Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

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**Exercise: MHT CET PYQ**

# Topic: Trigonometric Equation

Sub: Mathematics **MHT CET PYQ: Solution** Prof. Chetan Sir

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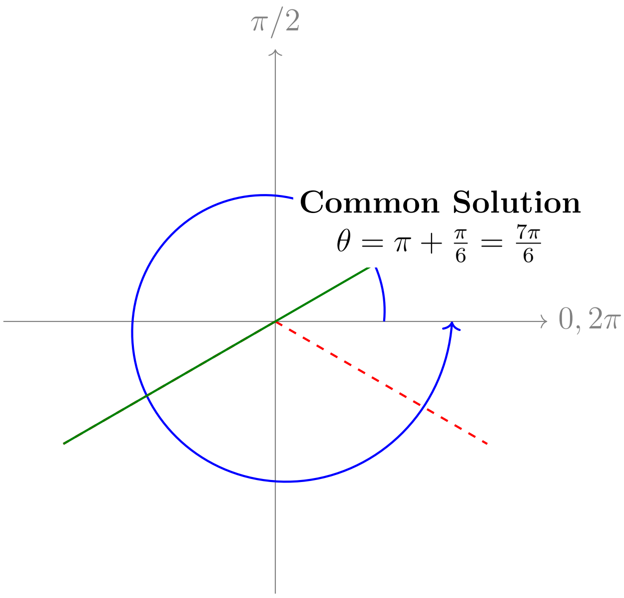
1. The common principal solution of the equations $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ is:

   (A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{7\pi}{6}$ (D) $\frac{11\pi}{6}$

   **Solution:** We need to find the common solution for $\sin\theta = -1/2$ and $\tan\theta = 1/\sqrt{3}$ in the interval $[0, 2\pi)$.

   - For $\sin\theta = -1/2$, sine is negative, so solutions are in Q3 and Q4.
   - For $\tan\theta = 1/\sqrt{3}$, tangent is positive, so solutions are in Q1 and Q3.

   The common quadrant for the solution is Quadrant III. In Q3, the angle is $\pi + \alpha$, where $\alpha$ is the reference angle. For $\sin\theta = 1/2$ and $\tan\theta = 1/\sqrt{3}$, the reference angle is $\alpha = \pi/6$. So, the common solution is $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.




   The correct option is **(C)**.

2. The principal solution of $(5 + 3\sin\theta)(2\cos\theta + 1) = 0$ are:

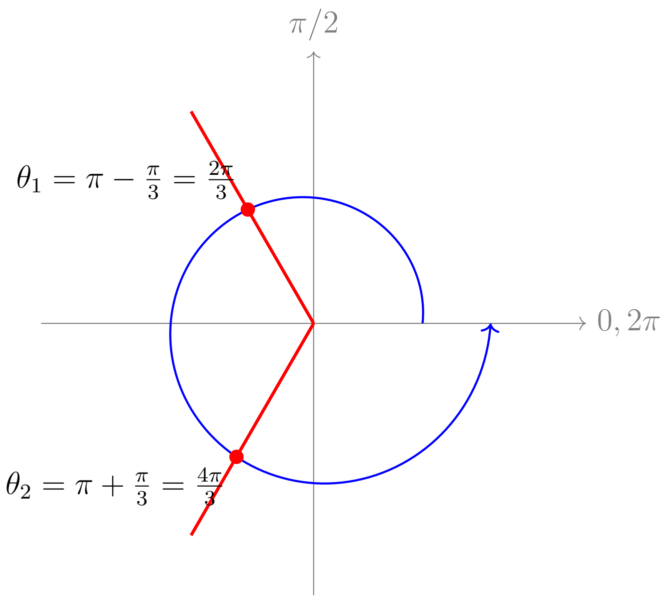
(A) $\frac{-\pi}{3}, \frac{2\pi}{3}$ (B) $\frac{2\pi}{3}, \frac{5\pi}{3}$ (C) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (D) $\frac{2pi}{3}, \frac{7\pi}{3}$

**Solution:** The equation $(5+3\sin\theta)(2\cos\theta+1) = 0$ implies either $5+3\sin\theta = 0$ or $2\cos\theta+1 = 0$.

- **Case 1:** $5 + 3\sin\theta = 0 \implies \sin\theta = -5/3$. This is not possible as $-1 \le \sin\theta \le 1$.
- **Case 2:** $2\cos\theta + 1 = 0 \implies \cos\theta = -1/2$. Cosine is negative in Q2 and Q3. The reference angle is $\pi/3$.

The principal solutions in $[0, 2\pi)$ are:

- Q2 solution: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.
- Q3 solution: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.




The correct option is **(C)**.

3. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is:

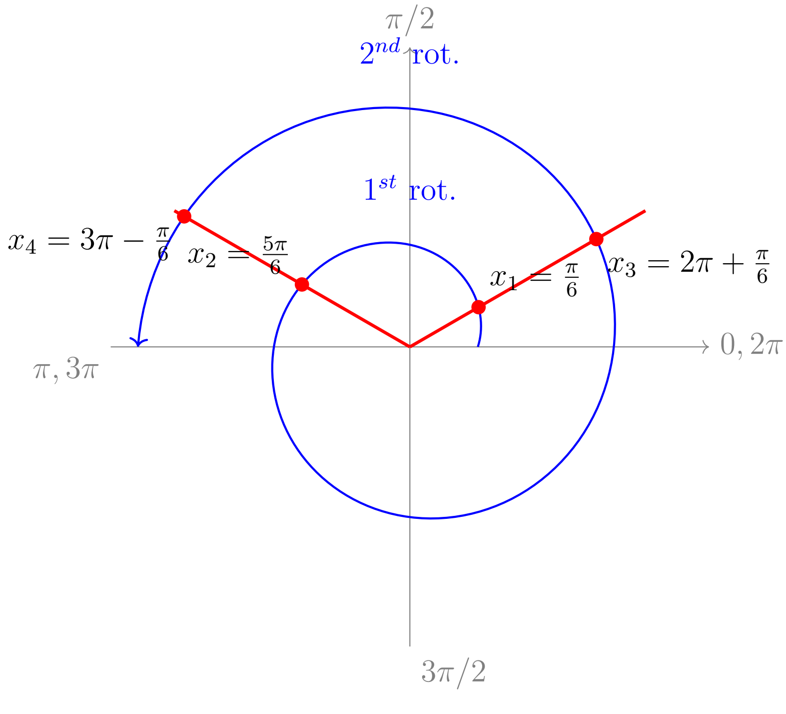
(A) 6 (B) 1 (C) 2 (D) 4

**Solution:** First, we solve the quadratic equation for $\sin x$. Let $y = \sin x$.

$$\begin{aligned} & 2y^2 + 5y - 3 = 0 \\ \implies & 2y^2 + 6y - y - 3 = 0 \\ \implies & 2y(y+3) - 1(y+3) = 0 \\ \implies & (2y-1)(y+3) = 0 \end{aligned}$$

This gives $\sin x = 1/2$ or $\sin x = -3$. Since $-1 \le \sin x \le 1$, the only valid solution is $\sin x = 1/2$. We need to find the number of solutions for $\sin x = 1/2$ in the interval $[0, 3\pi]$.

Solutions in $[0, 3\pi]$ are $x = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}$.
So there are 4 solutions.




The correct option is **(D)**.

4. The number of solutions of $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ in $0 \le x \le 2\pi$ are:

(A) 8 (B) 10 (C) 6 (D) 4

**Solution:**

$$16^{\sin^2 x} + 16^{1-\sin^2 x} = 10$$

Let $y = 16^{\sin^2 x}$. The equation becomes:

$$y + \frac{16}{y} = 10$$
$$\implies y^2 + 16 = 10y$$
$$\implies y^2 - 10y + 16 = 0$$
$$\implies (y-2)(y-8) = 0$$
$$\implies y = 2 \text{ or } y = 8.$$

**Case 1:** $16^{\sin^2 x} = 2$

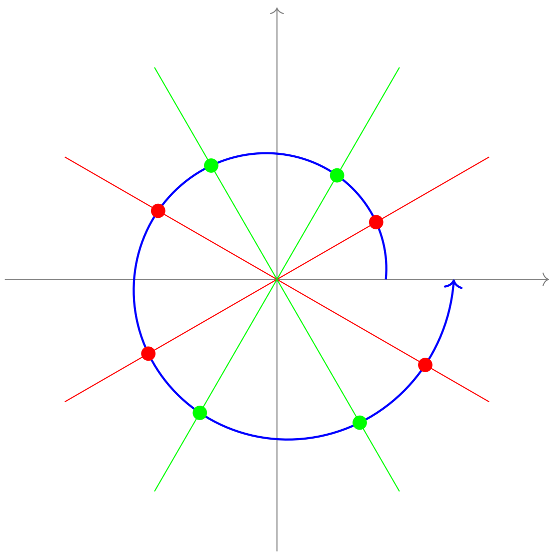
$$(2^4)^{\sin^2 x} = 2^1$$
$$4\sin^2 x = 1 \implies \sin^2 x = \frac{1}{4} \implies \sin x = \pm\frac{1}{2}.$$

This gives 4 solutions in $[0, 2\pi]$: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

**Case 2:** $16^{\sin^2 x} = 8$

$$(2^4)^{\sin^2 x} = 2^3$$
$$4\sin^2 x = 3 \implies \sin^2 x = \frac{3}{4} \implies \sin x = \pm\frac{\sqrt{3}}{2}.$$

This gives 4 solutions in $[0, 2\pi]$: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$. In total, there are $4 + 4 = 8$ distinct solutions.



The correct option is **(A)**.

