Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Exercise: Practice Question

Topic: Trigonometric Equation

Sub: Mathematics **Practice Question: Solution** Prof. Chetan Sir

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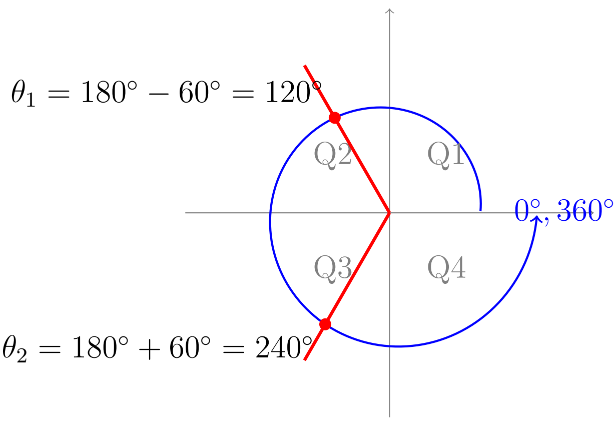
1. If $\cos\theta = \frac{-1}{2}$ and $0° < \theta < 360°$ then the values of θ are:

   (A) $120°$ and $300°$ (B) $60°$ and $120°$ (C) $120°$ and $240°$ (D) $60°$ and $240°$

   Solution:

   We are given $\cos\theta = -1/2$. Cosine is negative in Quadrants II and III. The reference angle for which $\cos\theta = 1/2$ is $60°$.



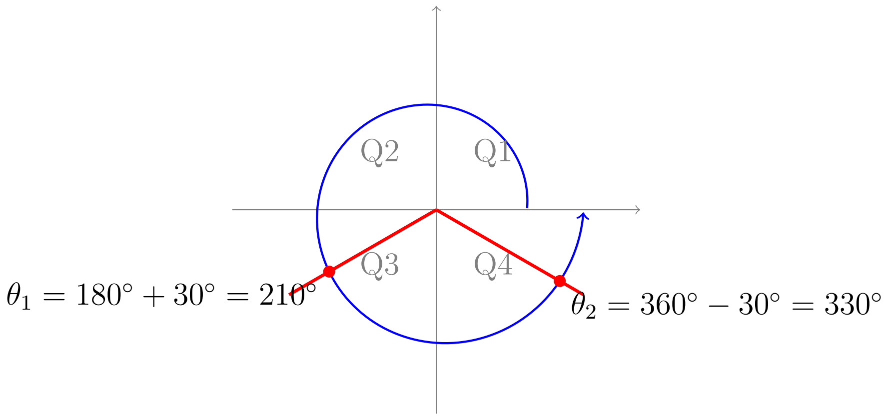
The diagram shows the two solutions in the required interval. The correct option is **(C)**.

2. Values of $\theta (0 < \theta < 360°)$ satisfying $cosec\ \theta + 2 = 0$ are:

   (A) $210°, 300°$ (B) $240°, 300°$ (C) $210°, 240°$ (D) $210°, 330°$

   Solution:

   $cosec\ \theta + 2 = 0 \implies \sin\theta = -1/2$. Sine is negative in Quadrants III and IV. The reference angle is $30°$.



   The correct option is **(D)**.

3. If $\sin\theta = \sqrt{3}\cos\theta,\ -\pi < \theta < 0$, then $\theta =$

   (A) $-\frac{5\pi}{6}$ (B) $-\frac{2\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

   Solution:

   $\sin\theta = \sqrt{3}\cos\theta \implies \tan\theta = \sqrt{3}.$
   The interval is $(-\pi, 0)$, which corresponds to Quadrants III and IV.
   Tangent is positive in Quadrant III.
   The reference angle is $\pi/3$.
   The angle in Q3 is found by measuring clockwise from the negative x-axis, or $-\pi + \pi/3$.
   $$\theta = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}.$$

The correct option is **(B)**.

4. The general solution of $\tan 3x = 1$ is:

(A) $x = n\pi + \frac{\pi}{4}$ (B) $x = \frac{n\pi}{3} + \frac{\pi}{12}$ (C) $x = n\pi$ (D) $x = n\pi \pm \frac{\pi}{4}$

Solution:

$$\tan 3x = 1 = \tan(\frac{\pi}{4}).$$
$$\text{The general solution for } \tan A = \tan B \text{ is } A = n\pi + B.$$
$$3x = n\pi + \frac{\pi}{4}.$$
$$x = \frac{n\pi}{3} + \frac{\pi}{12}.$$

The correct option is **(B)**.

5. General solution of $\tan 5\theta = \cot 2\theta$ is:

(A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$ (C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (D) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}$

Solution:

$$\tan 5\theta = \cot 2\theta = \tan\left(\frac{\pi}{2} - 2\theta\right).$$
$$5\theta = n\pi + \frac{\pi}{2} - 2\theta.$$
$$7\theta = n\pi + \frac{\pi}{2}.$$
$$\theta = \frac{n\pi}{7} + \frac{\pi}{14}.$$

The correct option is **(A)**.

6. The general value of θ satisfying $\sin^2\theta + \sin\theta = 2$ is:

(A) $n\pi + (-1)^n\frac{\pi}{6}$ (B) $n\pi + (-1)^n\frac{\pi}{4}$ (C) $n\pi + (-1)^n\frac{\pi}{2}$ (D) $n\pi + (-1)^n\pi$

Solution:

$$\sin^2\theta + \sin\theta - 2 = 0.$$
$$\text{Let } y = \sin\theta. \text{ Then } y^2 + y - 2 = 0.$$
$$(y+2)(y-1) = 0.$$
$$\text{This gives } y = -2 \text{ or } y = 1.$$
$$\sin\theta = -2 \text{ is not possible.}$$
$$\text{So, } \sin\theta = 1 = \sin(\frac{\pi}{2}).$$
$$\text{The general solution is } \theta = n\pi + (-1)^n\frac{\pi}{2}.$$

The correct option is **(C)**.

