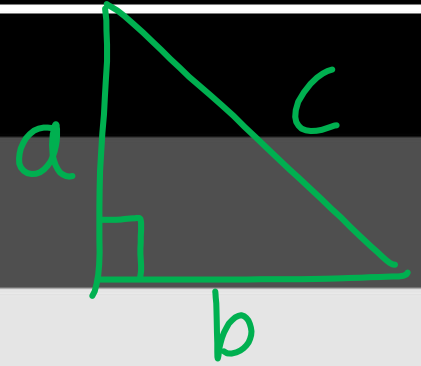


Find:

- $\sin\alpha = \frac{r}{p}$
- $\cos\alpha = \frac{q}{p}$
- $\tan\alpha = \frac{r}{q}$
- $\sin\beta = \frac{q}{p}$
- $\cos\beta = \frac{r}{p}$
- $\tan\beta = \frac{q}{r}$

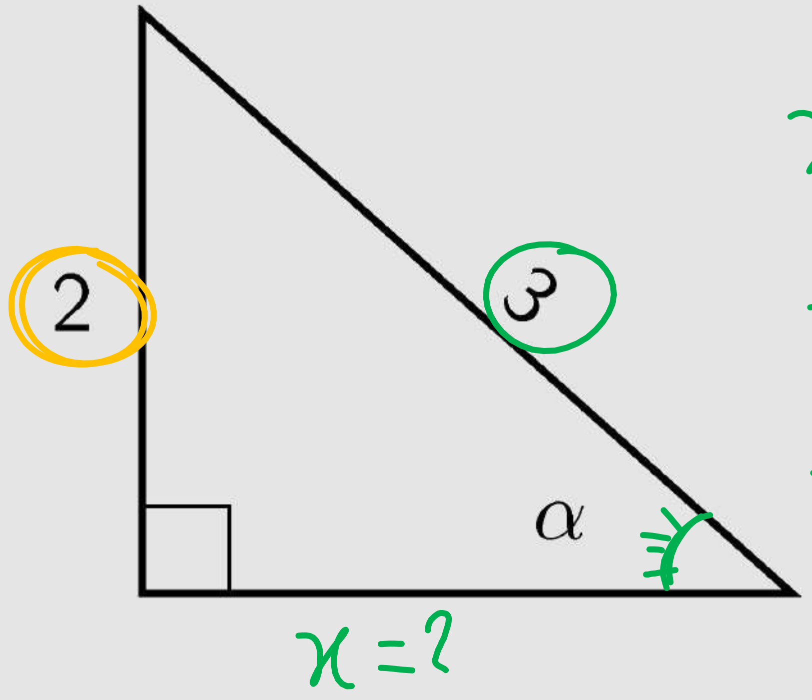
Example: Finding Trigonometric Ratios

Example 3



Pythagorus Thm

$a^2 + b^2 = c^2$



$x^2 + 2^2 = 3^2$

$x^2 = 3^2 - 2^2$

$x = \sqrt{3^2 - 2^2}$

$= \sqrt{9 - 4}$

$\boxed{x = \sqrt{5}}$

Find:

- $\sin\alpha = \frac{2}{3}$
- $\cos\alpha = \frac{\sqrt{5}}{3}$
- $\tan\alpha = \frac{2}{\sqrt{5}}$

Example 4

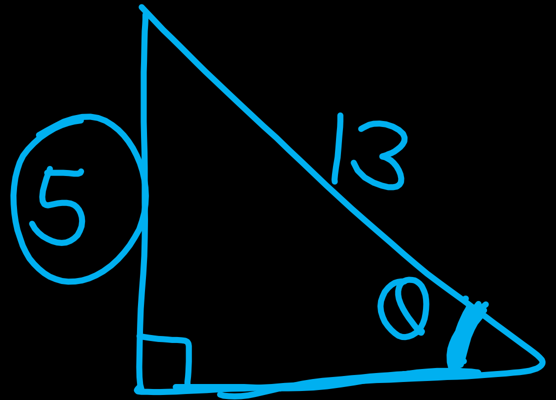
If $\boxed{\sin\theta = \frac{5}{13}}$, find value of $\cos\theta$, $\tan\theta$:

→ opp

→ Hypo

✓ Triangle Method:

M-2 Using Identities



$\sqrt{13^2 - 5^2} = \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

Basic Identities: Set-1

Basic Identities

1. $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$; $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sin\theta \operatorname{cosec}\theta = 1$

2. $\cos\theta = \frac{1}{\sec\theta}$; $\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \sec\theta = 1$

3. $\tan\theta = \frac{1}{\cot\theta}$; $\cot\theta = \frac{1}{\tan\theta}$; $\tan\theta . \cot\theta = 1$

4. $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Basic Identities: Set-2

Basic Identities

(5) $\sin^2\theta + \cos^2\theta = 1$

$\hookrightarrow \cos^2\theta = 1 - \sin^2\theta$

$\hookrightarrow \sin^2\theta = 1 - \cos^2\theta$

$\hookrightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\hookrightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

(6) $1 + \tan^2\theta = \sec^2\theta$

$\hookrightarrow \tan^2\theta = \sec^2\theta - 1$

$\hookrightarrow \sec^2\theta - \tan^2\theta = 1$

$a^2 - b^2 = (a-b)(a+b)$

$\hookrightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\hookrightarrow \sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$

$\hookrightarrow \sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

(7) $1 + \cot^2\theta = \csc^2\theta$

$\hookrightarrow \cot^2\theta = \csc^2\theta - 1$

& $\cot\theta = \pm\sqrt{\csc^2\theta - 1}$

$\hookrightarrow \csc^2\theta - \cot^2\theta = 1$

$\hookrightarrow (\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$

$\hookrightarrow \csc\theta + \cot\theta = \dfrac{1}{\csc\theta - \cot\theta}$

Expressing T-ratios in terms of each other

$1 + \tan^2\theta = \sec^2\theta$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

Table of Interrelations

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	1 2 $\sqrt{1-\cos^2\theta}$	1 3 $\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	1 4 $\frac{1}{\sqrt{1+\cot^2\theta}}$	1 5 $\sqrt{1-\frac{1}{\sec^2\theta}}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	2 3 $\frac{1}{\sqrt{1+\tan^2\theta}}$	2 4 $\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\sqrt{1-\frac{1}{\operatorname{cosec}^2\theta}}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$

1.2 $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$

1.3 $\sin\theta = \frac{\sin\theta}{\cos\theta} \cos\theta$

$= \tan\theta \cos\theta$

$= \tan\theta \cdot \frac{1}{\sec\theta}$

$= \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

* $1+\tan^2\theta = \sec^2\theta$

$\sqrt{1+\tan^2\theta} = \sec\theta$

** $1+\cot^2\theta = \csc^2\theta$

$\sqrt{1+\cot^2\theta} = \csc\theta$

$\cot\theta = ?$

1.4 $\sin\theta = \frac{1}{\csc\theta}$

$= \frac{1}{\sqrt{1+\cot^2\theta}}$

1.5 $\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\frac{1}{\sec^2\theta}}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = ?$

2.3 $\cos\theta = \dfrac{1}{\sec\theta}$

$= \dfrac{1}{\sqrt{1+\tan^2\theta}}$

2 4 $\cos\theta = \dfrac{\cos\theta}{\sin\theta}\, \sin\theta$ ✓

$= \cot\theta \, \dfrac{1}{\operatorname{cosec}\theta}$

$= \dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$

$1+\tan^2\theta = \sec^2\theta$

$\sqrt{1+\tan^2\theta} = \sec\theta$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$

2 6 $\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1 - \dfrac{1}{\operatorname{cosec}^2\theta}}$

31

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

32

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

36

$$\tan\theta = \frac{1}{\cot\theta}$$

$$= \frac{1}{\sqrt{\operatorname{cosec}^2\theta - 1}}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Question:1 on Identities

Ans: 1

Find the value of $(0^\circ < A < 90^\circ)$

$$\frac{\tan^2 A \cdot \sin^2 A}{\tan^2 A - \sin^2 A}$$

common leneka muken आता to common lena chahiye!

$$\text{LHS} = \frac{\tan^2 A \sin^2 A}{\tan^2 A - \sin^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A} \cdot \sin^2 A}{\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 A}{1}}$$

$$=$$

$$= \frac{\cancel{\sin^2 A}\ \frac{\sin^2 A}{\cos^2 A}}{\cancel{\sin^2 A}\left(\frac{1}{\cos^2 A} - 1\right)}$$

$$= \frac{\frac{\sin^2 A}{\cancel{\cos^2 A}}}{\frac{1-\cos^2 A}{\cancel{\cos^2 A}}}$$

$$= \frac{\cancel{\sin^2 A}}{\cancel{\sin^2 A}}$$

$$= 1$$

Question:2 on Identities

Prove that:

$$\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\sec\alpha}{1+\operatorname{cosec}\alpha}\right)=\tan\alpha$$

$\sec\alpha = \frac{1}{\cos\alpha}$, $\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$

सवालों Badlu sin/cos me

$$LHS = \left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\left(\frac{1+\frac{1}{\cos\alpha}}{1+\frac{1}{\sin\alpha}}\right)$$

$$= \left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)\frac{\left(\frac{\cos\alpha+1}{\cos\alpha}\right)}{\left(\frac{\sin\alpha+1}{\sin\alpha}\right)}$$

$$= \cancel{\left(\frac{1+\sin\alpha}{1+\cos\alpha}\right)}\left(\frac{\cancel{\cos\alpha+1}}{\cos\alpha}\right)\left(\frac{\sin\alpha}{\cancel{\sin\alpha+1}}\right)$$

$$LHS = \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= RHS \text{ //}$$

Question:3 on Identities

Prove that:

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

Rationlisation

$$LHS = \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$$

$$(a+b)(a-b) = a^2-b^2$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \qquad \sqrt{\frac{a^2}{b^2}} = \frac{a}{b}$$

$$= \sqrt{\left(\frac{1-\cos\theta}{\sin\theta}\right)^{\cancel{2}}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

split

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

$$= RHS$$

Question:4 on Identities

Prove that:

$$(\sec\theta + \operatorname{cosec}\theta)(\sin\theta + \cos\theta) - \sec\theta \cdot \operatorname{cosec}\theta = 2$$

सबको badla sin/cos

$$\text{LHS} = \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)\left(\sin\theta + \cos\theta\right) - \frac{1}{\cos\theta}\,\frac{1}{\sin\theta}$$

$$= \left(\frac{\sin\theta+\cos\theta}{\sin\theta\,\cos\theta}\right)\frac{(\sin\theta+\cos\theta)}{1} - \frac{1}{\sin\theta\,\cos\theta}$$

$$= \frac{(\sin\theta+\cos\theta)^2 - 1}{\sin\theta\,\cos\theta}$$

$$= \frac{\cancel{\sin^2\theta+\cos^2\theta} + 2\sin\theta\cos\theta \cancel{-1}}{\sin\theta.\cos\theta}$$

$$\text{LHS} = \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$$

$= 2$ //

$= \text{RHS}$

H.P

Question:5

If $\tan\theta + \sec\theta = 1.5$, find the values of $\sin\theta$, $\tan\theta$, and $\sec\theta$.

$\sec\theta + \tan\theta = \frac{3}{2}$ — (1)

$\sec\theta - \tan\theta = \frac{2}{3}$ — (2)

$2\sec\theta = \frac{3}{2} + \frac{2}{3}$

$2\sec\theta = \frac{13}{6}$

$$\boxed{\sec\theta = \frac{13}{12}}$$

13 → Hypo

12 → Adj

*$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

$= \frac{1}{3/2}$

$= 2/3$

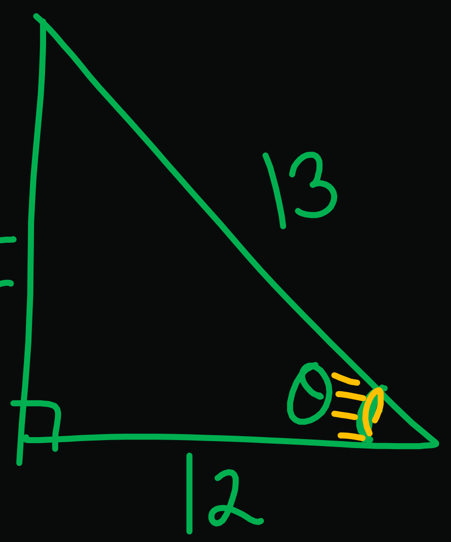
Triangle Method

$\sqrt{13^2 - 12^2}$

$\sqrt{169 - 144}$

$\sqrt{25}$

5



$\sin\theta = \frac{5}{13}$

$\tan\theta = \frac{5}{12}$

M-2

Using Identites

$1 + \tan^2\theta = \sec^2\theta$

Question:6

If $2\sin\theta = 2 - \cos\theta$, then find the value(s) of $\sin\theta$.

$$\rightarrow \quad 2\sin\theta = 2 - \cos\theta \quad \cancel{\rightarrow} \quad \sqrt{1-\sin^2\theta}$$

$$\cancel{\rightarrow} \quad (2\sin\theta)^2 = (2-\cos\theta)^2$$

$$\cos\theta = 2 - 2\sin\theta$$

Squaring

$$\cos^2\theta = 4 + 4\sin^2\theta - 8\sin\theta$$

$$1 - \sin^2\theta = 4 + 4\sin^2\theta - 8\sin\theta$$

$$\boxed{5\sin^2\theta - 8\sin\theta + 3 = 0}$$

$$15 \quad -5 \quad -3$$

$$\underbrace{5\sin^2\theta - 5\sin\theta}\, - 3\sin\theta + 3 = 0$$

$$5\sin\theta(\sin\theta - 1) - 3(\sin\theta - 1) = 0$$

$$(\sin\theta - 1)(5\sin\theta - 3) = 0$$

$$\boxed{\sin\theta = 1} \checkmark \qquad \boxed{\sin\theta = \frac{3}{5}} \checkmark$$

Question:7

If $\sin\theta + \sin^2\theta = 1$, then prove that:

$$\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^{8}\theta + \cos^{6}\theta - 1 = 0$$

$\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta$

$\boxed{\sin\theta = \cos^2\theta}$

LHS $= (\cos^2\theta)^6 + 3(\cos^2\theta)^5 + 3(\cos^2\theta)^4 + (\cos^2\theta)^3 - 1$

$= (\sin\theta)^6 + 3(\sin\theta)^5 + 3(\sin\theta)^4 + (\sin\theta)^3 - 1$

Common

$= \sin^3\theta\left[1\sin^3\theta + 3\sin^2\theta + 3\sin\theta + 1\right] - 1$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a^3b^3 = (ab)^3$

$= \sin^3\theta\left[(\sin\theta+1)^3\right] - 1$

$= [\sin\theta(\sin\theta+1)]^3 - 1$

$= [\sin^2\theta + \sin\theta]^3 - 1$

$= 1^3 - 1$

$= 0$

$=$ RHS

Question:8 JEE Main 2025 (January)

If $\sin x + \sin^2 x = 1$, where $x \in (0, \frac{\pi}{2})$, then the expression:

$$(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$$

is equal to: $= \cos^{12} x + \cos^{12} x + 3\left(\cos^{10} x + \cos^{10} x + \cos^8 x + \cos^8 x\right) + \cos^6 x + \cos^6 x$

(A) 3 (B) 4 (C) 2 (D) 1

$\sin x + \sin^2 x = 1$

$\sin x = 1 - \sin^2 x$

$\boxed{\sin x = \cos^2 x}$ — (*)

$\frac{\sin x}{\cos x} = \cos x$

$\boxed{\tan x = \cos x}$ — (**)

$= 2\cos^{12} x + 3\left(2\cos^{10} x + 2\cos^8 x\right) + 2\cos^6 x$

$= 2\sin^6 x + 6\sin^5 x + 6\sin^4 x + 2\sin^3 x$

common

$= 2\sin^3 x\left[\sin^3 x + 3\sin^2 x + 3\sin x + 1\right]$

$= 2\sin^3 x \cdot (\sin x + 1)^3$

$= 2\left(\sin x(\sin x + 1)\right)^3$

$= 2\left(\sin^2 x + \sin x\right)^3$

$= 2(1)^3$

$= 2$

Question:9

If $3\sec^4\theta + 8 = 10\sec^2\theta$, then find the positive value(s) of $\tan\theta$.

$3\sec^4\theta + 8 = 10\sec^2\theta$

let $\boxed{\sec^2\theta = t} \rightarrow \sec^4\theta = t^2$

$3t^2 + 8 = 10t$

$3t^2 - 10t + 8 = 0$ 24 → -6, -4

$3t^2 - 6t - 4t + 8 = 0$

$3t(t-2) - 4(t-2) = 0$

$t = 2 \quad t = \frac{4}{3}$

$\sec^2\theta = 2 \quad \sec^2\theta = \frac{4}{3}$

Using identity

$1 + \tan^2\theta = 2$

$\tan^2\theta = 2 - 1$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$1 + \tan^2\theta = \frac{4}{3}$

$\tan^2\theta = \frac{4}{3} - 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

Ans: positive values $\tan\theta = 1, \frac{1}{\sqrt{3}}$

M-2

$3\sec^4\theta + 8 = 10\sec^2\theta$

* $\sec^2\theta = 1 + \tan^2\theta$

$3(1+\tan^2\theta)^2 + 8 = 10(1+\tan^2\theta)$

$3[1 + \tan^4\theta + 2\tan^2\theta] + 8$

$= 10 + 10\tan^2\theta$

let $\tan^2\theta = t$

$\therefore$ Quadratic

Question:10

If $3\sin^4 x + 2\cos^4 x = \dfrac{6}{5}$, then find the value of $\tan^2 x$.

$$3\sin^4 x + 2\left(\cos^2 x\right)^2 = \frac{6}{5}$$

$$3\sin^4 x + 2\left(1-\sin^2 x\right)^2 = \frac{6}{5}$$

$$3\sin^4 x + 2\left[1+\sin^4 x - 2\sin^2 x\right] = \frac{6}{5}$$

$$** \; 5\sin^4 x - 4\sin^2 x + 2 - \frac{6}{5} = 0$$

let $\sin^2 x = t$

$\therefore \sin^4 x = t^2$

$$5t^2 - 4t + \frac{4}{5} = 0$$

$$25t^2 - 20t + 4 = 0$$

100

$-10 \quad -10$

$$25t^2 - 10t - 10t + 4 = 0$$

$$5t(5t-2) - 2(5t-2) = 0$$

$$(5t-2)(5t-2) = 0$$

$$(5t-2)^2 = 0$$

$$t = 2/5$$

$$\boxed{\sin^2 x = \frac{2}{5}} \; — \; 1$$

$$* \; \sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \frac{2}{5}$$

$$\cos^2 x = \frac{3}{5} \; — \; 2$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{2/5}{3/5} = \boxed{2/3}$$

Question:11 JEE Main 2021

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to:

(A) 350 (B) 500 (C) 400 (D) 250

$15\sin^4\alpha + 10(\cos^2\alpha)^2 = 6$

$15\sin^4\alpha + 10(1-\sin^2\alpha)^2 = 6$

let $\sin^2\alpha = t$

$15t^2 + 10(1-t)^2 - 6 = 0$

$15t^2 + 10(1+t^2-2t) - 6 = 0$

$\boxed{25t^2 - 20t + 4 = 0}$

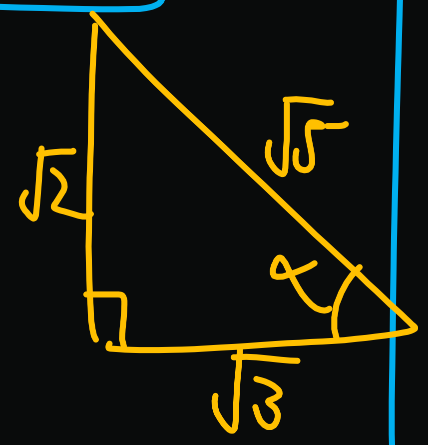
$25t^2 - 20t + 4 = 0$

$(5t-2)^2 = 0$

$t = \frac{2}{5}$

$\boxed{\sin^2\alpha = \frac{2}{5}}$

$\sin\alpha = \pm\frac{\sqrt{2}}{\sqrt{5}}$



$\text{Req} = 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$

$= 27\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^6 + 8\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^6$

$= \cancel{27}\cdot\frac{125}{\cancel{27}} + \cancel{8}\,\frac{125}{\cancel{8}}$

$= 125 + 125$

$= 250$

Question:12

If the following equation holds true:

$$\sec^4 x - \operatorname{cosec}^4 x - 2\sec^2 x + 2\operatorname{cosec}^2 x = \frac{15}{4}$$

then find the value of $\tan x$.

M-1

$$(\sec^4 x - 2\sec^2 x) - (\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x) = \frac{15}{4}$$

$(t^2 - 2t + 1) = (t-1)^2$

$$(\sec^4 x - 2\sec^2 x + 1) - (\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x + 1) = \frac{15}{4}$$

$$(\sec^2 x - 1)^2 - (\operatorname{cosec}^2 x - 1)^2 = \frac{15}{4}$$

$$(\tan^2 x)^2 - (\cot^2 x)^2 = \frac{15}{4}$$

$$\tan^4 x - \cot^4 x = \frac{15}{4}$$

$$\tan^4 x - \frac{1}{\tan^4 x} = \frac{15}{4}$$

Quad

let $\tan^4 x = t$

$$t - \frac{1}{t} = \frac{15}{4}$$

$$\frac{t^2 - 1}{t} = \frac{15}{4}$$

$$\tan^4 x - \frac{1}{\tan^4 x} = \frac{16-1}{4}$$

$$\boxed{\tan^4 x} - \frac{1}{\boxed{\tan^4 x}} = 4 - \frac{1}{4}$$

$$\tan^4 x = 4$$

Method-2

$$\sec^4 x - \csc^4 x - 2\sec^2 x + 2\csc^2 x = \frac{15}{4}$$

$$(1+\tan^2 x)^2 - (1+\cot^2 x)^2 - 2(1+\tan^2 x) + 2(1+\cot^2 x) = \frac{15}{4}$$

let $\tan^2 x = t$ $\therefore$ $\cot^2 x = \frac{1}{t}$

$$(1+t)^2 - \left(1+\frac{1}{t}\right)^2 - 2(1+t) + 2\left(1+\frac{1}{t}\right) = \frac{15}{4}$$

$$t^2 + \cancel{2t} + 1 - \left(\frac{t+1}{t}\right)^2 \cancel{-2} \cancel{-2t} \cancel{+2} + \frac{2}{t} = \frac{15}{4}$$

$$\frac{t^2+1}{1} - \frac{t^2+2t+1}{t^2} + \frac{2}{t} = \frac{15}{4}$$

$$\frac{t^4 + \cancel{t^2} - \cancel{t^2} - \cancel{2t} - 1 + \cancel{2t}}{t^2} = \frac{15}{4}$$

$$\frac{t^4-1}{t^2} = \frac{15}{4}$$

$$\frac{t^2-1}{t} = \frac{15}{4}$$

$$4t^2 - 4 = 15t$$

$$4t^2 - 15t - 4 = 0 \qquad -16$$

$$4t^2 - 16t + t - 4 = 0 \qquad -16 \quad +1$$

$$4t(t-4) + 1(t-4) = 0$$

$$(t-4)(4t+1) = 0$$

$t = 4$ | $t = -\frac{1}{4}$

$\tan^4 x = 4$ | $\tan^4 x = -\frac{1}{4}$ × Reject

$\tan^2 x = 2$

$\tan x = \pm\sqrt{2}$

$$\frac{t^4-1}{t^2}=\frac{15}{4}$$

$\tan^2 x = t$

$4t^4-4=15t^2$

$4t^4-15t^2-4=0$ $\quad -16$

$4t^4-16t^2+t^2-4=0$ $\quad -16+1$

$4t^2(t^2-4)+1(t^2-4)=0$

$(t^2-4)(4t^2+1)=0$

$t^2=4$ $\qquad t^2=-\frac{1}{4}$ Reject

$t^2=4$

$(\tan^2 x)^2=4$

$\tan^4 x=4$

$\tan^2 x=2$

$\boxed{\tan x=\pm\sqrt{2}}$

Question:13

The expression $4(\sin^6\theta + \cos^6\theta) - 6(\sin^4\theta + \cos^4\theta)$ is equal to:

(A) 0

(B) 1

(C) -2

(D) 2

$4(1-3\sin^2\theta\cos^2\theta) - 6(1-2\sin^2\theta\cdot\cos^2\theta) = 4 - \cancel{12 s^2 c^2} - 6 + \cancel{12 s^2 c^2}$

$= 4 - 6$

$= \boxed{-2}$

$\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

* $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$\boxed{\#\ \sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta}$

$\sin^4\theta + \cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2$

* $a^2 + b^2 = (a+b)^2 - 2ab$

$= (\sin^2\theta + \cos^2\theta)^2 - 2(\sin^2\theta)(\cos^2\theta)$

$\boxed{\#\ \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta}$

Question:14

Prove that:

$$\sin^8 x - \cos^8 x = (\sin^2 x - \cos^2 x)(1 - 2\sin^2 x \cos^2 x)$$

$LHS = \sin^8 x - \cos^8 x$

$= \boxed{(\sin^4 x)}^2 - (\cos^4 x)^2$

$= (\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)$

$= \left(\boxed{(\sin^2 x)}^2 - (\cos^2 x)^2\right)(1 - 2\sin^2 x \cdot \cos^2 x)$

$= (\sin^2 x - \cos^2 x)\cancel{(\sin^2 x + \cos^2 x)}(1 - 2\sin^2 x \cos^2 x)$

$= (\sin^2 x - \cos^2 x)(1 - 2\sin^2 x \cos^2 x)$

$= RHS$

Question:15

If $T_n = \sin^n x + \cos^n x$, prove that:

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

$T_1 = \sin^1 x + \cos^1 x$

$T_2 = \sin^2 x + \cos^2 x$

$T_3 = \sin^3 x + \cos^3 x$

$$LHS = \frac{T_3 - T_5}{T_1} = \frac{(\sin^3 x + \cos^3 x) - (\sin^5 x + \cos^5 x)}{(\sin x + \cos x)}$$

$$= \frac{(\sin^3 x - \sin^5 x) + (\cos^3 x - \cos^5 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^3 x\,(1 - \sin^2 x) + \cos^3 x\,(1 - \cos^2 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^3 x\,(\cos^2 x) + \cos^3 x\,(\sin^2 x)}{(\sin x + \cos x)}$$

$$= \frac{\sin^2 x \cos^2 x\,\cancel{(\sin x + \cos x)}}{\cancel{(\sin x + \cos x)}}$$

$$= \sin^2 x \cdot \cos^2 x$$

$$RHS = \frac{T_5 - T_7}{T_3}$$

$$= \frac{(\sin^5 x + \cos^5 x) - (\sin^7 x + \cos^7 x)}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^5 x(1 - \sin^2 x) + \cos^5 x(1 - \cos^2 x)}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^5 x \cos^2 x + \cos^5 x \sin^2 x}{(\sin^3 x + \cos^3 x)}$$

$$= \frac{\sin^2 x \cos^2 x \cancel{(\sin^3 x + \cos^3 x)}}{\cancel{(\sin^3 x + \cos^3 x)}}$$

$$RHS = \sin^2 x \cos^2 x$$

$\therefore LHS = RHS$

HP//

Question:16 JEE Main 2019

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \ldots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to:

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{12}$ (D) $\frac{5}{12}$

$$f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$$

$$Req = f_4(x) - f_6(x)$$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - 2\sin^2 x \cdot \cos^2 x\right) - \frac{1}{6}\left(1 - 3\sin^2 x \cos^2 x\right)$$

$$= \frac{1}{4} - \cancel{2\left(\frac{1}{4}\right)\sin^2 x \cos^2 x} - \frac{1}{6}$$

$$+ \cancel{\frac{1}{6}(3)\sin^2 x \cdot \cos^2 x}$$

$$= \frac{1}{4} - \frac{1}{6}$$

$$= \frac{3-2}{12}$$

$$= \boxed{\frac{1}{12}}$$

Question:17

Prove that:

$$(\operatorname{cosec} A + \cot A - 1)(\operatorname{cosec} A - \cot A + 1) = 2 \cot A$$

$$= \Big[\underbrace{(\operatorname{cosec} A)}_{a} + \underbrace{(\cot A - 1)}_{b}\Big]\Big[\underbrace{(\operatorname{cosec} A)}_{a} - \underbrace{(\cot A - 1)}_{b}\Big]$$

$$[a + b]\;[a - b]$$

$$= (\operatorname{cosec} A)^2 - (\cot A - 1)^2$$

$$= \operatorname{cosec}^2 A - [\cot^2 A + 1 - 2\cot A]$$

$$= \cancel{\operatorname{cosec}^2 A} - \cancel{\operatorname{cosec}^2 \theta} + 2\cot A$$

$$= 2\cot A$$

$$= \text{RHS}$$

Question:18 (Que.116)

Prove that:

$$\frac{(1-\sin\theta+\cos\theta)^2}{(1+\cos\theta)(1-\sin\theta)} = 2$$

$(a+b+c)^2 = (a+b)^2 + c^2 + 2(a+b)(c)$

$(a_1 + a_2)^2$

$$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc}$$

$(1-\sin\theta+\cos\theta)^2$

$(\underbrace{1}_{a}+\underbrace{(-\sin\theta)}_{b}+\underbrace{\cos\theta}_{c})^2 = 1^2+\underbrace{\sin^2\theta+\cos^2\theta}+2(-\sin\theta)+2(\cos\theta)+2(-\sin\theta)(\cos\theta)$

$= 2-2\sin\theta+2\cos\theta-2\sin\theta.\cos\theta$

$= 2\left(1-\sin\theta+\underbrace{\cos\theta-\sin\theta.\cos\theta}_{\text{common}}\right) = 2\left[(1-\sin\theta)+\cos\theta(1-\sin\theta)\right]$

$$(1+(-\sin\theta)+\cos\theta)^2 = 2\left[(1-\sin\theta)+\cos\theta(1-\sin\theta)\right]$$

$$= 2\left[(1-\sin\theta)(1+\cos\theta)\right]$$

$$LHS = \frac{2\cancel{(1-\sin\theta)}\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\cancel{(1-\sin\theta)}}$$

$$= 2$$

$$= RHS \;//$$

Question:19 (Que.115)

If $\operatorname{cosec}\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, prove that:

$$a^2b^2(a^2+b^2) = 1$$

$a = ?$

$a^3 = \operatorname{cosec}\theta - \sin\theta$

$a^3 = \frac{1}{\sin\theta} - \sin\theta$

$a^3 = \frac{1-\sin^2\theta}{\sin\theta}$

$a^3 = \frac{\cos^2\theta}{\sin\theta}$

$(\cancel{a^3})^{\cancel{1/3}} = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$

$a = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{1/3}$

$b^3 = \sec\theta - \cos\theta$

$b^3 = \frac{1}{\cos\theta} - \cos\theta$

$b^3 = \frac{1-\cos^2\theta}{\cos\theta}$

$b^3 = \frac{\sin^2\theta}{\cos\theta}$

$b = \left(\frac{\sin^2\theta}{\cos\theta}\right)^{1/3}$

Req $= a^2b^2(a^2+b^2)$

$= a^4b^2 + a^2b^4$

Req $= \left(\frac{\cos^2\theta}{\sin\theta}\right)^{4/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{4/3}$

$= \frac{(\cos\theta)^{\frac{8}{3}-\frac{2}{3}} \cdot \cancel{\sin^{4/3}\theta}}{\cancel{\sin^{4/3}\theta}} + \frac{\cancel{\cos^{4/3}\theta}\,(\sin\theta)^{\frac{8}{3}-\frac{2}{3}}}{\cancel{\cos^{4/3}\theta}}$

$= (\cos\theta)^2 + (\sin\theta)^2$

$= 1$

$=$ RHS

Question:20

Prove that:

$$\frac{\sin x + \cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$$LHS = \frac{\sin x + \cos x}{\cos^3 x}$$

Divide N^r & D^r by $\cos x$

$$= \frac{\dfrac{\sin x}{\cos x} + \cancel{\dfrac{\cos x}{\cos x}}}{\dfrac{\cos^3 x}{\cancel{\cos x}}}$$

$$= \frac{\tan x + 1}{\cos^2 x}$$

$$LHS = (\tan x + 1)\left(\frac{1}{\cos^2 x}\right)$$

$$= (\tan x + 1)(\sec^2 x)$$

$$= (\tan x + 1)(1 + \tan^2 x)$$

$$= \tan^3 x + \tan^2 x + \tan x + 1$$

$$= RHS$$

T-Ratios of Standard Angles

$\sin 30 < \cos 30$

Remember this Values

$\sin 60 > \cos 60$

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
$\cot\theta$	N D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N D
$\operatorname{cosec}\theta$	N D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

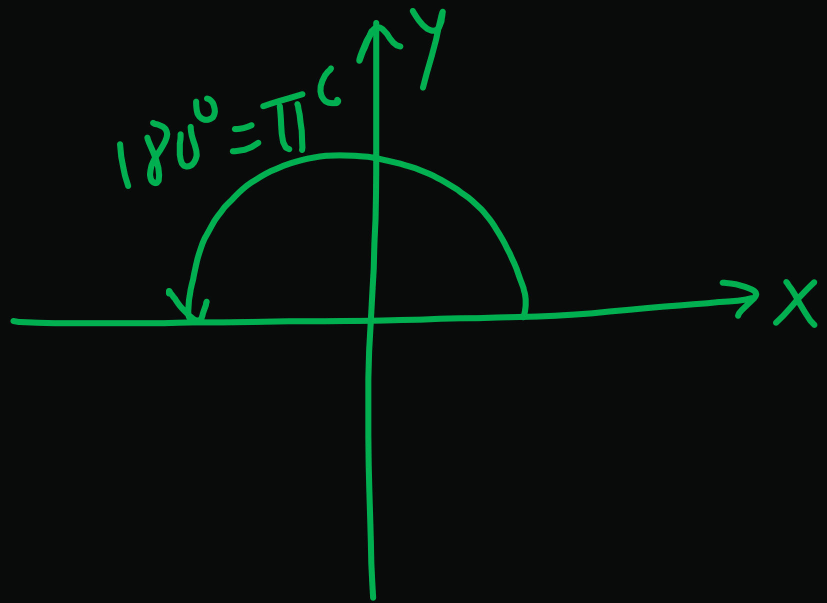
$\frac{1}{0} =$ N.D

Angle Measurement: Degrees & Radians

Conversion Formulas

$$\text{degree} \times \frac{\pi}{180^\circ} = \text{radian} \quad \longleftrightarrow \quad \text{radian} \times \frac{180^\circ}{\pi} = \text{degree}$$

$$\boxed{180^\circ = \pi^c}$$



① $30^\circ = (\ ?\ )^c$

$180^\circ \longrightarrow \pi^c$

$30^\circ \longrightarrow x$

$x \; 180 = 30\,\pi$

$x = 3\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{\pi}{6}$

② $60^\circ = 6\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{\pi}{3}$

③ $90^\circ = 9\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{\pi}{2}$

Examples:

④ $330^\circ = \cancel{330}^\circ \times \frac{\pi}{\cancel{180}} = \frac{11}{6}\pi$

$$\boxed{\text{Radian} \times \frac{180}{\pi} = \text{Degree}}$$

① $\frac{\pi}{4} = \frac{\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}} = 45^\circ$

② $\frac{2\pi}{3} = \frac{2\cancel{\pi}}{3} \times \frac{180}{\cancel{\pi}} = 120^\circ$