5. If $\sin(\frac{\pi}{4}\cot\theta) = \cos(\frac{\pi}{4}\tan\theta)$, then the general solution of $\theta$ is:

(A) $n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$ (B) $n\pi + (-1)^n\frac{\pi}{6}, n \in \mathbb{Z}$
(C) $2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z}$ (D) $2n\pi \pm 3\frac{\pi}{4}, n \in \mathbb{Z}$

**Solution:**

$$\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$$

We can write $\cos(A) = \sin(\frac{\pi}{2} - A)$.

$$\implies \sin\left(\frac{\pi}{4}\cot\theta\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\tan\theta\right)$$

$$\implies \frac{\pi}{4}\cot\theta + \frac{\pi}{4}\tan\theta = \frac{\pi}{2} + 2n\pi$$

Dividing by $\pi/4$ and considering the simplest case (n=0):

$$\cot\theta + \tan\theta = 2$$

$$\implies \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2$$

$$\implies \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = 2$$

$$\implies \frac{1}{\sin\theta\cos\theta} = 2 \implies 1 = 2\sin\theta\cos\theta \implies \sin(2\theta) = 1.$$

Alternatively, from $\tan\theta + \frac{1}{\tan\theta} = 2:$

$$\tan^2\theta - 2\tan\theta + 1 = 0$$

$$\implies (\tan\theta - 1)^2 = 0 \implies \tan\theta = 1.$$

The general solution for $\tan\theta = 1 = \tan(\frac{\pi}{4})$ is:

$$\theta = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}.$$

The correct option is **(A)**.

6. The possible values of $\theta \in (0, \pi)$ such that $\sin\theta + \sin(4\theta) + \sin(7\theta) = 0$ are:

(A) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (B) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(C) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{10}$ (D) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

**Solution:**

$$(\sin(7\theta) + \sin\theta) + \sin(4\theta) = 0$$
$$\text{Using } \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}:$$
$$2\sin(\frac{8\theta}{2})\cos(\frac{6\theta}{2}) + \sin(4\theta) = 0$$
$$2\sin(4\theta)\cos(3\theta) + \sin(4\theta) = 0$$
$$\sin(4\theta)(2\cos(3\theta) + 1) = 0$$

This gives two cases:

**Case 1:**

$$\sin(4\theta) = 0.$$
$$\text{Since } \theta \in (0, \pi), \text{ we have } 4\theta \in (0, 4\pi).$$
$$\text{Solutions for } \sin(4\theta) = 0 \text{ are } 4\theta = \pi, 2\pi, 3\pi.$$
$$\implies \theta = \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}.$$
$$\implies \theta \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}.$$

**Case 2:**

$$2\cos(3\theta) + 1 = 0 \implies \cos(3\theta) = -\frac{1}{2}.$$
$$\text{Since } \theta \in (0, \pi), \text{ we have } 3\theta \in (0, 3\pi).$$
$$\text{Solutions for } \cos(3\theta) = -1/2 \text{ are } 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi + \frac{2\pi}{3}.$$
$$\implies 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}.$$
$$\implies \theta \in \left\{\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}\right\}.$$

Combining the solutions from both cases, the set of all values is:
$\left\{\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}\right\}$.

The correct option is **(D)**.

7. If $\tan 3\theta = \cot\theta$, then $\theta =$

(A) $\frac{(2n+1)\pi}{8}, n \in \mathbb{Z}$ (B) $\frac{(2n+1)\pi}{4}, n \in \mathbb{Z}$
(C) $\frac{(n+2)\pi}{3}, n \in \mathbb{Z}$ (D) $n\pi, n \in \mathbb{Z}$

**Solution:**

$$\tan(3\theta) = \cot\theta$$
$$\text{Using } \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right):$$
$$\tan(3\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$$
$$\text{The general solution for } \tan A = \tan B \text{ is } A = n\pi + B.$$
$$\implies 3\theta = n\pi + \left(\frac{\pi}{2} - \theta\right), \quad n \in \mathbb{Z}$$
$$\implies 4\theta = n\pi + \frac{\pi}{2}$$
$$\implies 4\theta = \frac{2n\pi + \pi}{2} = \frac{(2n+1)\pi}{2}$$
$$\implies \theta = \frac{(2n+1)\pi}{8}, \quad n \in \mathbb{Z}.$$

The correct option is **(A)**.

8. The general solutions of the equation $\tan^2\theta + \sec 2\theta = 1$ are:

(A) $n\pi, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$ (B) $n\pi, n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z}$
(C) $\frac{n\pi}{4}, \frac{n\pi}{4} \pm \frac{\pi}{3}, n \in \mathbb{Z}$ (D) $n\pi, n \in \mathbb{Z}$

**Solution:**

$$\begin{aligned}
&\tan^2\theta + \sec 2\theta = 1\\
&\text{Using } \sec 2\theta = \frac{1+\tan^2\theta}{1-\tan^2\theta}:\\
&\tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1\\
&\text{Multiply by } (1-\tan^2\theta), \text{ assuming } \tan^2\theta \neq 1:\\
&\tan^2\theta(1-\tan^2\theta) + (1+\tan^2\theta) = (1-\tan^2\theta)\\
&\tan^2\theta - \tan^4\theta + 1 + \tan^2\theta = 1 - \tan^2\theta\\
&-\tan^4\theta + 3\tan^2\theta = 0\\
&\tan^2\theta(3-\tan^2\theta) = 0
\end{aligned}$$

This gives two cases:

**Case 1:**

$$\begin{aligned}
&\tan^2\theta = 0\\
\implies &\tan\theta = 0\\
\implies &\theta = n\pi, \quad n \in \mathbb{Z}.
\end{aligned}$$

**Case 2:**

$$\begin{aligned}
&3 - \tan^2\theta = 0\\
\implies &\tan^2\theta = 3\\
\implies &\tan\theta = \pm\sqrt{3}.
\end{aligned}$$

$$\text{The general solution is } \theta = n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

The complete set of solutions is $n\pi$ and $n\pi \pm \frac{\pi}{3}$.
The correct option is **(A)**.

9. If $0 \leq x \leq \pi$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ Then x takes the value:

(A) $\frac{\pi}{6}, \frac{\pi}{3}$ (B) $\frac{\pi}{3}, \frac{\pi}{4}$ (C) $\frac{5\pi}{6}, \frac{\pi}{2}$ (D) $\frac{2\pi}{3}, \frac{\pi}{4}$

**Solution:**

$$
\begin{aligned}
& 81^{\sin^2 x} + 81^{1-\sin^2 x} = 30 \\
& \text{Let } y = 81^{\sin^2 x}\text{. The equation becomes:} \\
& y + \frac{81}{y} = 30 \\
\implies & y^2 - 30y + 81 = 0 \\
\implies & (y-3)(y-27) = 0 \\
\implies & y = 3 \text{ or } y = 27.
\end{aligned}
$$

**Case 1:** $81^{\sin^2 x} = 3$

$$
\begin{aligned}
& (3^4)^{\sin^2 x} = 3^1 \\
& 4\sin^2 x = 1 \implies \sin^2 x = \frac{1}{4} \implies \sin x = \frac{1}{2} \quad (\text{since } x \in [0, \pi], \sin x \geq 0).
\end{aligned}
$$

Solutions in $[0, \pi]$ are $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

**Case 2:** $81^{\sin^2 x} = 27$

$$
\begin{aligned}
& (3^4)^{\sin^2 x} = 3^3 \\
& 4\sin^2 x = 3 \implies \sin^2 x = \frac{3}{4} \implies \sin x = \frac{\sqrt{3}}{2} \quad (\text{since } x \in [0, \pi]).
\end{aligned}
$$

Solutions in $[0, \pi]$ are $x = \frac{\pi}{3}, \frac{2\pi}{3}$.

The set of all possible values for x is $\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}\}$.
Option (A) lists two of these valid solutions.
The correct option is **(A)**.

10. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\sin(\frac{\pi}{4}+\theta) =$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2\sqrt{2}}$

**Solution:**

$$\begin{aligned}
& \tan(\pi\cos\theta) = \cot(\pi\sin\theta) \\
& \text{Using the identity } \cot(A) = \tan\left(\frac{\pi}{2} - A\right): \\
\implies & \tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right) \\
& \text{For } \tan(A) = \tan(B), \text{ the simplest solution is } A = B. \\
\\
& \text{Considering this principal case: when n=0} \\
\implies & (\pi\cos\theta) + (\pi\sin\theta) = \frac{\pi}{2} \\
& \text{Dividing by } \pi: \\
\implies & \cos\theta + \sin\theta = \frac{1}{2} \\
& \text{Multiply by } \frac{1}{\sqrt{2}} \text{ to convert to } R\sin(\theta+\alpha) \text{ form:} \\
\implies & \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{2\sqrt{2}} \\
\implies & \cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta = \frac{1}{2\sqrt{2}} \\
\implies & \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}.
\end{aligned}$$

From the options, $\frac{1}{2\sqrt{2}}$ is a possible value. The correct option is **(D)**.