7. General solution of the equation $\cot\theta - \tan\theta = 2$ is:

(A) $\theta = n\pi + \frac{\pi}{4}$ (B) $\theta = \frac{n\pi}{2} + \frac{\pi}{8}$ (C) $\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$ (D) $\theta = n\pi \pm \frac{\pi}{4}$

Solution:

$$\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = 2.$$
$$\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = 2.$$
$$\frac{\cos(2\theta)}{\frac{1}{2}\sin(2\theta)} = 2.$$
$$2\cot(2\theta) = 2 \implies \cot(2\theta) = 1.$$
$$2\theta = n\pi + \frac{\pi}{4}.$$
$$\theta = \frac{n\pi}{2} + \frac{\pi}{8}.$$

The correct option is **(B)**.

8. If $\sin^2\theta = \frac{1}{4}$, then the general value of θ is:

(A) $2n\pi \pm (-1)^n\frac{\pi}{6}$ (B) $\frac{n\pi}{2} \pm (-1)^n\frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{6}$ (D) $2n\pi \pm \frac{\pi}{6}$

Solution:

$$\sin^2\theta = \frac{1}{4} = \left(\frac{1}{2}\right)^2 = \sin^2\left(\frac{\pi}{6}\right).$$

The general solution for $\sin^2\theta = \sin^2\alpha$ is $\theta = n\pi \pm \alpha$.

$$\implies \theta = n\pi \pm \frac{\pi}{6}.$$

The correct option is **(C)**.

9. The general solution of the equation $4\cos^2 x + 6\sin^2 x = 5$ is:

(A) $x = n\pi \pm \frac{\pi}{2}$ (B) $x = n\pi \pm \frac{\pi}{4}$ (C) $x = n\pi \pm \frac{3\pi}{2}$ (D) $x = n\pi \pm \frac{3\pi}{4}$

Solution:

$$4\cos^2 x + 6(1 - \cos^2 x) = 5.$$
$$4\cos^2 x + 6 - 6\cos^2 x = 5.$$
$$-2\cos^2 x = -1 \implies \cos^2 x = \frac{1}{2}.$$
$$\cos^2 x = \left(\frac{1}{\sqrt{2}}\right)^2 = \cos^2\left(\frac{\pi}{4}\right).$$

$$\text{The general solution for } \cos^2 x = \cos^2 \alpha \text{ is } x = n\pi \pm \alpha.$$
$$\implies x = n\pi \pm \frac{\pi}{4}.$$

The correct option is **(B)**.

10. If $3(\sec^2\theta + \tan^2\theta) = 5$, then the general value of θ is:

(A) $2n\pi + \frac{\pi}{6}$ (B) $2n\pi \pm \frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{6}$ (D) $n\pi \pm \frac{\pi}{3}$

Solution:

$$3((1+\tan^2\theta)+\tan^2\theta) = 5.$$
$$3(1+2\tan^2\theta) = 5.$$
$$3 + 6\tan^2\theta = 5 \implies 6\tan^2\theta = 2.$$
$$\tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2 = \tan^2\left(\frac{\pi}{6}\right).$$
$$\text{The general solution is } \theta = n\pi \pm \frac{\pi}{6}.$$

The correct option is **(C)**.

11. The value of θ satisfying the given equation $\cos\theta + \sqrt{3}\sin\theta = 2$ is:

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

Solution:

$$\text{Divide by } \sqrt{1^2 + (\sqrt{3})^2} = 2.$$
$$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1.$$
$$\sin\frac{\pi}{6}\cos\theta + \cos\frac{\pi}{6}\sin\theta = 1.$$
$$\sin\left(\theta + \frac{\pi}{6}\right) = 1.$$
$$\theta + \frac{\pi}{6} = 2n\pi + \frac{\pi}{2}.$$
$$\theta = 2n\pi + \frac{\pi}{2} - \frac{\pi}{6} = 2n\pi + \frac{\pi}{3}.$$
$$\text{For n=0, a possible value is } \theta = \frac{\pi}{3}.$$

The correct option is **(A)**.

12. If $(2\cos x - 1)(3 + 2\cos x) = 0$, $0 \le x \le 2\pi$, then $x =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{3}, \frac{5\pi}{3}$ (C) $\frac{\pi}{3}, \frac{5\pi}{3}, \cos^{-1}(-\frac{3}{2})$ (D) $\frac{5\pi}{3}$

Solution:

This gives two cases:

Case 1: $2\cos x - 1 = 0 \implies \cos x = 1/2.$

Case 2: $3 + 2\cos x = 0 \implies \cos x = -3/2,$ which is not possible.

We only need to solve $\cos x = 1/2$ for $x \in [0, 2\pi]$.

The solutions are in Q1 and Q4. $x = \dfrac{\pi}{3}$ and $x = 2\pi - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$.

The correct option is **(B)**.

13. If $\tan\theta = -\frac{1}{\sqrt{3}}$, $\sin\theta = \frac{1}{2}$ and $\cos\theta = -\frac{\sqrt{3}}{2}$, then the principal value of θ will be:

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{7\pi}{6}$ (D) $-\frac{\pi}{6}$

Solution:

We need to find the quadrant where $\sin\theta > 0, \cos\theta < 0,$ and $\tan\theta < 0$.

$\sin\theta > 0$ in Q1 and Q2.

$\cos\theta < 0$ in Q2 and Q3.

The common quadrant is Quadrant II.