③ $\frac{5\pi}{3} = \frac{5 \times \cancel{180}}{\cancel{3}} = 300^\circ$

60

$45 \longrightarrow \frac{\pi}{4}$

$90 \longrightarrow \frac{\pi}{2}$

$135 \longrightarrow \frac{3\pi}{4}$

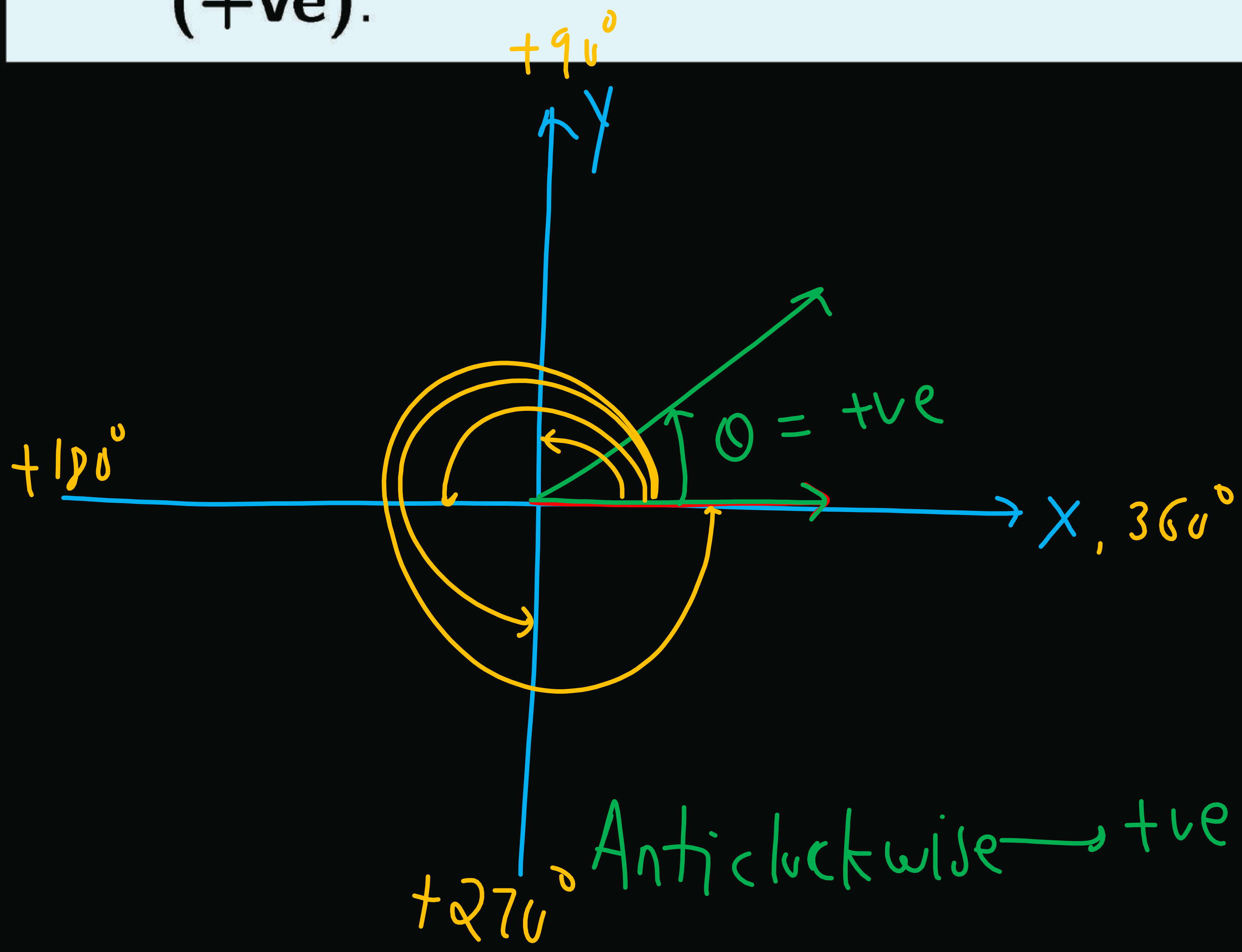
$180 \longrightarrow \pi$

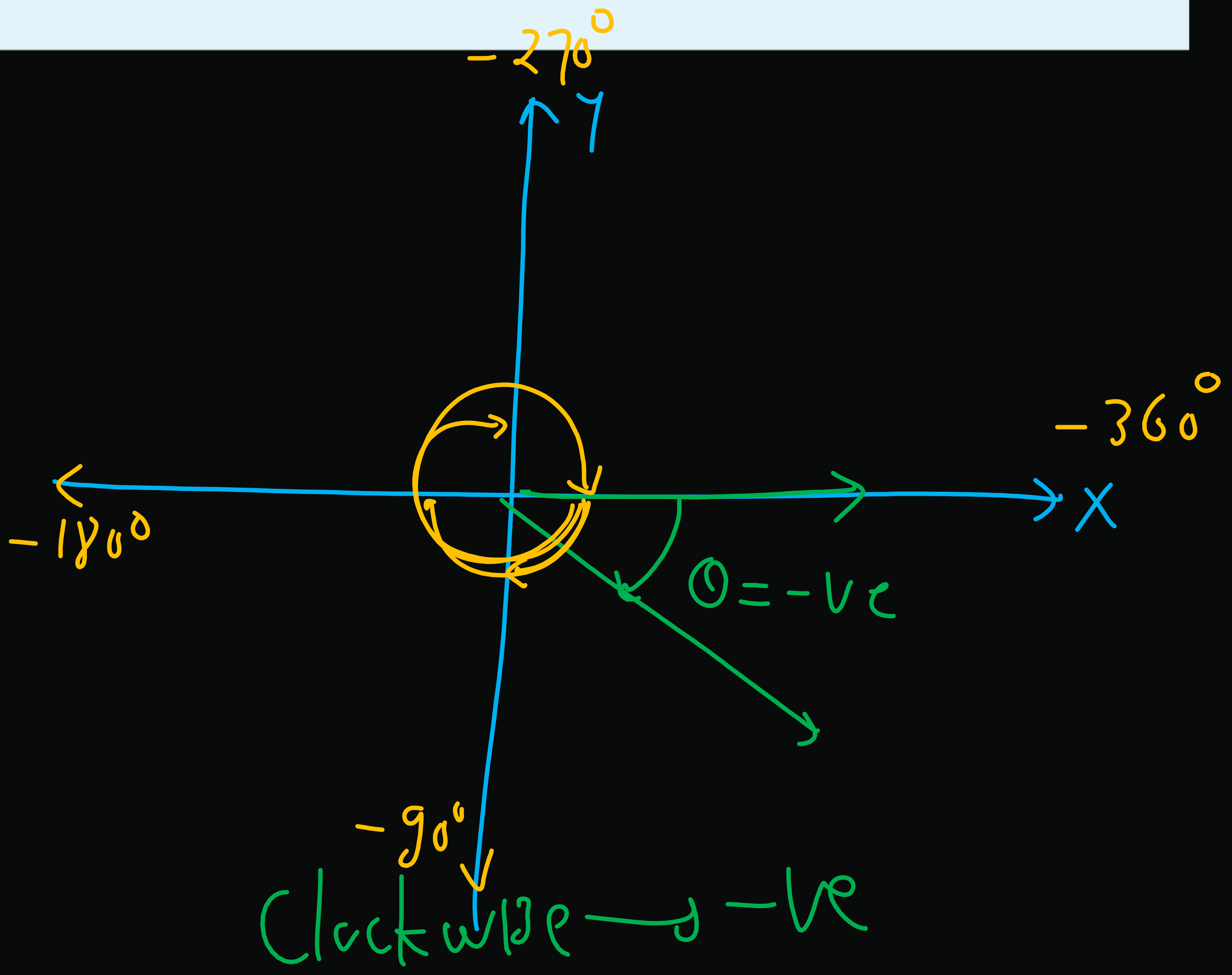
Some Important Angles

- $15^\circ = 15\times\frac{\pi}{180} = \frac{\pi}{12}$
- $18^\circ = 18\times\frac{\pi}{180} = \frac{\pi}{10}$
- $22.5^\circ = \frac{45^\circ}{2} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$
- $36^\circ = 2\times 18^\circ = 2\times\frac{\pi}{10} = \frac{\pi}{5}$
- $75^\circ = 90^\circ - 15^\circ = \frac{\pi}{2} - \frac{\pi}{12} = \frac{6\pi-\pi}{12} = \frac{5\pi}{12}$
- $72^\circ = \left(\frac{\pi}{10}\right)\times 4 = \frac{2\pi}{5}$
- $67.5^\circ = 90^\circ - 22.5^\circ = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}$
- $54^\circ = 3\times 18^\circ = 3\times\frac{\pi}{10} = \frac{3\pi}{10}$

Angle Convention

- **Anticlockwise** rotation from the positive x-axis is considered **positive (+ve)**.
- **Clockwise** rotation from the positive x-axis is considered **negative (-ve)**.

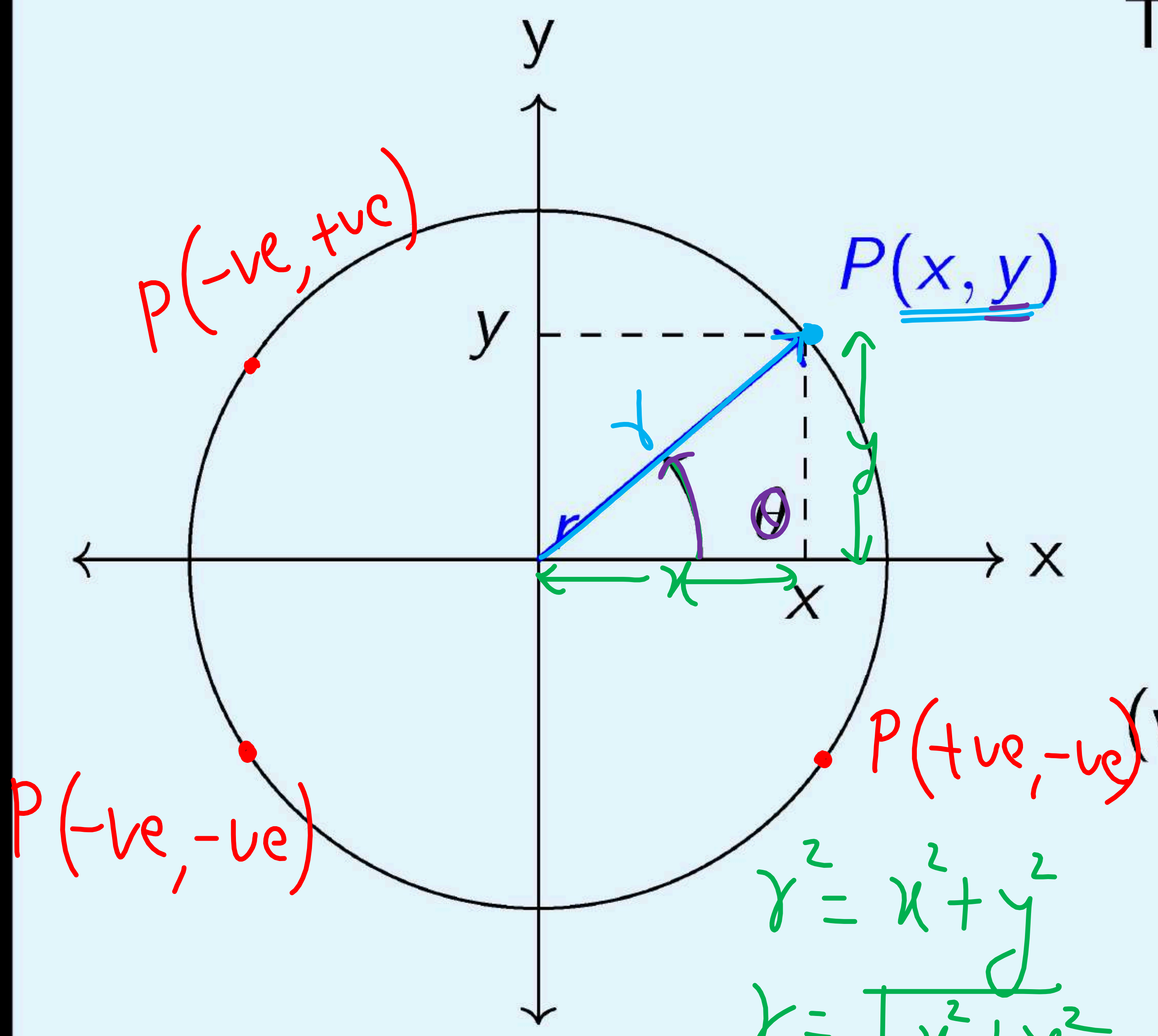




New Definition of Trigonometric Ratios

old defination $\sin\theta = \frac{Opp}{Hypo}$ $\cos\theta = \frac{Adj}{Hyp}$ $\tan\theta = \frac{Opp}{Adj}$

Let $P(x, y)$ be any point on a circle of radius r centered at the origin, and let θ be the angle formed with the positive x-axis.



The ratios are defined as:

- $\sin\theta = \dfrac{y\text{-coordinate}}{r}$
- $\cos\theta = \dfrac{x\text{-coordinate}}{r}$
- $\tan\theta = \dfrac{y\text{-coordinate}}{x\text{-coordinate}}$

(where $r = \sqrt{x^2 + y^2}$)

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

Limiting Cases:

ASTC Rule: Signs of T-Ratios in Quadrants

The sign of a trigonometric ratio depends on the quadrant in which the angle θ terminates.

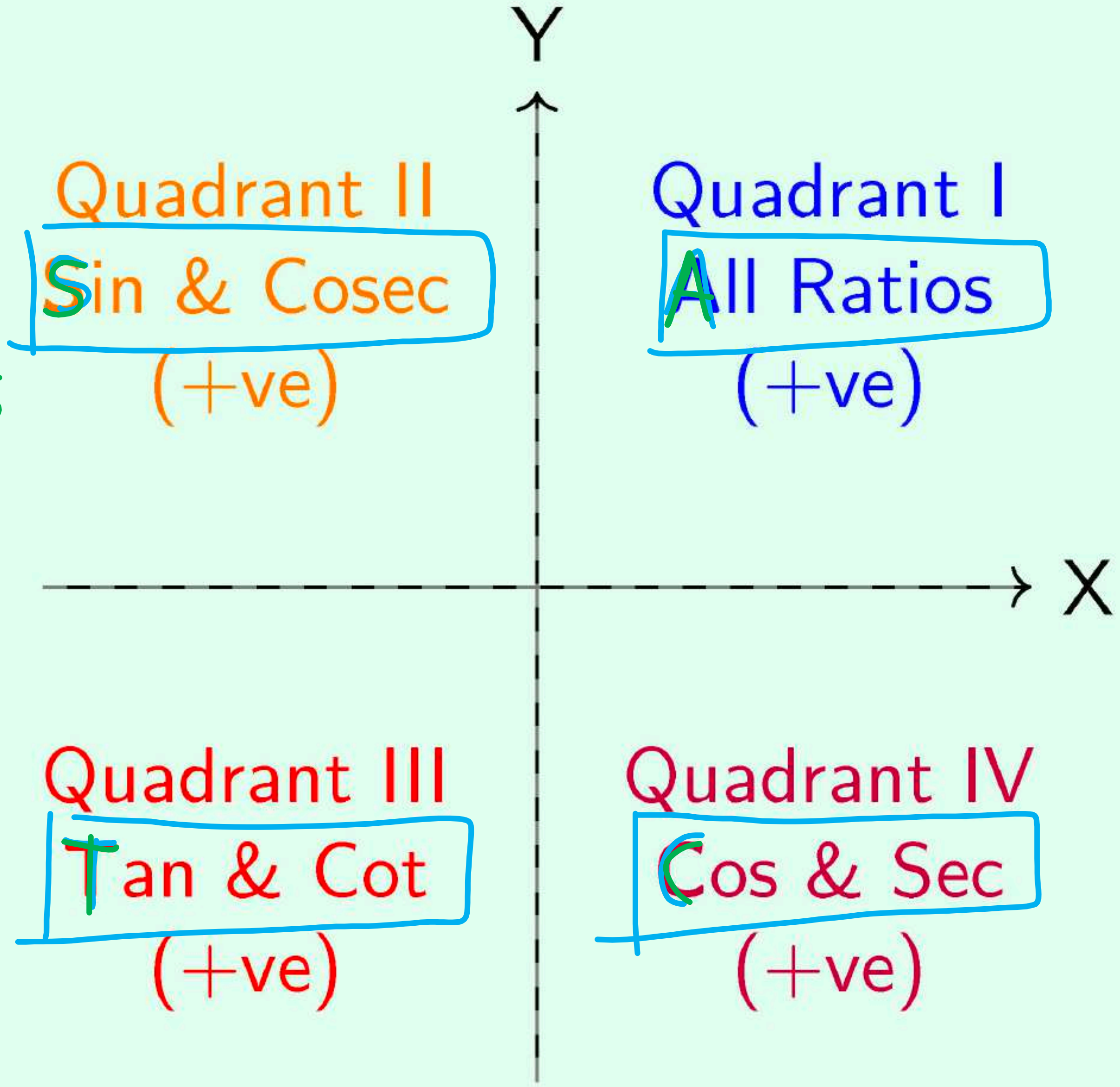
ASTC

A → All

S → students

T → Take

C → coffee



Proof: using new defination

I^{st} quad: x and y both co-ordinates are positive

$\sin\theta = \frac{y\text{-cordi}}{r} = \frac{+ve}{+ve} = +ve$

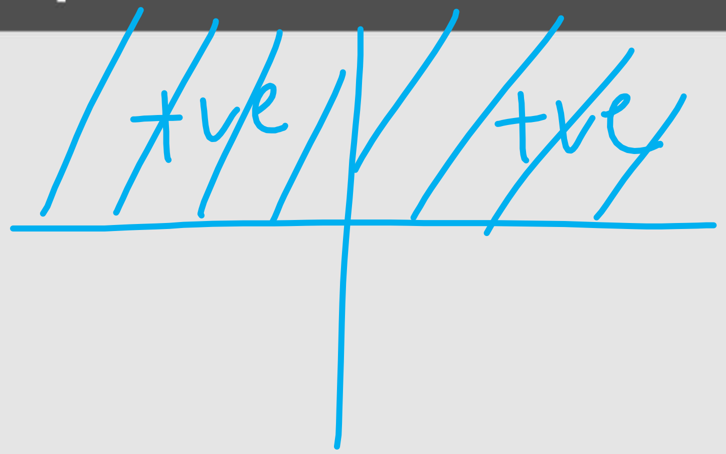
II^{nd} quad: x-coordinate → −ve
y-coordinate → +ve

$\sin\theta = \frac{y\text{-co}}{r} = \frac{+}{+} = +$

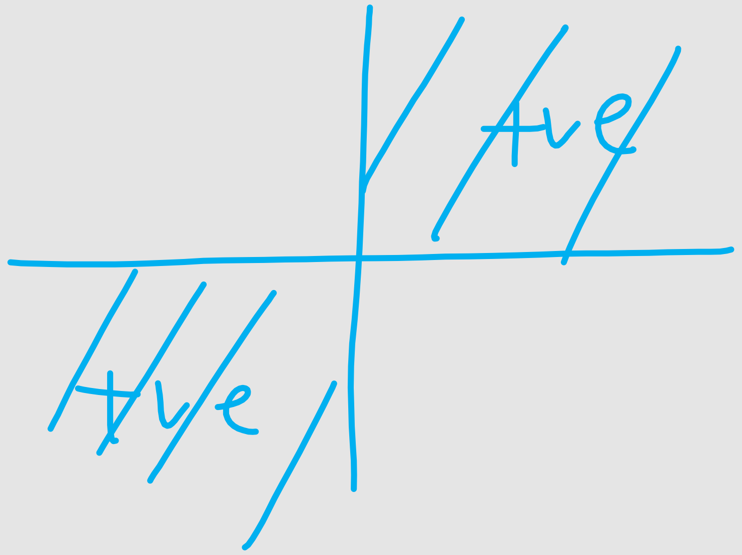
$\cos\theta = \frac{x\text{-co}}{r} = \frac{-ve}{+ve} = -ve$

ASCT: Different Perspective

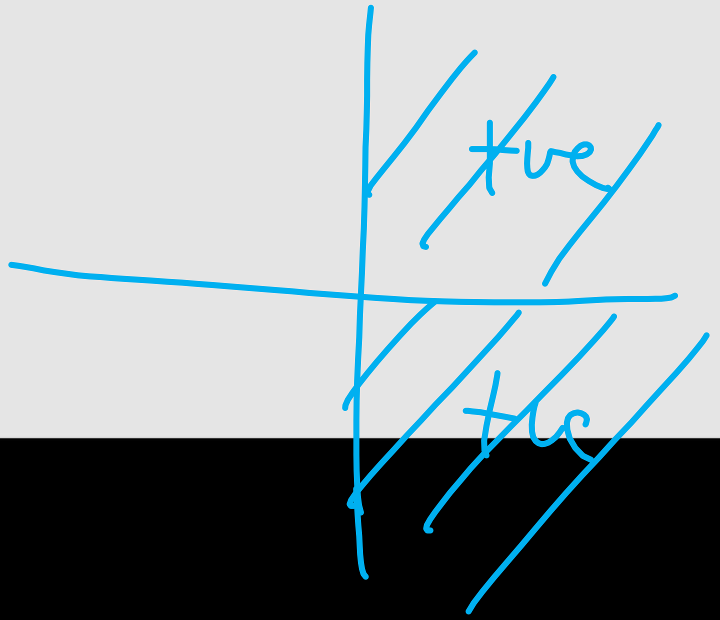
1. sin & cosec



2. tan & cot



3. cos & sec



Trigonometric Function of Allied Angle

1st Question: Change or NO Change?

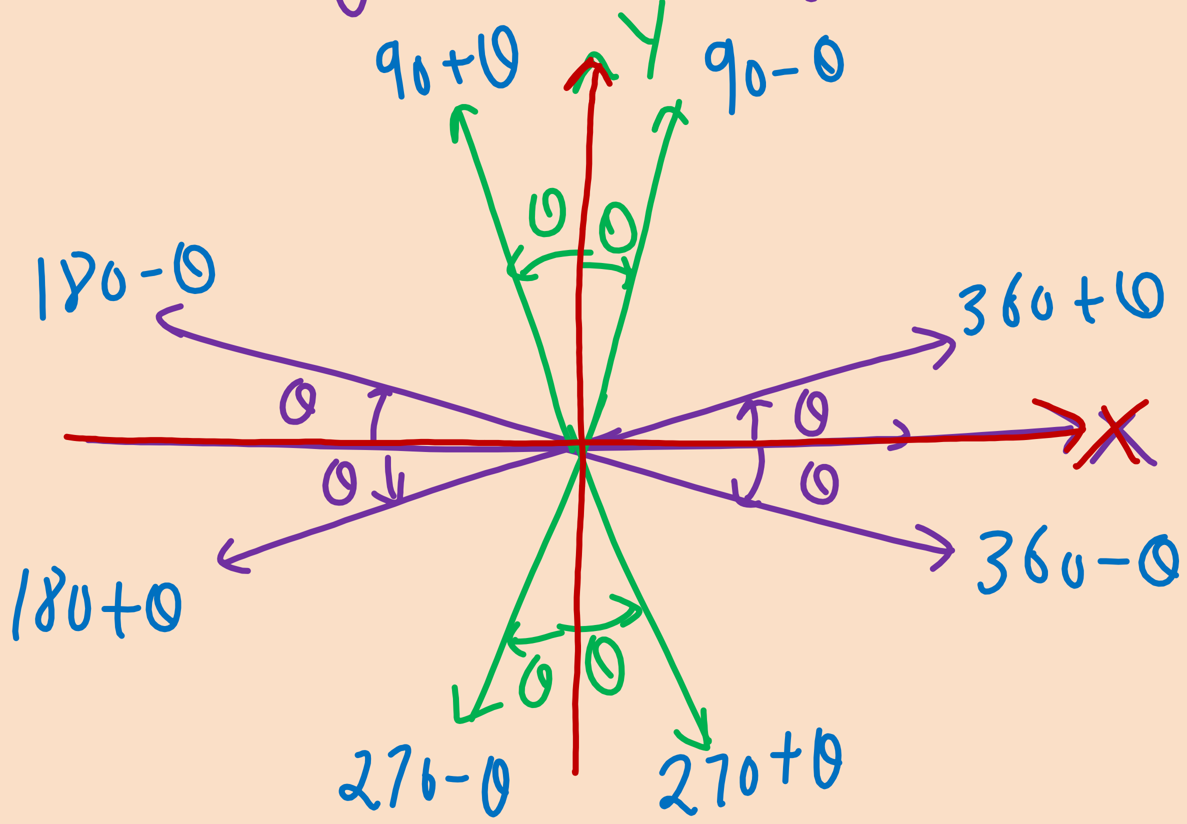
Change kya hoga!

sin ⇌ cos

tan ⇌ cot

sec ⇌ cosec

Change kab hoga!



$\left.\begin{array}{l} 90 \pm \theta \\ 270 \pm \theta \end{array}\right\}$ Change hoga

$\left.\begin{array}{l} 180 \pm \theta \\ 360 \pm \theta \end{array}\right\}$ Change nhi hoga.

अगर θ X-axis ke saath juda hai ⟹ No change

अगर θ X-axis ke saath juda hai ⟹ Change hoga!

2nd Question: Sign?

ASTC rule will decide the sign

$(90^\circ - \theta) \longrightarrow$ I^st Quad.

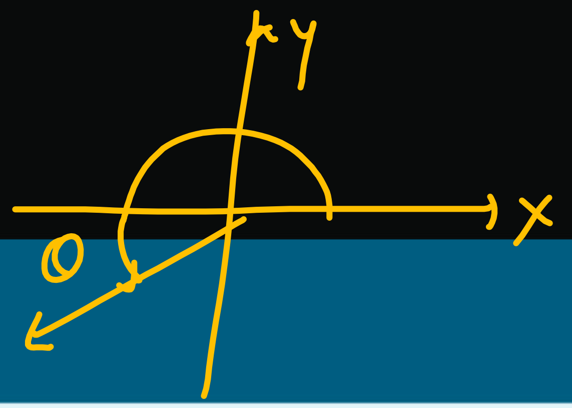
- $\sin(\underbrace{90^\circ - \theta}_{\text{Angle}}) = +\cos\theta$
- $\cos(90^\circ - \theta) = +\sin\theta$
- $\tan(90^\circ - \theta) = +\cot\theta$
- $\cot(90^\circ - \theta) = +\tan\theta$
- $\sec(90^\circ - \theta) = +\mathrm{cosec}\,\theta$
- $\mathrm{cosec}(90^\circ - \theta) = +\sec\theta$

$(90^\circ + \theta) \longrightarrow$ II^nd Quad

- $\sin(90^\circ + \theta) = +\cos\theta$
- $\cos(90^\circ + \theta) = -\sin\theta$
- $\tan(90^\circ + \theta) = -\cot\theta$
- $\cot(90^\circ + \theta) = -\tan\theta$
- $\sec(90^\circ + \theta) = -\mathrm{cosec}\,\theta$
- $\mathrm{cosec}(90^\circ + \theta) = +\sec\theta$

$(180^\circ - \theta) \longrightarrow \text{II}^{nd}$ quad

- $\sin(\underbrace{180^\circ - \theta}_{\text{II}^{nd}\text{ quad}}) = +\sin\theta$
- $\cos(\underbrace{180^\circ - \theta}) = -\cos\theta$
- $\tan(180^\circ - \theta) = -\tan\theta$
- $\cot(180^\circ - \theta) = -\cot\theta$
- $\sec(180^\circ - \theta) = -\sec\theta$
- $\mathrm{cosec}(180^\circ - \theta) = +\mathrm{cosec}\,\theta$



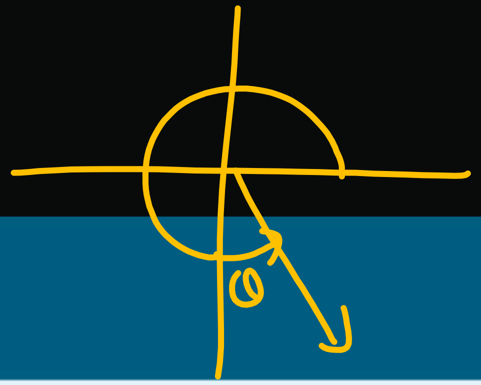
$(180^\circ + \theta) \longrightarrow \text{III}^{rd}$ quad.

- $\sin(\underbrace{180^\circ + \theta}_{\text{III}^{rd}\text{ quad}}) = -\sin\theta$
- $\cos(180^\circ + \theta) = -\cos\theta$
- $\tan(180^\circ + \theta) = +\tan\theta$
- $\cot(180^\circ + \theta) = +\cot\theta$
- $\sec(180^\circ + \theta) = -\sec\theta$
- $\mathrm{cosec}(180^\circ + \theta) = -\mathrm{cosec}\,\theta$

$(270^\circ - \theta) \longrightarrow$ IIIrd Quad

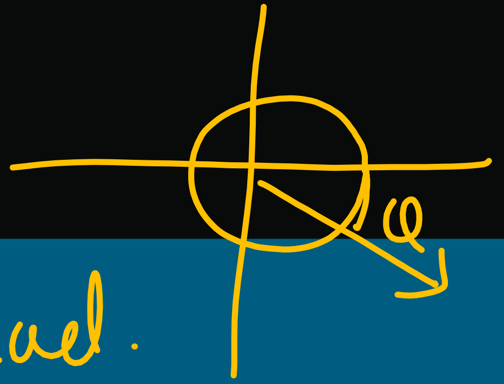
- $\sin(\underbrace{270^\circ - \theta}_{\text{III}^{rd}\text{ quad}}) = -\cos\theta$
- $\cos(270^\circ - \theta) = -\sin\theta$
- $\tan(270^\circ - \theta) = +\cot\theta$
- $\cot(270^\circ - \theta) = +\tan\theta$
- $\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$
- $\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$.

$(270^\circ + \theta) \longrightarrow$ IVth Quad



- $\sin(\underbrace{270^\circ + \theta}_{\text{IV}^{th}\text{ quad}}) = -\cos\theta$
- $\cos(270^\circ + \theta) = +\sin\theta$
- $\tan(270^\circ + \theta) = -\cot\theta$
- $\cot(270^\circ + \theta) = -\tan\theta$
- $\sec(270^\circ + \theta) = +\operatorname{cosec}\theta$
- $\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$

$(360° - \theta) \longrightarrow$ IVth Quad.



- $\sin(360° - \theta) = -\sin\theta$

  IVth quad

- $\cos(360° - \theta) = +\cos\theta$
- $\tan(360° - \theta) = -\tan\theta$
- $\cot(360° - \theta) = -\cot\theta$
- $\sec(360° - \theta) = +\sec\theta$
- $\text{cosec}(360° - \theta) = -\text{cosec}\,\theta$

$(360° + \theta) \longrightarrow$ Ist Quad

- $\sin(360° + \theta) = +\sin\theta$

  Ist quad

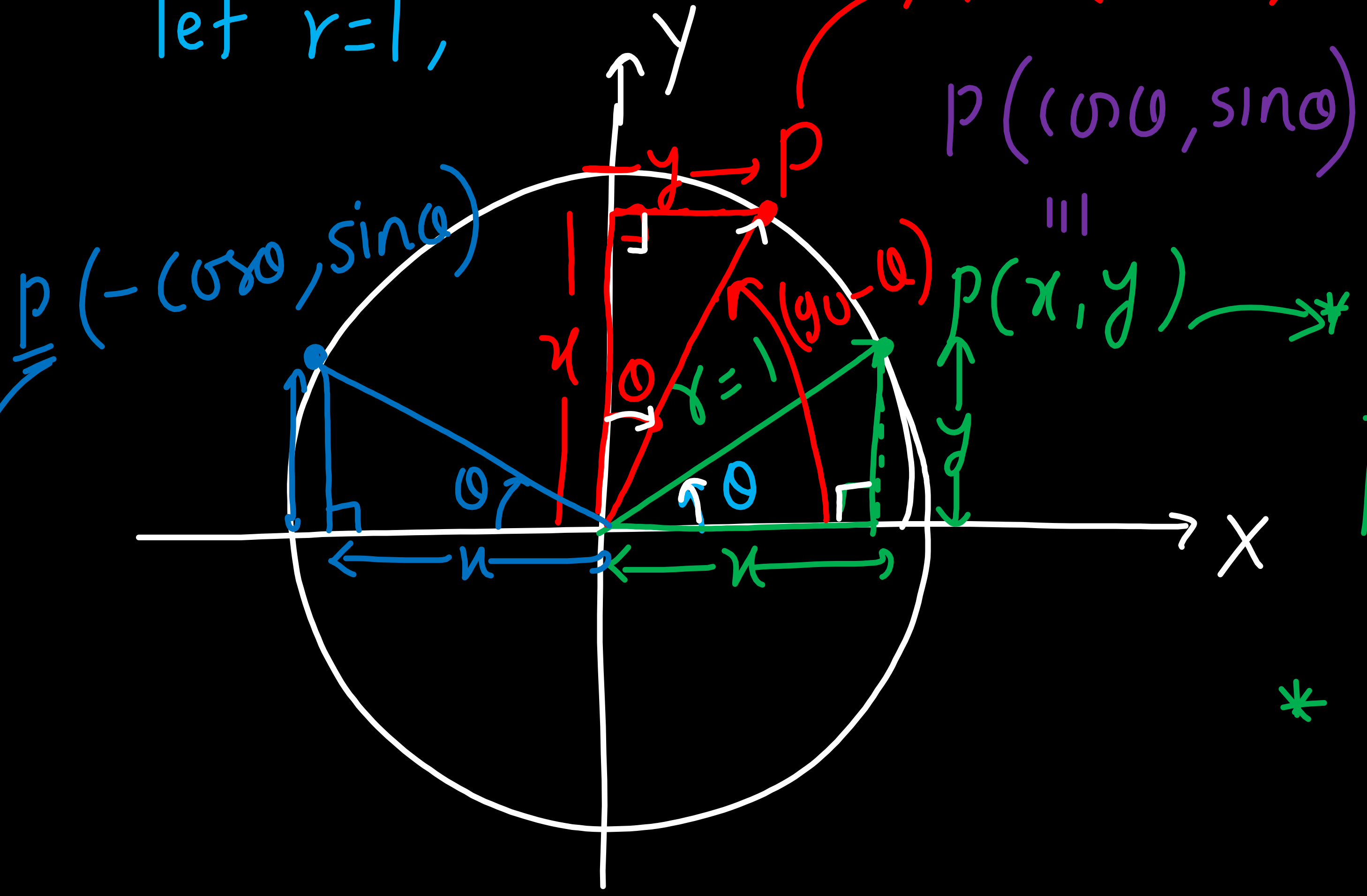
- $\cos(360° + \theta) = +\cos\theta$
- $\tan(360° + \theta) = +\tan\theta$
- $\cot(360° + \theta) = +\cot\theta$
- $\sec(360° + \theta) = +\sec\theta$
- $\text{cosec}(360° + \theta) = +\text{cosec}\,\theta$.