11. If $1 - \cos\theta = \sin\theta \cdot \sin\frac{\theta}{2}$, then the value of $\theta$ is:

(A) $2n\pi, 4n\pi, n \in \mathbb{Z}$ (B) $\frac{n\pi}{2}, \frac{n\pi}{3}, n \in \mathbb{Z}$
(C) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (D) $(2n-1)\frac{\pi}{4}, n \in \mathbb{Z}$

**Solution:**

$$1 - \cos\theta = \sin\theta \cdot \sin\frac{\theta}{2}$$

Using half-angle identities: $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$ and $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$.

$$\implies 2\sin^2\frac{\theta}{2} = \left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\sin\frac{\theta}{2}$$

$$\implies 2\sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\implies 2\sin^2\frac{\theta}{2} - 2\sin^2\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$\implies 2\sin^2\frac{\theta}{2}(1 - \cos\frac{\theta}{2}) = 0$$

This gives two cases:

**Case 1:** $\sin^2\frac{\theta}{2} = 0$

$$\implies \sin\frac{\theta}{2} = 0$$
$$\implies \frac{\theta}{2} = n\pi$$
$$\implies \theta = 2n\pi, \quad n \in \mathbb{Z}.$$

**Case 2:** $1 - \cos\frac{\theta}{2} = 0$

$$\implies \cos\frac{\theta}{2} = 1$$
$$\implies \frac{\theta}{2} = 2n\pi$$
$$\implies \theta = 4n\pi, \quad n \in \mathbb{Z}.$$

The correct option is **(A)**.

12. The number of values of x in interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is:

(A) 0 (B) 5 (C) 4 (D) 6

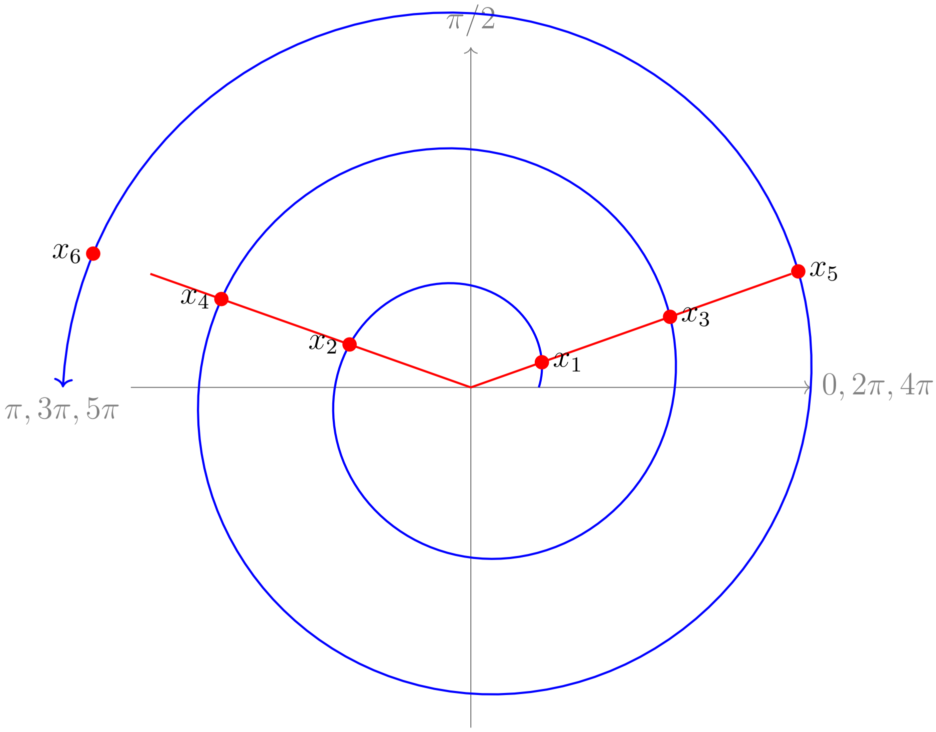
**Solution:**

$$\begin{aligned} & 3\sin^2 x - 7\sin x + 2 = 0 \\ \implies & 3\sin^2 x - 6\sin x - \sin x + 2 = 0 \\ \implies & 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0 \\ \implies & (3\sin x - 1)(\sin x - 2) = 0 \end{aligned}$$

This gives $\sin x = 2$ (which is impossible) or $\sin x = 1/3$.
We need to count the number of solutions for $\sin x = 1/3$ in the interval $[0, 5\pi]$.

- In $[0, 2\pi]$, there are 2 solutions.
- In $[2\pi, 4\pi]$, there are 2 solutions.
- In $[4\pi, 5\pi]$, which covers Q1 and Q2 of the next cycle, there are 2 solutions.

Total number of solutions is $2 + 2 + 2 = 6$.




The correct option is **(D)**.

13. The general solution of $\sin x + \cos x = 1$ is:

(A) $x = 2n\pi, n \in \mathbb{Z}$ (B) $x = 2n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$
(C) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in \mathbb{Z}$ (D) not existing

**Solution:**

$$\begin{aligned}
&\sin x + \cos x = 1 \\
&\text{Divide by } \sqrt{1^2+1^2} = \sqrt{2}: \\
&\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \\
&\text{Using } \cos(A-B) = \cos A \cos B + \sin A \sin B: \\
&\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \\
\implies &\cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4} \\
&\text{The general solution is } x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}.
\end{aligned}$$

**Case 1 (+ sign):** $x - \frac{\pi}{4} = 2n\pi + \frac{\pi}{4}$
$\implies x = 2n\pi + \frac{\pi}{2}.$

**Case 2 (- sign):** $x - \frac{\pi}{4} = 2n\pi - \frac{\pi}{4}$
$\implies x = 2n\pi.$

Now we check if option (C), $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, represents these solutions.

If n is even (let n=2k ):

$$\begin{aligned}
x &= 2k\pi + (-1)^{2k} \frac{\pi}{4} - \frac{\pi}{4} \\
&= 2k\pi + \frac{\pi}{4} - \frac{\pi}{4} \\
&= 2k\pi. \quad \text{(Matches Case 2)}
\end{aligned}$$

If n is odd (let n=2k+1 ):

$$\begin{aligned}
x &= (2k+1)\pi + (-1)^{2k+1} \frac{\pi}{4} - \frac{\pi}{4} \\
&= (2k+1)\pi - \frac{\pi}{4} - \frac{\pi}{4} \\
&= 2k\pi + \pi - \frac{\pi}{2} \\
&= 2k\pi + \frac{\pi}{2}. \quad \text{(Matches Case 1)}
\end{aligned}$$

Therefore, option (C) correctly represents both sets of solutions.
The correct option is **(C)**.

14. If for certain x, $3\cos x \neq 2\sin x$, then the general solution of, $\sin^2 x - \cos 2x = 2 - \sin 2x$ is:

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (B) $(2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$
(C) $n\pi + (-1)^n\frac{\pi}{3}, n \in \mathbb{Z}$ (D) $\frac{n\pi}{2} + 1, n \in \mathbb{Z}$

**Solution:**

Given the equation:
$\sin^2 x - \cos 2x = 2 - \sin 2x$

Convert all terms to forms of $\sin x$ and $\cos x$ :
$(1 - \cos^2 x) - (2\cos^2 x - 1) = 2 - 2\sin x \cos x$

Simplify the expression:
$1 - \cos^2 x - 2\cos^2 x + 1 = 2 - 2\sin x \cos x$
$2 - 3\cos^2 x = 2 - 2\sin x \cos x$
$-3\cos^2 x = -2\sin x \cos x$
$2\sin x \cos x - 3\cos^2 x = 0$

Factor out $\cos x$ :
$\cos x(2\sin x - 3\cos x) = 0$

This gives two possible cases for the solution:
**Case 1:** $\cos x = 0$
$$\implies x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}.$$

**Case 2:** $2\sin x - 3\cos x = 0$
$\implies 2\sin x = 3\cos x.$

The problem includes the constraint that $3\cos x \neq 2\sin x$.
This means we must exclude the solutions from Case 2.

Therefore, the only valid general solution is from Case 1.
$$x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}.$$

The correct option is **(A)**.

15. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval:

(A) $(\frac{\pi}{6}, \frac{5\pi}{6})$ (B) $(-1, \frac{5\pi}{6})$ (C) $(-1, 2)$ (D) $(\frac{\pi}{6}, 2)$

**Solution:**

We solve the two inequalities separately.

**Inequality 1:** $2\sin^2 x + 3\sin x - 2 > 0$

$(2\sin x - 1)(\sin x + 2) > 0.$

Since $-1 \le \sin x \le 1,$ the term $(\sin x + 2)$ is always positive.

Therefore, for the product to be positive, we must have:

$2\sin x - 1 > 0$

$\implies \sin x > \dfrac{1}{2}$

$\implies x \in \left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$ (in the principal cycle).

**Inequality 2:** $x^2 - x - 2 < 0$

$(x-2)(x+1) < 0.$

The expression is negative between its roots, $x = -1$ and $x = 2$.

$\implies x \in (-1, 2).$

**Intersection:**

We need the intersection of the two solution intervals: $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$ and $(-1, 2)$.

Using the approximations $\dfrac{\pi}{6} \approx 0.524$ and $\dfrac{5\pi}{6} \approx 2.618$,

we find the intersection of $(\approx 0.524, \approx 2.618)$ and $(-1, 2)$.

The common interval is $\left(\dfrac{\pi}{6}, 2\right)$.

The correct option is **(D)**.

16. The general solution of the equation $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is:

(A) $n\pi + (-1)^n \frac{\pi}{2} + \frac{\pi}{6}$ (B) $n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{6}$
(C) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}$ (D) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{3}$

**Solution:**

$$\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$$
$$\text{Divide by } \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2 :$$
$$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$
$$\text{Using } \sin(A+B) = \sin A\cos B + \cos A\sin B :$$
$$\sin\frac{\pi}{3}\cos\theta + \cos\frac{\pi}{3}\sin\theta = \frac{1}{\sqrt{2}}.$$
$$\implies \sin\left(\theta + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}.$$
$$\text{The general solution for } \sin A = \sin B \text{ is } A = n\pi + (-1)^n B.$$
$$\implies \theta + \frac{\pi}{3} = n\pi + (-1)^n\frac{\pi}{4}.$$
$$\implies \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}.$$

The correct option is **(C)**.