The reference angle for $\sin\theta = 1/2$ is $\pi/6$.

The angle in Q2 is $\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$.

The correct option is **(B)**.

14. If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, the general value of θ is:

(A) $n\pi \pm \frac{\pi}{3}$ (B) $n\pi \pm \frac{\pi}{6}$ (C) $2n\pi \pm \frac{\pi}{3}$ (D) $2n\pi \pm \frac{\pi}{6}$

Solution:

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}.$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}.$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}.$$

Since $\sin\theta \neq 0$ for the equation to be defined, we can multiply by $\sin\theta$.

$$\frac{1}{\cos\theta} = 2 \implies \cos\theta = \frac{1}{2}.$$

The general solution is $\theta = 2n\pi \pm \dfrac{\pi}{3}$.

The correct option is **(C)**.

15. The general value of θ satisfying the equation $\tan\theta + \tan(\frac{\pi}{2} - \theta) = 2$ is:

(A) $n\pi \pm \frac{\pi}{4}$ (B) $n\pi + \frac{\pi}{4}$ (C) $2n\pi \pm \frac{\pi}{4}$ (D) $n\pi + (-1)^n\frac{\pi}{4}$

Solution:

$$\tan\theta + \cot\theta = 2.$$
$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2.$$
$$\frac{1}{\sin\theta\cos\theta} = 2 \implies 2\sin\theta\cos\theta = 1.$$
$$\sin(2\theta) = 1.$$
$$2\theta = 2n\pi + \frac{\pi}{2}.$$
$$\theta = n\pi + \frac{\pi}{4}.$$

The correct option is **(B)**.

16. The most general value of θ which will satisfy both the equations $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ is:

(A) $n\pi + (-1)^n\frac{\pi}{6}$ (B) $n\pi + \frac{\pi}{6}$ (C) $2n\pi \pm \frac{\pi}{6}$ (D) $2n\pi + \frac{7\pi}{6}$

Solution:

From the conditions, $\sin\theta < 0$ and $\tan\theta > 0$.
This occurs only in Quadrant III.
The reference angle for both conditions is $\pi/6$.
The principal solution in Q3 is $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.
The most general solution is obtained by adding integer multiples of 2π.
$$\theta = 2n\pi + \frac{7\pi}{6}.$$

The correct option is **(D)**.

17. If $1 + \cot\theta = \operatorname{cosec}\theta$, then the general value of θ is:

(A) $n\pi + \frac{\pi}{2}$ (B) $2n\pi - \frac{\pi}{2}$ (C) $2n\pi + \frac{\pi}{2}$ (D) $2n\pi \pm \frac{\pi}{2}$

Solution:

$$1 + \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}.$$
Multiplying by $\sin\theta$ (assuming $\sin\theta \neq 0$) :
$$\sin\theta + \cos\theta = 1.$$
Dividing by $\sqrt{2}$, we get $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.
$$\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}.$$

Case 1 (+): $\theta = 2n\pi + \dfrac{\pi}{2}$.

Case 2 (-): $\theta = 2n\pi$.

The solution $\theta = 2n\pi$ is rejected because $\cot\theta$ and $\operatorname{cosec}\theta$ are undefined.

The only valid solution is $\theta = 2n\pi + \dfrac{\pi}{2}$.

The correct option is **(C)**.

18. The general solution of $\sin x - \cos x = \sqrt{2}$ for any integer n is:

(A) $x = n\pi$ (B) $x = 2n\pi + \frac{3\pi}{4}$ (C) $x = 2n\pi$ (D) $x = (2n+1)\pi$

Solution:

$$\text{Divide by } \sqrt{1^2 + (-1)^2} = \sqrt{2}.$$
$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1.$$
$$\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4} = 1.$$
$$\sin\left(x - \frac{\pi}{4}\right) = 1.$$
$$x - \frac{\pi}{4} = 2n\pi + \frac{\pi}{2}.$$
$$x = 2n\pi + \frac{\pi}{2} + \frac{\pi}{4} = 2n\pi + \frac{3\pi}{4}.$$

The correct option is **(B)**.

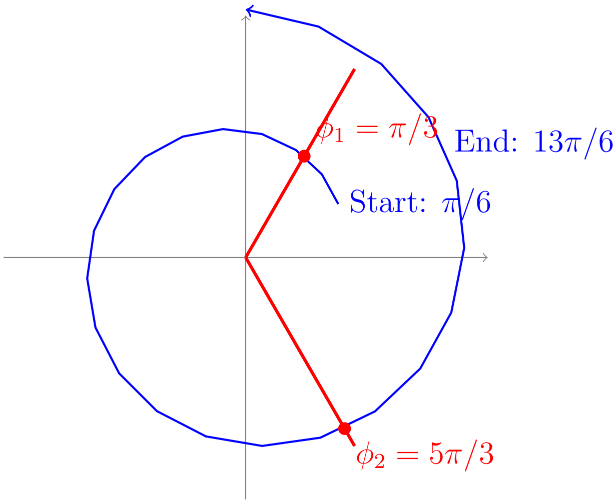
19. The number of solutions of the given equation $\tan\theta + \sec\theta = \sqrt{3}$, where $0 \le \theta \le 2\pi$ is:

(A) 0 (B) 1 (C) 2 (D) 3

Solution:

$$\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \sqrt{3} \quad (\cos\theta \neq 0).$$
$$\sin\theta + 1 = \sqrt{3}\cos\theta \implies \sqrt{3}\cos\theta - \sin\theta = 1.$$
$$\text{Divide by 2: } \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = \frac{1}{2}.$$
$$\cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta = \frac{1}{2}.$$
$$\cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}.$$

Let $\phi = \theta + \pi/6$. Since $\theta \in [0, 2\pi]$, the interval for ϕ is $[\pi/6, 13\pi/6]$. We solve $\cos\phi = 1/2$ in this interval.