Not necessary for JEE / Proof not important

Why? $\sin(90-\theta) = \cos\theta$

let $r=1$,

$P \equiv (\sin\theta, \cos\theta)$



* $\sin\theta = \dfrac{y\text{-coordinate}}{r=1}$

$\boxed{\sin\theta = y}$

* $\cos\theta = \dfrac{x\text{-coordinate}}{r=1}$

$\boxed{\cos\theta = x}$

* $\sin(90-\theta) = \dfrac{y\text{-coordinate}}{r=1}$

$\boxed{\sin(90-\theta) = \cos\theta}$

* $\cos(90-\theta) = \dfrac{x\text{-coordinate}}{r=1}$

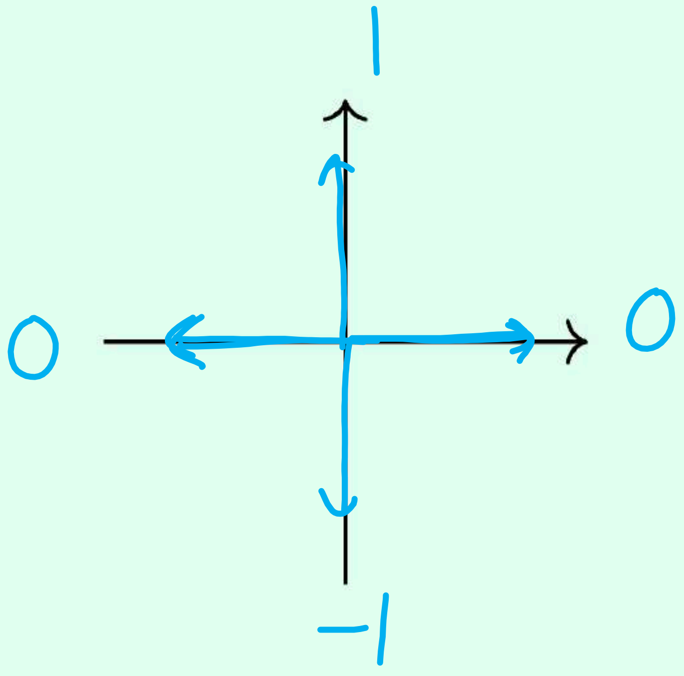
$\boxed{\cos(90-\theta) = \sin\theta}$

Both green & red triangles are congruent

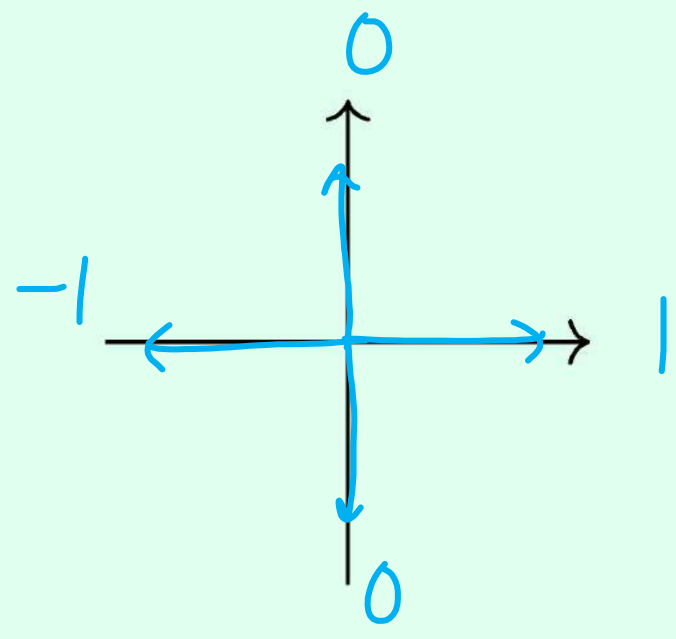
* $\sin(180-\theta) = \sin\theta$
* $\cos(180-\theta) = -\cos\theta$

Note: T-Ratio Values at Quadrantal Angles (0°, 90°, 180°, 270°)

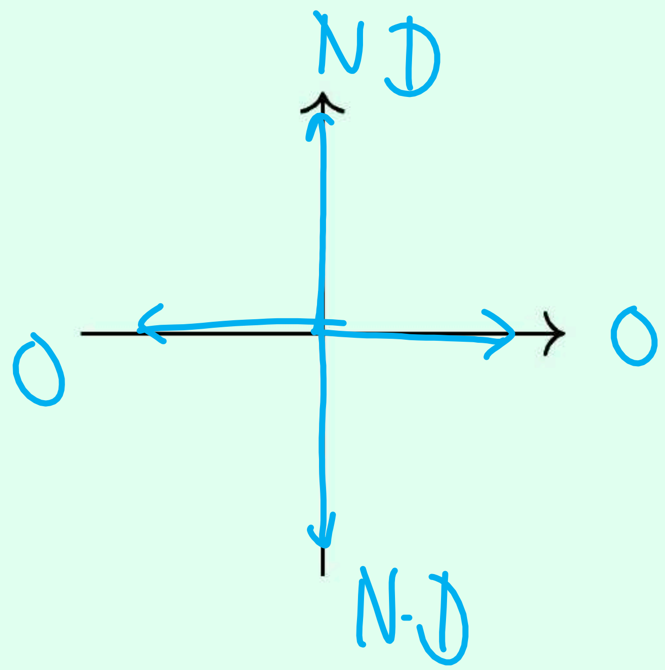
1) $\sin\theta$



2) $\cos\theta$

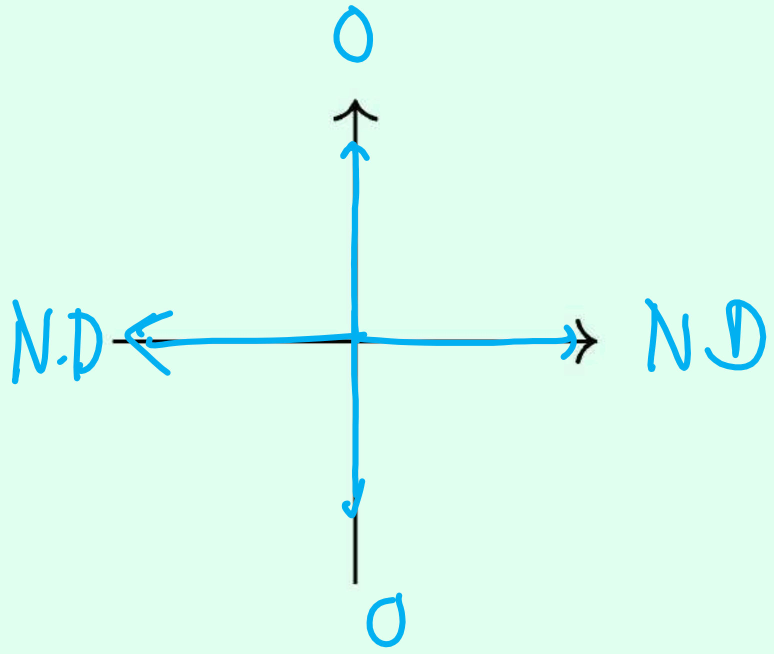


3) $\tan\theta$



*
$\sin\theta = 0 \Rightarrow$ on X-axis
$\cos\theta = 0 \Rightarrow$ on Y-axis
$\tan\theta = 0 \Rightarrow$ on X-axis
$\cot\theta = 0 \Rightarrow$ on Y-axis

4) $\cot\theta$



Example: 1

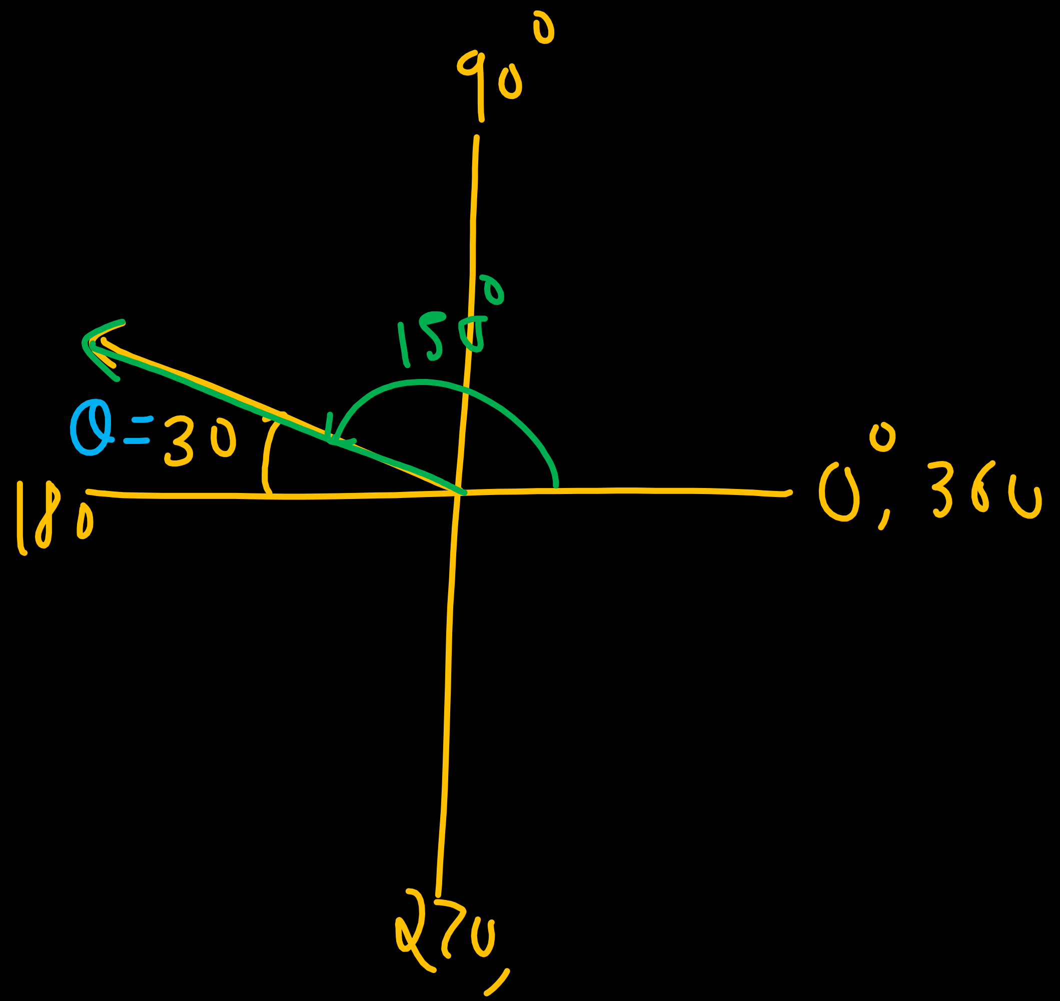
$\sin(150^\circ) =$

M-1

$\sin 150^\circ = \sin(180^\circ - 30^\circ) = +\sin 30$

$30^\circ = \theta$

$= \frac{1}{2}$

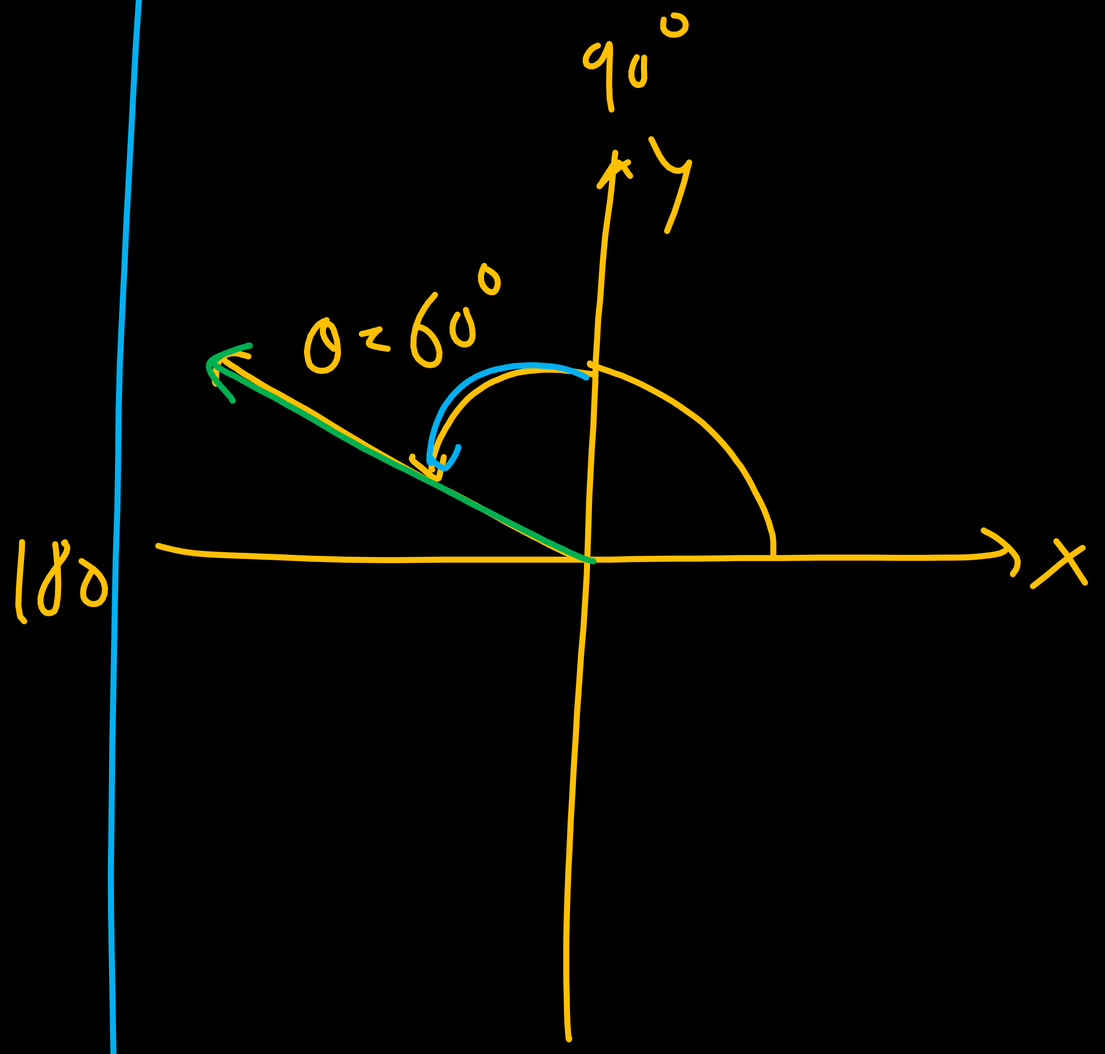


M-2

$\sin(150^\circ) = \sin(90^\circ + 60^\circ) = +\cos 60$

$60^\circ = \theta$

$= +\frac{1}{2}$



Example: 2

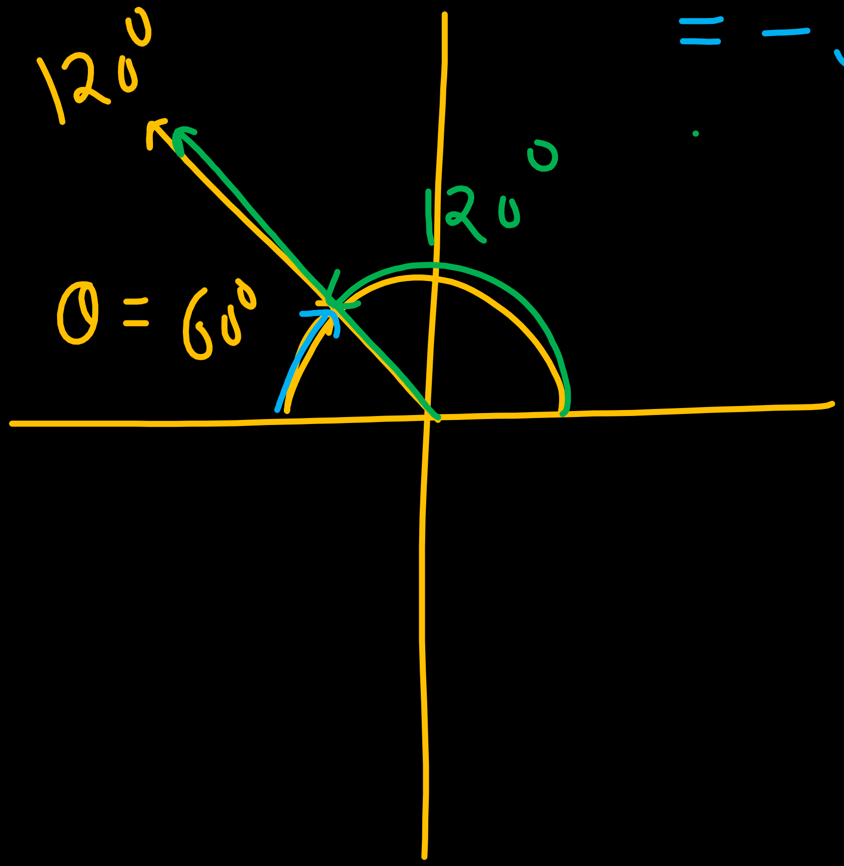
$\tan(120^\circ) =$

M-1

$$\tan 120^\circ = \tan(180^\circ - \underset{\theta}{60^\circ})$$

$$= -\tan 60$$

$$= -\sqrt{3}$$

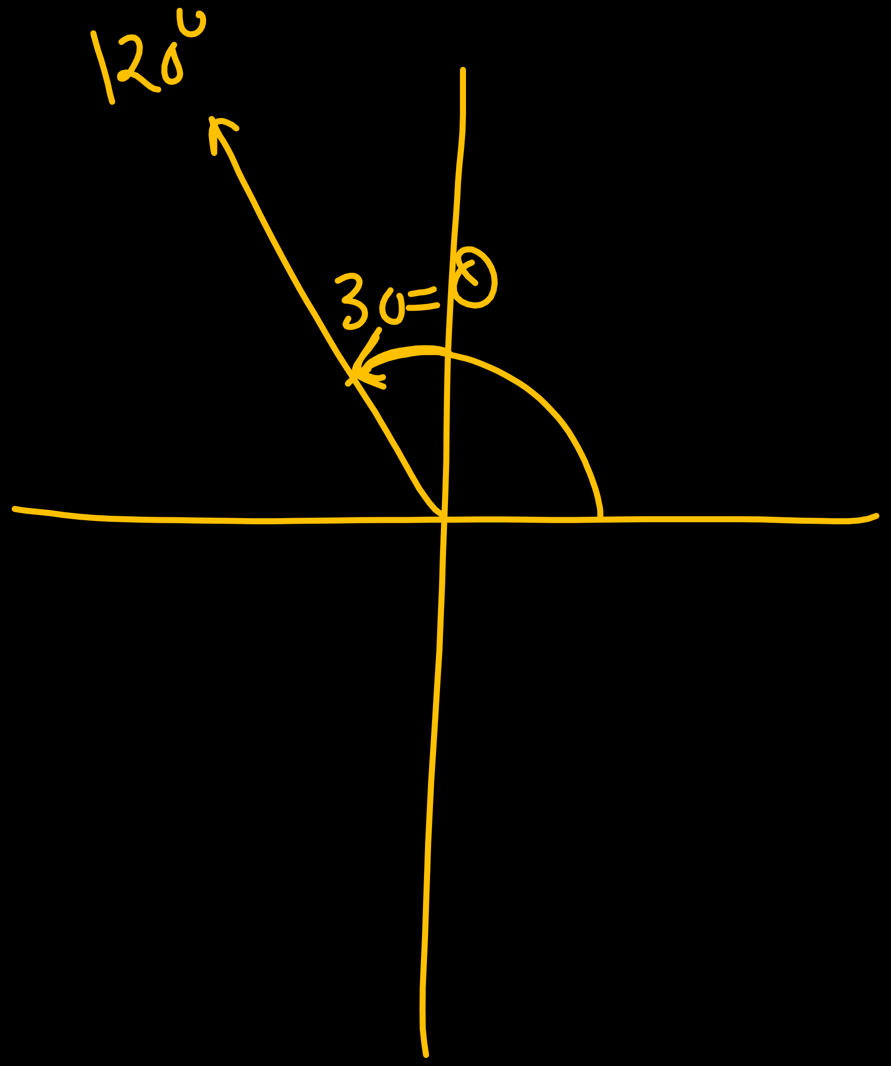


M-2

$$\tan 120^\circ = \tan(90^\circ + \underline{30^\circ})$$

$$= -\cot 30$$

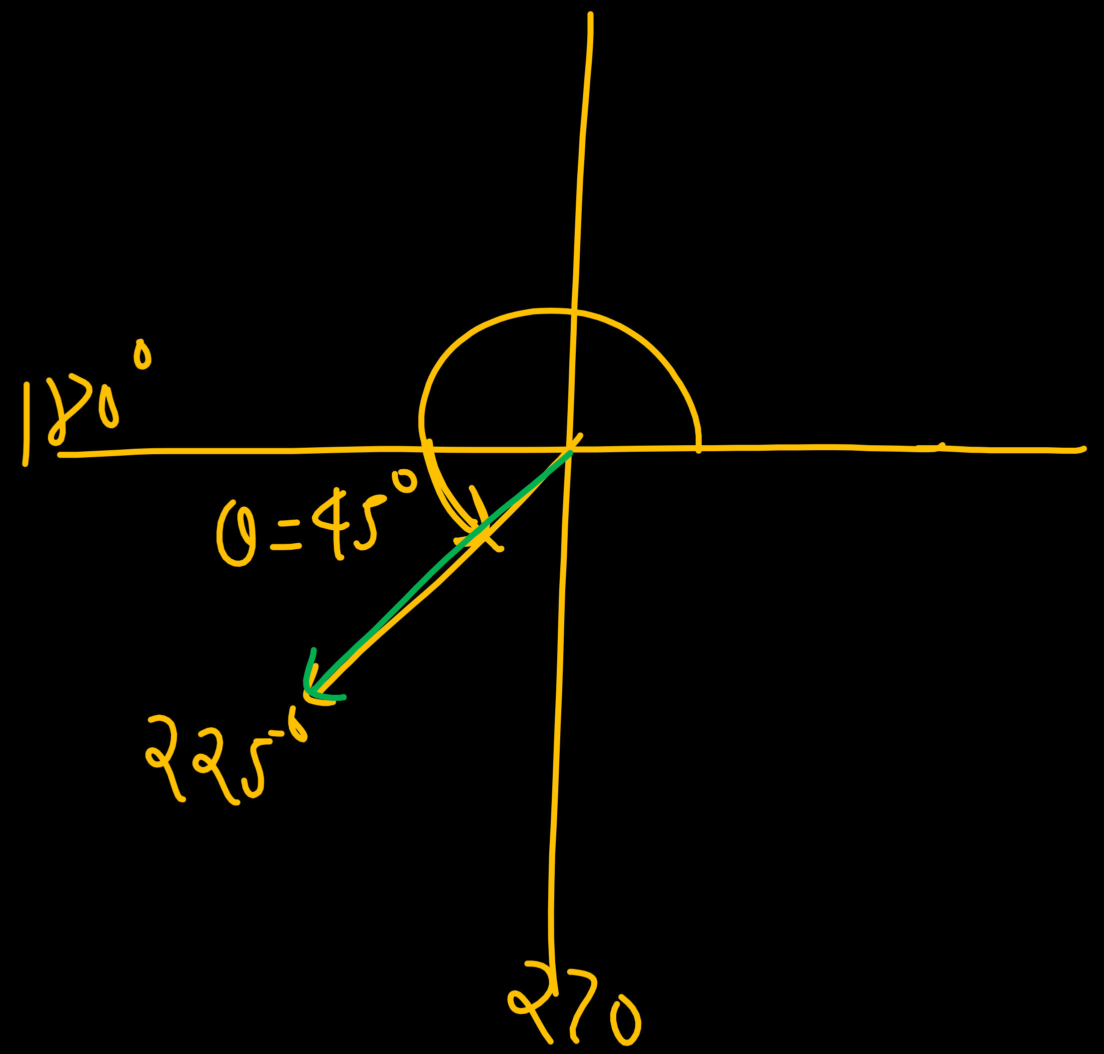
$$= -\sqrt{3}$$



Example: 3

$\cos(225°) =$

$\cos(225°) = \cos(\underbrace{180° + 45°}_{\text{III}^{rd} \text{ Quad}}) = -\cos 45 = -\frac{1}{\sqrt{2}}$

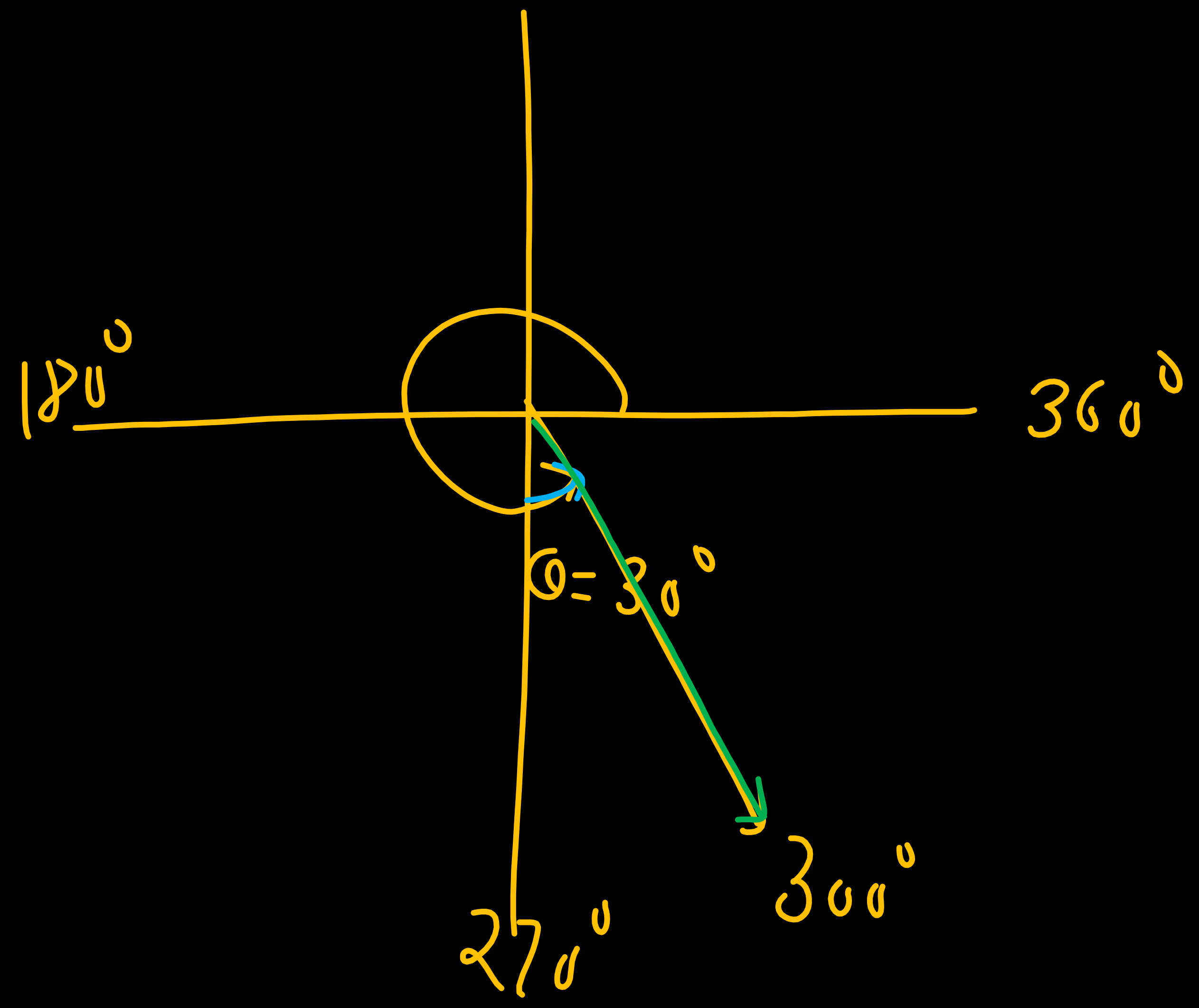


Example: 4

$\tan(300^\circ) =$

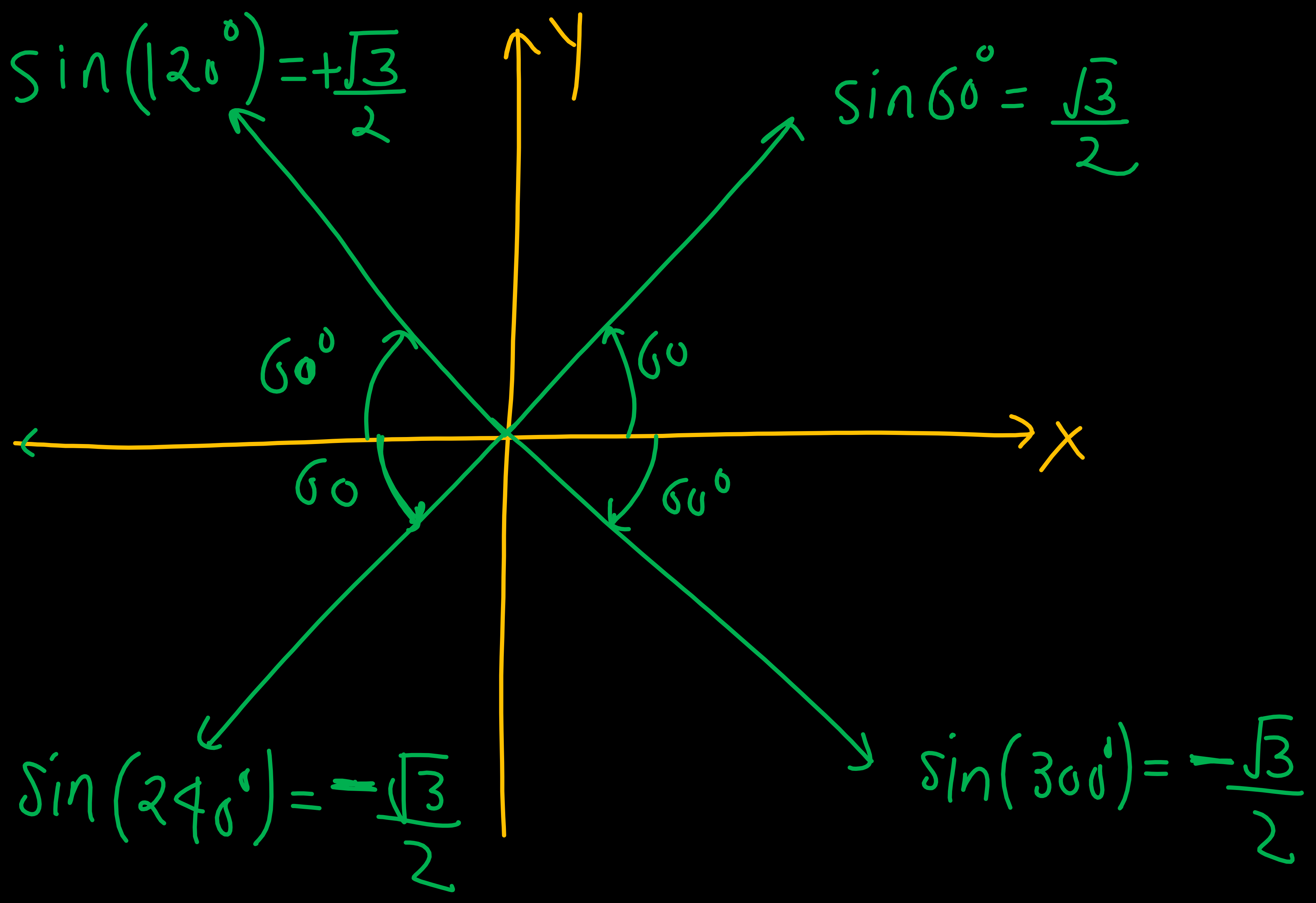
$$\tan 300^\circ = \underline{\tan}(\underbrace{270^\circ + 30^\circ}_{\text{IV}^{th}\text{ quar}}) = -\cot 30$$