17. Let $P = \{\theta | \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta | \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets, then:

(A) $P \subset Q$ and $Q - P \neq \phi$ (B) $Q \not\subset P$ (C) $P \not\subset Q$ (D) $P = Q$

**Solution:**
We simplify the defining condition for each set.
**For set P:**

$$\begin{aligned} & \sin\theta - \cos\theta = \sqrt{2}\cos\theta \\ \implies & \sin\theta = \sqrt{2}\cos\theta + \cos\theta \\ \implies & \sin\theta = (\sqrt{2}+1)\cos\theta \\ & \text{Assuming } \cos\theta \neq 0: \\ \implies & \tan\theta = \sqrt{2}+1. \end{aligned}$$

**For set Q:**

$$\begin{aligned} & \sin\theta + \cos\theta = \sqrt{2}\sin\theta \\ \implies & \cos\theta = \sqrt{2}\sin\theta - \sin\theta \\ \implies & \cos\theta = (\sqrt{2}-1)\sin\theta \\ & \text{Assuming } \sin\theta \neq 0: \\ \implies & \cot\theta = \sqrt{2}-1 \\ \implies & \tan\theta = \frac{1}{\sqrt{2}-1} \\ & \text{Rationalizing the denominator:} \\ \implies & \tan\theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1. \end{aligned}$$

Since both sets are defined by the same condition, $\tan\theta = \sqrt{2}+1$, the sets are equal. The correct option is **(D)**.

18. The number of values of x in the interval (0,5$\pi$) satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$:

(A) 0 (B) 5 (C) 6 (D) 10

**Solution:** This is the same equation as in Question 12, which simplifies to $\sin x = 1/3$.
The interval is now $(0, 5\pi)$, which excludes the endpoints 0 and $5\pi$.

Excluding the endpoints 0 and $5\pi$ does not remove any solutions. As determined in Q12, there are 6 solutions in the interval.

The correct option is **(C)**.

19. The solution set of the equation $\tan x + \sec x = 2\cos x$ in the interval $[0, 2\pi]$ is:

(A) $\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}\}$ (B) $\{\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}\}$
(C) $\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$ (D) $\{\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}\}$

**Solution:**

$$\begin{aligned}
&\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x \quad (\text{assuming } \cos x \neq 0)\\
\implies &\sin x + 1 = 2\cos^2 x\\
\implies &\sin x + 1 = 2(1 - \sin^2 x)\\
\implies &\sin x + 1 = 2 - 2\sin^2 x\\
\implies &2\sin^2 x + \sin x - 1 = 0\\
\implies &(2\sin x - 1)(\sin x + 1) = 0.
\end{aligned}$$

**Case 1:** $\sin x = 1/2.$

Solutions in $[0, 2\pi]$ are $x = \dfrac{\pi}{6}$ and $x = \dfrac{5\pi}{6}$.

For both, $\cos x \neq 0$, so they are valid.

**Case 2:** $\sin x = -1.$

The solution in $[0, 2\pi]$ is $x = \dfrac{3\pi}{2}$.

For this value, $\cos x = 0$, which makes the original equation undefined.

The, correct answer should be $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}$.

The correct option is **(C)**.

20. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k+1$ has a solution, is:

(A) 4 (B) 8 (C) 10 (D) 12

**Solution:** For an equation of the form $a\cos x + b\sin x = c$ to have a solution, we must have $|c| \leq \sqrt{a^2+b^2}$. Here, $a = 7$, $b = 5$, and $c = 2k+1$.

$$
\begin{aligned}
& -\sqrt{7^2+5^2} \leq 2k+1 \leq \sqrt{7^2+5^2} \\
\implies & -\sqrt{49+25} \leq 2k+1 \leq \sqrt{49+25} \\
\implies & -\sqrt{74} \leq 2k+1 \leq \sqrt{74} \\
& \text{Since } 8^2 = 64 \text{ and } 9^2 = 81, \sqrt{74} \approx 8.6. \\
\implies & -8.6 \leq 2k+1 \leq 8.6 \\
\implies & -8.6-1 \leq 2k \leq 8.6-1 \\
\implies & -9.6 \leq 2k \leq 7.6 \\
\implies & -4.8 \leq k \leq 3.8.
\end{aligned}
$$

The possible integer values for k are $\{-4, -3, -2, -1, 0, 1, 2, 3\}$. There are 8 such integer values. The correct option is **(B)**.

21. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\cos x + 2b\sin x = c$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots $\alpha$ and $\beta$ with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is:

(A) 0.1 (B) 0.5 (C) -0.5 (D) 1

**Solution:** Since $\alpha$ and $\beta$ are roots of the equation, they satisfy it.

$$\sqrt{3}a\cos\alpha + 2b\sin\alpha = c \quad \cdots(1)$$
$$\sqrt{3}a\cos\beta + 2b\sin\beta = c \quad \cdots(2)$$

Subtracting equation (2) from (1):

$$(\sqrt{3}a\cos\alpha + 2b\sin\alpha) - (\sqrt{3}a\cos\beta + 2b\sin\beta) = 0$$
$$\implies \sqrt{3}a(\cos\alpha - \cos\beta) + 2b(\sin\alpha - \sin\beta) = 0$$

Using sum-to-product formulas:

$$\sqrt{3}a\left(-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right) + 2b\left(2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right) = 0$$

Factor out $2\sin\dfrac{\alpha-\beta}{2}$ :

$$2\sin\frac{\alpha-\beta}{2}\left(-\sqrt{3}a\sin\frac{\alpha+\beta}{2} + 2b\cos\frac{\alpha+\beta}{2}\right) = 0$$

Since $\alpha \neq \beta, \sin\dfrac{\alpha-\beta}{2} \neq 0$. So we can divide by it.

$$\implies -\sqrt{3}a\sin\frac{\alpha+\beta}{2} + 2b\cos\frac{\alpha+\beta}{2} = 0$$

Given $\alpha + \beta = \dfrac{\pi}{3}$, so $\dfrac{\alpha+\beta}{2} = \dfrac{\pi}{6}$.

$$\implies -\sqrt{3}a\sin\frac{\pi}{6} + 2b\cos\frac{\pi}{6} = 0$$
$$\implies -\sqrt{3}a\left(\frac{1}{2}\right) + 2b\left(\frac{\sqrt{3}}{2}\right) = 0$$
$$\implies -\frac{\sqrt{3}}{2}a + \sqrt{3}b = 0$$
$$\implies \sqrt{3}b = \frac{\sqrt{3}}{2}a \implies b = \frac{1}{2}a$$
$$\implies \frac{b}{a} = \frac{1}{2} = 0.5.$$

The correct option is **(B)**.

22. The general solution of $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is:

(A) $x = n\pi + \frac{\pi}{4}$ (B) $x = 2n\pi + \frac{\pi}{4}$
(C) $x = n\pi + (-1)^n\frac{\pi}{4}$ (D) $x = \frac{n\pi}{2} + \frac{\pi}{8}$

**Solution:**

$$
\begin{aligned}
& \text{Rearrange the terms:} \\
& (\sin 3x + \sin x) - 3\sin 2x = (\cos 3x + \cos x) - 3\cos 2x \\
& \text{Using sum-to-product formulas:} \\
& 2\sin(\frac{3x+x}{2})\cos(\frac{3x-x}{2}) - 3\sin 2x = 2\cos(\frac{3x+x}{2})\cos(\frac{3x-x}{2}) - 3\cos 2x \\
\implies & 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x \\
\implies & \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3) \\
\implies & \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0 \\
\implies & (\sin 2x - \cos 2x)(2\cos x - 3) = 0 \\
& \text{Since } \cos x \neq 3/2, \text{ the factor } (2\cos x - 3) \text{ is never zero.} \\
& \text{Therefore, we must have } \sin 2x - \cos 2x = 0. \\
\implies & \sin 2x = \cos 2x \implies \tan 2x = 1. \\
& \tan 2x = \tan(\frac{\pi}{4}) \\
\implies & 2x = n\pi + \frac{\pi}{4}. \\
\implies & x = \frac{n\pi}{2} + \frac{\pi}{8}.
\end{aligned}
$$

The correct option is **(D)**.

23. The Solution set of the equation $sin^2\theta - cos\ \theta = \frac{1}{4}$ in the interval $[0,2\pi]$ is:

(A) $\{\frac{\pi}{6}, \frac{5\pi}{6}\}$ (B) $\{\frac{\pi}{3}, \frac{5\pi}{3}\}$ (C) $\{\frac{\pi}{3}, \frac{2\pi}{3}\}$ (D) $\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

**Solution:**

$$\begin{aligned}
&(1-\cos^2\theta) - \cos\theta = \frac{1}{4} \\
\implies &1 - \cos^2\theta - \cos\theta - \frac{1}{4} = 0 \\
\implies &-\cos^2\theta - \cos\theta + \frac{3}{4} = 0 \\
&\text{Multiply by -4:} \\
\implies &4\cos^2\theta + 4\cos\theta - 3 = 0 \\
\implies &4\cos^2\theta + 6\cos\theta - 2\cos\theta - 3 = 0 \\
\implies &2\cos\theta(2\cos\theta+3) - 1(2\cos\theta+3) = 0 \\
\implies &(2\cos\theta - 1)(2\cos\theta+3) = 0
\end{aligned}$$

This gives $\cos\theta = 1/2$ or $\cos\theta = -3/2$ (impossible).

The solutions for $\cos\theta = 1/2$ in $[0, 2\pi]$ are in Q1 and Q4.