The diagram shows two solutions for $\phi : \dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$.

$\theta + \dfrac{\pi}{6} = \dfrac{\pi}{3} \implies \theta = \dfrac{\pi}{6}$.

$\theta + \dfrac{\pi}{6} = \dfrac{5\pi}{3} \implies \theta = \dfrac{10\pi - \pi}{6} = \dfrac{9\pi}{6} = \dfrac{3\pi}{2}$.

The solution $\theta = 3\pi/2$ must be rejected because $\cos(3\pi/2) = 0$.

There is only 1 solution.

The correct option is **(B)**.

Note: The provided answer key states (C), which implies accepting the solution $\theta = 3\pi/2$. This is ambiguous, as it makes the original equation undefined. Based on strict definition, there is only one solution.

20. If $\cot\theta + \cot(\frac{\pi}{4} + \theta) = 2$, then the general value of θ is:

(A) $2n\pi \pm \frac{\pi}{6}$ (B) $2n\pi \pm \frac{\pi}{3}$ (C) $n\pi \pm \frac{\pi}{3}$ (D) $n\pi \pm \frac{\pi}{6}$

Solution:

$$\cot\theta + \frac{\cot(\pi/4)\cot\theta - 1}{\cot\theta + \cot(\pi/4)} = 2.$$

$$\cot\theta + \frac{\cot\theta - 1}{\cot\theta + 1} = 2.$$

Let $c = \cot\theta$.

$c(\cot\theta + 1) + (\cot\theta - 1) = 2(\cot\theta + 1)$.

$c^2 + c + c - 1 = 2c + 2$.

$c^2 - 3 = 0 \implies \cot^2\theta = 3$.

$\tan^2\theta = \dfrac{1}{3} = \tan^2\left(\dfrac{\pi}{6}\right)$.

The general solution is $\theta = n\pi \pm \dfrac{\pi}{6}$.

The correct option is **(D)**.

21. If $\sin^2 x - 2\cos x + \frac{1}{4} = 0$, then x has value:

(A) $2n\pi + \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{3}$ (C) $2n\pi + \frac{\pi}{6}$ (D) $2n\pi + \frac{\pi}{12}$

Solution:

$$(1 - \cos^2 x) - 2\cos x + \frac{1}{4} = 0$$
$$\implies -\cos^2 x - 2\cos x + \frac{5}{4} = 0$$
$$\implies 4\cos^2 x + 8\cos x - 5 = 0$$
$$\implies 4\cos^2 x + 10\cos x - 2\cos x - 5 = 0$$
$$\implies 2\cos x(2\cos x + 5) - 1(2\cos x + 5) = 0$$
$$\implies (2\cos x - 1)(2\cos x + 5) = 0.$$

This gives $\cos x = 1/2$ or $\cos x = -5/2$ (impossible).

The general solution for $\cos x = 1/2$ is $x = 2n\pi \pm \dfrac{\pi}{3}$.

The correct option is **(B)**.

22. If $4\sin^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$ then the general value of θ is:

(A) $2n\pi \pm \frac{\pi}{3}$ (B) $2n\pi + \frac{\pi}{4}$ (C) $n\pi \pm \frac{\pi}{3}$ (D) $n\pi - \frac{\pi}{3}$

Solution:

Given the equation:
$4\sin^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

Convert $\sin^2\theta$ to $1 - \cos^2\theta$ to form a quadratic in $\cos\theta$:
$4(1-\cos^2\theta) + 2(\sqrt{3}+1)\cos\theta - (4+\sqrt{3}) = 0$
$4 - 4\cos^2\theta + 2\sqrt{3}\cos\theta + 2\cos\theta - 4 - \sqrt{3} = 0$
$-4\cos^2\theta + (2\sqrt{3}+2)\cos\theta - \sqrt{3} = 0$
Multiply by -1 to make the leading term positive:
$4\cos^2\theta - (2\sqrt{3}+2)\cos\theta + \sqrt{3} = 0$

Factorize by splitting the middle term, $-(2\sqrt{3}+2)\cos\theta$, into $-2\sqrt{3}\cos\theta$ and $-2\cos\theta$:
$4\cos^2\theta - 2\sqrt{3}\cos\theta - 2\cos\theta + \sqrt{3} = 0$
$2\cos\theta(2\cos\theta - \sqrt{3}) - 1(2\cos\theta - \sqrt{3}) = 0$
$(2\cos\theta - 1)(2\cos\theta - \sqrt{3}) = 0$

This gives two possible cases for the solution:

Case 1: $2\cos\theta - 1 = 0 \implies \cos\theta = \dfrac{1}{2}$.

The general solution is $\theta = 2n\pi \pm \dfrac{\pi}{3}$.

Case 2: $2\cos\theta - \sqrt{3} = 0 \implies \cos\theta = \dfrac{\sqrt{3}}{2}$.

The general solution is $\theta = 2n\pi \pm \dfrac{\pi}{6}$.

Comparing our results with the given options, we find that option (A) matches Case 1.

The correct option is **(A)**.