$$= -\sqrt{3}$$



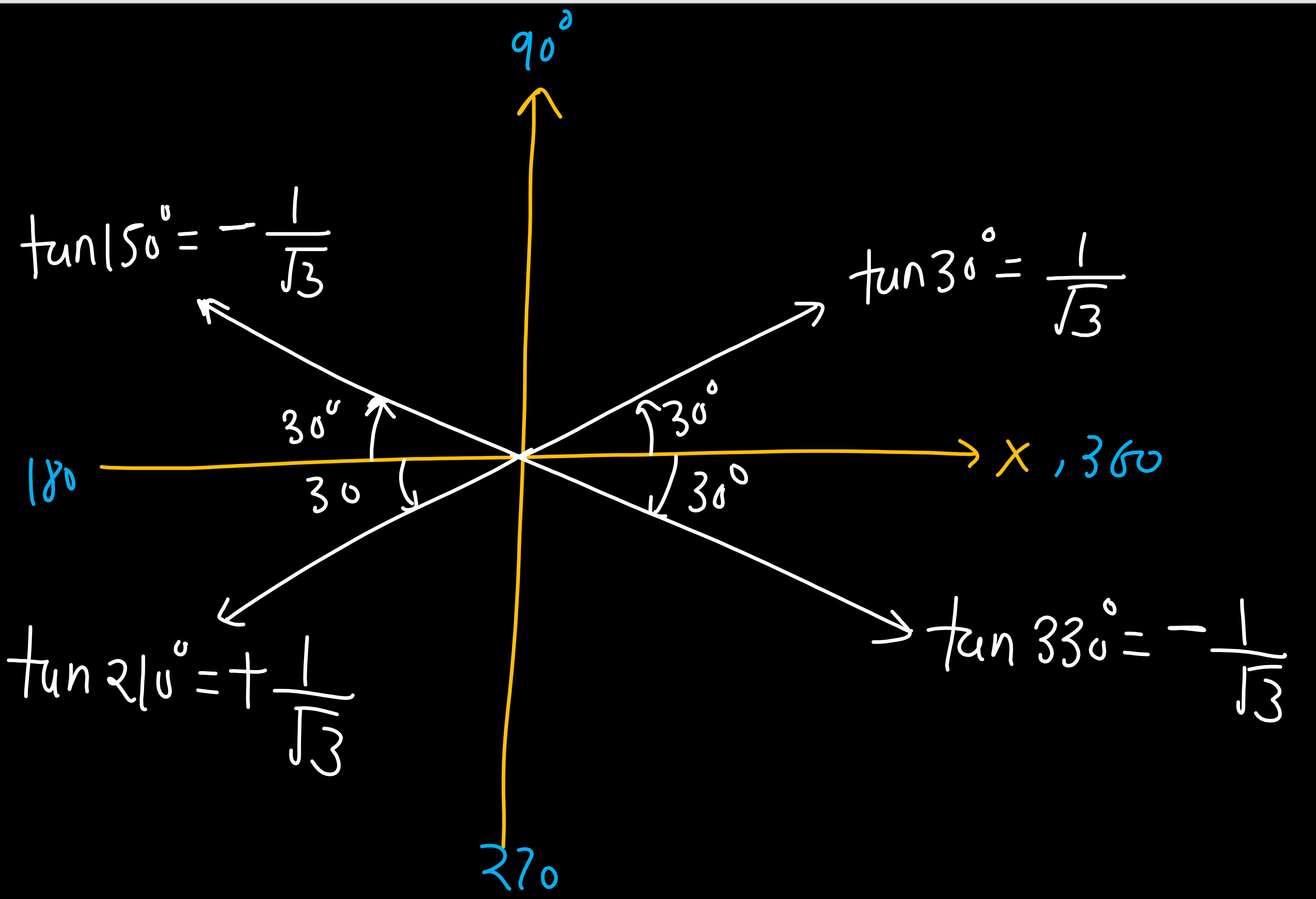
Example: 5

$\sin(120^\circ) =$



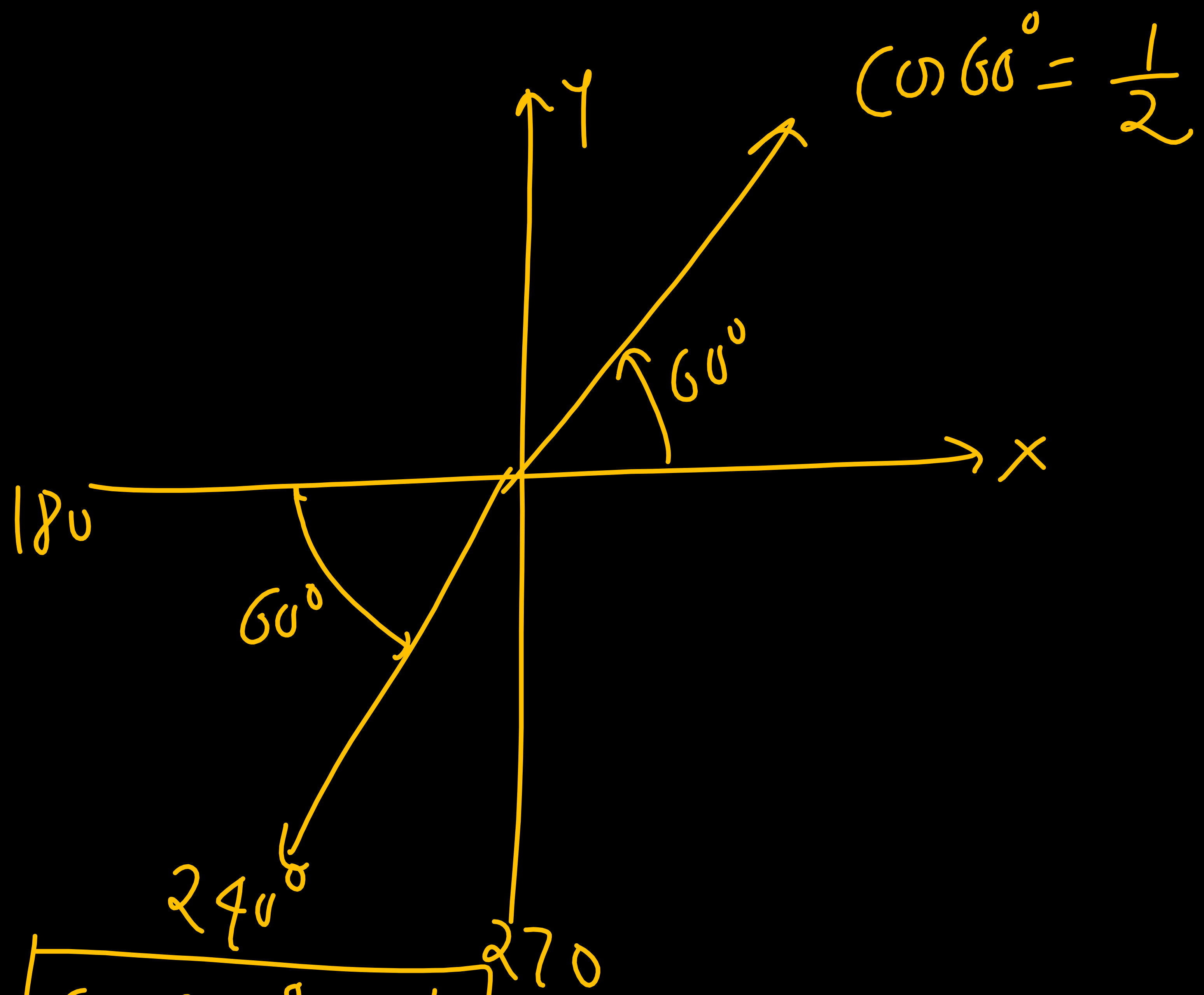
Example: 6

$\tan(150^\circ) =$



Example: 7

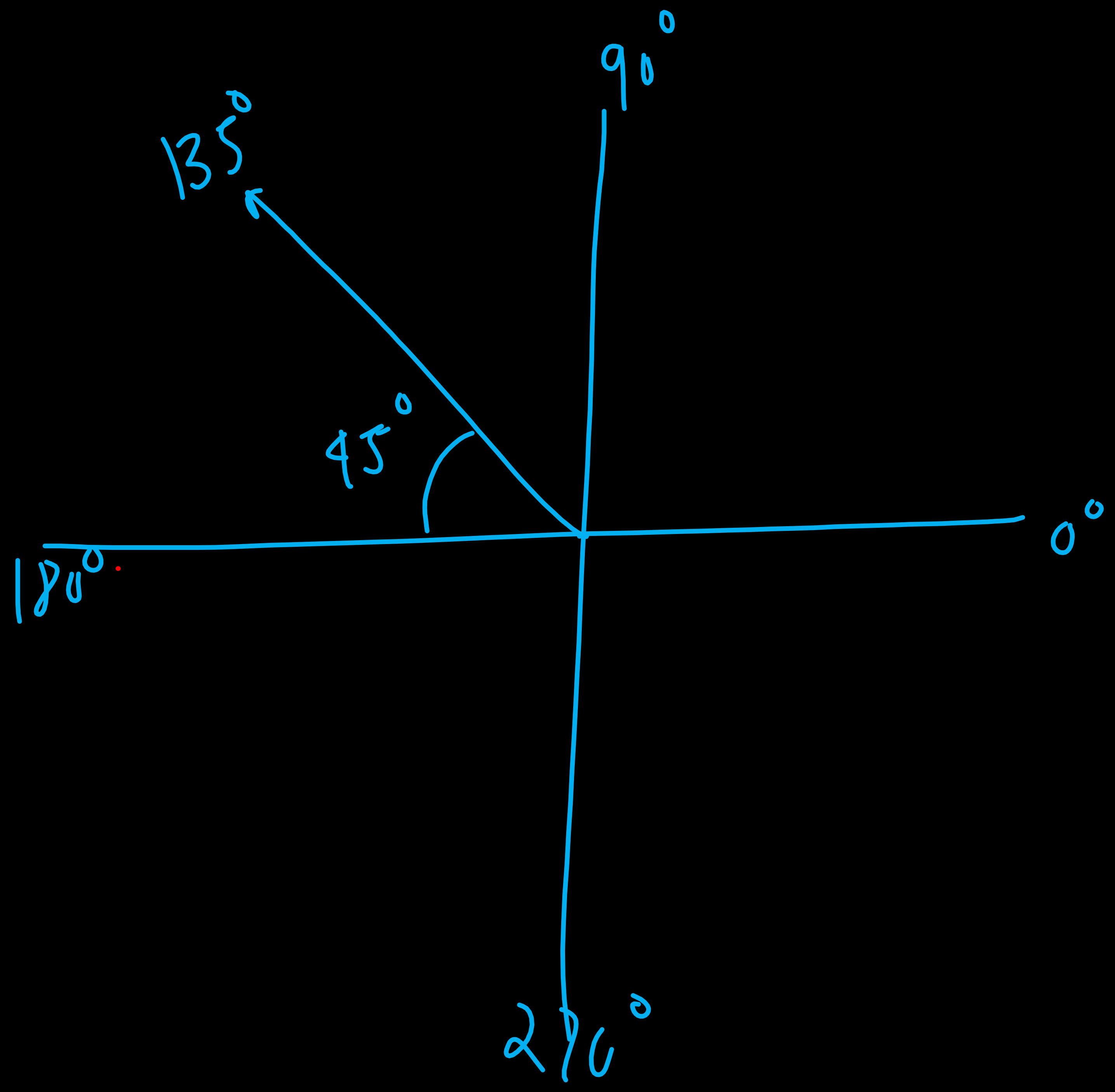
$\cos(240^\circ) =$



$$\boxed{\cos 240^\circ = -\frac{1}{2}}$$

Example: 8

$\sec(135^\circ) =$

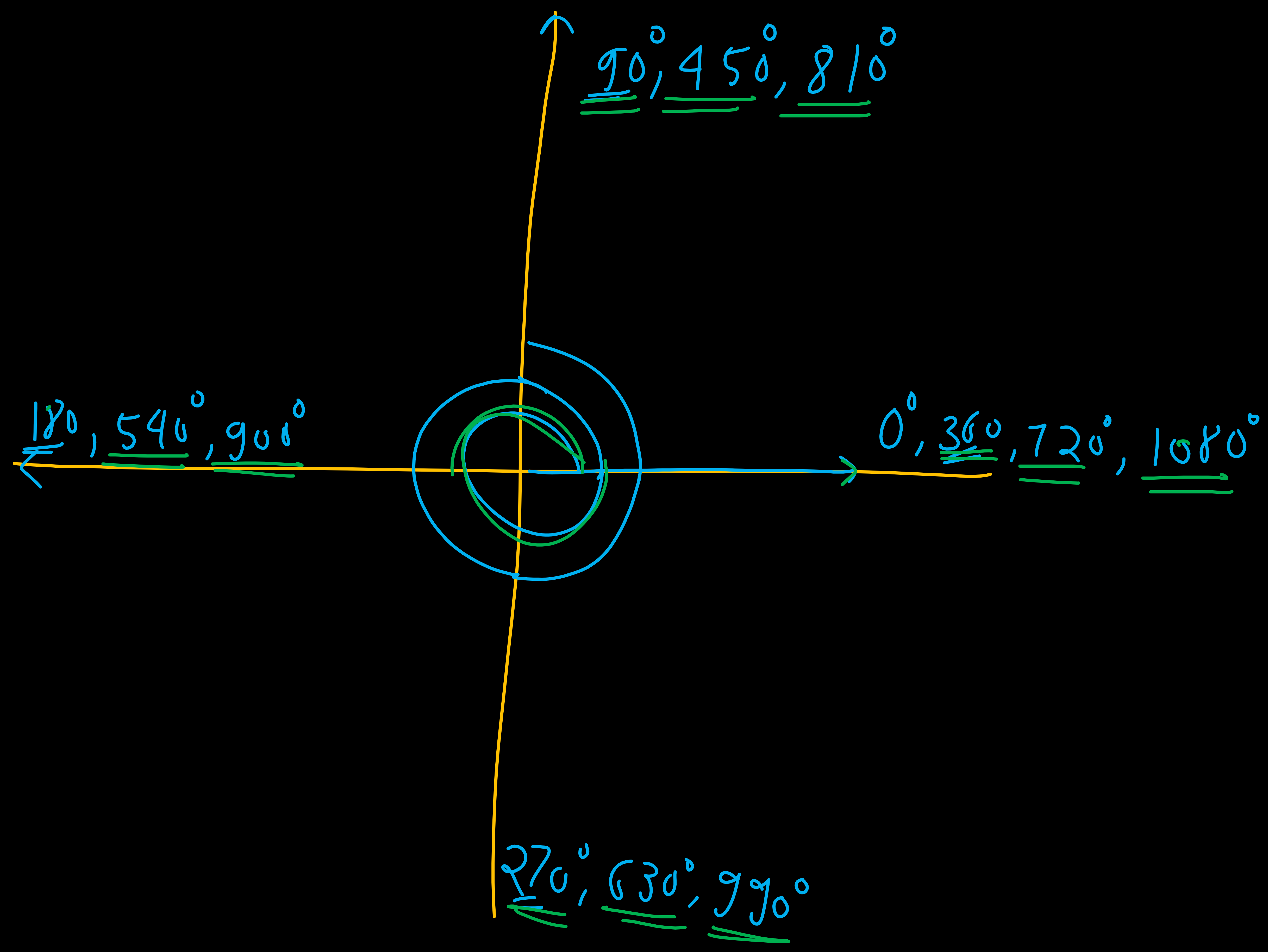


$$\sec(135^\circ) = \sec(180^\circ - 45^\circ)$$

$$= -\sec 45$$

$$= -\sqrt{2}$$

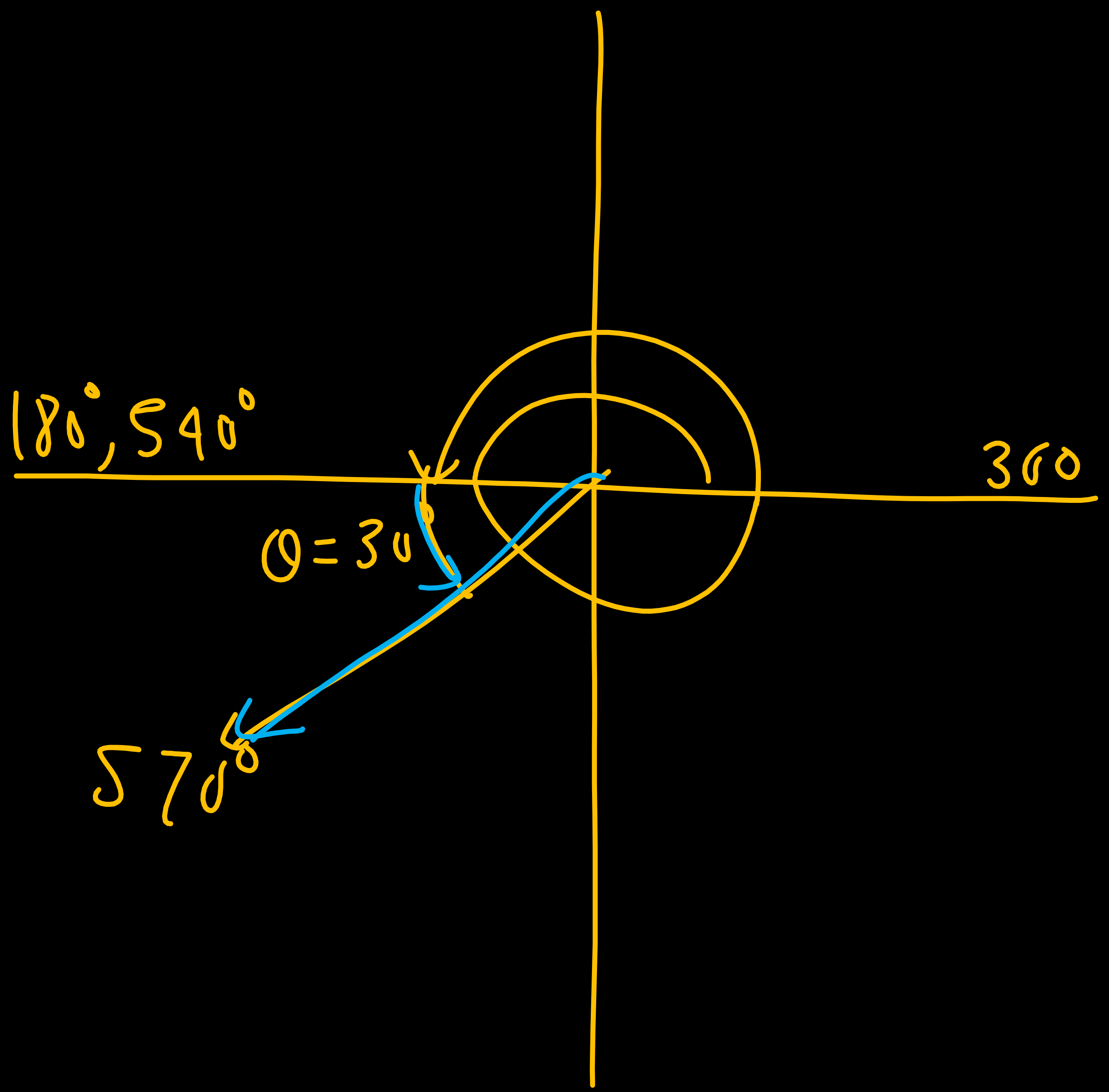
Note



Example: 9

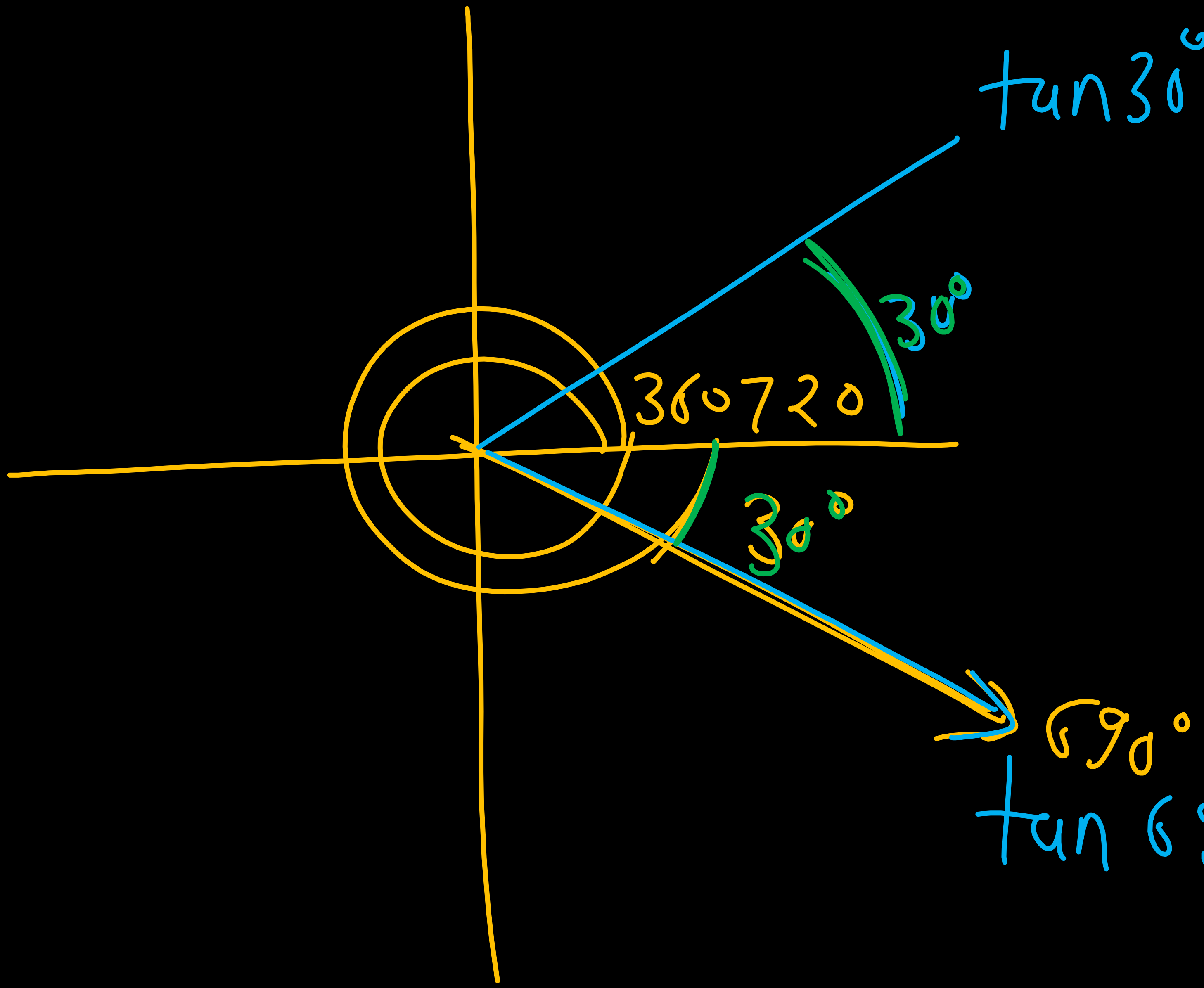
$\cos(570^\circ) =$

$$\cos(570^\circ) = \cos(540^\circ + 30^\circ) = -\cos 30^\circ$$



Example: 10

$\tan(690^\circ) =$



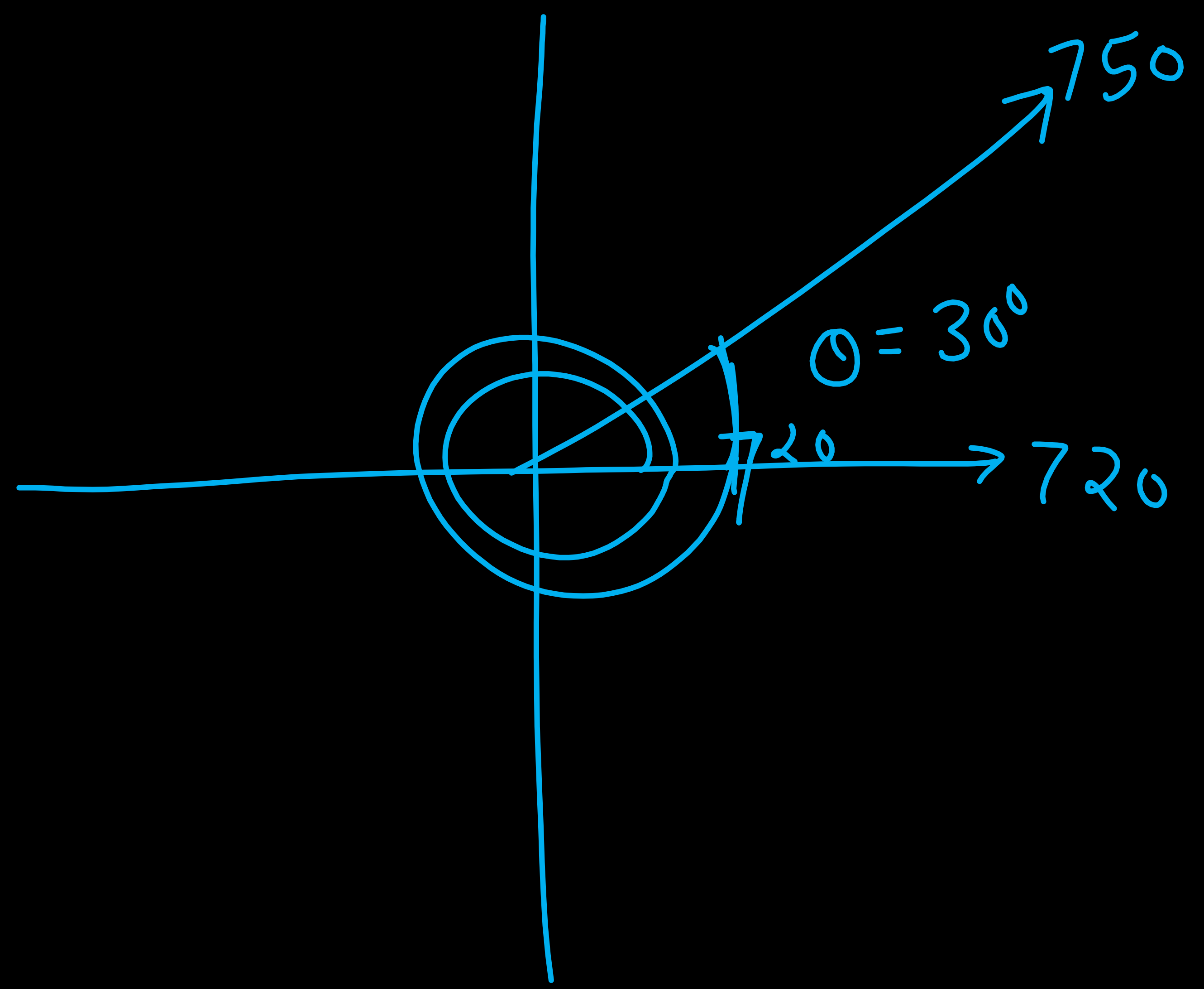
$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\tan 690^\circ = -\frac{1}{\sqrt{3}}$

Example: 11

$\text{cosec}(750^\circ) =$

$\text{cosec}(750^\circ) = \text{cosec}(720^\circ + 30^\circ) = + \text{cosec}\, 30^\circ$

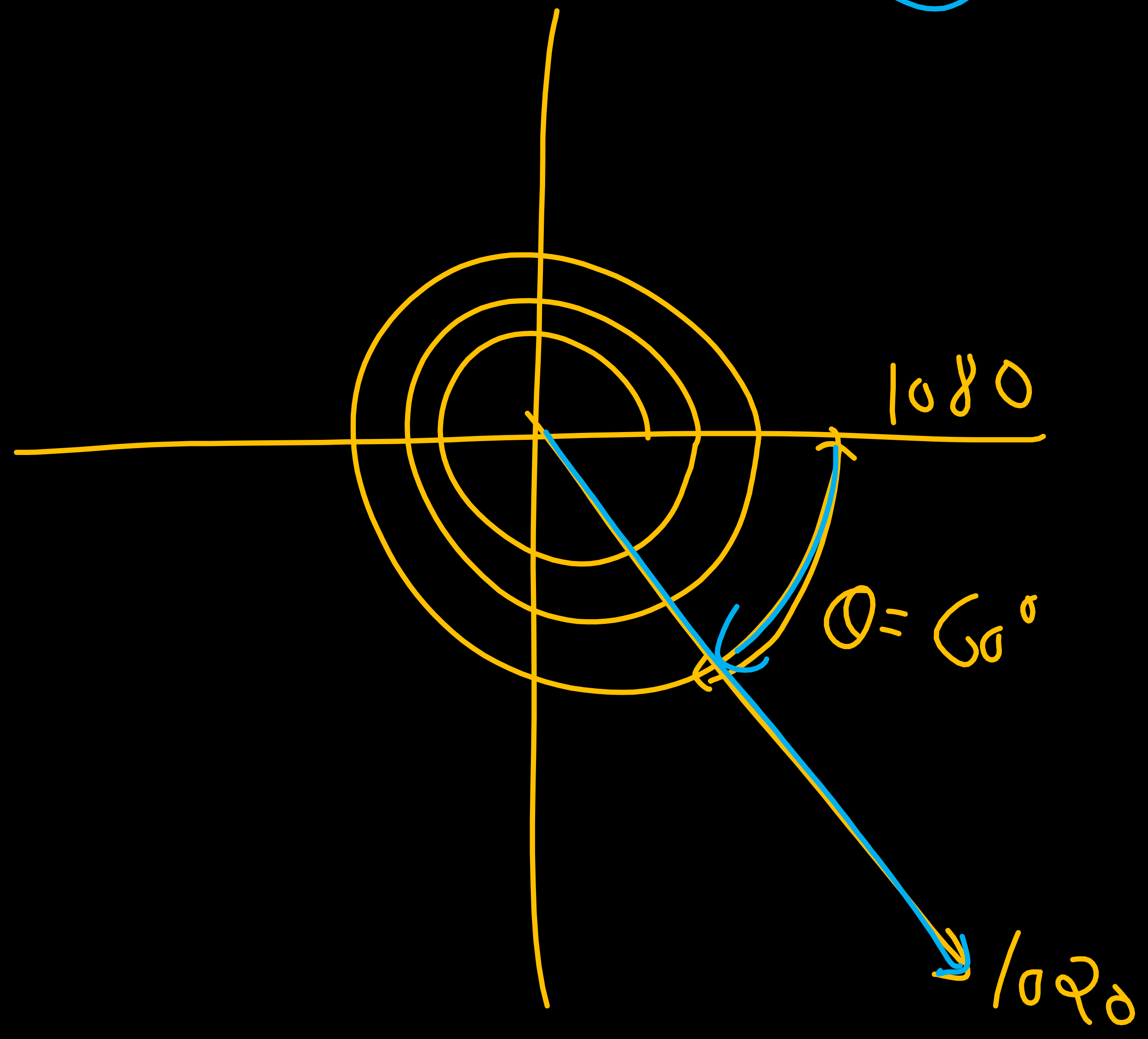


Example: 12

$\sin(1020^\circ) =$

$$\sin(1020^\circ) = \sin(\underline{1080^\circ - 60^\circ}) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

IVth quad



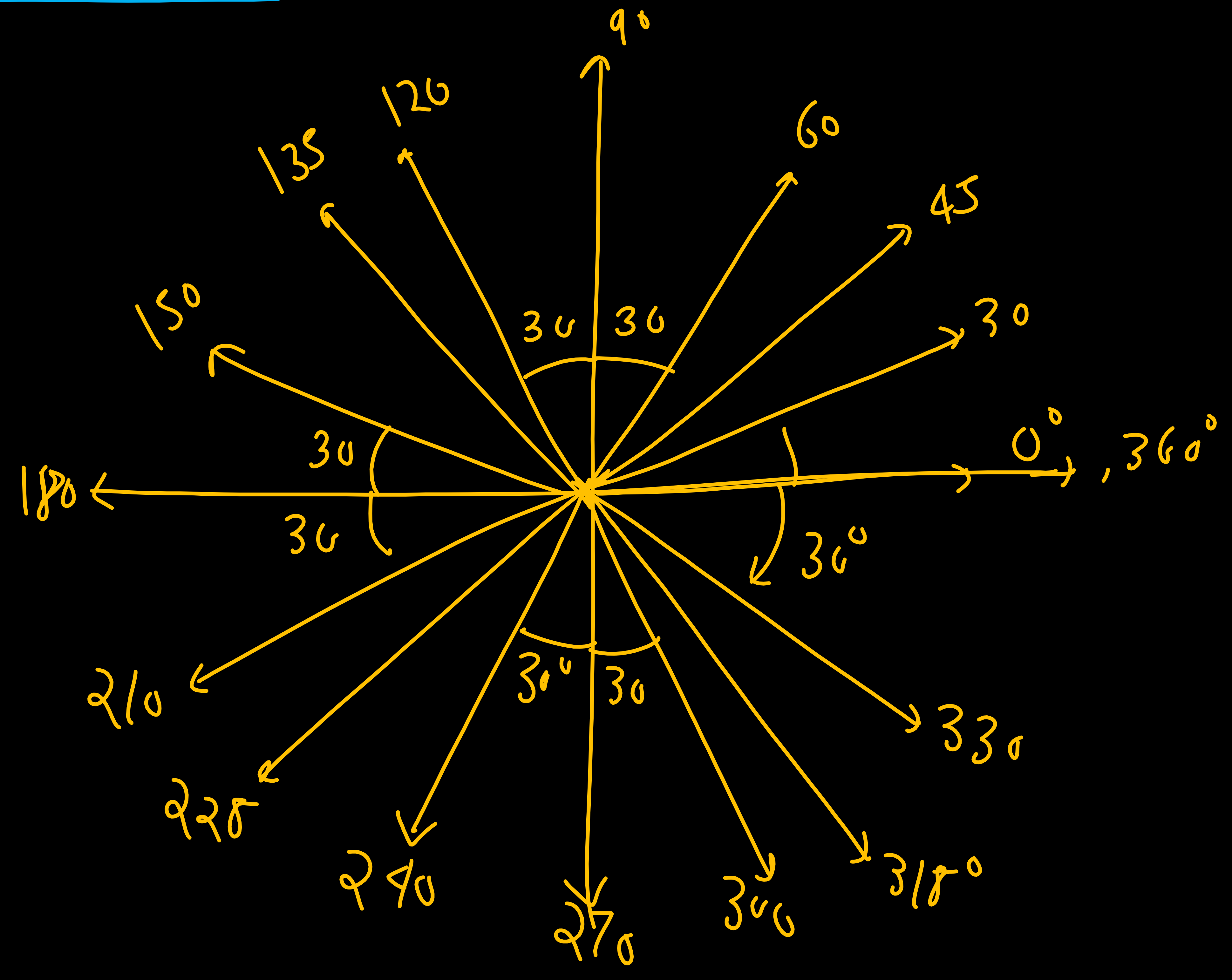
Write down this table in classnotes by finding each value

Trigonometric Ratios Table (0 to 2π)

Ratio	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$\csc\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	ND
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND

$\sin(210^\circ) = \sin(180+30) = -\sin 30 = -\frac{1}{2}$

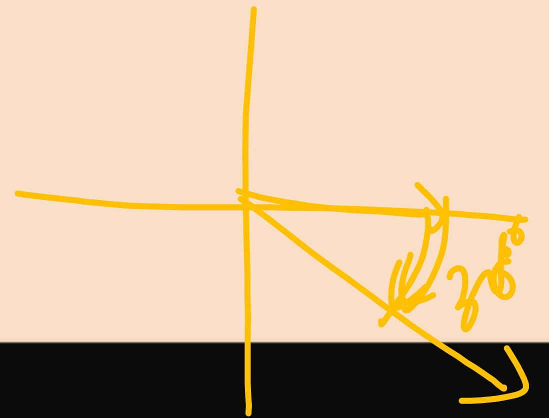
IMP Angles for tables

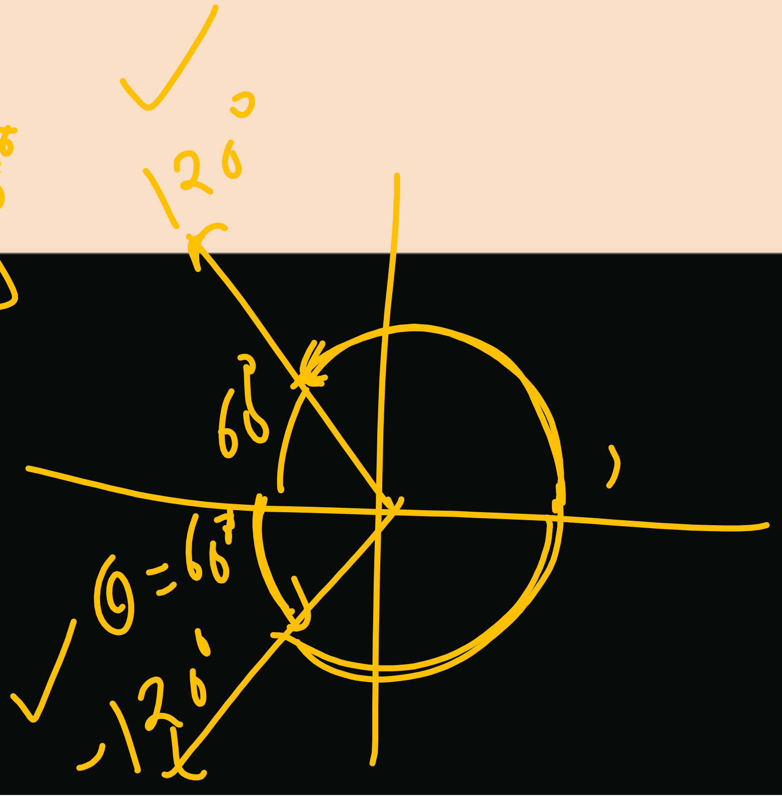


Formulas for $(-\theta)$

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\sec(-\theta) = \sec\theta$
- $\cot(-\theta) = -\cot\theta$
- $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\,\theta$

Proof: $\cos(-30^\circ) = \cos(0^\circ - 30^\circ) = +\cos 30$





$\cos(120^\circ) = \cos(-120^\circ)$

Example: 13

$\sin(-150^\circ) =$

$$\sin(-150^\circ) = -\left[\underline{\underline{\sin 150^\circ}}\right]$$

$$= -\left[\sin(\underline{180^\circ - 30^\circ})\right]$$

$$= -\left[+\sin 30\right]$$

$$= -\frac{1}{2}$$

Example: 14

$\cos(-225^\circ) =$

$$\cos(-225^\circ) = \cos 225^\circ$$

$$= \cos(\underline{\underline{180}} + 45)$$

$$= -\cos 45$$

$$= -\frac{1}{\sqrt{2}}$$

Example: 15

$\tan(-330^\circ) =$

$$\tan(-330^\circ) = -\tan(330^\circ)$$
$$= -\left[\tan(360^\circ - 30^\circ)\right]$$
$$= -\left[-\tan 30^\circ\right]$$
$$= \tan 30^\circ$$
$$= \frac{1}{\sqrt{3}}$$