- Q1 solution: $\theta = \frac{\pi}{3}$.
- Q4 solution: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

The solution set is $\{\frac{\pi}{3}, \frac{5\pi}{3}\}$. The correct option is **(B)**.

24. The number of all values of $\theta$ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ is:

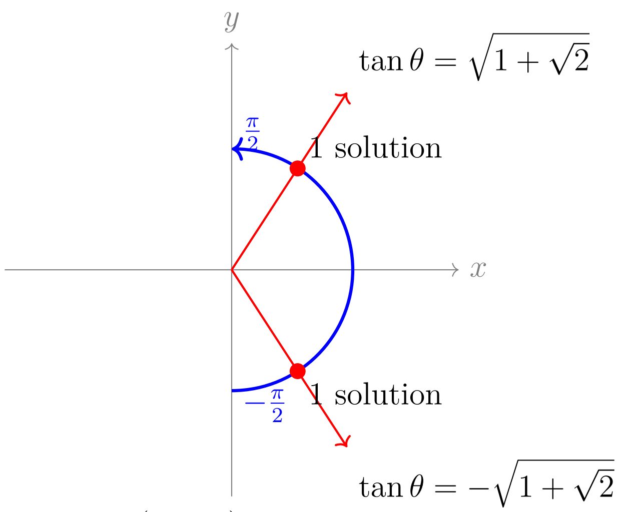
(A) 1 (B) 0 (C) 2 (D) infinitely many

**Solution:**

$$(1 - \tan^2\theta)\sec^2\theta + 2\tan^2\theta = 0$$
Using $\sec^2\theta = 1 + \tan^2\theta$ :
$$(1 - \tan^2\theta)(1 + \tan^2\theta) + 2\tan^2\theta = 0$$
$$1 - \tan^4\theta + 2\tan^2\theta = 0$$
$$\implies \tan^4\theta - 2\tan^2\theta - 1 = 0$$
Let $t = \tan^2\theta$. The equation is $t^2 - 2t - 1 = 0$.

Using the quadratic formula: $t = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$$t = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}.$$
Since $t = \tan^2\theta, t$ must be non-negative. So we must take the positive root.
$$t = \tan^2\theta = 1 + \sqrt{2}.$$
$$\implies \tan\theta = \pm\sqrt{1 + \sqrt{2}}.$$




The interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ covers Q4 and Q1.
There are two distinct solutions in this interval.

In the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, the tangent function takes all real values.
There is one solution for $\tan\theta = +\sqrt{1+\sqrt{2}}$ (in Q1) and one solution for $\tan\theta = -\sqrt{1+\sqrt{2}}$ (in Q4).

Thus, there are 2 solutions.
The correct option is **(C)**.

25. If $\theta$ and $\alpha$ are not odd multiples of $\frac{\pi}{2}$ then $\tan\theta = \tan\alpha$ implies general solution is:

(A) $\theta = \alpha + \frac{n\pi}{2}, n \in \mathbb{Z}$ (B) $\theta = \alpha + \frac{3n\pi}{2}, n \in \mathbb{Z}$
(C) $\theta = n\pi + \alpha, n \in \mathbb{Z}$ (D) $\theta = \frac{n\pi}{4} + \alpha, n \in \mathbb{Z}$

**Solution:** This question asks for the standard formula for the general solution of a tangent equation. The period of the tangent function is $\pi$. If $\tan\theta = \tan\alpha$, it means that $\theta$ and $\alpha$ have the same terminal side, or their terminal sides are separated by an integer multiple of $\pi$. Therefore, the general solution is:

$$\theta = n\pi + \alpha, \quad n \in \mathbb{Z}$$

The correct option is **(C)**.

26. The general solution of $2\sqrt{3}\cos^2\theta = \sin\theta$ is:

(A) $n\pi + (-1)^n\frac{\pi}{3}, n \in \mathbb{Z}$ (B) $n\pi + (-1)^n\frac{\pi}{6}, n \in \mathbb{Z}$
(C) $n\pi \pm (-1)^n\frac{\pi}{4}, n \in \mathbb{Z}$ (D) $n\pi + (-1)^n\frac{2\pi}{3}, n \in \mathbb{Z}$

**Solution:**

$$2\sqrt{3}(1 - \sin^2\theta) = \sin\theta$$
$$\implies 2\sqrt{3} - 2\sqrt{3}\sin^2\theta = \sin\theta$$
$$\implies 2\sqrt{3}\sin^2\theta + \sin\theta - 2\sqrt{3} = 0$$

This is a quadratic in $\sin\theta$. Using the formula $\sin\theta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ :

$$\sin\theta = \frac{-1 \pm \sqrt{1^2 - 4(2\sqrt{3})(-2\sqrt{3})}}{2(2\sqrt{3})}$$
$$\sin\theta = \frac{-1 \pm \sqrt{1 + 16(3)}}{4\sqrt{3}} = \frac{-1 \pm \sqrt{49}}{4\sqrt{3}} = \frac{-1 \pm 7}{4\sqrt{3}}.$$

Two possible values:

$$\sin\theta = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}.$$
$$\sin\theta = \frac{-8}{4\sqrt{3}} = \frac{-2}{\sqrt{3}} \quad \text{(impossible, as it's } < -1\text{)}.$$

So, we must have $\sin\theta = \dfrac{\sqrt{3}}{2} = \sin\left(\dfrac{\pi}{3}\right)$.

The general solution is $\theta = n\pi + (-1)^n\dfrac{\pi}{3}, \quad n \in \mathbb{Z}$.

The correct option is **(A)**.

27. The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi]$ are:

(A) 6 (B) 4 (C) 3 (D) 2

**Solution:** The equation is valid for $\cos x \neq 0$, which means $x \neq \pi/2, 3\pi/2$.

$$\begin{aligned} &\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x \\ \implies &\sin x + 1 = 2\cos^2 x \\ \implies &\sin x + 1 = 2(1 - \sin^2 x) \\ \implies &2\sin^2 x + \sin x - 1 = 0 \\ \implies &(2\sin x - 1)(\sin x + 1) = 0. \end{aligned}$$

This gives two cases:

**Case 1:** $\sin x = 1/2$.
The solutions in $[0, 2\pi]$ are $x = \pi/6$ and $x = 5\pi/6$.
For these values, $\cos x \neq 0$, so they are valid solutions.

**Case 2:** $\sin x = -1$.
The solution in $[0, 2\pi]$ is $x = 3\pi/2$.
However, this value was excluded because it makes the original equation's denominator zero.

Thus, there are only 2 valid solutions.
The correct option is **(D)**.

28. If the general solution of the equation $\frac{\tan 3x - 1}{\tan 3x + 1} = \sqrt{3}$ is $x = \frac{n\pi}{p} + \frac{7\pi}{q}$, $n, p, q \in \mathbb{Z}$, then $\frac{p}{q}$ is:

(A) 12 (B) $\frac{1}{12}$ (C) 3 (D) 36

**Solution:**

$$\frac{\tan 3x - 1}{1 + \tan 3x} = \sqrt{3}$$

Recognize that $1 = \tan(\pi/4)$.

$$\frac{\tan 3x - \tan(\pi/4)}{1 + \tan 3x \tan(\pi/4)} = \sqrt{3}$$

Using the identity for $\tan(A - B)$ :

$$\tan\left(3x - \frac{\pi}{4}\right) = \sqrt{3}$$

$$\tan\left(3x - \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{3}\right)$$

The general solution is:

$$3x - \frac{\pi}{4} = n\pi + \frac{\pi}{3}$$

$$\implies 3x = n\pi + \frac{\pi}{3} + \frac{\pi}{4}$$

$$\implies 3x = n\pi + \frac{4\pi + 3\pi}{12} = n\pi + \frac{7\pi}{12}$$

$$\implies x = \frac{n\pi}{3} + \frac{7\pi}{36}.$$

Comparing with $x = \frac{n\pi}{p} + \frac{7\pi}{q}$, we have $p = 3$ and $q = 36$.

$$\implies \frac{p}{q} = \frac{3}{36} = \frac{1}{12}.$$

The correct option is **(B)**.

29. The solution set of $8\cos^2\theta + 14\cos\theta + 5 = 0$, in the interval $[0, 2\pi]$, is:

(A) $\{\frac{\pi}{3}, \frac{2\pi}{3}\}$ (B) $\{\frac{\pi}{3}, \frac{4\pi}{3}\}$ (C) $\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$ (D) $\{\frac{2\pi}{3}, \frac{5\pi}{3}\}$

**Solution:**

We solve the quadratic equation for $\cos\theta$.

$$8\cos^2\theta + 14\cos\theta + 5 = 0$$
$$8\cos^2\theta + 10\cos\theta + 4\cos\theta + 5 = 0$$
$$2\cos\theta(4\cos\theta + 5) + 1(4\cos\theta + 5) = 0$$
$$(2\cos\theta + 1)(4\cos\theta + 5) = 0$$

This gives two possibilities:

- $2\cos\theta + 1 = 0 \implies \cos\theta = -1/2$.
- $4\cos\theta + 5 = 0 \implies \cos\theta = -5/4$ (impossible, as it's $< -1$).

We need to find solutions for $\cos\theta = -1/2$ in $[0, 2\pi]$. Cosine is negative in Q2 and Q3. The reference angle is $\pi/3$.

- Q2 solution: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.
- Q3 solution: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

The solution set is $\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$. The correct option is **(C)**.