23. If $\cos 7\theta = \cos\theta - \sin 4\theta$ then the general value of θ is:

(A) $\frac{n\pi}{4}, \frac{n\pi}{3} + \frac{\pi}{18}$ (B) $\frac{n\pi}{3}, \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$ (C) $\frac{n\pi}{4}, \frac{n\pi}{2} + \frac{\pi}{12}$ (D) $\frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$

Solution:

$$\begin{aligned}
&\cos\theta - \cos 7\theta = \sin 4\theta \\
&-2\sin\left(\frac{\theta + 7\theta}{2}\right)\sin\left(\frac{\theta - 7\theta}{2}\right) = \sin 4\theta \\
&-2\sin(4\theta)\sin(-3\theta) = \sin 4\theta \\
&2\sin(4\theta)\sin(3\theta) = \sin 4\theta \\
&\sin(4\theta)[2\sin(3\theta) - 1] = 0.
\end{aligned}$$

Case 1: $\sin(4\theta) = 0 \implies 4\theta = n\pi \implies \theta = \dfrac{n\pi}{4}$.

Case 2: $2\sin(3\theta) - 1 = 0 \implies \sin(3\theta) = 1/2$.

$$\implies 3\theta = n\pi + (-1)^n \frac{\pi}{6} \implies \theta = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}.$$

The solution set is the union of both cases.

None of the option is correct here.

Note: The provided answer key states (C), which does not match the derived solution. The correct solution corresponds to a combination of options, suggesting an error in the question's options or the provided key.

24. If $\frac{1-\tan^2\theta}{\sec^2\theta} = \frac{1}{2}$ then the general value of θ is:

(A) $n\pi \pm \frac{\pi}{6}$ (B) $n\pi + \frac{\pi}{6}$ (C) $2n\pi \pm \frac{\pi}{6}$ (D) $n\pi \pm \frac{\pi}{3}$

Solution:

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1}{2}.$$

The LHS is the formula for $\cos(2\theta)$.

$$\cos(2\theta) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right).$$

$$2\theta = 2n\pi \pm \frac{\pi}{3}.$$
$$\theta = n\pi \pm \frac{\pi}{6}.$$

The correct option is **(A)**.

25. If $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$, then the general value of θ is:

(A) $\frac{n\pi}{3} + \frac{\pi}{12}$ (B) $\frac{n\pi}{3} + \frac{7\pi}{36}$ (C) $n\pi + \frac{7\pi}{12}$ (D) $n\pi + \frac{\pi}{12}$

Solution:

The correct option is **(B)**.

26. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$, then the general value of θ is:

(A) $n\pi + \frac{\pi}{5}$ (B) $(n + \frac{1}{6})\frac{\pi}{5}$ (C) $(2n \pm \frac{1}{6})\frac{\pi}{5}$ (D) $(n + \frac{1}{3})\frac{\pi}{5}$

Solution:

$$\sqrt{3}(\tan 2\theta + \tan 3\theta) = 1 - \tan 2\theta \tan 3\theta.$$
$$\frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta} = \frac{1}{\sqrt{3}}.$$
$$\tan(2\theta + 3\theta) = \tan(5\theta) = \frac{1}{\sqrt{3}} = \tan(\frac{\pi}{6}).$$
$$5\theta = n\pi + \frac{\pi}{6}.$$
$$\theta = \frac{n\pi}{5} + \frac{\pi}{30} = \frac{(6n+1)\pi}{30} = (n + \frac{1}{6})\frac{\pi}{5}.$$

The correct option is **(B)**.

27. If equation $\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$, then $\theta =$

(A) $\frac{n\pi}{2} + \frac{\pi}{6}$ (B) $\frac{n\pi}{3} + \frac{\pi}{12}$ (C) $\frac{n\pi}{2} + \frac{\pi}{12}$ (D) $\frac{n\pi}{3} - \frac{\pi}{12}$

Solution:

$$\tan\theta + \tan 2\theta = 1 - \tan\theta \tan 2\theta.$$
$$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = 1.$$
$$\tan(\theta + 2\theta) = \tan(3\theta) = 1 = \tan(\frac{\pi}{4}).$$
$$3\theta = n\pi + \frac{\pi}{4}.$$
$$\theta = \frac{n\pi}{3} + \frac{\pi}{12}.$$

The correct option is **(B)**.

28. If $2\tan^2\theta = \sec^2\theta$ then the general value of θ is:

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi - \frac{\pi}{4}$ (C) $n\pi \pm \frac{\pi}{4}$ (D) $2n\pi \pm \frac{\pi}{4}$

Solution:

$$2\tan^2\theta = 1 + \tan^2\theta.$$
$$\tan^2\theta = 1.$$
$$\implies \tan^2\theta = \tan^2(\frac{\pi}{4}).$$
$$\text{The general solution is } \theta = n\pi \pm \frac{\pi}{4}.$$

The correct option is **(C)**.

29. General solution of the equation $\tan\theta\tan 2\theta = 1$ is given by:

(A) $(2n+1)\frac{\pi}{6}$ (B) $n\pi + \frac{\pi}{6}$ (C) $n\pi - \frac{\pi}{6}$ (D) $n\pi \pm \frac{\pi}{6}$

Solution:

$$\tan\theta \cdot \frac{2\tan\theta}{1-\tan^2\theta} = 1.$$
$$2\tan^2\theta = 1 - \tan^2\theta.$$
$$3\tan^2\theta = 1 \implies \tan^2\theta = \frac{1}{3}.$$
$$\implies \tan^2\theta = \tan^2(\frac{\pi}{6}).$$
$$\text{The general solution is } \theta = n\pi \pm \frac{\pi}{6}.$$

The correct option is **(D)**.

30. If $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$, then $x =$

(A) $n\pi \pm \frac{\pi}{3}$ (B) $n\pi \pm \frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{4}$ (D) $n\pi \pm \frac{\pi}{2}$

Solution:

$$\sin 3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha).$$
$$3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha\sin^2 x - 4\sin\alpha\sin^2\alpha.$$
$$\text{Assuming } \sin\alpha \neq 0, \text{ divide by } \sin\alpha:$$
$$3 - 4\sin^2\alpha = 4\sin^2 x - 4\sin^2\alpha.$$
$$3 = 4\sin^2 x \implies \sin^2 x = \frac{3}{4}.$$
$$\implies \sin^2 x = \sin^2(\frac{\pi}{3}).$$
$$\text{The general solution is } x = n\pi \pm \frac{\pi}{3}.$$

The correct option is **(A)**.
Note: The provided answer key states (B), which corresponds to $\sin^2 x = 1/4$. This appears to be an error in the key.