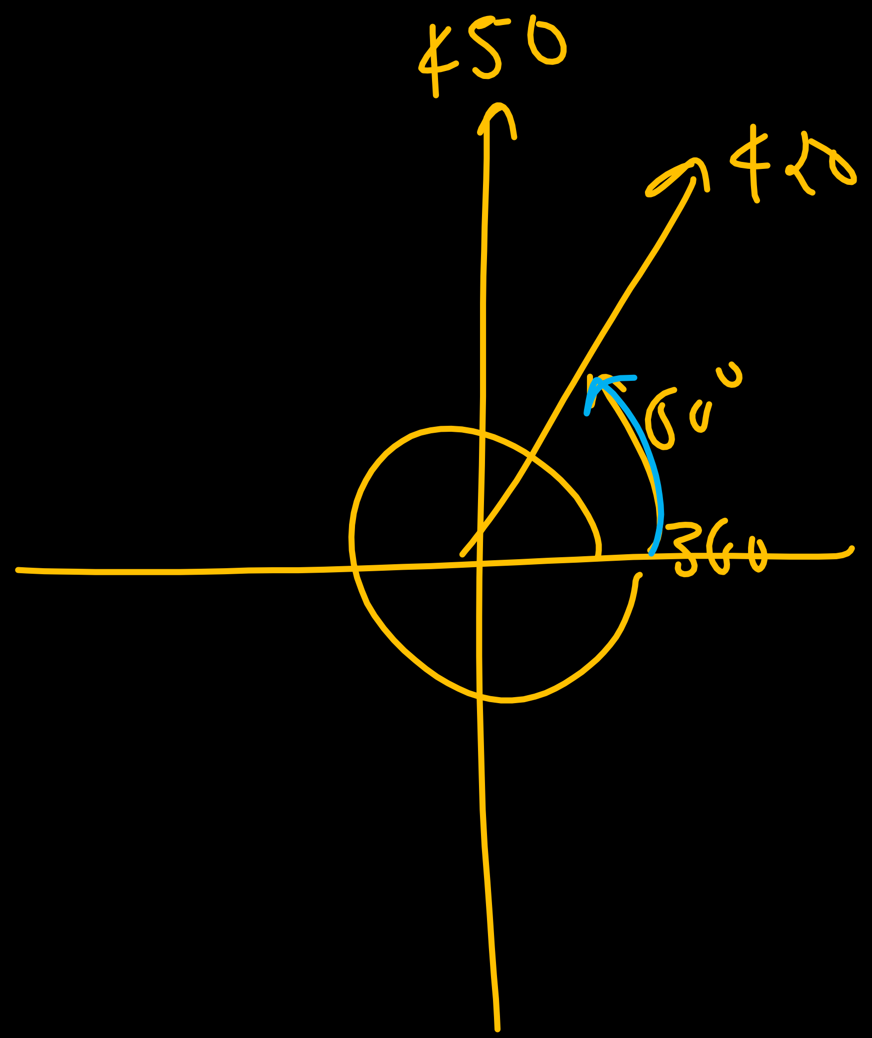
Example: 16

$\sec(-420°) =$

$$\sec(-420°) = \sec(420°)$$

$$= \sec(\underline{360° + 60°}) = +\sec 60$$

$$= 2$$



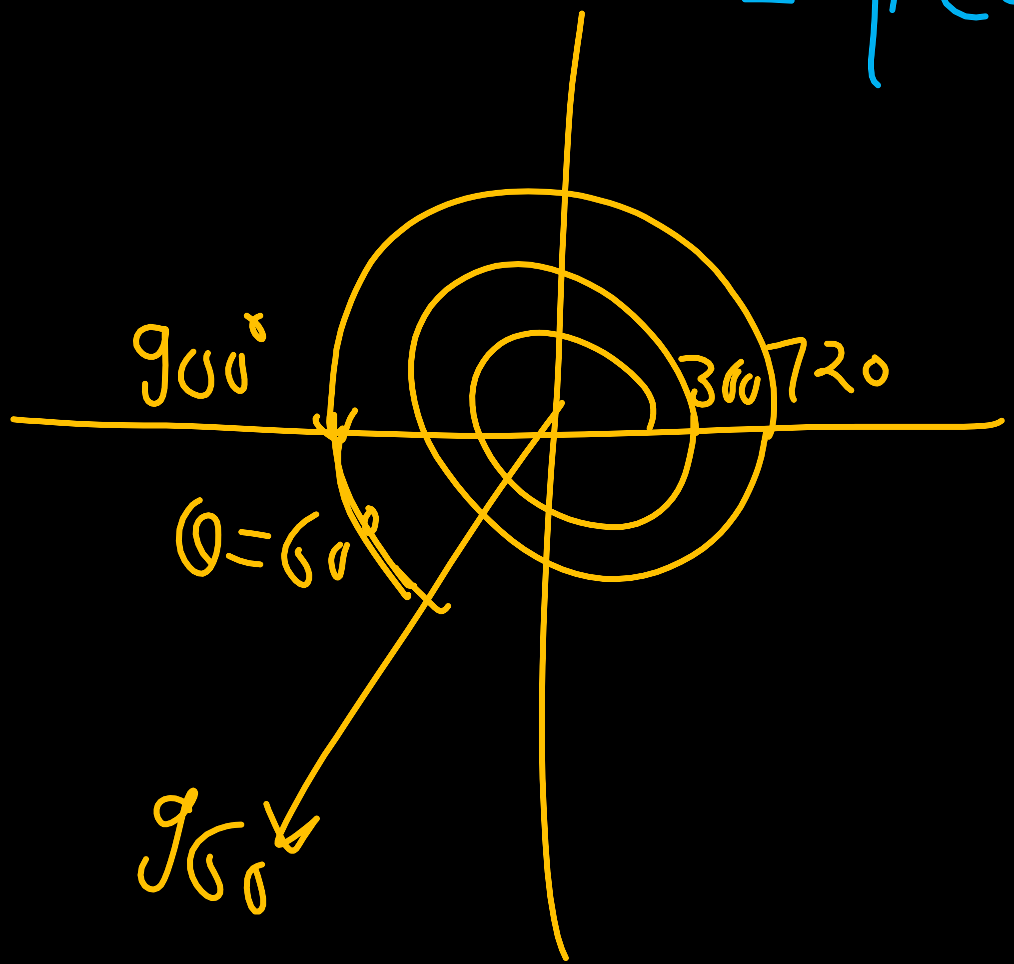
Example: 17

$\cot(-960^\circ) =$

$$\cot(-960^\circ) = -\cot(960^\circ)$$

$$= -\left[\cot(900+60)\right]$$

$$= -\left[+\cot 60\right] = -\frac{1}{\sqrt{3}}$$



Example: 18

$\cos(\frac{7\pi}{6}) =$

<u>M-1</u> $\cos\left(\frac{7\pi}{6}\right) = \cos\left(\frac{7 \times \cancel{180}^{30}}{\cancel{6}}\right) = \cos(210^\circ)$

<u>M-2</u> $\cos\left(\frac{7\pi}{6}\right) = \cos\left(\frac{6\pi + \pi}{6}\right)$

$= \cos\left(\frac{\cancel{6}\pi}{\cancel{6}} + \frac{\pi}{6}\right)$

$= \cos\left(\pi + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

Example: 19

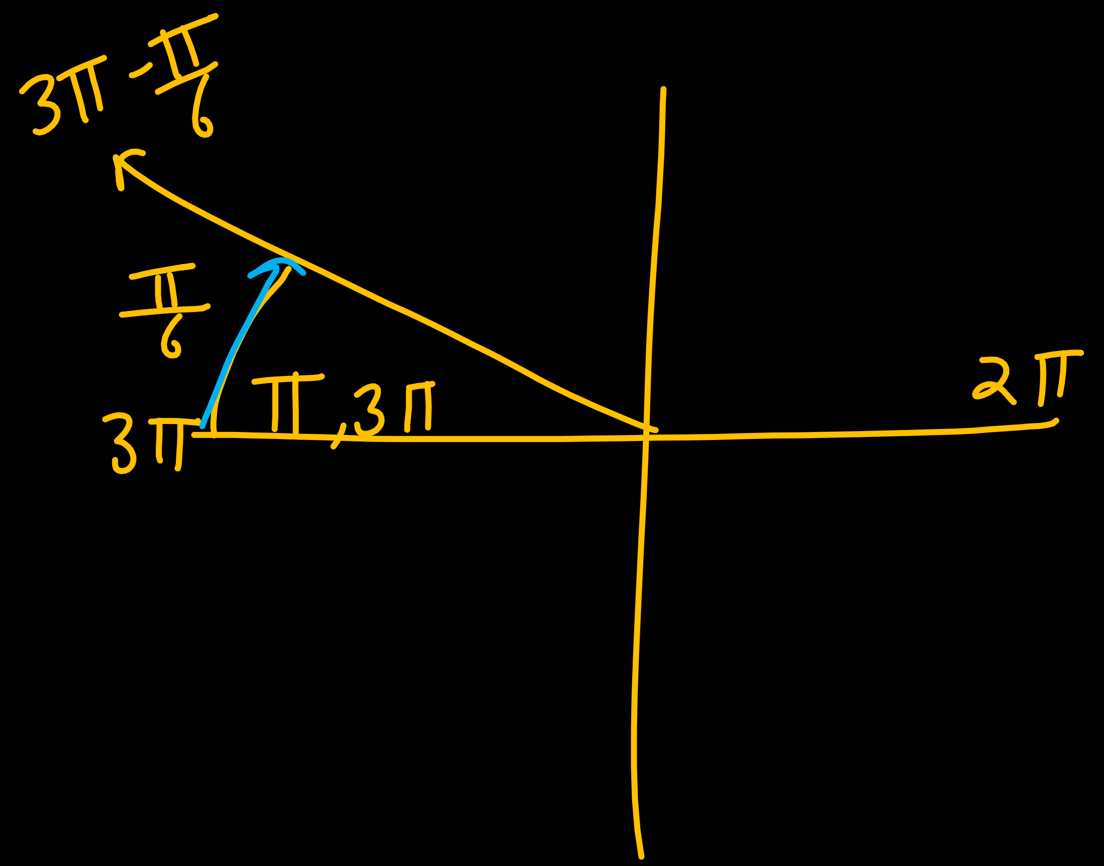
$\tan(\frac{17\pi}{6}) =$

$$\tan\left(\frac{17\pi}{6}\right) = \tan\left(\frac{18\pi - \pi}{6}\right)$$

$$= \tan\left(3\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{6}$$

$$= -\frac{1}{\sqrt{3}}$$



Example: 20

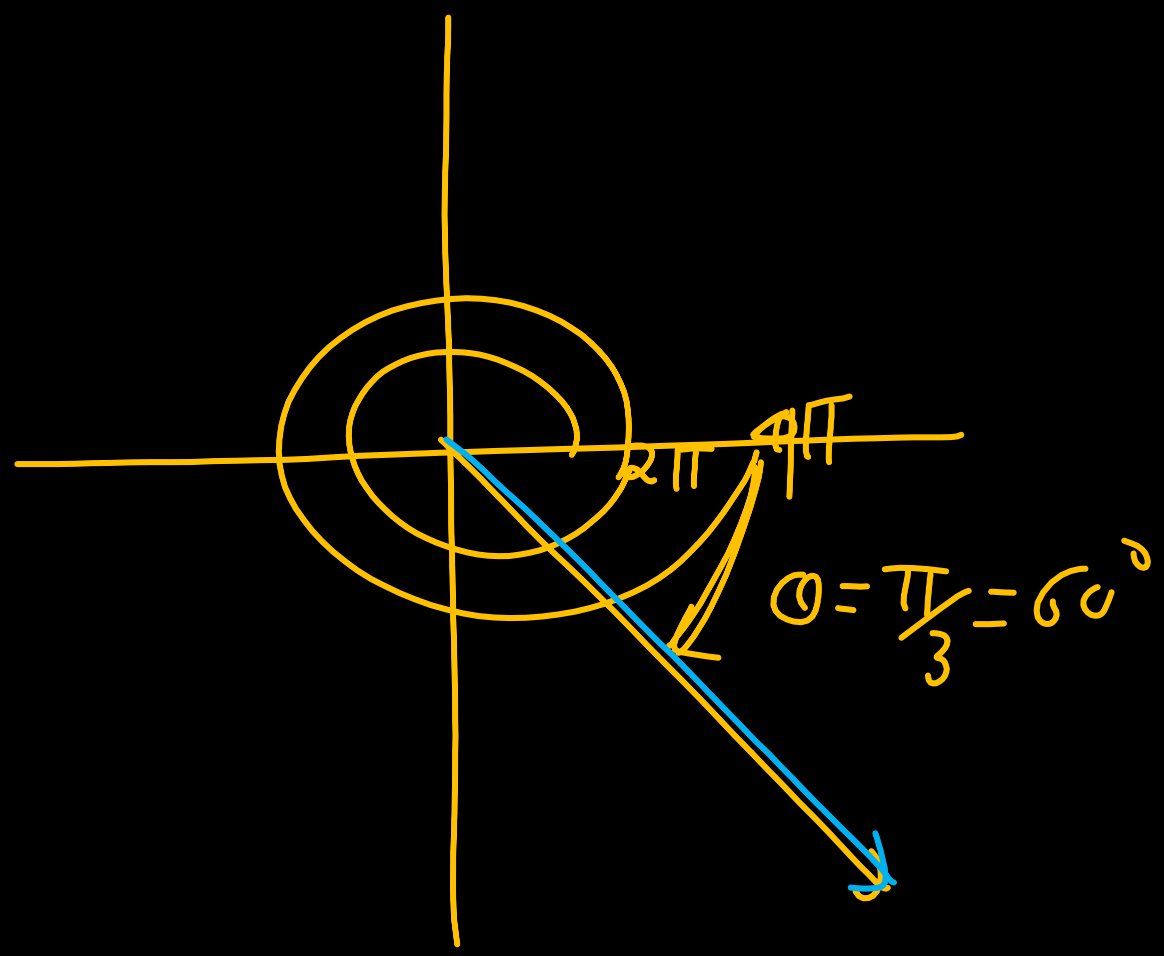
$\sin\left(\frac{11\pi}{3}\right) =$

$$\sin\left(\frac{11\pi}{3}\right) = \sin\left(\frac{12\pi-\pi}{3}\right)$$

$$= \sin\left(4\pi - \frac{\pi}{3}\right)$$

$$= -\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{2}$$



Example: 21

$\tan(\frac{-4\pi}{3}) =$

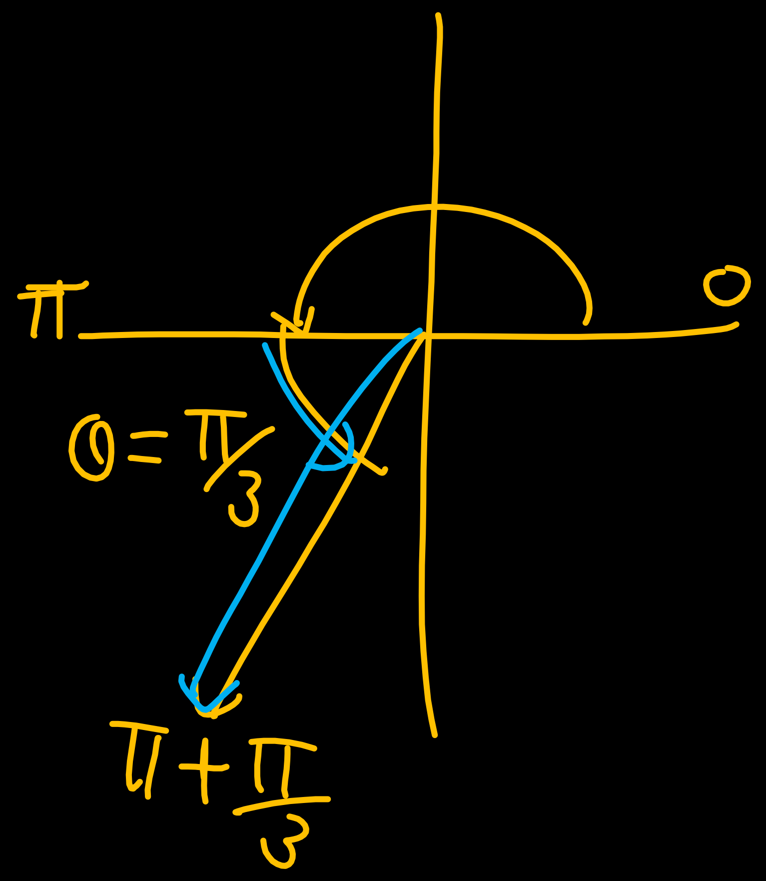
$$\tan\left(-\frac{4\pi}{3}\right) = -\tan\left(\frac{4\pi}{3}\right)$$

$$= -\tan\left(\frac{3\pi+\pi}{3}\right)$$

$$= -\left[\tan\left(\pi+\frac{\pi}{3}\right)\right]$$

$$= -\left[+\tan\frac{\pi}{3}\right]$$

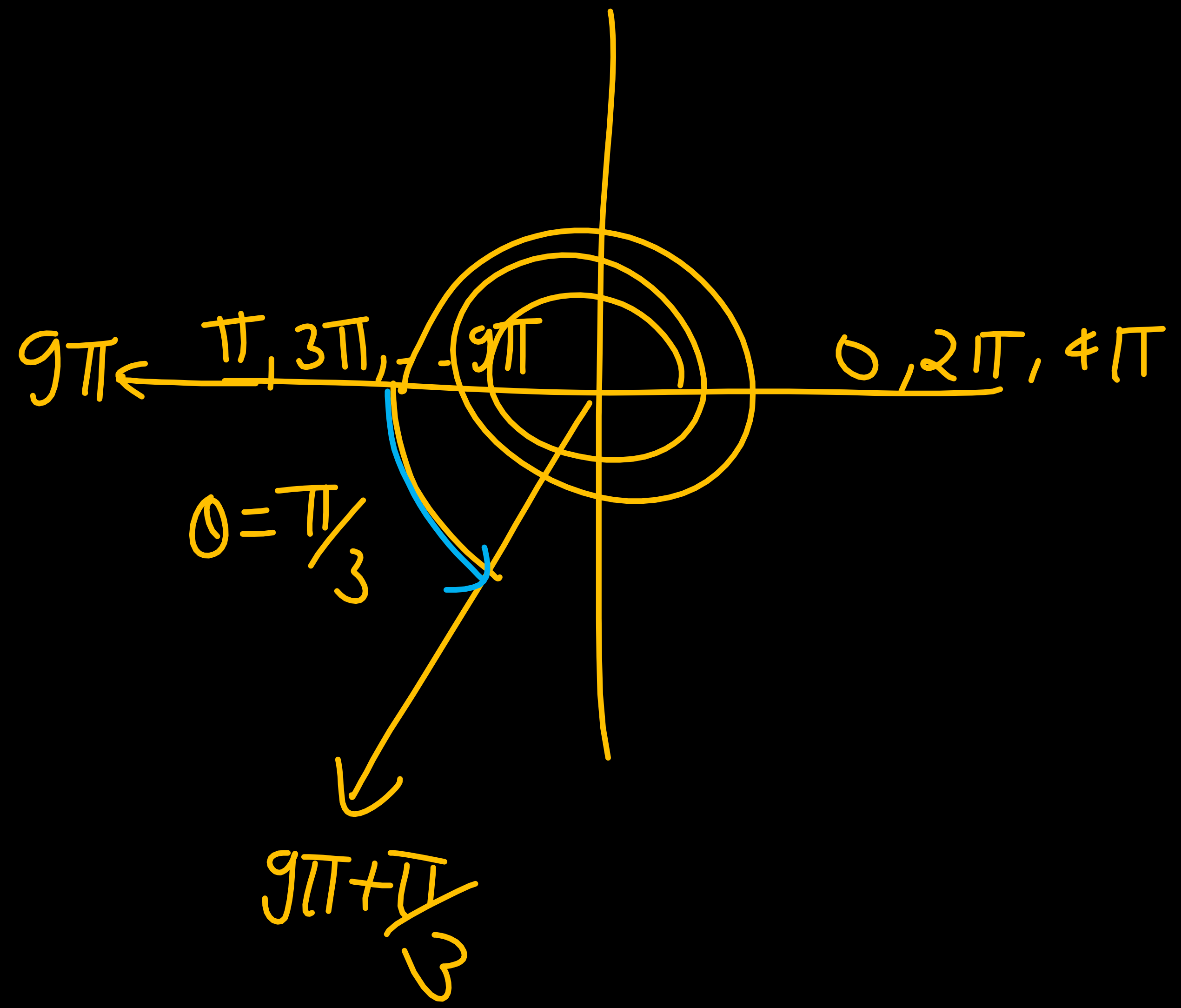
$$= -\sqrt{3}$$



Example: 22

$\sin(\frac{28\pi}{3}) =$

$$\sin\left(\frac{28\pi}{3}\right) = \sin\left(\frac{27\pi+\pi}{3}\right)$$
$$= \sin\left(9\pi + \frac{\pi}{3}\right)$$
$$= -\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{3}}{2}$$



Note

1 $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ → Remainder after dividing by 4 = 1

General Form: $(4n+1)\frac{\pi}{2}$

Y

Even multiples of π

$0, 2\pi, 4\pi, \ldots$

General Form: $2n\pi$

X

Odd multiples of π

$\pi, 3\pi, 5\pi, \ldots$

General Form: $(2n+1)\pi$

$\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$ → Remainder after dividing by 4 $= \boxed{3}$

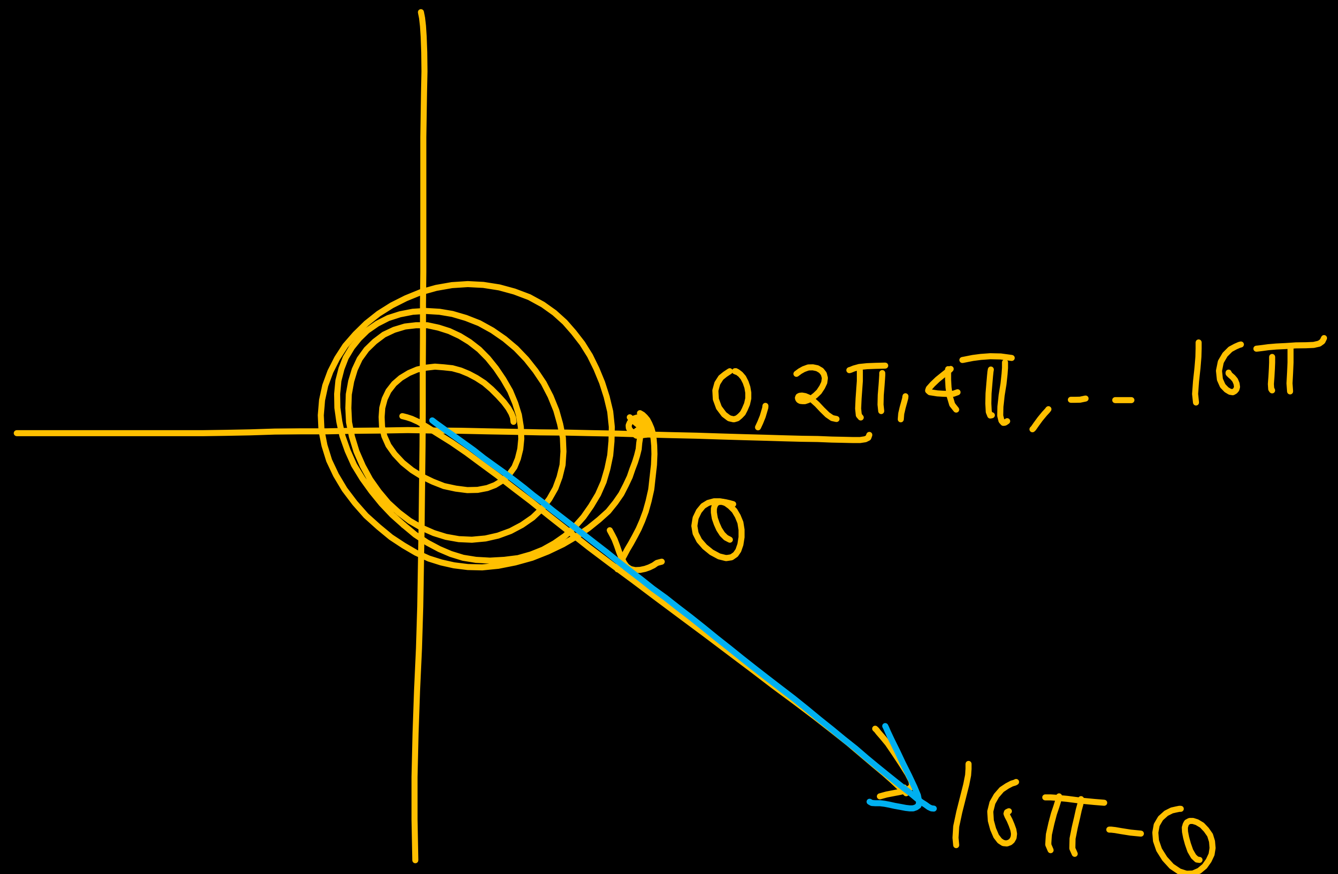
General Form: $(4n-1)\frac{\pi}{2}$

- Complete X-axis: $n\pi$
- Complete Y-axis: $(2n+1)\frac{\pi}{2}$ or $n\pi + \frac{\pi}{2}$ → odd multiple of $\frac{\pi}{2}$

Example: 23

$\cos(16\pi - \theta) =$

$\cos(16\pi - \theta) = +\cos\theta$

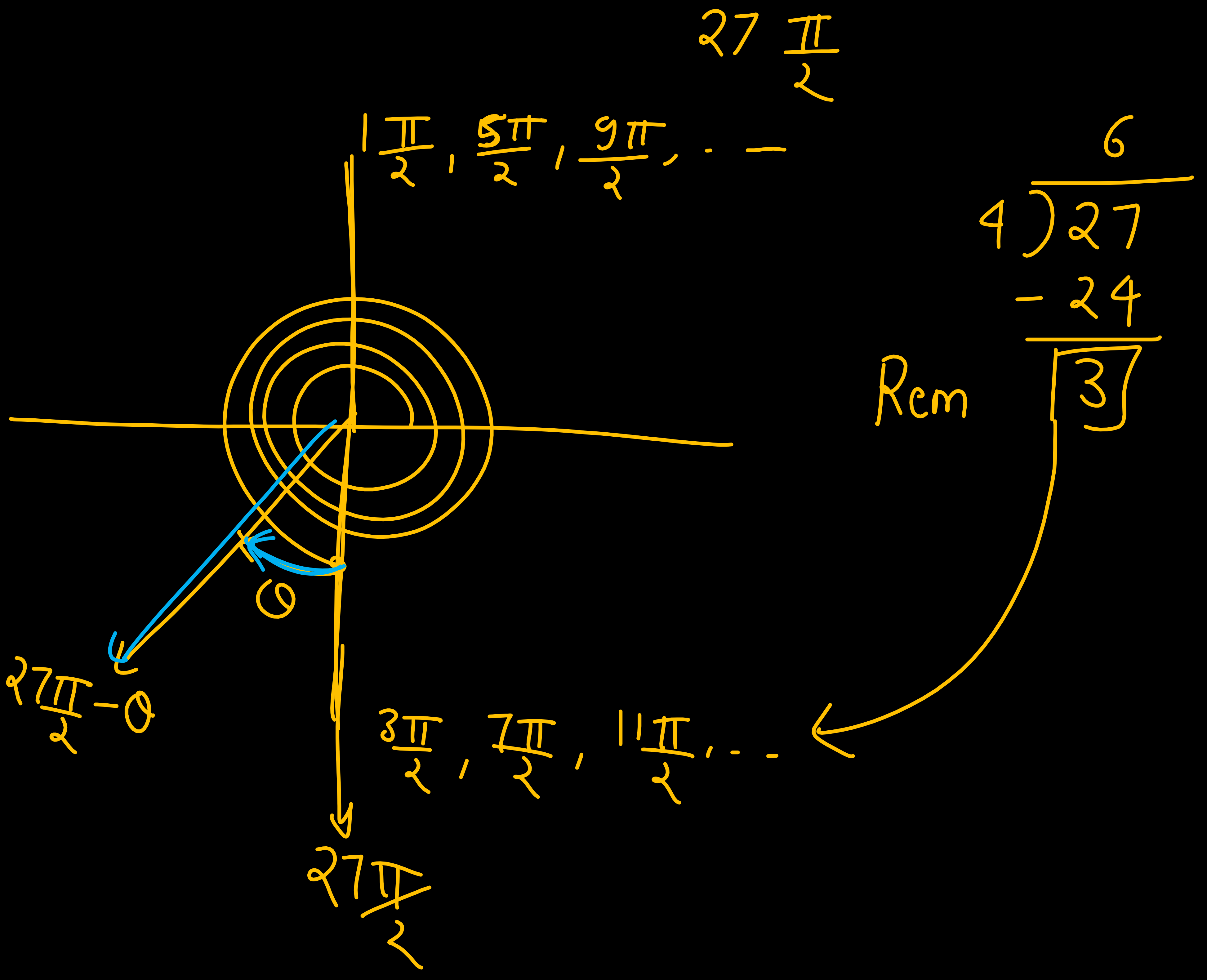


Example: 24

$\sin(\frac{27\pi}{2} - \theta) =$

$\sin\left(\underline{\frac{27\pi}{2} - \theta}\right) = -\cos\theta$

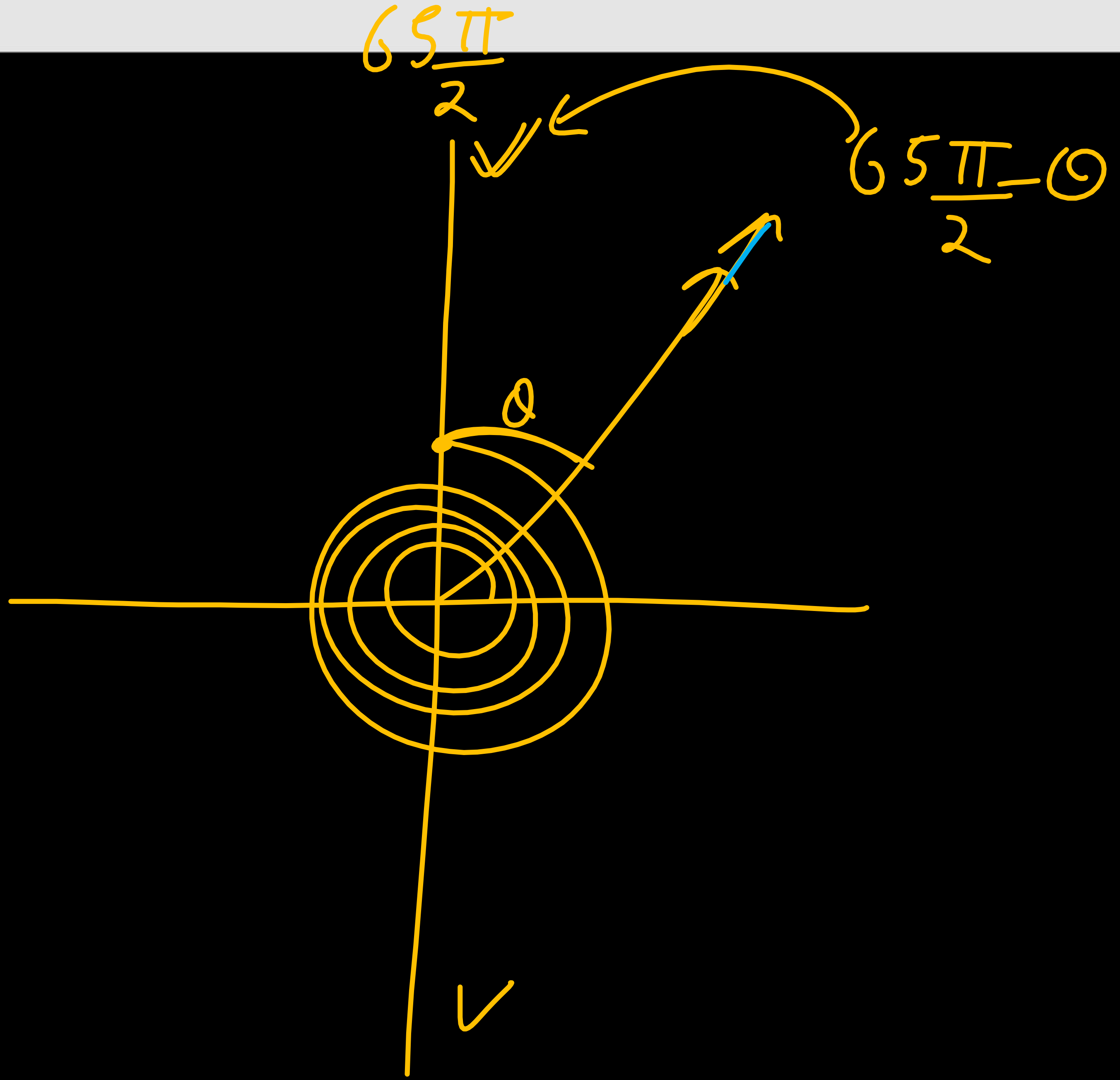
3^{rd} quad



Example: 25

$\tan(\frac{65\pi}{2} - \theta) =$

$$\tan\left(\frac{65\pi}{2} - \theta\right) = +\cot\theta$$



$$\begin{array}{r} 16 \\ 4\,\overline{)\,65} \\ -\,64 \\ \hline \end{array}$$

Rem: $\boxed{1}$

Example: 26

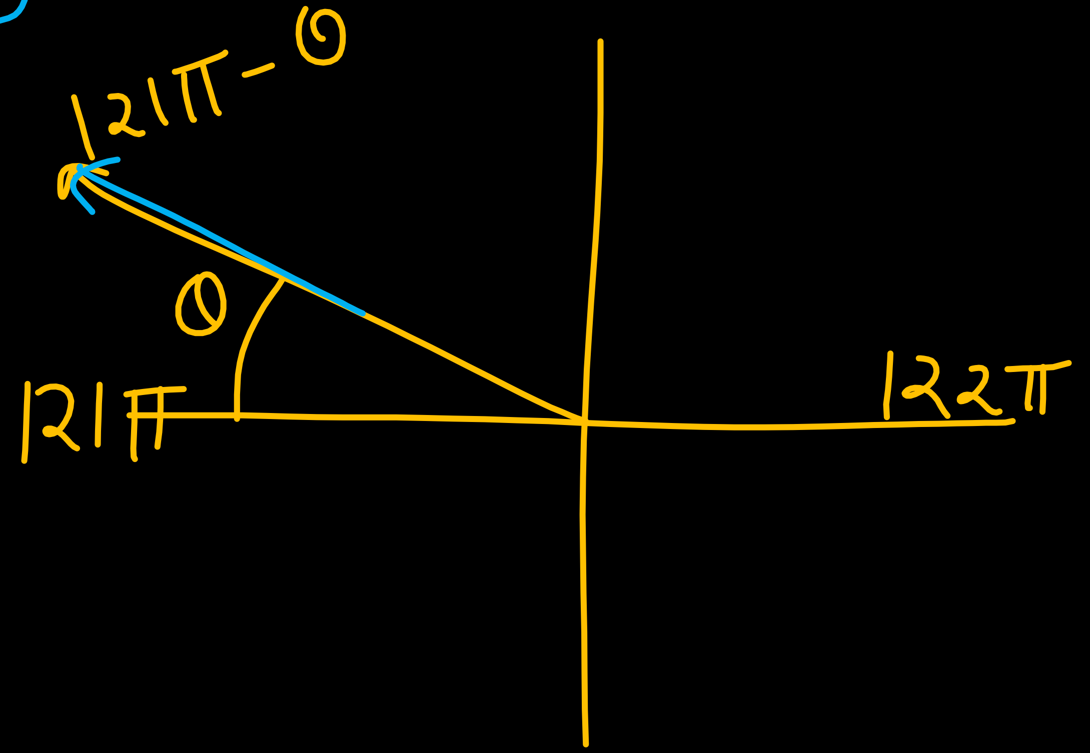
$\tan(\theta - 121\pi) =$

$$\tan(\theta - 121\pi) = \tan\left(-(121\pi - \theta)\right) = -\left[\tan(121\pi - \theta)\right]$$

take out "−"

$$= -\left[-\tan\theta\right]$$

$$= \tan\theta$$



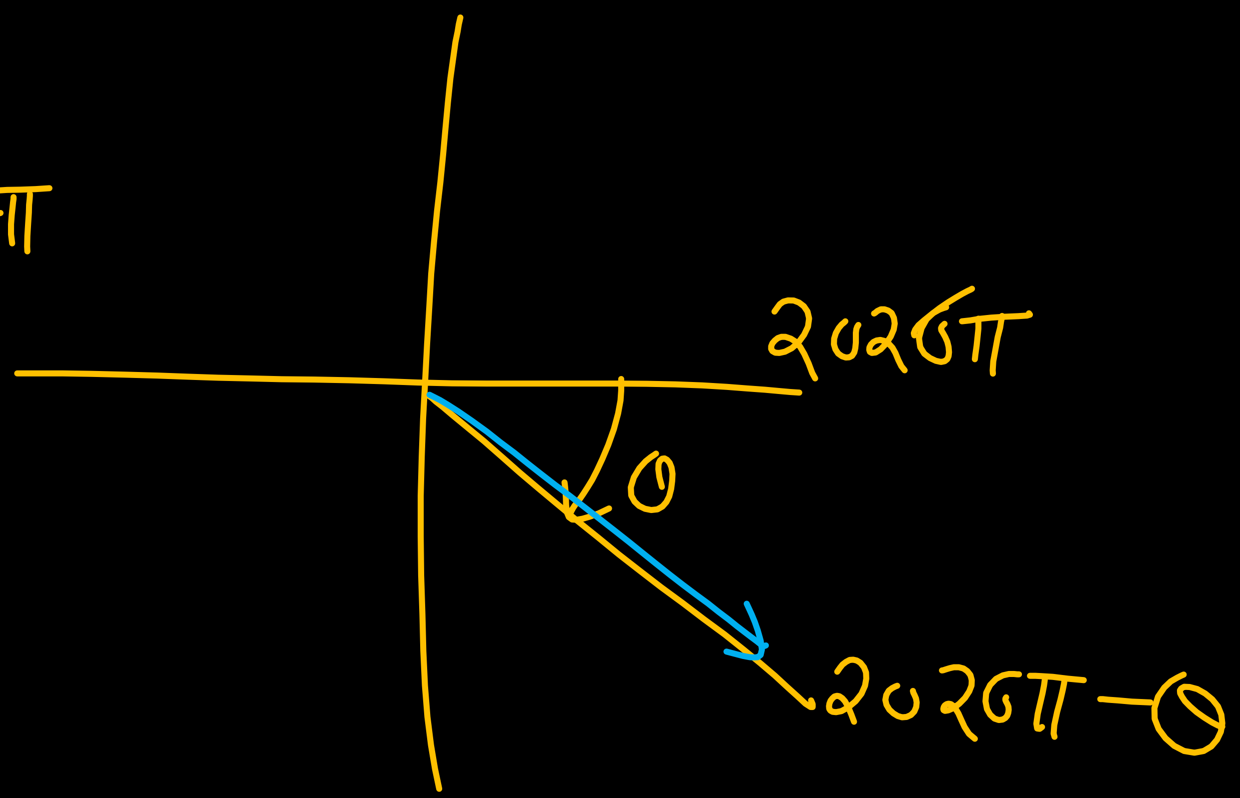
Example: 27

$\cos(\theta - 2026\pi) =$

$$\cos(\theta - 2026\pi) = \cos\left(-(2026\pi - \theta)\right)$$

$$= \cos(\underbrace{2026\pi}_{\text{Even multiple of } \pi} - \theta)$$

$$= +\cos\theta$$



Questions on Allied Angles

Question: 1

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \cdots + \sin^2 90^\circ$$

Complementary angles ko saath me rakho

$$\left(\sin^2 5^\circ + \underbrace{\sin^2 85^\circ}_{\cos^2 5^\circ}\right) + \left(\sin^2 10^\circ + \underbrace{\sin^2 80^\circ}_{\cos^2 10^\circ}\right) + \left(\sin^2 15^\circ + \underbrace{\sin^2 75^\circ}_{\cos^2 15^\circ}\right) + \cdots$$

$$+ \left(\sin^2 40^\circ + \underbrace{\sin^2 50^\circ}_{\cos^2 40^\circ}\right) + \left(\sin^2 45^\circ\right) + \left(\sin^2 90^\circ\right)$$

$$\Rightarrow \underbrace{1 + 1 + 1 + \cdots + 1}_{8\text{-times}} + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2 = 8 + \frac{1}{2} = \boxed{\frac{17}{2}}$$