30. The principal solutions of the equation $\sec x + \tan x = 2\cos x$ are:

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{\pi}{6}, \frac{\pi}{2}$ (C) $\frac{\pi}{6}, \frac{2\pi}{3}$ (D) $\frac{\pi}{6}, \frac{\pi}{12}$

**Solution:** This is the same equation as Q27. We must have $\cos x \neq 0$.

$$\begin{aligned} &\frac{1}{\cos x} + \frac{\sin x}{\cos x} = 2\cos x \\ \implies &1 + \sin x = 2\cos^2 x \\ \implies &1 + \sin x = 2(1 - \sin^2 x) \\ \implies &2\sin^2 x + \sin x - 1 = 0 \\ \implies &(2\sin x - 1)(\sin x + 1) = 0. \end{aligned}$$

This gives $\sin x = 1/2$ or $\sin x = -1$.

The principal solutions (in $[0, 2\pi)$) for $\sin x = 1/2$ are $x = \pi/6$ and $x = 5\pi/6$. Both are valid.

The principal solution for $\sin x = -1$ is $x = 3\pi/2$, which is rejected as $\cos(3\pi/2) = 0$.

The correct option is **(A)**.

31. The solutions of $\sin x + \sin 5x = \sin 3x$ in $(0, \frac{\pi}{2})$ are:

(A) $\frac{\pi}{4}, \frac{\pi}{10}$ (B) $\frac{\pi}{6}, \frac{\pi}{3}$ (C) $\frac{\pi}{4}, \frac{\pi}{12}$ (D) $\frac{\pi}{8}, \frac{\pi}{16}$

**Solution:**

$$(\sin 5x + \sin x) - \sin 3x = 0$$

$$\text{Using } \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} :$$

$$2\sin(\frac{6x}{2})\cos(\frac{4x}{2}) - \sin 3x = 0$$

$$2\sin 3x\cos 2x - \sin 3x = 0$$

$$\sin 3x(2\cos 2x - 1) = 0$$

**Case 1:** $\sin 3x = 0$.

$\implies 3x = n\pi \implies x = \frac{n\pi}{3}$.

For $n = 1, x = \pi/3$. This is in the interval $(0, \pi/2)$.

(For $n = 0, x = 0$, which is not in the interval).

**Case 2:** $2\cos 2x - 1 = 0 \implies \cos 2x = 1/2$.

$\implies 2x = 2n\pi \pm \frac{\pi}{3} \implies x = n\pi \pm \frac{\pi}{6}$.

For $n = 0$, taking the $+$ sign gives $x = \pi/6$.

This is in the interval $(0, \pi/2)$.

The solutions are $\frac{\pi}{6}$ and $\frac{\pi}{3}$.

The correct option is **(B)**.

32. The general solution of the equation $3\sec^2\theta = 2\,\mathrm{cosec}\,\theta$ is:

(A) $n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$ (B) $2n\pi + (-1)^n \frac{\pi}{12}, n \in \mathbb{Z}$

(C) $n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$ (D) $n\pi + (-1)^n \frac{\pi}{3}, n \in \mathbb{Z}$

**Solution:**

$$\frac{3}{\cos^2\theta} = \frac{2}{\sin\theta}$$
$$\implies 3\sin\theta = 2\cos^2\theta$$
$$\implies 3\sin\theta = 2(1-\sin^2\theta)$$
$$\implies 3\sin\theta = 2 - 2\sin^2\theta$$
$$\implies 2\sin^2\theta + 3\sin\theta - 2 = 0$$
$$\implies (2\sin\theta - 1)(\sin\theta + 2) = 0.$$

This gives $\sin\theta = 1/2$ or $\sin\theta = -2$ (impossible).

The general solution for $\sin\theta = 1/2 = \sin(\pi/6)$ is $\theta = n\pi + (-1)^n\frac{\pi}{6}$.

The correct option is **(C)**.

33. If general solution of $\cos^2\theta - 2\sin\theta + \frac{1}{4} = 0$ is $\theta = \frac{n\pi}{A} + (-1)^n\frac{\pi}{B}$, $n \in \mathbb{Z}$ then $A + B$ has the value:

(A) 7 (B) 6 (C) 1 (D) -7

**Solution:**

$$\begin{aligned}
&(1-\sin^2\theta) - 2\sin\theta + \frac{1}{4} = 0 \\
\implies &-\sin^2\theta - 2\sin\theta + \frac{5}{4} = 0 \\
&\text{Multiply by -4:} \\
\implies &4\sin^2\theta + 8\sin\theta - 5 = 0 \\
\implies &4\sin^2\theta + 10\sin\theta - 2\sin\theta - 5 = 0 \\
\implies &2\sin\theta(2\sin\theta+5) - 1(2\sin\theta+5) = 0 \\
\implies &(2\sin\theta - 1)(2\sin\theta+5) = 0.
\end{aligned}$$

This gives $\sin\theta = 1/2$ or $\sin\theta = -5/2$ (impossible).
The general solution for $\sin\theta = 1/2 = \sin(\pi/6)$ is $\theta = n\pi + (-1)^n\frac{\pi}{6}$.

Comparing this with the given form $\theta = \frac{n\pi}{A} + (-1)^n\frac{\pi}{B}$, we can see that $A = 1$ and $B = 6$.

Therefore, $A + B = 1 + 6 = 7$.

The correct option is **(A)**.

34. The number of possible solutions of $\sin\theta + \sin 4\theta + \sin 7\theta = 0$ for $\theta \in (0, \pi)$:

(A) 3 (B) 4 (C) 6 (D) 8

**Solution:**

The equation is $\sin\theta + \sin 4\theta + \sin 7\theta = 0$.
Rearranging and using the sum-to-product formula:
$(\sin 7\theta + \sin\theta) + \sin 4\theta = 0$

$$\implies 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + \sin 4\theta = 0$$
$$\implies 2\sin 4\theta\cos 3\theta + \sin 4\theta = 0$$
$$\implies \sin 4\theta(2\cos 3\theta + 1) = 0$$

This gives two cases for solutions in $(0, \pi)$ :

**Case 1:** $\sin 4\theta = 0$

$$\implies 4\theta = n\pi$$
$$\implies \theta = \frac{n\pi}{4}.$$

For $n = 1, 2, 3,$ we get $\theta \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$. (3 solutions)

**Case 2:** $2\cos 3\theta + 1 = 0 \implies \cos 3\theta = -1/2$.

The solutions for $3\theta$ in the interval $(0, 3\pi)$ are $\frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{8\pi}{3}$.

$$\implies \theta \in \left\{\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}\right\}. \quad \text{(3 solutions)}$$

The total number of distinct solutions is $3 + 3 = 6$.

The correct option is **(C)**.

35. The principal solutions of $\tan 3\theta = -1$ are:

(A) $\{\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}\}$

(B) $\{\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{\pi}{16}, \frac{19\pi}{12}, \frac{23\pi}{24}\}$

(C) $\{\frac{\pi}{4}, \frac{\pi}{12}\}$

(D) $\{\frac{\pi}{4}, \frac{\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}, \frac{19\pi}{4}, \frac{23\pi}{12}\}$

**Solution:**

The question asks for solutions $\theta$ in the interval $[0, 2\pi)$.
Let $\phi = 3\theta$. If $\theta \in [0, 2\pi)$, then $\phi \in [0, 6\pi)$.

We solve $\tan\phi = -1$.
The general solution is $\phi = n\pi + \dfrac{3\pi}{4}$.

We find values of $\phi$ in $[0, 6\pi)$ :

$n = 0: \quad \phi = \dfrac{3\pi}{4}$

$n = 1: \quad \phi = \pi + \dfrac{3\pi}{4} = \dfrac{7\pi}{4}$

$n = 2: \quad \phi = 2\pi + \dfrac{3\pi}{4} = \dfrac{11\pi}{4}$

$n = 3: \quad \phi = 3\pi + \dfrac{3\pi}{4} = \dfrac{15\pi}{4}$

$n = 4: \quad \phi = 4\pi + \dfrac{3\pi}{4} = \dfrac{19\pi}{4}$

$n = 5: \quad \phi = 5\pi + \dfrac{3\pi}{4} = \dfrac{23\pi}{4}$

Now, we find $\theta = \phi/3$ :

$$\theta \in \left\{\frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$$
$$= \left\{\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}.$$

All these values are in $[0, 2\pi)$.

The correct option is **(A)**.

36. The value of $\theta$, satisfying both the equation $\cos\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$ in $[0, 2\pi]$, is:

(A) $(\frac{\pi}{4})$ (B) $(\frac{5\pi}{4})$ (C) $(\frac{7\pi}{4})$ (D) $(\frac{3\pi}{4})$

**Solution:**

We need the quadrant where cosine is positive and tangent is negative.

- $\cos\theta > 0$ in Quadrants I and IV.
- $\tan\theta < 0$ in Quadrants II and IV.

The common quadrant is Quadrant IV.

The reference angle for both conditions is $\frac{\pi}{4}$.