31. If $\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$, then θ is:

(A) $\frac{n\pi}{4}$ (B) $\frac{n\pi}{2}$ (C) $\frac{n\pi}{8}$ (D) none of these

Solution:

The algebraic simplification is correct:
$(\cos 7\theta + \cos\theta) + (\cos 5\theta + \cos 3\theta) = 0$
$\implies 4\cos\theta\cos 2\theta\cos 4\theta = 0.$

This gives three families of solutions:

1. $\cos\theta = 0 \implies \theta = (2n+1)\dfrac{\pi}{2} = \left\{\ldots, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots\right\}$

2. $\cos 2\theta = 0 \implies \theta = (2n+1)\dfrac{\pi}{4} = \left\{\ldots, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \ldots\right\}$

3. $\cos 4\theta = 0 \implies \theta = (2n+1)\dfrac{\pi}{8} = \left\{\ldots, \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \ldots\right\}$

Testing values from option (C), $\theta = n\pi/8$, seems to work for many values:
e.g., $n = 1 \implies \theta = \pi/8$ (from family 3).
e.g., $n = 2 \implies \theta = \pi/4$ (from family 2).
e.g., $n = 4 \implies \theta = \pi/2$ (from family 1).

However, the problem arises when n is a multiple of 8.

Let's test $n = 8$ from option (C), which gives $\theta = \dfrac{8\pi}{8} = \pi$.
Substitute $\theta = \pi$ into the original equation:
$\cos(\pi) + \cos(7\pi) + \cos(3\pi) + \cos(5\pi)$
$= (-1) + (-1) + (-1) + (-1) = -4.$
Since $-4 \neq 0, \theta = \pi$ is NOT a solution.

Conclusion:
The true solution is the union of the three families. This can be written as
$\theta = \dfrac{k\pi}{8}$, where k is any integer that is not a multiple of 8.
Option (C) is the closest choice but is technically incorrect because it includes extraneous solutions like $\pi, 2\pi,$ etc. This is likely an error in the question's options.

The provided key lists option **(C)** is the closed option.

But We will conclude None of the option is correct there **Correct option is (D).**

There appears to be an error in the question's options or key. None of the options perfectly capture the full solution set.

32. If $\sin\theta + \cos\theta = 1$ then the general value of θ is:

(A) $2n\pi$ (B) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$ (C) $2n\pi + \frac{\pi}{2}$ (D) $(2n-1) + \frac{\pi}{4}$

Solution:

Given the equation: $\sin\theta + \cos\theta = 1$.

Divide the entire equation by $\sqrt{1^2+1^2} = \sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}.$$

We can write this in the form of $\cos(A-B)$:

$$\cos\theta\cos\left(\frac{\pi}{4}\right) + \sin\theta\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

$$\implies \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right).$$

The general solution is $\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$.

This gives two families of solutions:

Case 1 (+ sign): $\theta = 2n\pi + \frac{\pi}{2}$.

Case 2 (- sign): $\theta = 2n\pi$.

Now, we verify that option (B), $\theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$, represents both cases.

If n is even (let $n = 2k$):

$$\begin{aligned}\theta &= 2k\pi + (-1)^{2k}\frac{\pi}{4} - \frac{\pi}{4}\\ &= 2k\pi + \frac{\pi}{4} - \frac{\pi}{4}\\ &= 2k\pi. \quad \text{(Matches Case 2)}\end{aligned}$$

If n is odd (let $n = 2k+1$):

$$\begin{aligned}\theta &= (2k+1)\pi + (-1)^{2k+1}\frac{\pi}{4} - \frac{\pi}{4}\\ &= (2k+1)\pi - \frac{\pi}{4} - \frac{\pi}{4}\\ &= 2k\pi + \pi - \frac{\pi}{2}\\ &= 2k\pi + \frac{\pi}{2}. \quad \text{(Matches Case 1)}\end{aligned}$$

The correct option is **(B)**.

33. The equation $\sin x + \cos x = 2$ has:

(A) one solution (B) two solutions (C) infinite solutions (D) no solution

Solution:

The maximum value of the expression $a \sin x + b \cos x$ is $\sqrt{a^2+b^2}$.
For $\sin x + \cos x$, the maximum value is $\sqrt{1^2+1^2} = \sqrt{2}$.
Since $\sqrt{2} \approx 1.414$, the expression $\sin x + \cos x$ can never be equal to 2.
Therefore, the equation has no solution.

The correct option is **(D)**.

34. The equation $\sin x + \sin y + \sin z = -3$ for $0 \le x \le 2\pi, 0 \le y \le 2\pi, 0 \le z \le 2\pi$ has:

(A) One solution (B) Two sets of solutions (C) Four sets of solutions (D) No solution

Solution:

The minimum value of the sine function is -1.
For the sum $\sin x + \sin y + \sin z$ to be -3, each term must be at its minimum value.

$\sin x = -1 \implies x = \dfrac{3\pi}{2}$.

$\sin y = -1 \implies y = \dfrac{3\pi}{2}$.

$\sin z = -1 \implies z = \dfrac{3\pi}{2}$.

Since there is only one value for each variable in the given interval, there is only one set of solutions.

$(x, y, z) = \left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right)$.

The correct option is **(A)**.