Question: 2

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot \cdots \cdot \tan 89^\circ$$

Complementary angles ko saath me rakho

$(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \cdot \tan 88^\circ)(\tan 3^\circ \tan 87^\circ) \ldots (\tan 44^\circ \tan 46^\circ)(\tan 45^\circ)$

$(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ)(\tan 3^\circ \cot 3^\circ) \ldots (\tan 44^\circ \cot 44^\circ)(1)$

$\tan\theta \cot\theta = 1$

$\cancel{\tan\theta} \dfrac{1}{\cancel{\tan\theta}} = 1$

$1 \times 1 \times 1 \times \ldots \times 1 = \boxed{1}$

Question: 3

CCMM

$$\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$$

Complementary angles ko saath me rakho

$\frac{\pi}{16}+\frac{7\pi}{16}=\frac{8\pi}{16}=\frac{\pi}{2}$

$\frac{3\pi}{16}+\frac{5\pi}{16}=\frac{8\pi}{16}=\frac{\pi}{2}$

$$\left(\cos^2\frac{\pi}{16}+\cos^2\frac{7\pi}{16}\right)+\left(\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}\right)$$

$$\left(\cos^2\frac{\pi}{16}+\sin^2\frac{\pi}{16}\right)+\left(\cos^2\frac{3\pi}{16}+\sin^2\left(\frac{3\pi}{16}\right)\right)$$

$$1+1$$

$$\boxed{2}$$

Question: 4

$$\cot\frac{\pi}{20}\cdot\cot\frac{3\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\frac{7\pi}{20}\cdot\cot\frac{9\pi}{20}$$

$$\frac{\pi}{20}+\frac{9\pi}{20}=\frac{10\pi}{20}=\frac{\pi}{2}$$

$$\cos\left(\frac{\pi}{4}\right)$$

$$\left(\cot\frac{\pi}{20}\ \cot\frac{9\pi}{20}\right)\left(\cot\frac{3\pi}{20}\ \cot\frac{7\pi}{20}\right)\ \cot\left(\frac{\pi}{4}\right)$$

$\tan\theta\ \cot\theta=1$ $$\left(\cot\frac{\pi}{20}\ \tan\left(\frac{\pi}{20}\right)\right)\left(\cot\frac{3\pi}{20}\ \tan\left(\frac{3\pi}{20}\right)\right)\ 1$$

$$1\times 1\times 1=\boxed{1}$$

Question: 5

$$\cos 10^\circ + \cos 20^\circ + \cos 30^\circ + \cdots + \cos 180^\circ$$

Supplementary angles ko saath me rakho

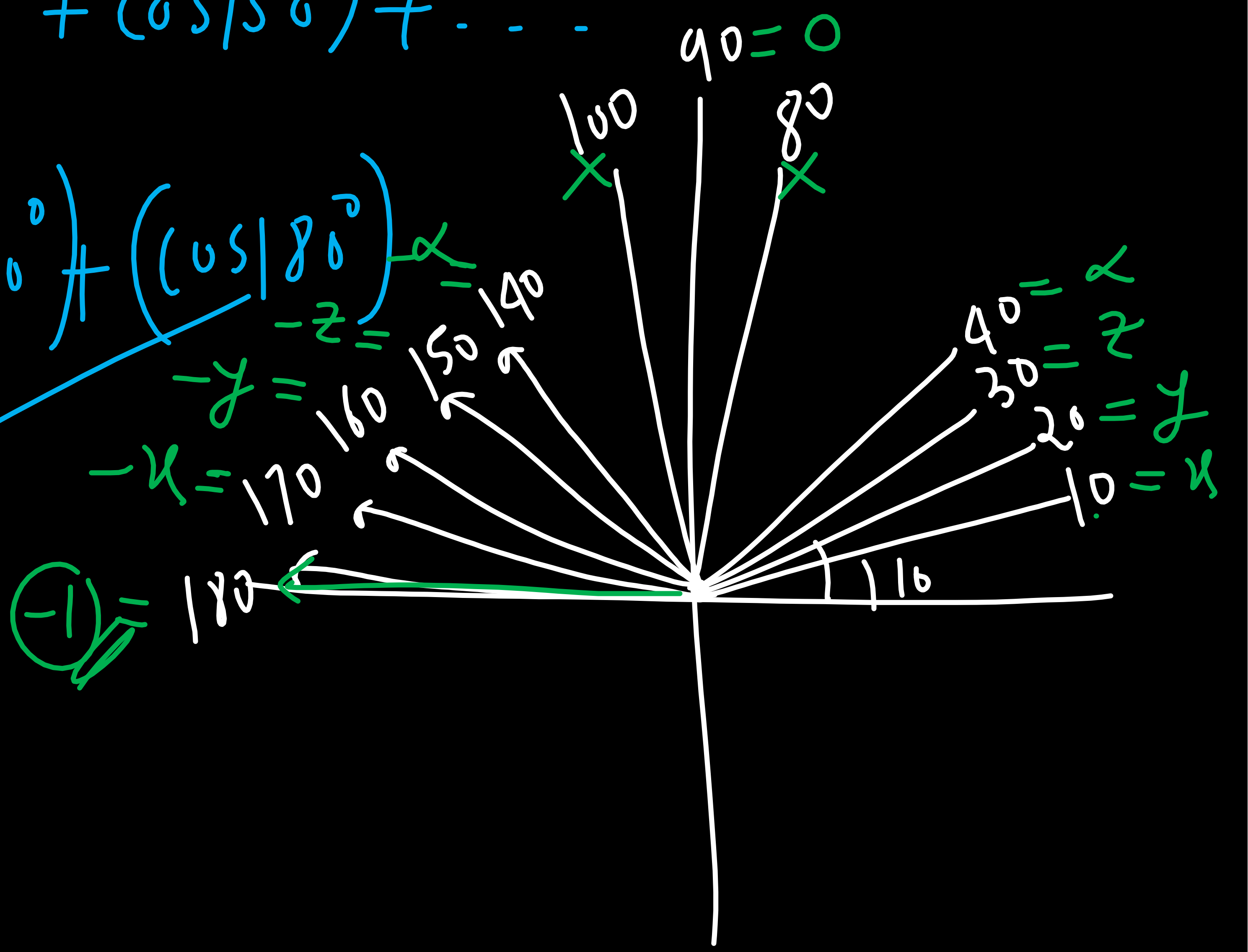
$$(\cos 10^\circ + \cos 170^\circ) + (\cos 20^\circ + \cos 160^\circ) + (\cos 30^\circ + \cos 150^\circ) + \dots$$

$$+ (\cos 80^\circ + \cos 100^\circ) + (\cos 90^\circ) + (\cos 180^\circ)$$

$$A + B = 180^\circ \Rightarrow \cos A + \cos B = 0$$

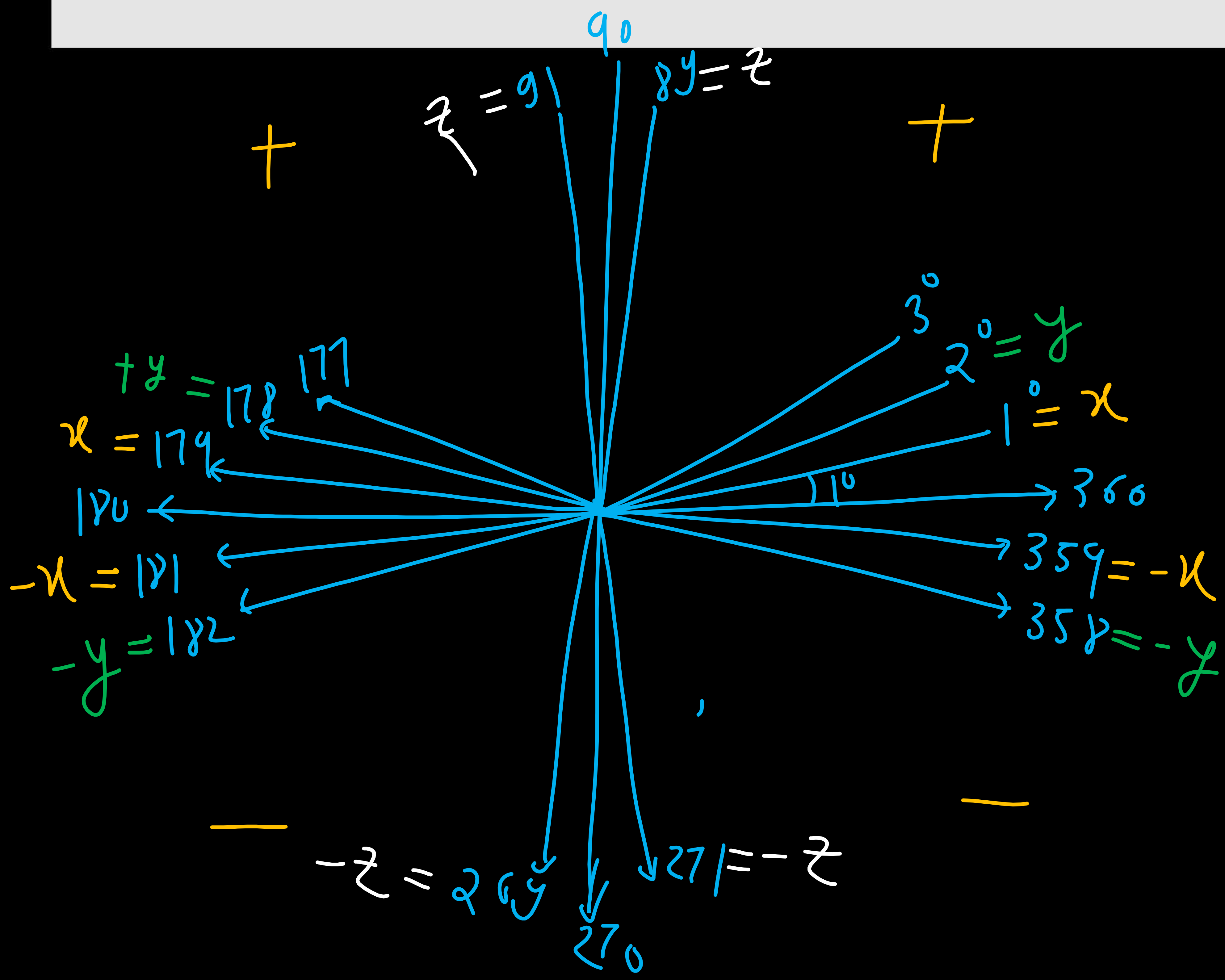
$$0 + 0 + 0 + \dots + 0 + 0 - 1$$

$$\boxed{-1}$$



Question: 6

$$\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 360^\circ$$



$$(\sin 1^\circ + \sin 359^\circ) = 0$$
$$+ (\sin 2^\circ + \sin 358^\circ)$$
$$+ (\sin 3^\circ + \sin 357^\circ)$$
$$\vdots$$
$$+ (\sin(89^\circ) + \sin 271)$$
$$+ (\underset{1}{\sin 90} + \underset{-1}{\sin 270})$$
$$+ (\sin 91 + \sin 269)$$
$$+ (\sin 92 + \sin 268)$$
$$+$$
$$+ (\sin 179 + \sin 181)$$
$$+ \sin 180^\circ$$
$$+ \sin 360^\circ$$

Ans zero

Question: 7

$$\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}$$

$$\frac{\pi}{11}+\frac{10\pi}{11}=\frac{11\pi}{11}=\pi$$

$$\tan A+\tan B=0 \Rightarrow A+B=\pi$$

$$\left(\tan\frac{\pi}{11}+\tan\frac{10\pi}{11}\right)+\left(\tan\frac{2\pi}{11}+\tan\frac{9\pi}{11}\right)+\left(\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}\right)$$

zero + zero + zero

$\boxed{\text{zero}}$

Question: 8 Evaluate the following expression:

$$\frac{\cancel{\sin\frac{11\pi}{17}}\ \cancel{\cos\frac{10\pi}{13}}\ \tan\frac{\pi}{7}}{\cancel{\cos\frac{3\pi}{13}}\ \cancel{\sin\frac{6\pi}{17}}\ \tan\frac{6\pi}{7}} = (-1)\times(-1) = \boxed{1}$$

$A + B = \pi$

$\sin A = \sin B$

$\tan A = -\tan B$

$\cos A = -\cot B$

$\frac{11\pi}{17} + \frac{6\pi}{17} = \pi$

$\frac{10\pi}{13} + \frac{3\pi}{13} = \pi$

$\frac{\pi}{7} + \frac{6\pi}{7} = \pi$

Question: 9 (HW) CC MM Ans:2

$$\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}$$

$\sin 10 = \cos 80$

$$\left(\sin^2\frac{\pi}{18}+\sin^2\frac{4\pi}{9}\right)+\left(\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}\right)$$

$\frac{\pi}{2}$

$$\left(\sin^2\frac{\pi}{18}+\cos^2\frac{\pi}{18}\right)+\left(\sin^2\frac{\pi}{9}+\cos^2\left(\frac{\pi}{9}\right)\right)$$

$1+1$

2

$\frac{\pi}{18}+\frac{7\pi}{18}=\frac{8\pi}{18}=\frac{4\pi}{9}$ ✗

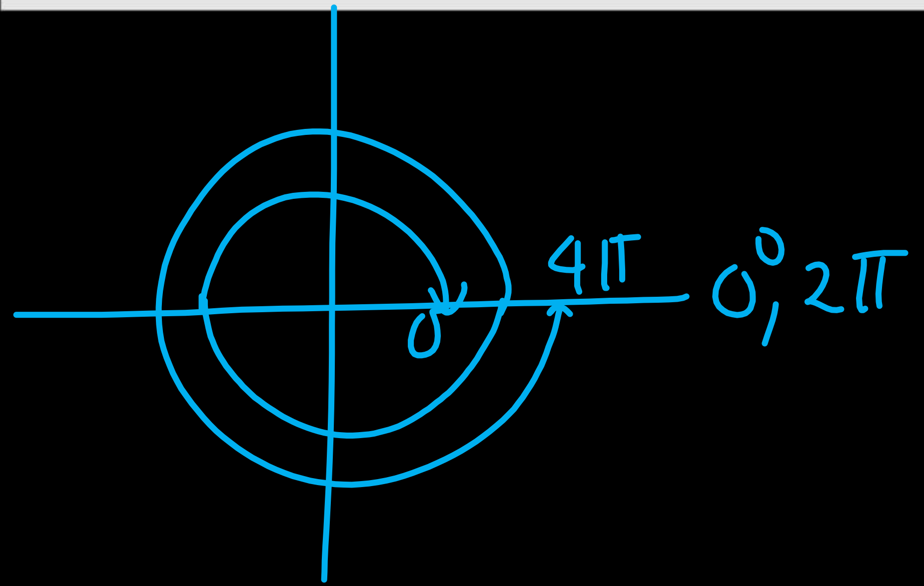
CC : $\frac{\pi}{18}+\frac{4\pi}{9}=\frac{9\pi}{18}=\frac{\pi}{2}$ ✓

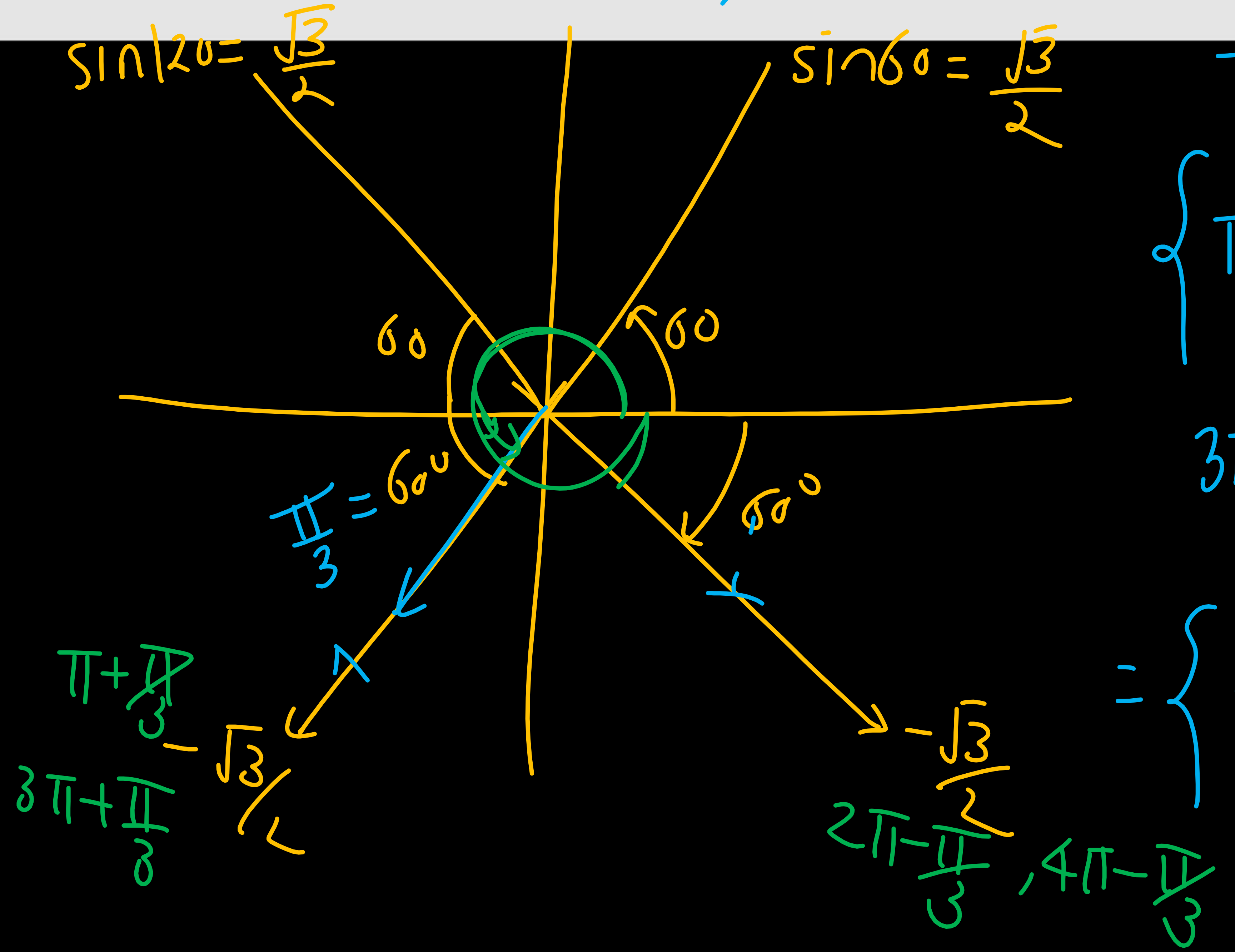
MM $\frac{\pi}{9}+\frac{7\pi}{18}=\frac{9\pi}{18}=\frac{\pi}{2}$ ✓

Question: 10 ~~(Triangle Method Question)~~

If $\theta \in (0, 4\pi)$, find all possible values of θ for which:

$$\sin\theta = -\frac{\sqrt{3}}{2}$$





Total values 4

$$\left\{\pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3},\ 3\pi + \frac{\pi}{3},\ 4\pi - \frac{\pi}{3}\right\}$$

$$= \left\{\frac{4\pi}{3},\ \frac{5\pi}{3},\ \frac{10\pi}{3},\ \frac{11\pi}{3}\right\}$$

Question: 11 (Triangle Method Question)

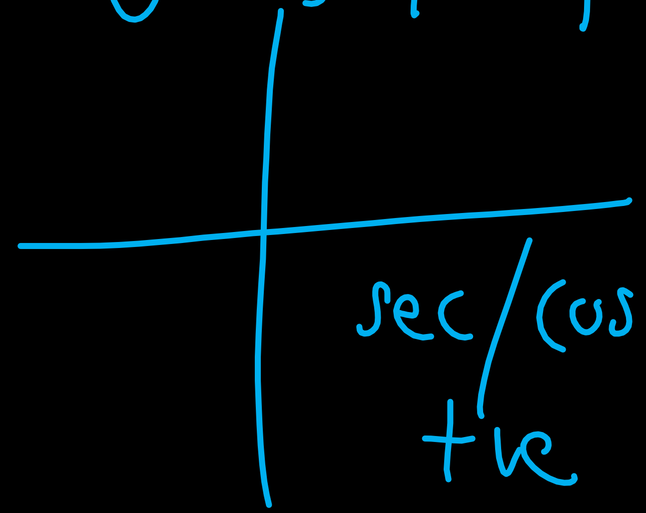
If $\boxed{\sec\theta = \sqrt{2}}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the value of the expression:

$$X = \frac{1 + \tan\theta + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$$

(A) 1 (B) -1 (C) 0 (D) 2

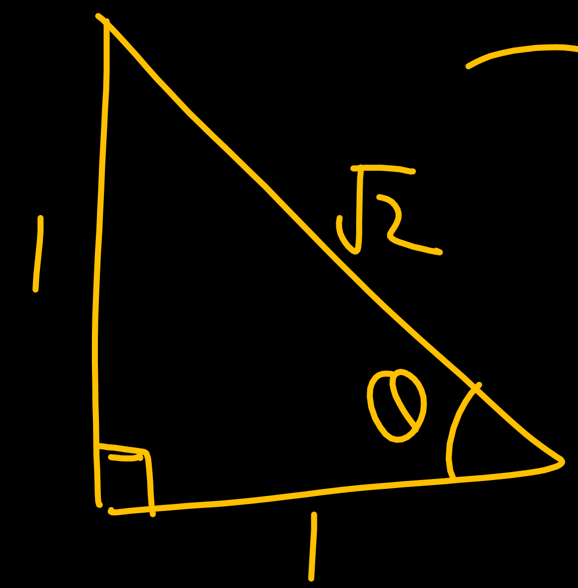
$270^\circ < \theta < 360^\circ$

$\theta \rightarrow 4^{th}$ Quad



$\sec\theta = \frac{\sqrt{2}}{1}$

$\cos\theta = \frac{1}{\sqrt{2}}$



$\tan\theta = -1$

$\cot\theta = -1$

$\operatorname{cosec}\theta = \frac{-\sqrt{2}}{1}$

$$X = \frac{\cancel{1} + \cancel{(-1)} + (-\sqrt{2})}{\cancel{1} + \cancel{(-1)} - (-\sqrt{2})}$$

$$= \frac{-\cancel{\sqrt{2}}}{+\cancel{\sqrt{2}}}$$

$$\boxed{X = -1}$$

Question: 12 JEE Mains 2024 (Triangle Method Question)

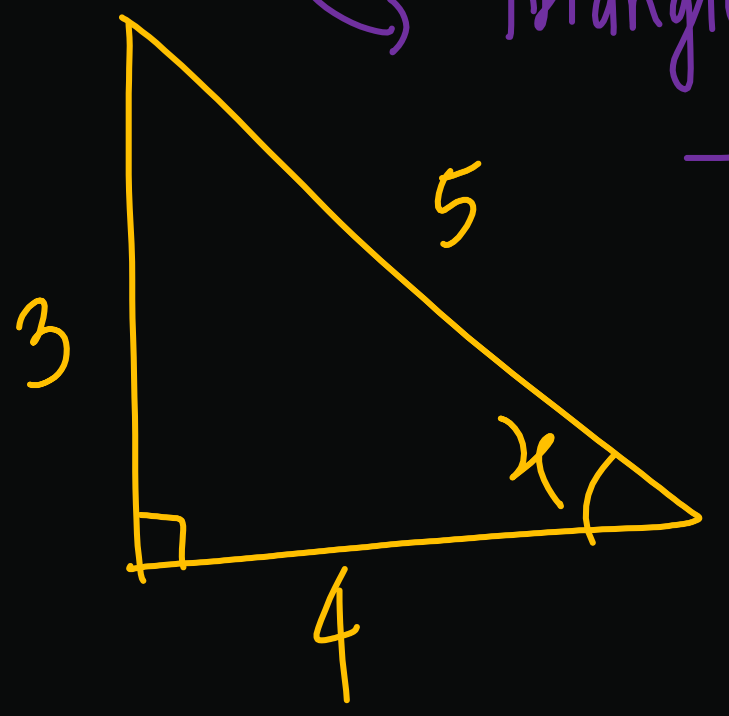
If $\boxed{\sin x = -\frac{3}{5},}$ where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to:

(A) 109 (B) 108 (C) 19 (D) 18

$x \longrightarrow 3^{rd}$ Quad $\tan x = ?$ $\cos x = ?$

$\sin x = -\frac{3}{5}$

triangle banate samay -ve sign ko consider na karo.



$\tan x = +\frac{3}{4}$, $\cos x = -\frac{4}{5}$

$$\text{Req} = 80(\tan^2 x - \cos x)$$

$$= 80\left(\left(\frac{3}{4}\right)^2 - \left(-\frac{4}{5}\right)\right)$$

$$= 80\left(\frac{9}{16} + \frac{4}{5}\right)$$

$$= \cancel{80}\left(\frac{45+64}{\cancel{80}}\right)$$

$$= 109$$

True or False?

- $\sin 1 > 0 \longrightarrow$ True $\qquad \sin 1 = \sin 1^c \approx \underline{\sin 57.3^\circ} > 0$
- $\sin 2 > 0 \longrightarrow$ True $\qquad \sin 2 = \sin 114.6^\circ > 0$
- $\sin 3 > 0 \longrightarrow$ True
- $\sin 4 > 0 \longrightarrow$ False
- $\sin 7 > 0 \longrightarrow$ True $\qquad 7 \times 60 = 420 = 360 + 60$
  $\longrightarrow$ 1st Quad
- $\cos 1 > 0 \longrightarrow$ True
- $\cos 2 > 0 \longrightarrow$ False
- $\cos 3 > 0 \longrightarrow$ False
- $\tan 5 > 0 \longrightarrow$ False

Note:

- If no unit (like the degree symbol $^\circ$) is specified for an angle, it is assumed to be measured in **radians**.
- 1 **radian** $\approx 57.3^\circ$.

$\sin 1^\circ \neq \sin 1$

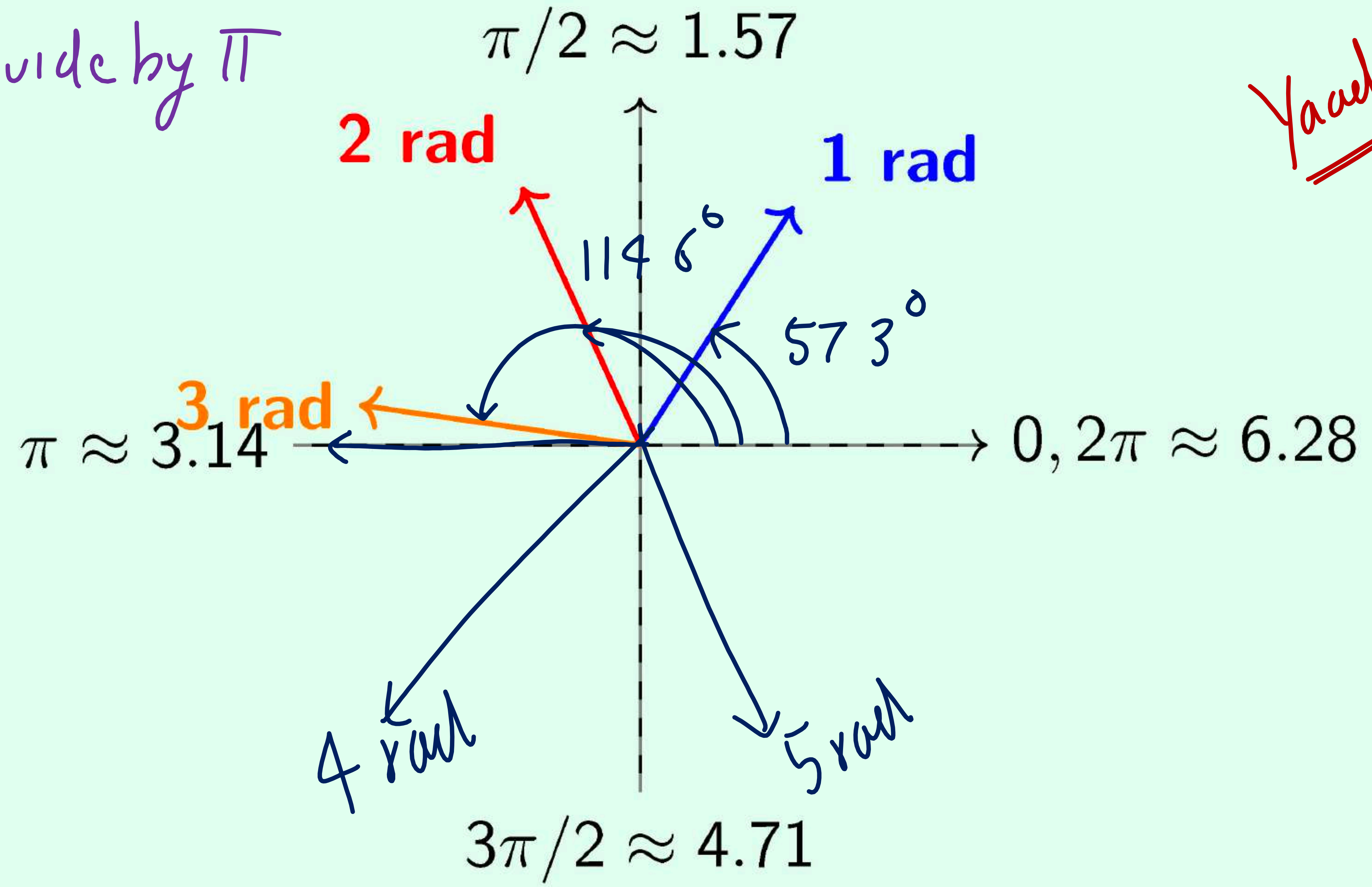
no unit

$\pi^c = 180^\circ$ ... divide by π

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$1^c = \left(\frac{180}{3.14}\right)^\circ$

$\boxed{1^c \approx 57.3^\circ}$



Yaad Rakhe

$\boxed{\begin{array}{l} \pi \approx 3.14 \\ \frac{\pi}{2} \approx 1.57 \end{array}}$

Important Concept: Complementary Angles

$A + B = 90^\circ$ or $\frac{\pi}{2}$

If $A + B = 90^\circ$ (or $\pi/2$ radians)

- $\sin A = \cos B$
- $\cos A = \sin B$
- $\boxed{\tan A = \cot B} \implies \tan A \tan B = 1$