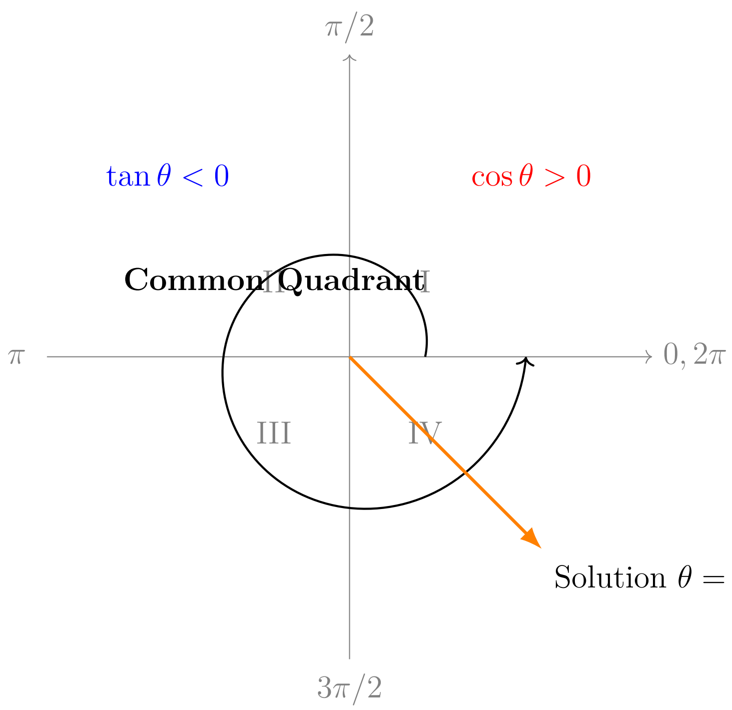
The angle in Quadrant IV with this reference angle is:

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

Verification:

$$\cos\left(\frac{7\pi}{4}\right) = \cos\left(2\pi - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \text{(Correct, positive)}$$

$$\tan\left(\frac{7\pi}{4}\right) = \tan\left(2\pi - \frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1 \quad \text{(Correct, negative)}$$




The correct option is **(C)**.

37. If $3\sin\theta = 2\sin 3\theta$ and $0 < \theta < \pi$ then $\sin\theta =$

(A) $\frac{\sqrt{2}}{\sqrt{5}}$ (B) $\frac{\sqrt{3}}{2\sqrt{2}}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{\sqrt{3}}{\sqrt{5}}$

**Solution:**

$$\begin{aligned}
&3\sin\theta = 2(3\sin\theta - 4\sin^3\theta) \\
&\implies 3\sin\theta = 6\sin\theta - 8\sin^3\theta \\
&\implies 8\sin^3\theta - 3\sin\theta = 0 \\
&\implies \sin\theta(8\sin^2\theta - 3) = 0.
\end{aligned}$$

Since $0 < \theta < \pi$, we know that $\sin\theta \neq 0$.
Therefore, we must have $8\sin^2\theta - 3 = 0$.

$$\implies \sin^2\theta = \frac{3}{8}$$

$$\implies \sin\theta = \pm\sqrt{\frac{3}{8}}.$$

Since $0 < \theta < \pi, \sin\theta$ must be positive.

$$\implies \sin\theta = \sqrt{\frac{3}{8}} = \frac{\sqrt{3}}{\sqrt{8}} = \frac{\sqrt{3}}{2\sqrt{2}}.$$

The correct option is **(B)**.

38. If $2\sin(\theta+\frac{\pi}{3})=\cos(\theta-\frac{\pi}{6})$ then $\tan\theta=$

(A) $\frac{-1}{\sqrt{3}}$ (B) $-\sqrt{3}$ (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

**Solution:** We use the sum and difference formulas for sine and cosine.

$$\begin{aligned}
& 2\left(\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\right)=\cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6} \\
\implies & 2\left(\sin\theta\cdot\frac{1}{2}+\cos\theta\cdot\frac{\sqrt{3}}{2}\right)=\cos\theta\cdot\frac{\sqrt{3}}{2}+\sin\theta\cdot\frac{1}{2} \\
\implies & \sin\theta+\sqrt{3}\cos\theta=\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta \\
& \text{Group sin and cos terms:} \\
\implies & \sin\theta-\frac{1}{2}\sin\theta=\frac{\sqrt{3}}{2}\cos\theta-\sqrt{3}\cos\theta \\
\implies & \frac{1}{2}\sin\theta=-\frac{\sqrt{3}}{2}\cos\theta \\
& \text{Assuming } \cos\theta\neq 0: \\
\implies & \frac{\sin\theta}{\cos\theta}=-\sqrt{3} \\
\implies & \tan\theta=-\sqrt{3}.
\end{aligned}$$

The correct option is **(B)**.

39. The principal solutions of $\sqrt{3}\sec x + 2 = 0$ are:

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{5\pi}{6}, \frac{7\pi}{6}$ (C) $\frac{\pi}{3}, \frac{2\pi}{3}$ (D) $\frac{2\pi}{3}, \frac{4\pi}{3}$

**Solution:**

$$\begin{aligned} &\sqrt{3}\sec x + 2 = 0 \\ \implies &\sec x = -2/\sqrt{3} \\ \implies &\cos x = -\sqrt{3}/2. \end{aligned}$$

Cosine is negative in Q2 and Q3. The reference angle is $\alpha$ such that $\cos\alpha = \sqrt{3}/2$, so $\alpha = \pi/6$.

- Q2 solution: $x = \pi - \pi/6 = 5\pi/6$.
- Q3 solution: $x = \pi + \pi/6 = 7\pi/6$.

The principal solutions (in $[0, 2\pi)$) are $5\pi/6$ and $7\pi/6$.

The correct option is **(B)**.

40. If $x \in (0, \frac{\pi}{2})$ and satisfies the equation $\cos(4x) = \frac{1}{2}$, then the values of x are:

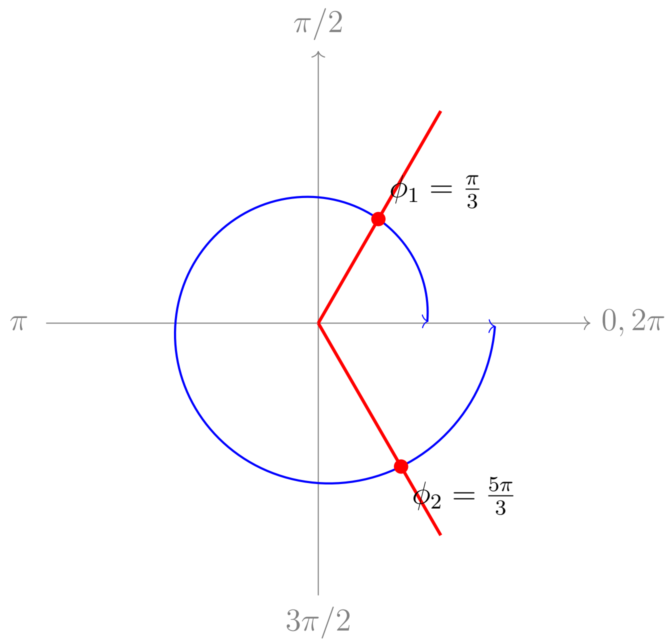
(A) $\frac{\pi}{12}, \frac{5\pi}{12}$ (B) $\frac{\pi}{8}, \frac{3\pi}{8}$ (C) $\frac{\pi}{8}, \frac{\pi}{4}$ (D) $\frac{\pi}{6}, \frac{\pi}{12}$

**Solution:**

Let $\phi = 4x$.

Since $x \in (0, \pi/2)$, the interval for $\phi$ is $(0, 2\pi)$.

We solve $\cos\phi = 1/2$ for $\phi \in (0, 2\pi)$.




The diagram shows the two solutions for $\phi$ in the interval $(0, 2\pi)$.

Now we find the corresponding values of $x = \phi/4$ :

$$4x = \frac{\pi}{3} \implies x = \frac{\pi}{12}.$$

$$4x = \frac{5\pi}{3} \implies x = \frac{5\pi}{12}.$$

Both values, $\frac{\pi}{12}$ and $\frac{5\pi}{12}$, are in the interval $(0, \pi/2)$.

The correct option is **(A)**.

41. If $3\cos x \neq 2\sin x$, then the general solution of $\sin^2 x - \cos 2x = 2 - \sin 2x$ is:

(A) $x = n\pi + \frac{\pi}{2}$ (B) $x = (n + \frac{1}{2})\pi$ (C) $x = n\pi$ (D) $x = (2n + 1)\pi$

**Solution:**

This is a repeat of Question 14.
The solution was found by testing $x = (2n + 1)\frac{\pi}{2}$.

The option (B) is $x = (n + \frac{1}{2})\pi = \frac{(2n+1)\pi}{2}$. This is the same set of solutions.

The correct option is **(B)**.

42. If $3\sin^2 x - 8\sin x + 4 = 0, x \in [\frac{\pi}{2}, \frac{3\pi}{2}]$, then $\tan x =$

(A) $-\frac{\sqrt{5}}{2}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$

**Solution:** We solve the quadratic equation for $\sin x$.

$$\begin{aligned} &3\sin^2 x - 8\sin x + 4 = 0 \\ \implies &3\sin^2 x - 6\sin x - 2\sin x + 4 = 0 \\ \implies &3\sin x(\sin x - 2) - 2(\sin x - 2) = 0 \\ \implies &(3\sin x - 2)(\sin x - 2) = 0. \end{aligned}$$

This gives $\sin x = 2/3$ or $\sin x = 2$ (impossible).

We have $\sin x = 2/3$. The interval for $x$ is $[\pi/2, 3\pi/2]$, which covers Q2 and Q3. Since $\sin x = 2/3$ is positive, $x$ must be in Q2. In Q2, cosine is negative.

$$\begin{aligned} \cos^2 x &= 1 - \sin^2 x = 1 - (2/3)^2 = 1 - 4/9 = 5/9. \\ \cos x &= -\sqrt{5/9} = -\frac{\sqrt{5}}{3}. \\ \tan x &= \frac{\sin x}{\cos x} = \frac{2/3}{-\sqrt{5}/3} = -\frac{2}{\sqrt{5}}. \end{aligned}$$

The correct option is **(C)**.