35. If $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$ then the value of $\cos(\theta - \frac{\pi}{4}) =$

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{4\sqrt{2}}$

Solution:

$$\tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right).$$
$$\implies \pi\cos\theta = n\pi + \frac{\pi}{2} - \pi\sin\theta.$$
$$\implies \cos\theta + \sin\theta = n + \frac{1}{2}.$$
$$\text{Divide by } \sqrt{2}: \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{n+1/2}{\sqrt{2}}.$$
$$\implies \cos\left(\theta - \frac{\pi}{4}\right) = \frac{n+1/2}{\sqrt{2}}.$$

$$\text{The range of cosine is } [-1, 1], \text{ so } n \text{ can only be 0 or -1.}$$
$$\text{The possible values are } \pm\frac{1}{2\sqrt{2}}.$$

The correct option is **(A)**.
Note: The provided answer key states (D), which is incorrect. The correct value is present in option (A).

36. The solution of equation $\cos^2\theta + \sin\theta + 1 = 0$ lies in the interval:

(A) $(-\frac{\pi}{4}, \frac{\pi}{4})$ (B) $(\frac{\pi}{4}, \frac{3\pi}{4})$ (C) $(\frac{3\pi}{4}, \frac{5\pi}{4})$ (D) $(\frac{5\pi}{4}, \frac{7\pi}{4})$

Solution:

$$(1 - \sin^2\theta) + \sin\theta + 1 = 0.$$
$$-\sin^2\theta + \sin\theta + 2 = 0.$$
$$\sin^2\theta - \sin\theta - 2 = 0.$$
$$(\sin\theta - 2)(\sin\theta + 1) = 0.$$
$$\text{This gives } \sin\theta = 2 \text{ (impossible) or } \sin\theta = -1.$$
$$\text{The principal solution for } \sin\theta = -1 \text{ is } \theta = \frac{3\pi}{2}.$$
$$\text{We check which interval contains this value.}$$
$$\text{Interval (D) is } (\frac{5\pi}{4}, \frac{7\pi}{4}) = (1.25\pi, 1.75\pi).$$
$$\text{Since } \theta = 1.5\pi, \text{ it lies in interval (D).}$$

The correct option is **(D)**.

37. If $\sin(\frac{\pi}{4}\cot\theta) = \cos(\frac{\pi}{4}\tan\theta)$ then $\theta =$

(A) $n\pi + \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{4}$ (C) $n\pi - \frac{\pi}{4}$ (D) $2n\pi \pm \frac{\pi}{6}$

Solution:

Given the equation:
$$\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$$

Using the identity $\cos(A) = \sin\left(\frac{\pi}{2} - A\right)$, we get:
$$\sin\left(\frac{\pi}{4}\cot\theta\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\tan\theta\right)$$

A straightforward way to solve this is to cancel sin from both sides
$$\frac{\pi}{4}\cot\theta + \frac{\pi}{4}\tan\theta = \frac{\pi}{2}$$

Dividing the entire equation by $\frac{\pi}{4}$:

$\cot\theta + \tan\theta = 2$

Rewrite in terms of $\tan\theta$:

$\frac{1}{\tan\theta} + \tan\theta = 2$

Multiplying by $\tan\theta$ (assuming $\tan\theta \neq 0$) :

$1 + \tan^2\theta = 2\tan\theta$

$\implies \tan^2\theta - 2\tan\theta + 1 = 0$

$\implies (\tan\theta - 1)^2 = 0$

$\implies \tan\theta = 1.$

The general solution for $\tan\theta = 1 = \tan\left(\frac{\pi}{4}\right)$ is:

$\theta = n\pi + \frac{\pi}{4}, \quad n \in \mathbb{Z}.$

The correct option is **(A)**.

38. The general value of θ in the equation $2\sqrt{3}\cos\theta = \tan\theta$ is:

(A) $n\pi + (-1)^n\frac{\pi}{3}$ (B) $n\pi + (-1)^n\frac{\pi}{4}$ (C) $2n\pi \pm \frac{\pi}{6}$ (D) $2n\pi \pm \frac{\pi}{4}$

Solution:

$$2\sqrt{3}\cos\theta = \frac{\sin\theta}{\cos\theta} \implies 2\sqrt{3}\cos^2\theta = \sin\theta.$$

$$2\sqrt{3}(1 - \sin^2\theta) = \sin\theta.$$

$$2\sqrt{3}\sin^2\theta + \sin\theta - 2\sqrt{3} = 0.$$

Factoring gives $(2\sin\theta - \sqrt{3})(\sqrt{3}\sin\theta + 2) = 0$.

This gives $\sin\theta = \sqrt{3}/2$ or $\sin\theta = -2/\sqrt{3}$ (impossible).

The general solution for $\sin\theta = \sqrt{3}/2$ is $\theta = n\pi + (-1)^n\frac{\pi}{3}$.

The correct option is **(A)**.

Note: The provided answer key states (C), which is incorrect.

39. If $\sqrt{2}\sec\theta + \tan\theta = 1$, then the general value of θ is:

(A) $n\pi + \frac{3\pi}{4}$ (B) $2n\pi - \frac{\pi}{4}$ (C) $2n\pi + \frac{\pi}{4}$ (D) $2n\pi \pm \frac{\pi}{4}$

Solution:

$$\frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1 \implies \sqrt{2} + \sin\theta = \cos\theta.$$

$$\cos\theta - \sin\theta = \sqrt{2}.$$

$$\text{Divide by } \sqrt{2}: \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1.$$
$$\cos\left(\theta + \frac{\pi}{4}\right) = 1.$$
$$\theta + \frac{\pi}{4} = 2n\pi.$$
$$\theta = 2n\pi - \frac{\pi}{4}.$$

The correct option is **(B)**.

