Trigonometry Formula Sheet

Prof. Chetan Sir

Right-Angled Triangle Ratios

$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$ $\text{cosec}\,\theta = \frac{\text{Hyp}}{\text{Opp}}$ $\sec\theta = \frac{\text{Hyp}}{\text{Adj}}$ $\cot\theta = \frac{\text{Adj}}{\text{Opp}}$

Fundamental Identities

$\sin\theta = \frac{1}{\text{cosec}\,\theta}$, $\cos\theta = \frac{1}{\sec\theta}$, $\tan\theta = \frac{1}{\cot\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \text{cosec}^2\,\theta$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$, $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Allied & Negative Angle Formulas

$\sin(90^\circ - \theta) = \cos\theta$ $\sin(180^\circ - \theta) = \sin\theta$ $\sin(270^\circ - \theta) = -\cos\theta$ $\sin(360^\circ - \theta) = -\sin\theta$
$\cos(90^\circ - \theta) = \sin\theta$ $\cos(180^\circ - \theta) = -\cos\theta$ $\cos(270^\circ - \theta) = -\sin\theta$ $\cos(360^\circ - \theta) = \cos\theta$
$\tan(90^\circ - \theta) = \cot\theta$ $\tan(180^\circ - \theta) = -\tan\theta$ $\tan(270^\circ - \theta) = \cot\theta$ $\tan(360^\circ - \theta) = -\tan\theta$

$\sin(90^\circ + \theta) = \cos\theta$ $\sin(180^\circ + \theta) = -\sin\theta$ $\sin(270^\circ + \theta) = -\cos\theta$ $\sin(-\theta) = -\sin\theta$
$\cos(90^\circ + \theta) = -\sin\theta$ $\cos(180^\circ + \theta) = -\cos\theta$ $\cos(270^\circ + \theta) = \sin\theta$ $\cos(-\theta) = \cos\theta$
$\tan(90^\circ + \theta) = -\cot\theta$ $\tan(180^\circ + \theta) = \tan\theta$ $\tan(270^\circ + \theta) = -\cot\theta$ $\tan(-\theta) = -\tan\theta$

Complementary & Supplementary Angle Concepts

Complementary Angles ($A + B = 90^\circ$)

- $\sin A = \cos B$
- $\cos A = \sin B$
- $\tan A = \cot B$ $\tan A \tan B = 1$

Supplementary Angles ($A + B = 180^\circ$)

- $\sin A = \sin B$
- $\cos A + \cos B = 0$
- $\tan A + \tan B = 0$

Formula Set 1: Compound Angle Formulas

$\sin(A + B) = \sin A\cos B + \cos A\sin B$ $\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$ $\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(45^\circ + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$ $\tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$

$\cot(A + B) = \frac{\cot A\cot B - 1}{\cot B + \cot A}$ $\cot(A - B) = \frac{\cot A\cot B + 1}{\cot B - \cot A}$

Two Very Important Identities

1. $$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

2. $$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

Formula Set 2: Transformation Formulas

Product to Sum

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

Sum to Product

$\sin C+\sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C-\sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\cos C+\cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\cos C-\cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Formula Set 3: Multiple Angle Formulas (2θ)

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1 \implies \mathbf{1+\cos(2\theta) = 2\cos^2\theta} \implies \cos^{\mathbf{2}}\theta = \frac{\mathbf{1+\cos(2\theta)}}{\mathbf{2}}$

$\cos(2\theta) = 1 - 2\sin^2\theta \implies \mathbf{1-\cos(2\theta) = 2\sin^2\theta} \implies \sin^{\mathbf{2}}\theta = \frac{\mathbf{1-\cos(2\theta)}}{\mathbf{2}}$

$\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$ $\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$1+\sin(2\theta) = (\sin\theta+\cos\theta)^2$

$1-\sin(2\theta) = (\sin\theta-\cos\theta)^2$

Formula Set 4: Triple Angle Formulas (3θ)

1. $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$
2. $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$
3. $\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$
4. $\tan(A+B+C) = \frac{\sum\tan A - \prod\tan A}{1-\sum\tan A\tan B}$

Three Important Results

1. $\sin\theta\cdot\sin(60^\circ-\theta)\cdot\sin(60^\circ+\theta) = \frac{1}{4}\sin(3\theta)$
2. $\cos\theta\cdot\cos(60^\circ-\theta)\cdot\cos(60^\circ+\theta) = \frac{1}{4}\cos(3\theta)$
3. $\tan\theta\cdot\tan(60^\circ-\theta)\cdot\tan(60^\circ+\theta) = \tan(3\theta)$

Important Values

$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\tan 15^\circ = \cot 75^\circ = 2-\sqrt{3}$

$\tan 75^\circ = \cot 15^\circ = 2+\sqrt{3}$

$\tan 22.5^\circ = \cot 67.5^\circ = \sqrt{2}-1$

$\tan 67.5^\circ = \cot 22.5^\circ = \sqrt{2}+1$

$\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$

$\cos 36^\circ = \sin 54^\circ = \frac{\sqrt{5}+1}{4}$