43. The general solution of $\frac{1-\cos 2x}{1+\cos 2x} = 3$ is:

(A) $x = 2n\pi \pm \frac{\pi}{3}$ (B) $x = n\pi \pm \frac{\pi}{6}$ (C) $x = 2n\pi \pm \frac{\pi}{6}$ (D) $x = n\pi \pm \frac{\pi}{3}$

**Solution:**
Using the half-angle identities, $1 - \cos 2x = 2\sin^2 x$ and $1 + \cos 2x = 2\cos^2 x$.

$$\begin{aligned} &\frac{2\sin^2 x}{2\cos^2 x} = 3 \\ \implies &\tan^2 x = 3 \\ \implies &\tan x = \pm\sqrt{3}. \end{aligned}$$

This is equivalent to the condition $\tan^2 x = (\tan(\pi/3))^2$.
The general solution for $\tan^2 x = \tan^2 \alpha$ is $x = n\pi \pm \alpha$.
Here, $\alpha = \pi/3$. So, $x = n\pi \pm \frac{\pi}{3}$.
The correct option is **(D)**.

44. If $2\cos^2\theta + 3\cos\theta = 2$ then permissible value of $\cos\theta$ is:

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

**Solution:**

$$\begin{aligned} & 2\cos^2\theta + 3\cos\theta - 2 = 0 \\ \implies & 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0 \\ \implies & 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0 \\ \implies & (2\cos\theta - 1)(\cos\theta + 2) = 0. \end{aligned}$$

This gives $\cos\theta = 1/2$ or $\cos\theta = -2$.
Since the range of cosine is $[-1, 1]$, $\cos\theta = -2$ is not possible.
The only permissible value is $\cos\theta = 1/2$.

The correct option is **(C)**.

45. The number of solutions of $\sin x + \sin 3x + \sin 5x = 0$ in the interval $[\frac{\pi}{2}, \frac{3\pi}{2}]$ is:

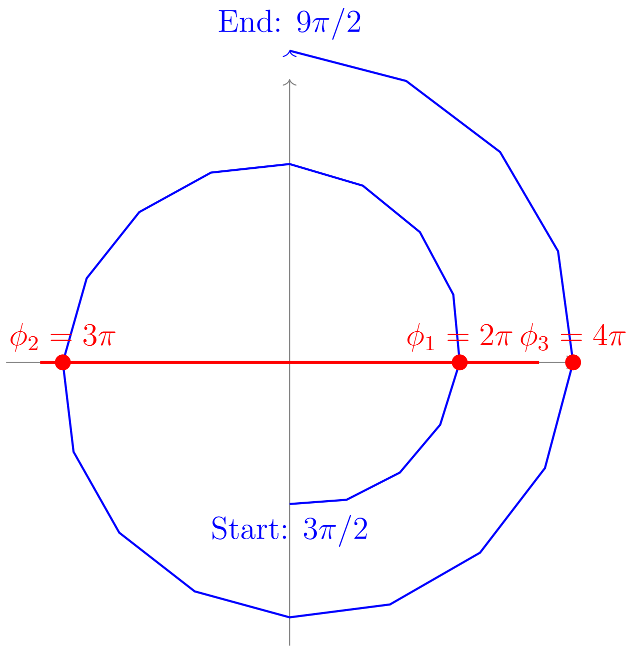
(A) 2 (B) 3 (C) 4 (D) 5

**Solution:**

$$
\begin{aligned}
& (\sin 5x + \sin x) + \sin 3x = 0 \\
\implies & 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0 \\
\implies & 2\sin 3x \cos 2x + \sin 3x = 0 \\
\implies & \sin 3x(2\cos 2x + 1) = 0.
\end{aligned}
$$

This gives two cases to solve for $x \in [\frac{\pi}{2}, \frac{3\pi}{2}]$.

**Case 1:** $\sin 3x = 0$. Let $\phi = 3x$. Since $x \in [\pi/2, 3\pi/2]$, the interval for $\phi$ is $[3\pi/2, 9\pi/2]$. We find the solutions for $\sin\phi = 0$ in this interval.

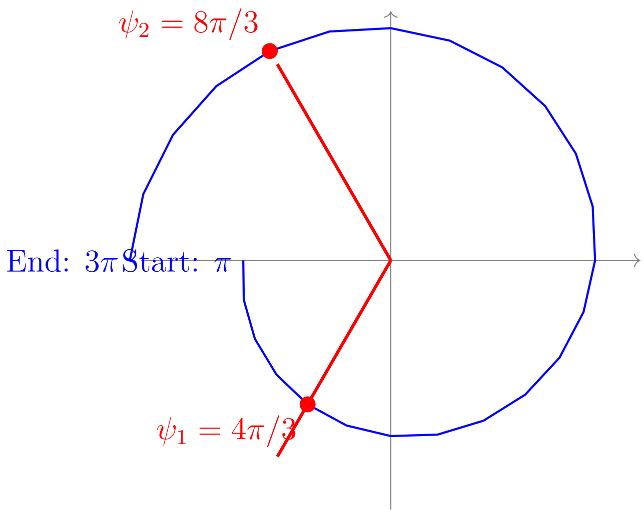



The diagram shows 3 solutions for $\phi$ in $[3\pi/2, 9\pi/2]$.

$3x \in \{2\pi, 3\pi, 4\pi\}$

$\implies x \in \{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\}$.

**Case 2:** $2\cos 2x + 1 = 0 \implies \cos 2x = -1/2$. Let $\psi = 2x$. Since $x \in [\pi/2, 3\pi/2]$, the interval for $\psi$ is $[\pi, 3\pi]$. We find solutions for $\cos\psi = -1/2$ in this interval.




The diagram shows 2 solutions for $\psi$ in $[\pi, 3\pi]$.

$2x \in \{\frac{4\pi}{3}, \frac{8\pi}{3}\}$

$\implies x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$.

These solutions were already found in Case 1. The set of distinct solutions is $\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\}$. There are a total of **3** solutions.

The distinct solutions are $\{2\pi/3, \pi, 4\pi/3\}$. There are 3 solutions. The correct option is **(B)**.

46. The general solution of the equation $\tan^2 x = 1$ is:

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi - \frac{\pi}{4}$ (C) $n\pi \pm \frac{\pi}{4}$ (D) $2n\pi \pm \frac{\pi}{4}$

**Solution:**

$$\begin{aligned} &\tan^2 x = 1 \\ \implies &\tan x = \pm 1. \end{aligned}$$

This is equivalent to $\tan x = \tan(\pm\pi/4)$.
The general solution for $\tan^2 x = \tan^2 \alpha$ is $x = n\pi \pm \alpha$. Here, $\alpha = \pi/4$.
So, the general solution is $x = n\pi \pm \frac{\pi}{4}$.

The correct option is **(C)**.

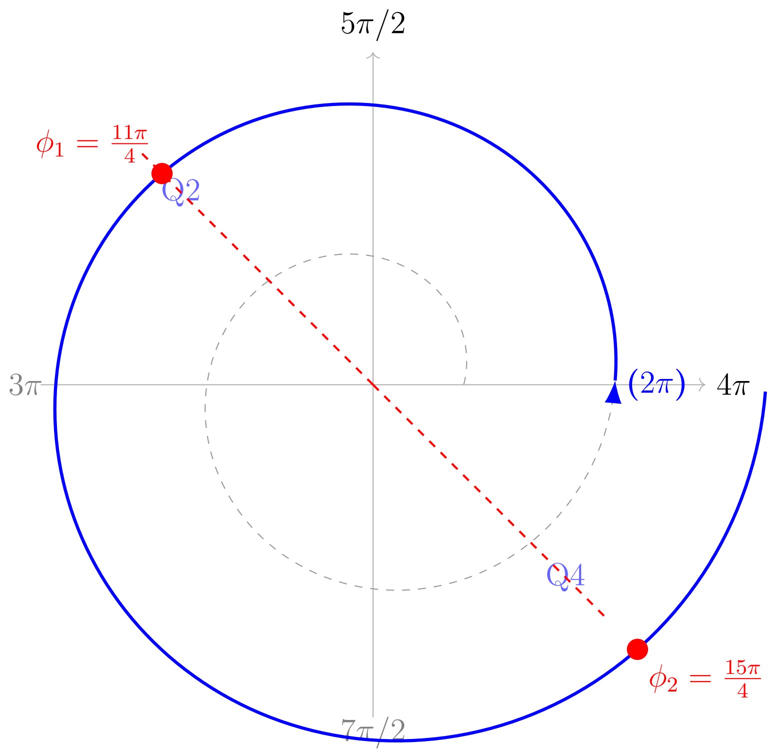
47. The solution of the equation $\sin 2x + \cos 2x = 0$, where $\pi < x < 2\pi$ are:

(A) $\frac{7\pi}{8}, \frac{11\pi}{8}$ (B) $\frac{9\pi}{8}, \frac{13\pi}{8}$ (C) $\frac{11\pi}{8}, \frac{15\pi}{8}$ (D) $\frac{15\pi}{8}, \frac{19\pi}{8}$

**Solution:**

$$\sin 2x + \cos 2x = 0$$
$$\implies \tan 2x = -1.$$

To find the solutions, we let $\phi = 2x$. Since $x \in (\pi, 2\pi)$, the interval for $\phi$ becomes $(2\pi, 4\pi)$. The problem is now to find the solutions for $\tan \phi = -1$ within this new interval.




From the diagram, the solutions for $\phi$ are $\frac{11\pi}{4}$ and $\frac{15\pi}{4}$.

Now, we solve for $x = \phi/2$:

$2x = \frac{11\pi}{4} \implies x = \frac{11\pi}{8}.$

$2x = \frac{15\pi}{4} \implies x = \frac{15\pi}{8}.$

Both values lie within the required interval $(\pi, 2\pi)$.

The correct option is **(C)**.