$\sin\left(\frac{\pi}{9}\right) = \cos\left(\frac{7\pi}{18}\right)$

$\frac{\pi}{2} - \frac{\pi}{9} = \frac{7\pi}{18}$

$\cos\left(\frac{3\pi}{10}\right) = \sin\left(\frac{2\pi}{10}\right)$

$A + B = 90^\circ$

$\sin \underbrace{A = \sin(90 - B) = \cos B}$

$\tan A = \tan(90 - B) = \cot B$

$\sin 30^\circ = \cos 60^\circ$

$\sin 60^\circ = \cos 30^\circ$

$\sin 15^\circ = \cos 75^\circ$

$\sin 10^\circ = \cos 80^\circ$

$\sin 1^\circ = \cos 89^\circ$

$\cos 70^\circ = \sin 20^\circ$

$\cos(3^\circ) = \sin(87^\circ)$

$\tan 30 = \cot 60$

$\cot 50 = \tan 40$

*$\tan\left(\frac{5\pi}{12}\right) =$

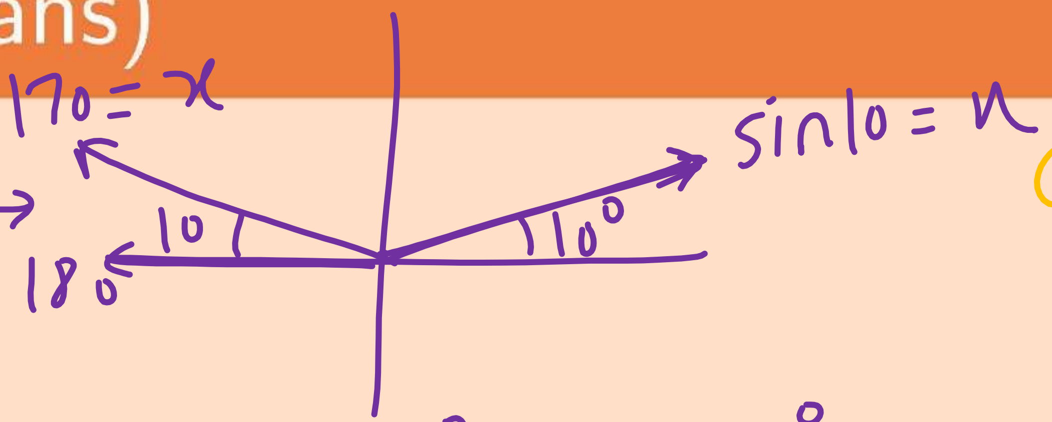
$\cot\left(\frac{\pi}{12}\right)$

Important Concept: Supplementary Angles

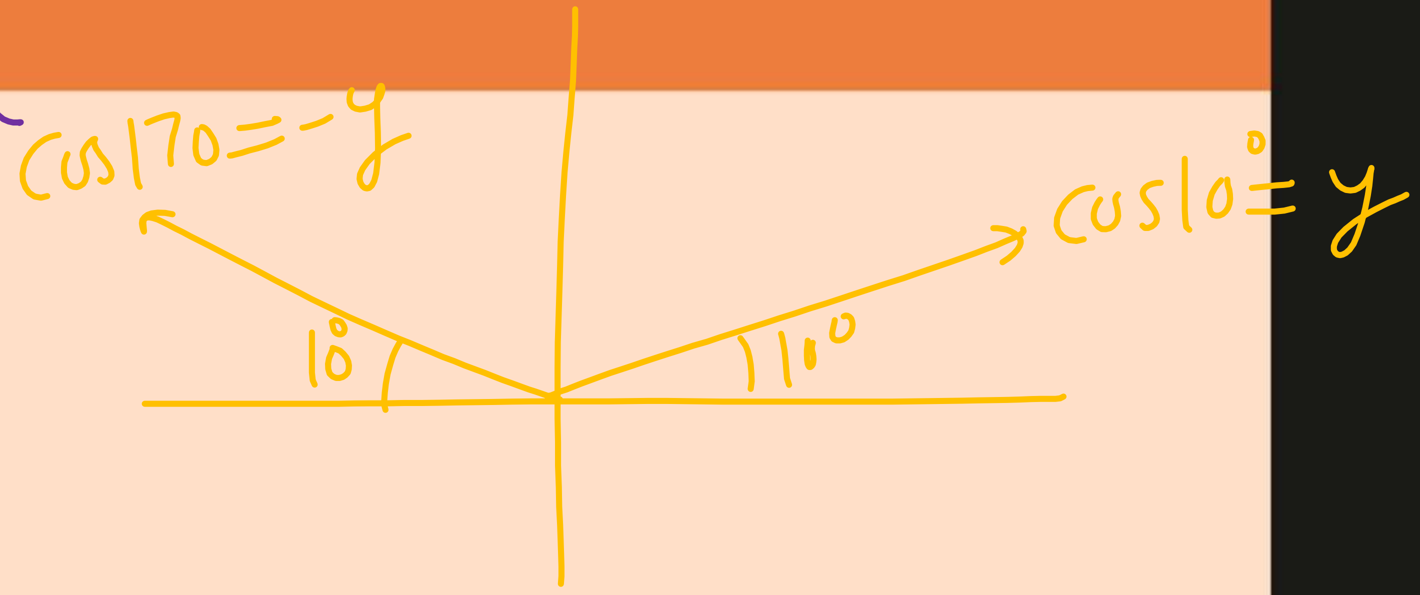
$A+B=180^\circ \,/\, \pi^c$

If $A+B=180^\circ$ (or π radians)

- $\sin A = \sin B$
- $\cos A + \cos B = 0$
- $\tan A + \tan B = 0$



$\sin 10^\circ = \sin 170^\circ$



$\cos 10^\circ + \cos 170^\circ = 0$

$y + (-y) = 0$

$\sin A = \sin(180-B) = \sin B$

$\cos A = \cos(180-B) = -\cos B$

$\tan A = \tan(180-B) = -\tan B$

eg

$\sin 10^\circ = \sin 170^\circ$

$\sin 60 = \sin 120^\circ$

$\sin 89^\circ = \sin 91^\circ$

$\cos 20 + \cos 160 = 0$

$\cos 35^\circ + \cos(145^\circ) = 0$

$\sin\left(\frac{3\pi}{16}\right) = \sin\left(\frac{13\pi}{16}\right)$

$\cos\left(\frac{7\pi}{9}\right) + \cos\left(\frac{2\pi}{9}\right) = 0$

$\tan\left(\frac{5\pi}{12}\right) + \tan\left(\frac{7\pi}{12}\right) = 0$

Trigonometric Ratios & Identities: Section 3

Formulas

Section- 2 Allied Angle

HW: Assignment- 02

mathbyiiserite

Section-1 Basic Identities

HW Assignment-01

Formula Set-1: Compound Angle Formulas

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Formula Set-1: Compound Angle Formulas (Continued)

5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\longrightarrow$ $\tan(45+\theta) = \dfrac{1+\tan\theta}{1-\tan\theta}$

6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ $\longrightarrow$ $\tan(45-\theta) = \dfrac{1-\tan\theta}{1+\tan\theta}$

optional

7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

optional

8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Proof of $\tan(A+B)$ & $\cot(A+B)$

Optional # Not IMP

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{\frac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \frac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B}}{\frac{\cancel{\cos A \cos B}}{\cancel{\cos A \cos B}} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\checkmark \cot(A+B) = \frac{1}{\tan(A+B)}$$

$$= \frac{1 - \tan A \cdot \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \frac{1}{\cot A} \frac{1}{\cot B}}{\frac{1}{\cot A} + \frac{1}{\cot B}}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

Compound Angle Formula Proof:

Not IMP

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

let $PY = x$

opposite sides are equal

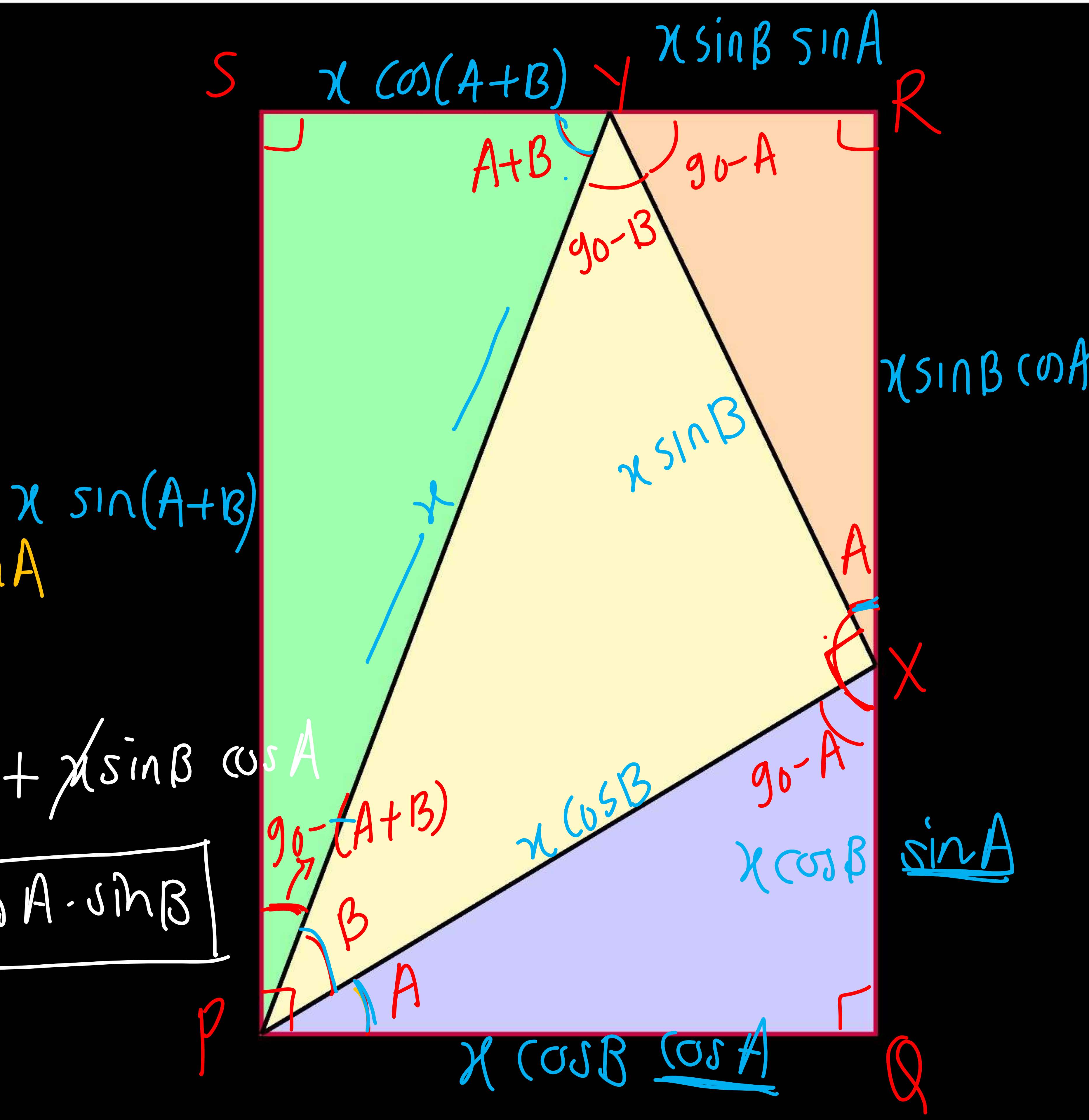
$PQ = SR$

$\cancel{x} \cos B \cos A = \cancel{x} \cos(A+B) + \cancel{x} \sin B \sin A$

$\boxed{\cos(A+B) = \cos A . \cos B - \sin A . \sin B}$

$PS = QR$ $\quad \cancel{x} \sin(A+B) = \cancel{x} \cos B . \sin A + \cancel{x} \sin B \cos A$

$\boxed{\sin(A+B) = \sin A \cos B + \cos A . \sin B}$



Simplify

1. $\sin(\underbrace{45^\circ + \theta}_{A})\cos(\underbrace{15^\circ + \theta}_{B}) - \cos(\underbrace{45^\circ + \theta}_{A})\sin(\underbrace{15^\circ + \theta}_{B})$ $= \sin(A - B)$

$= \sin\left((45 + \cancel{\theta}) - (15 + \cancel{\theta})\right)$

$= \sin 30 = \frac{1}{2}$

2. $\cos(\underbrace{45^\circ - A}_{X})\cos(\underbrace{45^\circ - B}_{Y}) - \sin(\underbrace{45^\circ - A}_{X})\sin(\underbrace{45^\circ - B}_{Y})$

$\cos(X+Y) = \cos(45 - A + 45 - B) = \cos(90 - (A+B)) = \sin(A+B)$

3. $\sin 99^\circ \cos 21^\circ + \cos 99^\circ \sin 21^\circ$

$\sin(99+21) = \sin(120) = \frac{\sqrt{3}}{2}$

4. $\cos 50^\circ \cos 10^\circ - \sin 50^\circ \sin 10^\circ$

$\cos(50+10) = \cos 60 = \frac{1}{2}$

Prove that: 1

$$\tan A \cot\left(\frac{A}{2}\right) - 1 = \sec A$$

Prove that: 2 (HOMEWORK)

$$1 + \tan A \tan\left(\frac{A}{2}\right) = \sec A$$

$$LHS = \frac{\sin A}{\cos A}\,\frac{\cos(A/2)}{\sin(A/2)} - 1$$

$$= \frac{\sin A \cos(A/2) - \cos(A)\sin(A/2)}{\cos A \sin(A/2)}$$

$$= \frac{\sin(A - A/2)}{\cos A \sin(A/2)} = \frac{\cancel{\sin(A/2)}}{\cos A \cancel{\sin(A/2)}}$$

$$= \sec A$$

$$= RHS$$

Question: Find the value

1. $\sin 15^\circ$ 2. $\cos 15^\circ$ 3. $\tan 15^\circ$ 4. $\sin 75^\circ$ 5. $\cancel{\sin 75^\circ}$ tan75 6. $\cos 75^\circ$

① $\sin 15^\circ$

$$\sin 15 = \sin(45-30)$$
$$= \sin 45 \cos 30 - \cos 45 \sin 30$$
$$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \frac{1}{2}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

④ $\sin(75) = \sin(90-15) = \cos 15^\circ$

⑤ $\cos(75) = \cos(90-15) = \sin 15$

⑥ $\tan(75) = \cancel{\frac{\sin 75}{\cos 75}}$

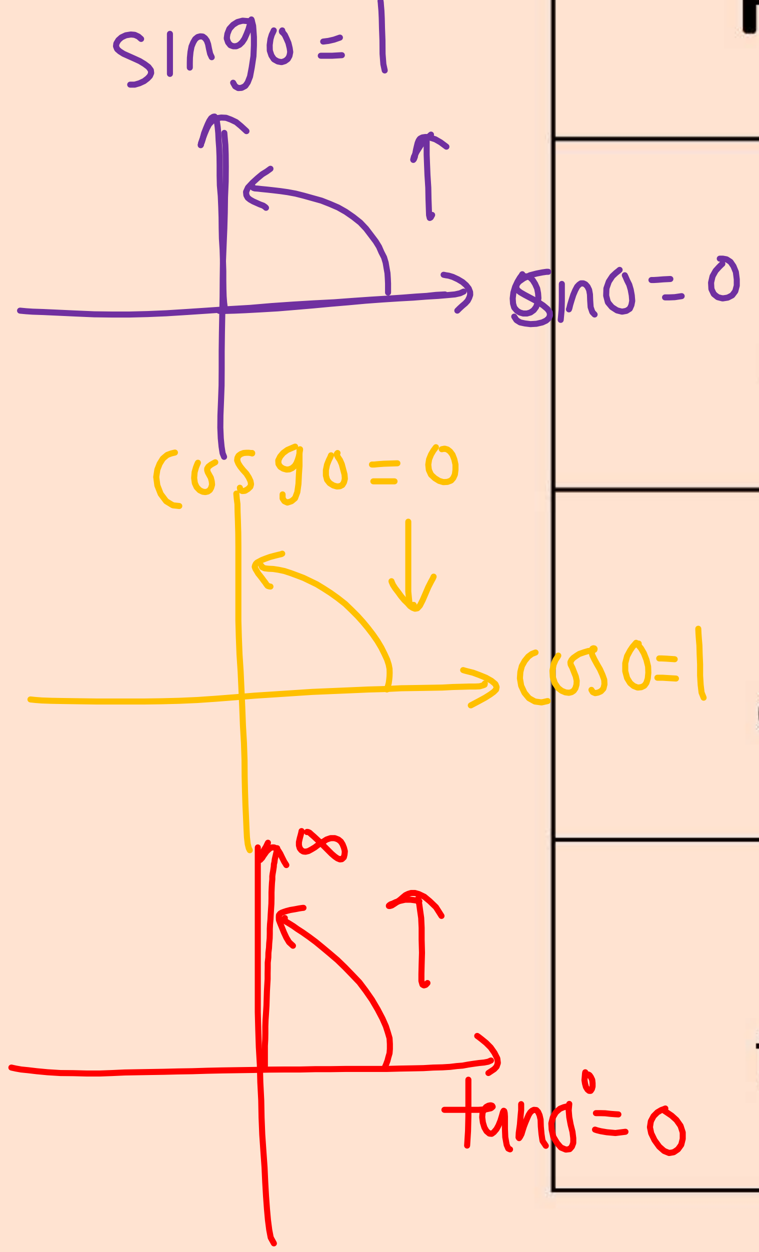
② $\cos 15^\circ = \cos(45-30)$

$$= \cos 45 \cos 30 + \sin 45 \sin 30$$
$$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

③ $\tan 15^\circ = \tan(45-30^\circ)$ OR $\tan 15^\circ = \frac{\sin 15}{\cos 15}$

$$2-\sqrt{3} = \frac{4-2\sqrt{3}}{2} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\frac{\sqrt{3}-1}{\cancel{2\sqrt{2}}}}{\frac{\sqrt{3}+1}{\cancel{2\sqrt{2}}}}$$

Important Values Yaad Rakhe



Ratio	$\boxed{15^\circ = \frac{\pi}{12}}$	$\boxed{75^\circ = \frac{5\pi}{12}}$
$\sin\theta \longrightarrow$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
$\cos\theta$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
$\tan\theta$	$2-\sqrt{3}$	$2+\sqrt{3}$

Question: 1 [JEE Main 2023] Ans:(A)

If $\tan 15^\circ + \dfrac{1}{\tan 75^\circ} + \dfrac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \dfrac{1}{a}\right)$ is:

(A) 4

(B) $4 - 2\sqrt{3}$

(C) 2

(D) $5 - \dfrac{3}{2}\sqrt{3}$

① $\tan 15^\circ = 2 - \sqrt{3}$

② $\tan 75 = 2 + \sqrt{3}$

③ $\tan(105^\circ) = \tan(180 - 75)$
$= -\tan 75$
$= -(2 + \sqrt{3})$

④ $\tan 195^\circ = \tan(180^\circ + 15^\circ)$
$= \tan 15 = 2 - \sqrt{3}$

$2 - \sqrt{3} + \cancel{\left(\dfrac{1}{2+\sqrt{3}}\right)} + \cancel{\dfrac{1}{-(2+\sqrt{3})}} + 2 - \sqrt{3} = 2a$

$\cancel{2}(2 - \sqrt{3}) = \cancel{2}a$

$a = 2 - \sqrt{3}$

Key $= a + \dfrac{1}{a} = 2 - \sqrt{3} + \dfrac{1}{2 - \sqrt{3}} = 2 - \cancel{\sqrt{3}} + 2 + \cancel{\sqrt{3}} = 4$

Question: 2

$\sin\alpha = \dfrac{3}{5}$ and $\cos\beta = \dfrac{9}{41}$, where α and β are acute angles (in the first quadrant).

less than 90°

Find the values of:

1. $\sin(\alpha - \beta)$
2. $\cos(\alpha - \beta)$

① $\sin(\alpha-\beta) = \sin\alpha\ \cos\beta - \cos\alpha\ \sin\beta$

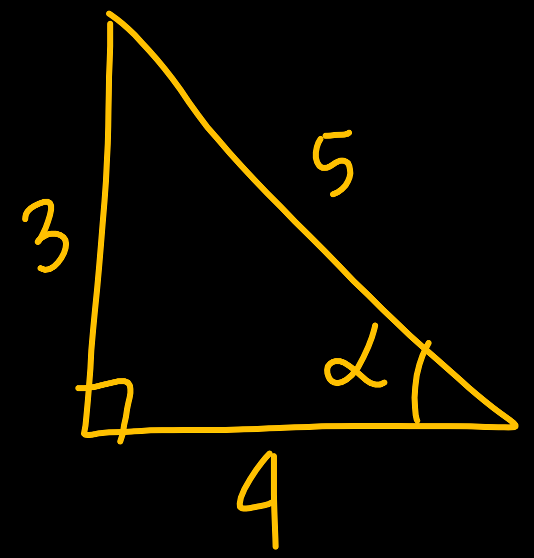
$$= \left(\frac{3}{5}\right)\left(\frac{9}{41}\right) - \left(\frac{4}{5}\right)\left(\frac{40}{41}\right)$$

Do Calculation by yourself

② $\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$

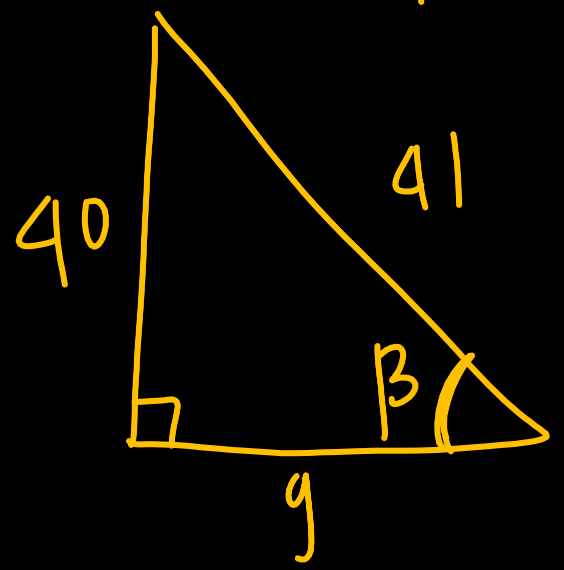
put the values.

$\sin\alpha = \dfrac{3}{5}$



$\cos\alpha = \dfrac{4}{5}$

$\cos\beta = \dfrac{9}{41}$



$\sin\beta = \dfrac{40}{41}$

Question: 3 [JEE Main 2022] Ans:(A)

$\alpha \to 3^{rd}$ quad $\quad \beta \to 2^{nd}$ quad

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are:

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) -7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

$$\text{Req} = \tan(\alpha+\beta)$$

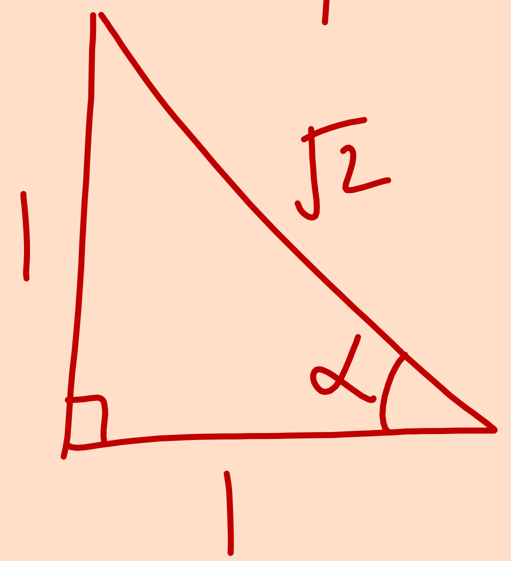
$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{1 + \left(-\frac{4}{3}\right)}{1 - (1)\left(-\frac{4}{3}\right)}$$

$$= \frac{-\frac{1}{\cancel{3}}}{\frac{7}{\cancel{3}}}$$

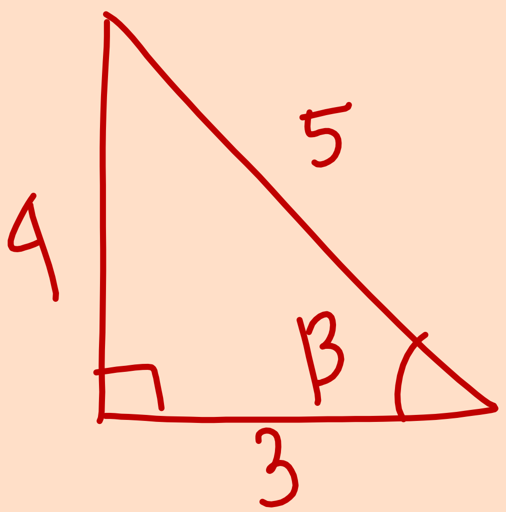
$$= -\frac{1}{7}$$

$\cot\alpha = \frac{1}{1}$



$\tan\alpha = 1$

$\sec\beta = -\frac{5}{3}$

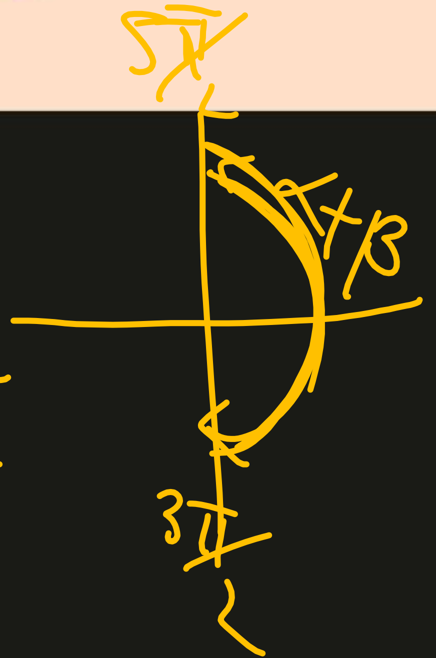


$\tan\beta = -\frac{4}{3}$

$$\pi < \alpha < \frac{3\pi}{2}$$

$$\frac{\pi}{2} < \beta < \pi$$

$$\frac{3\pi}{2} < \alpha+\beta < \frac{5\pi}{2}$$



Remark-1

Yaad Rakho!

1. $\tan(45^\circ + \theta) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

2. $\tan(45^\circ - \theta) = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

Proof $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$A = 45^\circ$, $B = \theta$

$\tan(45+\theta) = \dfrac{\tan 45 + \tan\theta}{1 - \tan 45 \cdot \tan\theta}$

$= \dfrac{1 + \tan\theta}{1 - \tan\theta}$

Do it by yourself!

$\tan(A-B) = \underline{\qquad\qquad}$

put $A = 45^\circ$, $B = \theta$

Question: 4

If $A + B = \dfrac{\pi}{2}$, then prove that:

$$\tan A \tan B = 1$$

$\rightarrow$

$$A + B = \frac{\pi}{2}$$

take 'tan' on both sides

$$\tan(A+B) = \tan\left(\frac{\pi}{2}\right)$$

$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = ND = \frac{\square}{0}$$

Niche zero aana chahiye!

$$1 - \tan A \cdot \tan B = 0$$

$$\boxed{\tan A \cdot \tan B = 1}$$

Question: 5

If $A + B = \frac{\pi}{4}$ then prove that:

$$(1 + \tan A)(1 + \tan B) = 2$$

$$A + B = \frac{\pi}{4}$$

$$\tan(A+B) = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \cdot \tan B$$

$$\tan A + \underline{\tan B + \tan A \tan B} = 1$$

$$(1 + \tan A) + \tan B\,(1 + \tan A) = 1 + 1$$

$$(1 + \tan A)(1 + \tan B) = 2$$

Question: 6 HOMEWORK

If $A + B = 45^\circ$, then prove that:

$$(\cot A - 1)(\cot B - 1) = 2$$

$A + B = 45^\circ$

take "cot" on both side

Question: 7 [MHT-CET 2019] HOMEWORK

Ans | 2

If $A - B = \frac{\pi}{4}$, then the value of $(1 + \tan A)(1 - \tan B)$?

$A - B = \frac{\pi}{4}$

$\tan(A-B) = \tan\left(\frac{\pi}{4}\right)$

$\frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\tan A - \tan B = 1 + \tan A \cdot \tan B$

$\tan A - \underline{\tan B - \tan A \cdot \tan B} = 1$

$(1 + \tan A) - \tan B\,(1 + \tan A) = 1 + 1$

$(1 + \tan A)(1 - \tan B) = 2$

Question: 8

Prove that:

$$\tan 5\theta \cdot \tan 3\theta \cdot \tan 2\theta = \tan 5\theta - \tan 3\theta - \tan 2\theta$$

tan ka question, start hoga ~~&~~ tan ke formula से

** $5\theta = 3\theta + 2\theta$

$\tan(5\theta) = \tan(3\theta + 2\theta)$

$$\tan 5\theta = \frac{\tan 3\theta + \tan 2\theta}{1 - \tan 3\theta \cdot \tan 2\theta}$$

$\tan 5\theta - \tan 5\theta \tan 3\theta \cdot \tan 2\theta = \tan 3\theta + \tan 2\theta$

Rearrange to get final Ans:

Question: 9

Find the value of the following expression:

$$\tan \underline{125}^\circ + \tan \underline{100}^\circ + \tan \underline{100}^\circ \tan \underline{125}^\circ$$

$225^\circ = 100^\circ + 125^\circ$

$\underline{25} = 125 - 100$ ✗

$\tan(225) = \tan(100 + 125)$

$$1 = \frac{\tan 100 + \tan 125}{1 - \tan 100 \cdot \tan 125}$$

Rearrange to get final and l

Ans: 1

Question: 10 HOMEWORK

Prove that:

$$\tan 80^\circ = \tan 10^\circ + 2\tan 70^\circ$$

Formulate

$80 = 70 + 10$

$A + B = \frac{\pi}{2}$

$\tan A \tan B = 1$

$\tan 80 \tan 10 = 1$

$\tan(80) = \tan(70+10)$

$$\tan 80 = \frac{\tan 70 + \tan 10}{1 - \tan 70 \tan 10}$$

$\tan 80 - \tan 80 \tan 70 \tan 10 = \tan 70 + \tan 10$

$\tan 80 - \tan 70 = \tan 70 + \tan 10$

$\tan 80 = 2\tan 70 + \tan 10$

Question: 11

If $\tan\theta = \dfrac{1}{2}$ and $\tan\phi = \dfrac{1}{3}$, find the value of $\tan(2\theta + \phi)$.

$$\text{Req} = \tan(2\theta + \phi)$$

$$= \frac{\tan 2\theta + \tan\phi}{1 - \tan 2\theta \ \tan\phi}$$

$$= \frac{\frac{4}{3} + \frac{1}{3}}{1 - \left(\frac{4}{3}\right)\left(\frac{1}{3}\right)} = \frac{\frac{\cancel{5}}{3}}{\frac{\cancel{5}}{9}} = 3$$

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta \cdot \tan\theta}$$

$$= \frac{\frac{1}{2} + \frac{1}{2}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{1}{\frac{3}{4}}$$

$$= \frac{4}{3}$$

Question: 12 [AIEEE 2010]

If $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$, where α and β both lie between 0 and $\frac{\pi}{4}$, then find the value of $\tan(2\alpha)$.

$0<\alpha, \beta<\frac{\pi}{4}$

Chalaki !

$2\alpha=(\alpha-\beta)+(\alpha+\beta)$

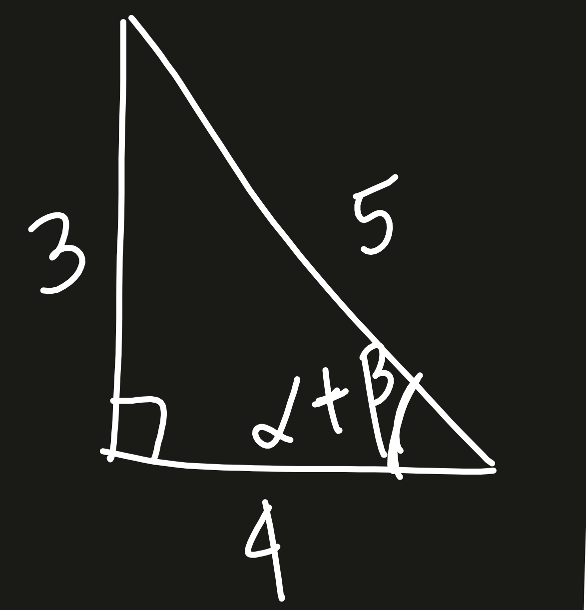
take 'tan'

$\tan(2\alpha)=\tan[\underbrace{(\alpha-\beta)}_{A}+\underbrace{(\alpha+\beta)}_{B}]$

$=\frac{\tan(\alpha-\beta)+\tan(\alpha+\beta)}{1-\tan(\alpha-\beta)\cdot\tan(\alpha+\beta)}$

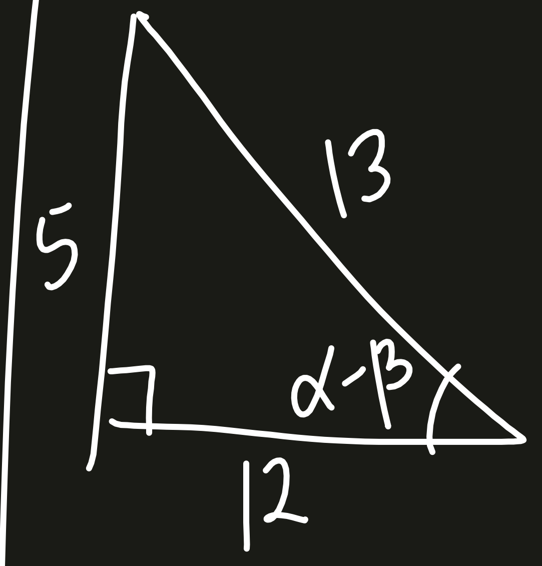
$=\frac{\frac{5}{12}+\frac{3}{4}}{1-\left(\frac{5}{12}\right)\left(\frac{3}{4}\right)} \Rightarrow$ Ans:

$\cos(\alpha+\beta)=\frac{4}{5}$



$\tan(\alpha+\beta)=\frac{3}{4}$

$\sin(\alpha-\beta)=\frac{5}{13}$



$\tan(\alpha-\beta)=\frac{5}{12}$

Question: 13 HOMEWORK

If $\cos(\theta - \alpha) = a$ and $\sin(\theta - \beta) = b$, then prove that:

$$\cos^2(\alpha - \beta) + 2ab\sin(\alpha - \beta) = a^2 + b^2$$

$$\alpha - \beta = \alpha - \theta - \beta + \theta$$

$$\alpha - \beta = (\theta - \beta) - (\theta - \alpha) \quad \text{——} \circledast$$

take "cos"

$$\cos(\alpha - \beta) = \cos\big(\underbrace{(\theta - \beta)}_{A} - \underbrace{(\theta - \alpha)}_{B}\big)$$

$$= \cos(\theta - \beta)\cdot\cos(\theta - \alpha) + \sin(\theta - \beta)\cdot\sin(\theta - \alpha)$$

$$\cos(\alpha - \beta) = \sqrt{1 - b^2}\cdot a + b\cdot\sqrt{1 - a^2}$$

take "sin" from $\circledast$

$$\sin(\alpha - \beta) = \sin\big(\underbrace{(\theta - \beta)}_{A} - \underbrace{(\theta - \alpha)}_{B}\big)$$

$$= \sin(\theta - \beta)\cdot\cos(\theta - \alpha) - \cos(\theta - \beta)\sin(\theta - \alpha)$$

$$= b\,a - \sqrt{1 - b^2}\sqrt{1 - a^2}$$

$$LHS = \cos^2(\alpha-\beta) + 2ab\sin(\alpha-\beta)$$

$$= \left(\sqrt{1-b^2}\cdot a + \sqrt{1-a^2}\cdot b\right)^2 + 2ab\left(ab - \sqrt{1-a^2}\sqrt{1-b^2}\right)$$

$$= (1-b^2)a^2 + (1-a^2)b^2 + \cancel{2ab\sqrt{1-a^2}\sqrt{1-b^2}} + 2a^2b^2 - \cancel{2ab\sqrt{1-a^2}\sqrt{1-b^2}}$$

$$= a^2 - \cancel{a^2b^2} + b^2 - \cancel{a^2b^2} + \cancel{2a^2b^2}$$

$$= a^2+b^2$$

$$= RHS$$

IMP Question

Question 1: Find the Value

$$1\cos 15^\circ + 1\sin 15^\circ$$

$\sin\theta \pm \cos\theta$

$\cos\theta \pm \sin\theta$

(Hint: Multiply and divide by $\sqrt{2}$.)

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 15 + \frac{1}{\sqrt{2}}\sin 15\right)$

$\sqrt{2}\left(\cos 45 \cos 15 + \sin 45 \sin 15\right) = \sqrt{2}\cos(45-15)$

$= \sqrt{2}\cos 30^\circ$

$= \sqrt{2}\,\frac{\sqrt{3}}{2}$

Question 2: Find the Value

$\sqrt{3}\sin\theta \pm \cos\theta$

$\sqrt{3}\cos\theta \pm \sin\theta$

$$\frac{1}{4}(\sqrt{3}\cos 23^\circ - 1\sin 23^\circ)$$

(Hint: Multiply and divide by 2.)

$\frac{2}{4}\left(\frac{\sqrt{3}}{2}\cdot\cos 23 - \frac{1}{2}\sin 23\right) = \frac{1}{2}\left(\sin 60 \cos 23 - \cos 60 \sin 23\right)$

$= \frac{1}{2}\sin(60-23) = \frac{1}{2}\sin 37^\circ = \frac{1}{2}\left(\frac{3}{5}\right)$

$= \frac{3}{10}$

Question 3: Simplify the Expression

Divide by $\cos 8^\circ$

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \frac{\frac{\cos 8^\circ}{\cos 8^\circ} - \frac{\sin 8}{\cos 8}}{\frac{\cos 8}{\cos 8} + \frac{\sin 8}{\cos 8}} = \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ} = \tan(45-8)$$

$= \tan 37 = \frac{3}{4}$

(Hint: Divide the numerator and denominator by $\cos 8^\circ$.)

eg $\cos^2 15 - \sin^2 15 = \cos(15+15)\cos(15-15)$

$= \cos 30 \cdot \cos 0^\circ$

$= \sqrt{3}/2 \quad 1$

Two Very Important Identities

Yaad Rakhe!

1. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$

2. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$

Proof

① $\sin(A+B)\,(\sin(A-B))$

$= (\underbrace{\sin A\cos B}_{x} + \underbrace{\cos A\sin B}_{y})(\underbrace{\sin A\cos B}_{x} - \underbrace{\cos A\sin B}_{y})$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A\,(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \cancel{\sin^2 A\cdot\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$

$= \boxed{\sin^2 A - \sin^2 B}$

$= (1-\cos^2 A) - (1-\cos^2 B) = \boxed{\cos^2 B - \cos^2 A}$ //

② $LHS = \cos(A+B)\cos(A-B)$

$= (\cos A\cos B - \sin A\cdot\sin B)$

$(\cos A\cos B + \sin A\cdot\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A\,(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \cancel{\sin^2 B.\cos^2 A} - \sin^2 B + \cancel{\sin^2 B.\cos^2 A}$

$= \cos^2 A - \sin^2 B$ //

$= \cos^2 B - \sin^2 A$ //

Formula Set-2 : Transformation Formulas

Transformation Formulas: Product to Sum

①+②

1. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

①−②

2. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

③+④

3. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

③−④

4. $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

① $\sin(A+B) = \sin A\cos B + \cos A\sin B$

② $\sin(A-B) = \sin A\cos B - \cos A\sin B$

③ $\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

④ $\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$

Formula Set-2 (continue)

Transformation Formulas: Sum to Product

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$A+B=C$

$A-B=D$

$2A = C+D$

$\boxed{A = \frac{C+D}{2}}$

$2B = C-D$

$\boxed{B = \frac{C-D}{2}}$

$\sin C + \cos D$ ✗

Question 1: Express as a Sum

$$2\sin(3x)\cos(x)$$

Product $\longrightarrow$ Sum

$$2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$$

$$= \sin(3x+x) + \sin(3x-x)$$

$$= \sin 4x + \sin 2x$$

Question 2: Prove that

Sum → Product

$$\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan(2A)$$

$$LHS = \frac{\cancel{2}\sin\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)}{\cancel{2}\cos\left(\frac{A+3A}{2}\right)\cdot\cos\left(\frac{A-3A}{2}\right)}$$

$$= \frac{\sin(2A)\,\cancel{\cos(-A)}}{\cos(2A)\,\cancel{\cos(-A)}}$$

$$= \tan(2A)$$

$$= RHS$$

Question 3: Prove that

$$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \tan 6\theta$$

$$LHS = \frac{2\sin(4\theta)\cos(-\theta) + 2\sin(8\theta)\cos(-\theta)}{2\cos(4\theta)\cos(-\theta) + 2\cos(8\theta)\cos(-\theta)}$$

$$= \frac{\cancel{2\cos(-\theta)}\left[\sin 4\theta + \sin 8\theta\right]}{\cancel{2\cos(-\theta)}\left[\cos 4\theta + \cos 8\theta\right]}$$

$$= \frac{\cancel{2}\sin(6\theta)\cdot\cancel{\cos(-2\theta)}}{\cancel{2}\cdot\cos(6\theta)\cdot\cancel{\cos(-2\theta)}} = \tan 6\theta$$

Question 4: Prove that

$$\frac{\sin A + 2\sin(3A) + \sin(5A)}{\sin(3A) + 2\sin(5A) + \sin(7A)} = \frac{\sin(3A)}{\sin(5A)}$$

$$LHS = \frac{\sin A + \sin 3A + \sin 3A + \sin 5A}{\sin 3A + \sin 5A + \sin 5A + \sin 7A}$$

$=$ Do it by yourself!

Question 5: Prove that

$$\sin(\beta+\gamma-\alpha)+\sin(\gamma+\alpha-\beta)+\sin(\alpha+\beta-\gamma)-\sin(\alpha+\beta+\gamma)=4\sin\alpha\sin\beta\sin\gamma$$

$\sin C + \sin D$ $\qquad\qquad$ $\sin C - \sin D$

$$LHS = 2\sin\left(\frac{\cancel{\beta}+\gamma-\cancel{\alpha}+\gamma+\cancel{\alpha}-\cancel{\beta}}{2}\right)\cdot\cos\left(\frac{(\beta+\cancel{\gamma}-\alpha)-(\cancel{\gamma}+\alpha-\beta)}{2}\right)$$

$$= 2\sin\gamma\cos\left(\frac{2\beta-2\alpha}{2}\right) + 2\cdot\cos\left(\frac{\alpha+\beta-\cancel{\gamma}+\alpha+\beta+\cancel{\gamma}}{2}\right)\sin\left(\frac{\cancel{\alpha}+\cancel{\beta}-\gamma-\cancel{\alpha}-\cancel{\beta}-\gamma}{2}\right)$$

$$+\,2\cos\left(\frac{2\alpha+2\beta}{2}\right)\sin(-\gamma)$$

$$= 2\sin\gamma\cos(\beta-\alpha)+2\cos(\alpha+\beta)\cdot\sin(-\gamma)$$

$$= 2\sin\gamma\left[\cos(\beta-\alpha)-\cos(\alpha+\beta)\right] = 2\sin\gamma$$

$$= 2\sin\gamma\left[\cos(\underbrace{\beta-\alpha}_{C}) - \cos(\underbrace{\alpha+\beta}_{D})\right]$$

$$= 2\sin\gamma\left((-2)\sin\left(\frac{\beta-\cancel{\alpha}+\cancel{\alpha}+\beta}{2}\right)\sin\left(\frac{\cancel{\beta}-\alpha-\alpha-\cancel{\beta}}{2}\right)\right)$$

$$= -4\sin\gamma \cdot \sin\beta\,\sin(-\alpha)$$

$$= 4\sin\alpha \cdot \sin\beta\,\sin\gamma \quad //$$

Question 6: Prove that

$$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ = -\frac{3}{4}$$

Product $\longrightarrow$ Sum

Multiple & divide by 2

$$LHS = \frac{1}{2}\Big(2\cos 20 \cos 100 + 2\cos 100 \cos 140 - \big(2\cos 140 \cos 200\big)\Big)$$

$$= \frac{1}{2}\Big(\cos 120 + \cos 80 + \cos(240) + \cos(40) - \big(\cos 340 + \cos 60\big)\Big)$$

$-\frac{1}{2}$ $\quad -\frac{1}{2}$ $\quad \frac{1}{2}$

$$= \frac{1}{2}\Big(-\frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \cos 80 + \cos 40 - \cos 20\Big)$$

$^*\cos(340) = \cos(360 - 20) = \cos 20$

$$= \frac{1}{2}\left(-\frac{3}{2} + \cos 80 + \cos 40 - \cos 20\right)$$

Sum ⟶ Product

$$= \frac{1}{2}\left(-\frac{3}{2} + \cancel{2}\cos(60)\cos(20) - \cos(20)\right)$$

$$\frac{1}{\cancel{2}}$$

$$= \frac{1}{2}\left(-\frac{3}{2} + \cancel{\cos 20} - \cancel{\cos 20}\right)$$

$$= \frac{1}{2}\left(-\frac{3}{2}\right)$$

$$= \frac{-3}{4}$$

Question 7: Find the Value

$$\cos 20^\circ + \cos 100^\circ + \cos 140^\circ$$

Sum ⟶ Product

$$\cos(140) = \cos(180 - 40)$$
$$= -\cos 40$$

$$2\cos(60^\circ) \cdot \cos(-40^\circ) - \cos(40^\circ)$$

$$\cancel{2}\left(\frac{1}{\cancel{2}}\right) \cos(40) - \cos(40)$$

$$\cos 40 - \cos 40$$

zero //

Question 8: Prove that

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

Product ✓

Sum ✗

$$LHS = \cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+\cos\left(\frac{3\pi}{13}\right)+\cos\left(\frac{5\pi}{13}\right)$$

π π

$A+B=\pi \Rightarrow \cos A+\cos B=0$

Check for Complementary OR Supplementary

$$= 0+0$$

$$= 0$$

Question 9: Prove that

$$\cos^2 73^\circ + \cos^2 47^\circ + \cos 73^\circ \cos 47^\circ = \frac{3}{4}$$

Do $\cos^2$ dikh jaye, ek ko badlo sin mel

Product $\longrightarrow$ Sum

$$LHS = \cos^2 73 + 1 - \sin^2 47 + \frac{1}{2}\left(2\cos 73^\circ . \cos 47^\circ\right)$$

$$= \left(\cos^2 73 - \sin^2 47\right) + 1 + \frac{1}{2}\left(\cos(73+47) + \cos(73-47)\right)$$

Two IMP Identities

$$\left[\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)\right]$$

$$= \cos(73+47).\cos(73-47) + 1 + \frac{1}{2}\left(\cos 120^\circ + \cos 26^\circ\right)$$

$$= \cos 120^\circ \cos 26^\circ + 1 + \frac{\cos 120^\circ}{2} + \frac{\cos 26^\circ}{2}$$

$$LHS = \cancel{-\frac{1}{2}\cos 2\theta} + 1 + \frac{\left(-\frac{1}{2}\right)}{2} + \cancel{\frac{\cos 2\theta}{2}}$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$= RHS \;//$$

Question 10: Prove that

$$\frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} = \sqrt{3}$$

Badlo sin $\rightarrow$

Badlo cos $\rightarrow$

$$LHS = \frac{\sin 70 + \sin 50}{\cos 70 + \cos 50}$$

$$= \frac{\cancel{2} \sin 60 \; \cancel{\cos(10)}}{\cancel{2} \cos(60) \cdot \cancel{\cos(10)}}$$

$$= \tan 60$$

$$= \sqrt{3}$$

Sum ke formulas

$\sin C \pm \sin D$ ✓

$\cos C \pm \cos D$ ✓

$\sin C \pm \cos D$ ✗

Question 11: Prove that

$$\cot 70^\circ + 4\cos 70^\circ = \sqrt{3}$$

cot ko badlenge cos/sin

$$LHS = \frac{\cos 70}{\sin 70} + 4\cos 70$$

$$= \frac{\cos 70 + 4\sin 70\cos 70}{\sin 70}$$

Product → Sum

$$= \frac{\cos 70 + 2(2\sin 70\cos 70)}{\sin 70}$$

$$= \frac{\cos 70 + 2[\sin 140 + \cancel{\sin 0}]}{\sin 70}$$

$*$ $\sin 140^\circ = \sin(180 - 40) = \sin 40$

$$= \frac{\cos 70^\circ + 2\sin 40^\circ}{\sin 70}$$

$$= \frac{\cos 70 + \sin 40 + \sin 40}{\sin 70}$$

$$= \frac{\sin 20 + \sin 40 + \sin 40}{\sin 70}$$

$$= \frac{2\sin(30)\cos(-10^\circ) + \sin 40}{\sin 70}$$

$$= \frac{\cos 10^\circ + \sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{\cos 10^\circ + \cos 50^\circ}{\sin 70^\circ}$$

$$= \frac{2 \cdot \cos(30^\circ)\ \cos(-20^\circ)}{\sin 70^\circ}$$

$$= \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right) \frac{\cancel{\cos 20}}{\cancel{\cos 20}}$$

$$= \sqrt{3}$$

$$= \text{RHS}$$

Question 12: Prove that

$*$ $2\sin A\cdot\sin B = \cos(A-B)-\cos(A+B)$

$$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$$

M-1

$LHS = \frac{1}{2}\sin 10\ \underbrace{\sin 50\ \sin 70}$

$= \frac{1}{2}\cdot\frac{1}{2}\sin 10\left(2\sin 50\sin 70\right)$

$= \frac{1}{4}\sin 10\left[\cos(-20^\circ)-\cos(120^\circ)\right]$

$= \frac{1}{4}\sin 10\left[\cos 20+\frac{1}{2}\right]$

$= \frac{1}{4}\sin 10\ \frac{\left[2\cos 20+1\right]}{2}$

M-2 Using formula

$= \frac{1}{8}\sin 10\left(2\cos 20+1\right)$

$= \frac{1}{8}\left(2\sin 10\cos 20+\sin 10\right)$

$= \frac{1}{8}\left(\sin 30+\cancel{\sin(-10^\circ)}+\cancel{\sin 10}\right)$

$= \frac{1}{8}\left(\frac{1}{2}\right)=\frac{1}{16}$

Question 13: JEE Main 2025 (23 Jan Shift 1)

The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$ is:

(A) $\frac{2}{3}$ (B) 1 (C) 0 (D) $\frac{3}{2}$

cot ko badlenge cos/sin

$$LHS = \sin 70\left(\frac{\cos 10}{\sin 10}\cdot\frac{\cos 70}{\sin 70} - 1\right)$$

$$= \cancel{\sin 70}\left(\frac{\cos 10\cos 70 - \sin 10 . \sin 70}{\sin 10 . \cancel{\sin 70}}\right) \rightarrow \cos(A+B)$$

$$= \frac{\cos(10+70)}{\sin 10}$$

$$= \frac{\cancel{\cos 80}}{\cancel{\sin 10}} = 1 \qquad [\because \cos 80 = \sin 10]$$

Homework

Assignment - 03

S L Loney

Que 1

Que 2 → Sum to Product

Que 3 → Product to Sum

Trigonometric Ratios of Multiple and Submultiple angle

Angle $\longrightarrow \theta$

Multiple angle $\longrightarrow 2\theta, 3\theta, 4\theta, \ldots$

Submultiple angle $\longrightarrow \frac{\theta}{2}, \frac{\theta}{4}, \frac{\theta}{8}, \ldots$

Formula Set:3 "2θ wale Formulas"

① ▶ $\sin(2\theta) = 2\sin\theta\cos\theta$

---

2 1 ▶ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

2 2 ▶ $\cos 2\theta = 2\cos^2\theta - 1 \implies \mathbf{1 + \cos 2\theta = 2\cos^2\theta} \implies \cos^2\theta = \dfrac{\mathbf{1 + \cos 2\theta}}{\mathbf{2}}$

2 3 ▶ $\cos 2\theta = 1 - 2\sin^2\theta \implies \mathbf{1 - \cos 2\theta = 2\sin^2\theta} \implies \sin^2\theta = \dfrac{\mathbf{1 - \cos 2\theta}}{\mathbf{2}}$

---

③ ▶ $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

---

*In terms of tan

▶ $\sin(2\theta) = \dfrac{2\tan\theta}{1 + \tan^2\theta}$ ▶ $\cos(2\theta) = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$ ▶ $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

▶ $1 + \sin(2\theta) = (\sin\theta + \cos\theta)^2$

▶ $1 - \sin(2\theta) = (\sin\theta - \cos\theta)^2$

Proofs:

① $\sin(A+B) = \sin A \cos B + \cos A \cdot \sin B$

put $A = B = \theta$

$\sin(2\theta) = \sin\theta \cos\theta + \cos\theta \sin\theta$

$$\boxed{\sin 2\theta = 2 \sin\theta \cos\theta}$$

② $\cos(A+B) = \cos A \cos B - \sin A \sin B$

put $A = B = \theta$

2.1 $\boxed{\cos(2\theta) = \cos^2\theta - \sin^2\theta}$

$= \cos^2\theta - (1 - \cos^2\theta)$

2.2 $\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$= 2(1 - \sin^2\theta) - 1$

2.3 $\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$

③ $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

put $A = B = \theta$

$$\boxed{\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

In terms of $\tan\theta$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$RHS = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\sec^2\theta}$$

$$= 2\,\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cos^{\cancel{2}}\theta$$

$$= 2\sin\theta\cos\theta$$

$$= \sin 2\theta \,//$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$RHS = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cancel{\cos^2\theta}\ \cancel{\sec^2\theta}}$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \cos 2\theta \,//$$

$$(1+\sin 2\theta) = (\sin\theta + \cos\theta)^2$$

$$LHS = 1+\sin 2\theta$$

$$= \underbrace{\sin^2\theta + \cos^2\theta}_{A^2+B^2} + \underbrace{2\sin\theta\cos\theta}_{2A\cdot B} = (A+B)^2$$

$$= (\sin\theta + \cos\theta)^2$$

$$1-\sin 2\theta = (\sin\theta - \cos\theta)^2$$

$$LHS = 1-\sin 2\theta$$

$$= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$

$$= (\sin\theta - \cos\theta)^2$$

** Use Cases :

① $\sin 2\theta = 2\sin\theta\cos\theta$

$$\boxed{\sin 2\square = 2\sin\square\cos\square}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\sin\frac{\theta}{2} = 2\sin\frac{\theta}{4}\cos\frac{\theta}{4}$$

$$\sin\frac{\theta}{4} = 2\cdot\sin\frac{\theta}{8}\cos\frac{\theta}{8}$$

$$\sin 4\theta = 2\sin 2\theta\cos 2\theta$$

$$\sin\left(\frac{A+B}{2}\right) = 2\sin\left(\frac{A+B}{4}\right)\cdot\cos\left(\frac{A+B}{4}\right)$$

② $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$$

$$* \boxed{\cos\square = \cos^2\frac{\square}{2} - \sin^2\frac{\square}{2}}$$

$$\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$$

2 2 $\cos 2\theta = 2\cos^2\theta - 1$

$$\cos\theta = 2\cos^2\theta/2 - 1$$

$$1+\cos 2\theta = 2\cos^2\theta$$

$$1+\cos\square = 2\cos^2\frac{\square}{2}$$

$$1+\cos\theta = 2\cos^2\theta/2$$

$$1-\cos\theta = 2\sin^2\theta/2$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan\theta/2 = \frac{2\tan\theta/4}{1-\tan^2\theta/4}$$

Question 1

Given that $0 < A < \frac{\pi}{4}$, find the value for each of the following:

1. If $\cos A = \frac{1}{3}$, find $\cos(2A)$.
2. If $\sin A = \frac{2}{5}$, find $\cos(2A)$.
3. If $\sin A = \frac{3}{5}$, find $\sin(2A)$.
4. If $\tan A = \frac{1}{3}$, find $\tan(2A)$.
5. If $\tan A = \frac{1}{7}$, find $\sin(2A)$.
6. If $\cos A = \frac{4}{5}$, find $\tan^2\left(\frac{A}{2}\right)$.

① $\cos A = \frac{1}{3} \quad \cos 2A = ?$

$$\boxed{2\,2 \quad \cos 2\theta = 2\cos^2\theta - 1}$$

$$\cos 2A = 2\cos^2 A - 1$$
$$= 2\left(\frac{1}{3}\right)^2 - 1$$
$$= \frac{2}{9} - 1$$
$$= \frac{-7}{9}$$

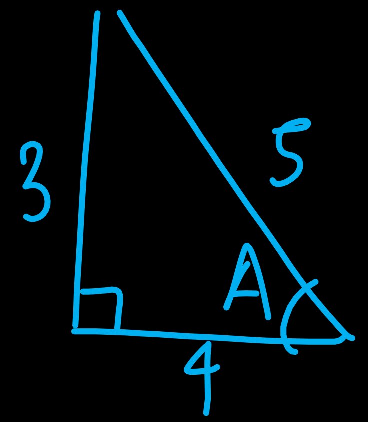
② $\sin A = \frac{2}{5} \quad \cos 2A.$

$2\,3 \quad \cos 2\theta = 1 - 2\sin^2\theta$

$$\cos 2A = 1 - 2\sin^2 A = 1 - 2\left(\frac{2}{5}\right)^2 = \frac{17}{25} //$$

③ $\sin A = \frac{3}{5} \quad \sin 2A = ?$

$$\sin 2A = 2\sin A \cos A$$
$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$
$$= \frac{24}{25} //$$



④ $\tan A = \frac{1}{3} \quad \tan 2A = ?$

$$\rightarrow \tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2\left(\frac{1}{3}\right)}{1 - \frac{1}{9}} = \frac{2/3}{8/9} = \frac{3}{4} //$$

(5) $\tan A = \frac{1}{7}$ $\quad \sin 2A = ?$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$= \frac{2\left(\frac{1}{7}\right)}{1+\frac{1}{49}}$$

$$= \frac{\cancel{\frac{2}{7}}}{\cancel{\frac{50}{49}}}$$

25 7

$$= \frac{7}{25}$$ //

(6) $\cos A = \frac{4}{5}$ $\quad \tan^2 \frac{A}{2}$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\cos A = \frac{1-\tan^2 A/2}{1+\tan^2 A/2}$$ $= t$

$$\frac{4}{5} = \frac{1-t}{1+t}$$

$$4+4t = 5-5t$$

$$9t = 1$$

$$t = \frac{1}{9}$$ $\quad \tan^2 A/2 = \frac{1}{9}$

Question 2: Find the Value

If $\sin\alpha = -\dfrac{3}{5}$ where $\pi < \alpha < \dfrac{3\pi}{2}$, then $\cos\dfrac{\alpha}{2}$ is equal to:

(A) $\dfrac{1}{\sqrt{10}}$

(B) $-\dfrac{1}{\sqrt{10}}$

(C) $\dfrac{3}{\sqrt{10}}$

(D) $-\dfrac{3}{\sqrt{10}}$

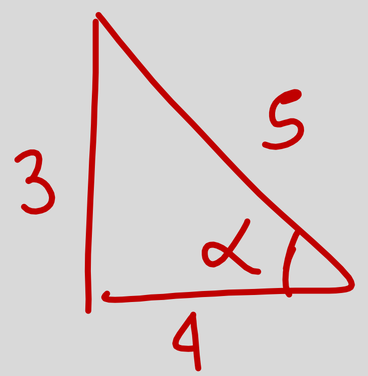
$\alpha \longrightarrow 3^{rd}$ Quad

$$\begin{array}{ccc} \sin(\alpha) & \longrightarrow & \cos(\alpha) \\ \downarrow & \swarrow & \\ \cos\left(\frac{\alpha}{2}\right) & & \end{array}$$

$\sin\alpha = -\frac{3}{5}$

$\downarrow$

$\cos\alpha = -\frac{4}{5}$



① $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$\sqrt{1-t^2}$ $\quad t$

② $\cos 2\theta = 2\cos^2\theta - 1$

$\cos\alpha = 2\cos^2\left(\alpha/2\right) - 1$

$-\frac{4}{5} = 2\cos^2 \alpha/2 - 1$

$$2\cos^2 \alpha/2 = -\frac{4}{5} + 1$$

$$2\cos^2 \alpha/2 = \frac{1}{5}$$

$$\cos^2 \alpha/2 = \frac{1}{10}$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{10}} \qquad \boxed{\cos\left(\frac{\alpha}{2}\right) = \frac{-1}{\sqrt{10}}} \checkmark$$

Reject

$\frac{\alpha}{2}$ lies in 2^{nd} Quad.

Given $\pi < \alpha < \frac{3\pi}{2}$

Divide by 2

$$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4}$$

2^{nd} Quad

Question 3

If $\pi < \theta < \frac{3\pi}{2}$ and $\cos\theta = -\frac{3}{5}$, then $\tan\left(\frac{\theta}{4}\right)$ is equal to:

(A) $\frac{\sqrt{5}-1}{2}$

(B) $\frac{\sqrt{5}+1}{2}$

(C) $-\frac{\sqrt{5}+1}{4}$

(D) $-\frac{\sqrt{5}-1}{4}$

$\cos(\theta)$

step-1

$\tan\left(\frac{\theta}{2}\right)$

Step-2

$\tan\left(\frac{\theta}{4}\right) = ?$

Step-1

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos\theta = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$ let $\tan\theta/2 = t$

$-\frac{3}{5} = \frac{1-t^2}{1+t^2}$

$-3-3t^2 = 5-5t^2$

$2t^2 = 8$

$t^2 = 4$

$t = \pm 2$

$\tan\frac{\theta}{2} = \pm 2$

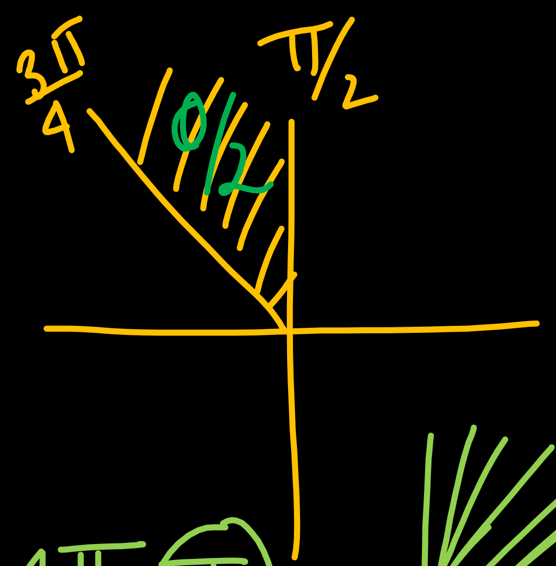
$\tan\left(\frac{\theta}{2}\right) = 2$ → Reject

$\boxed{\tan\left(\frac{\theta}{2}\right) = -2}$ ✓

$\frac{\theta}{2}$ lies in 2nd quad

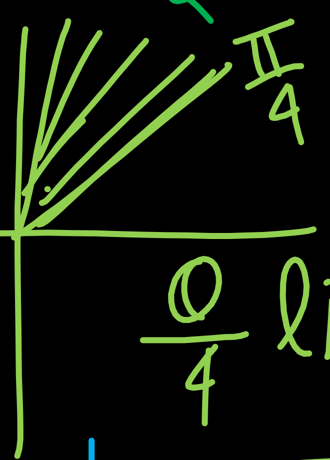
Given $\pi < \theta < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$



$\frac{\theta}{2}$ lies in 2nd quad

$\frac{\pi}{4} < \frac{\theta}{4} < \frac{3\pi}{8} < \frac{4\pi}{8} = \frac{\pi}{2}$



$\frac{\theta}{4}$ lies in 1st quad

Step- 2 $\tan\left(\frac{\theta}{2}\right)$ → $\tan\left(\frac{\theta}{4}\right)$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan\theta/2 = \frac{2\tan\theta/4}{1-\tan^2\theta/4}$

$-\cancel{2} = \frac{\cancel{2}\tan\theta/4}{1-\tan^2\theta/4}$

$\tan^2\theta/4 - 1 = \tan\theta/4$

$t^2 - t - 1 = 0$

$t = \frac{1 \pm \sqrt{1+4}}{2}$

$t = \frac{1 \pm \sqrt{5}}{2}$

✓ $\boxed{\tan\frac{\theta}{4} = \frac{1+\sqrt{5}}{2}}$

× $\boxed{\tan\frac{\theta}{4} = \frac{1-\sqrt{5}}{2}}$

Reject

Question 4

If $\sin\alpha - \cos\alpha = \frac{1}{5}$, then the possible values of $\tan\left(\frac{\alpha}{2}\right)$ are:

(A) -3 or $\frac{1}{2}$

(B) 3 or $-\frac{1}{2}$

(C) -2 or $\frac{1}{3}$

(D) 2 or $-\frac{1}{3}$

$\sin\alpha - \cos\alpha = \frac{1}{5}$

* Using in terms of $\tan\theta$ wale formulas.

$$\left(\frac{2\tan\alpha/2}{1+\tan^2\alpha/2}\right) - \left(\frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}\right) = \frac{1}{5}$$

$$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} = \frac{1}{5}$$

$$\frac{t^2+2t-1}{1+t^2} = \frac{1}{5}$$

$5t^2+10t-5 = t^2+1$

$4t^2+10t-6=0$

$2t^2+5t-3=0$ $\quad -6$

$2t^2+6t-t-3=0$ $\quad +6 \; -1$

$2t(t+3)-1(t+3)=0$

$t=-3 \quad t=\frac{1}{2}$

$\tan\frac{\alpha}{2}=-3 \quad \tan\frac{\alpha}{2}=\frac{1}{2}$

Question 5

If $\tan\theta = t$, prove that:

$$\tan(2\theta) + \sec(2\theta) = \frac{1+t}{1-t}$$

$$LHS = \tan 2\theta + \frac{1}{\cos 2\theta}$$

$$= \frac{2\tan\theta}{1-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}$$

$$= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$$

$$= \frac{t^2+2t+1}{1-t^2}$$

$$= \frac{(t+1)^2}{1^2-t^2}$$

$$= \frac{(t+1)\cancel{(t+1)}}{(1-t)\cancel{(1+t)}}$$

$$= \frac{1+t}{1-t}$$

$$= RHS$$

Question 6

$2\sin\square\cos\square = \sin 2\square$

Prove that:

$$8\sin\frac{x}{8}\cos\frac{x}{8}\cos\frac{x}{4}\cos\frac{x}{2} = \sin x$$

$\frac{x}{8}\times 2 = \frac{x}{4}$

$LHS = 8\underbrace{\sin\frac{x}{8}}_{\sin\square}\underbrace{\cos\frac{x}{8}}_{\cos\square}\cos\frac{x}{4}\cos\frac{x}{2}$

$\sin\frac{x}{4} = 2\sin\frac{x}{8}\cos\frac{x}{8}$

$= 4\left(2\sin\frac{x}{8}\cos\frac{x}{8}\right)\cdot\cos\frac{x}{4}\cos\frac{x}{2}$

$\sin\frac{x}{2} = 2\sin\frac{x}{4}\cos\frac{x}{4}$

$= 4\underbrace{\sin\frac{x}{4}}_{\sin\square}\cdot\underbrace{\cos\frac{x}{4}}_{\cos\square}\cdot\cos\frac{x}{2}$

$\frac{x}{4}\times 2 = \frac{x}{2}$

$= 2\left(2\sin\frac{x}{4}\cdot\cos\frac{x}{4}\right)\cdot\cos\frac{x}{2}$

$= 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}$

$\frac{x}{2}\times 2 = x$

$= \sin x$

$= RHS$ //

Question 7

Prove that:

$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta} \rightarrow \frac{\frac{\sin 8\theta}{\cos 8\theta}}{\frac{\sin 2\theta}{\cos 2\theta}} = \frac{\sin 8\theta}{\cos 8\theta} \cdot \frac{\cos 2\theta}{\sin 2\theta}$$

$$\text{LHS} = \frac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{\cos 4\theta} - 1}$$

$$= \left(\frac{1 - \cos 8\theta}{\cos 8\theta}\right)\left(\frac{\cos 4\theta}{1 - \cos 4\theta}\right)$$

$$= \left(\frac{2\sin^2 4\theta}{\cos 8\theta}\right)\left(\frac{\cos 4\theta}{2\sin^2 2\theta}\right)$$

$$= \frac{(2\sin 4\theta\cos 4\theta)\,\sin 4\theta}{\cos 8\theta \cdot (2\sin^2 2\theta)}$$

$$= \frac{\sin 8\theta}{\cos 8\theta} \cdot \frac{(\cancel{2}\,\cancel{\sin 2\theta}\cos 2\theta)}{\cancel{2}\,\sin^{\cancel{2}} 2\theta}$$

$$= \tan 8\theta \left(\frac{\cos 2\theta}{\sin 2\theta}\right)$$

$$= \frac{\tan 8\theta}{\tan 2\theta} = \text{RHS}$$

$1 + \cos 2\theta = 2\cos^2\theta$

$1 - \cos 2\theta = 2\sin^2\theta$

$* \; 1 - \cos 8\theta = 2\sin^2 4\theta$

$* \; 1 - \cos 4\theta = 2\sin^2 2\theta$

$* \; \sin 8\theta = 2\sin 4\theta\cos 4\theta$

Question 8

Find the exact value of the expression:

$$\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ$$

$$LHS = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= \frac{\sqrt{3}\cos 20 - \sin 20^{*}}{\sin 20^\circ \cdot \cos 20^\circ}$$

$\rightarrow 2\sin\square \cdot \cos\square = \sin 2\square$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20 - \frac{1}{2}\sin 20\right)}{\frac{1}{2}\left(2\sin 20 \cdot \cos 20\right)}$$

$*$ $\sqrt{3}\sin\theta \pm \cos\theta$, $\sqrt{3}\cos\theta \pm \sin\theta$ } M & D by 2

Before Two IMP Identities

$$LHS = 4\,\frac{\left(\overset{\sin 60}{\frac{\sqrt{3}}{2}}\cos 20 - \overset{\cos 60}{\frac{1}{2}}\sin 20\right)}{2\sin 20\cos 20} \longrightarrow \sin A\cdot\cos A - \cos A\sin A = \sin(A-B)$$

$$= 4\,\frac{\sin(60-20)}{\sin 40}$$

$$= 4\cdot\frac{\cancel{\sin 40}}{\cancel{\sin 40}}$$

$$= 4$$

IMP

Question 9: Prove the Identity

1. $\dfrac{1+\cos 2A}{\sin 2A} = \cot A$

2. $\dfrac{1-\cos 2A}{\sin 2A} = \tan A$

3. $\dfrac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$

4. $\dfrac{1-\cos\theta}{\sin\theta} = \tan\left(\dfrac{\theta}{2}\right)$

5. $\dfrac{\sin\theta}{1+\cos\theta} = \tan\left(\dfrac{\theta}{2}\right)$

IMP

*6. $\dfrac{1-\sin(\theta)}{\cos(\theta)} = \tan\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$

*7. $\dfrac{1+\sin(\theta)}{\cos(\theta)} = \tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)$

① $LHS = \dfrac{\cancel{2}\cos^{\cancel{2}} A}{\cancel{2}\cdot\sin A\cdot\cancel{\cos A}} = \dfrac{\cos A}{\sin A} = \cot A$

② $LHS = \dfrac{\cancel{2}\sin^{\cancel{2}} A}{\cancel{2}\,\cancel{\sin A}\cos A} = \dfrac{\sin A}{\cos A} = \tan A$

③ $LHS = \dfrac{2\sin^2 A}{2\cos^2 A} = \tan^2 A$

④ $LHS = \dfrac{\cancel{2}\sin^{\cancel{2}}\theta/2}{\cancel{2}\,\cancel{\sin\theta/2}\cos\theta/2} = \dfrac{\sin\theta/2}{\cos\theta/2} = \tan\theta/2$

⑤ $LHS = \dfrac{\cancel{2}\sin\theta/2\cdot\cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2} = \dfrac{\sin\theta/2}{\cos\theta/2} = \tan\theta/2$

⑥ $\dfrac{1-\sin\theta}{\cos\theta} = \tan\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$

$$LHS = \frac{\sin^2\theta/2 + \cos^2\theta/2 - 2\sin\theta/2\cos\theta/2}{\cos^2\theta/2 - \sin^2\theta/2}$$

$$= \frac{\left(\cos\theta/2 - \sin\theta/2\right)^{2}}{\cancel{\left(\cos\theta/2 - \sin\theta/2\right)}\left(\cos\theta/2 + \sin\theta/2\right)}$$

$$= \frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$$

Divide by "$\cos\theta/2$"

$$= \frac{\dfrac{\cos\theta/2}{\cos\theta/2} - \dfrac{\sin\theta/2}{\cos\theta/2}}{\dfrac{\cos\theta/2}{\cos\theta/2} + \dfrac{\sin\theta/2}{\cos\theta/2}}$$

$$= \frac{1-\tan\theta/2}{1+\tan\theta/2} \longrightarrow \text{Remark-1}$$

$$= \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$= RHS$ //

Question 10: Prove the Identity

Prove that:

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$$

Question 11: JEE Main 2021 (26 Aug Shift 2)

The value of the expression:

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

(A) $\frac{1}{4\sqrt{2}}$ (B) $\frac{1}{8}$ (C) $\frac{1}{8\sqrt{2}}$ (D) $\frac{1}{4}$