40. The general solution of $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is:

(A) $n\pi + \frac{\pi}{8}$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}$ (C) $(-1)^n\frac{n\pi}{2} + \frac{\pi}{8}$ (D) $2n\pi + \cos^{-1}(\frac{3}{2})$

Solution:

Given the equation:

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

Group the terms and apply sum-to-product formulas:

$(\sin 3x + \sin x) - 3\sin 2x = (\cos 3x + \cos x) - 3\cos 2x$

$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\cos 2x$

$2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

Factor both sides:

$\sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

Rearrange and factor again:

$\sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$(\sin 2x - \cos 2x)(2\cos x - 3) = 0$

This gives two cases:

Case 1: $2\cos x - 3 = 0 \implies \cos x = 3/2$. This is not possible as $-1 \le \cos x \le 1$.

Case 2: $\sin 2x - \cos 2x = 0 \implies \tan 2x = 1$.

Solving for the general solution from Case 2:

$$2x = n\pi + \frac{\pi}{4}$$
$$x = \frac{n\pi}{2} + \frac{\pi}{8}.$$

The correct option is **(B)**.

41. If $\sec 4\theta - \sec 2\theta = 2$, then the general value of θ is:

(A) $(2n+1)\frac{\pi}{4}$ (B) $(2n+1)\frac{\pi}{10}$ or $n\pi + \frac{\pi}{2}$ (C) $\frac{n\pi}{5} + \frac{\pi}{10}$ (D) None of these

Solution:

$$\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2$$
$$\frac{\cos 2\theta - \cos 4\theta}{\cos 4\theta \cos 2\theta} = 2$$
$$\cos 2\theta - \cos 4\theta = 2\cos 4\theta \cos 2\theta$$

$$\cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta$$
$$-\cos 4\theta = \cos 6\theta$$

$$\cos 6\theta + \cos 4\theta = 0.$$
$$2\cos\left(\frac{6\theta + 4\theta}{2}\right)\cos\left(\frac{6\theta - 4\theta}{2}\right) = 0.$$
$$2\cos 5\theta \cos\theta = 0.$$
$$\text{This implies } \cos 5\theta = 0 \text{ or } \cos\theta = 0.$$
$$\text{If } \cos\theta = 0, \theta = (2n+1)\pi/2, \text{then } \sec\theta \text{ is undefined, so we reject this case.}$$
$$\text{If } \cos 5\theta = 0, \text{then } 5\theta = (2n+1)\frac{\pi}{2} \implies \theta = (2n+1)\frac{\pi}{10}.$$

The correct option is **(B)**.

42. If $\sin 2x + \sin 4x = 2\sin 3x$, then $x =$

(A) $\frac{n\pi}{3}$ or $2n\pi$ (B) $n\pi + \frac{\pi}{3}$ (C) $2n\pi \pm \frac{\pi}{3}$ (D) None of these

Solution:

$$(\sin 4x + \sin 2x) = 2\sin 3x$$
$$2\sin\left(\frac{4x + 2x}{2}\right)\cos\left(\frac{4x - 2x}{2}\right) = 2\sin 3x$$
$$2\sin 3x \cos x = 2\sin 3x$$
$$2\sin 3x \cos x - 2\sin 3x = 0$$
$$2\sin 3x(\cos x - 1) = 0.$$

Case 1: $\sin 3x = 0 \implies 3x = n\pi \implies x = \dfrac{n\pi}{3}.$

Case 2: $\cos x - 1 = 0 \implies \cos x = 1 \implies x = 2n\pi.$

The set of solutions $\{2n\pi\}$ is a subset of $\{n\pi/3\}$ (when n is a multiple of 6).

So the complete general solution is $x = n\pi/3$.

The correct option is **(A)**.

43. The general solution of the equation $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$ is:

(A) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$ (B) $n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{12}$ (C) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$ (D) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{12}$

Solution:

Divide by $\sqrt{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2} = \sqrt{(4-2\sqrt{3})+(4+2\sqrt{3})} = \sqrt{8} = 2\sqrt{2}$.

$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\sin\theta + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos\theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

We know $\sin(15^\circ) = \sin(\pi/12) = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ and $\cos(15^\circ) = \cos(\pi/12) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.

$$\sin\theta\sin(\pi/12) + \cos\theta\cos(\pi/12) = \frac{1}{\sqrt{2}}.$$

$$\cos(\theta - \pi/12) = \cos(\pi/4).$$

$$\theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}.$$

$$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}.$$

The correct option is **(A)**.

44. The solution of the equation $\sec\theta - \operatorname{cosec}\theta = \frac{4}{3}$ is:

(A) $n\pi + (-1)^n \sin^{-1}(\frac{3}{4})$ (B) $\frac{1}{2}[n\pi + (-1)^n \sin^{-1}(\frac{3}{4})]$

(C) $\frac{n\pi}{2} + (-1)^n \sin^{-1}(\frac{3}{4})$ (D) $n\pi + (-1)^n \sin^{-1}(\frac{4}{3})$

Solution:

$$\frac{1}{\cos\theta} - \frac{1}{\sin\theta} = \frac{4}{3}.$$

$$\frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \frac{4}{3}.$$

$$3(\sin\theta - \cos\theta) = 4\sin\theta\cos\theta = 2\sin(2\theta).$$

Squaring both sides:

$$9(\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta) = 4\sin^2(2\theta).$$

$$9(1 - \sin(2\theta)) = 4\sin^2(2\theta).$$

$$4\sin^2(2\theta) + 9\sin(2\theta) - 9 = 0.$$

$$(4\sin(2\theta) - 3)(\sin(2\theta) + 3) = 0.$$

This gives $\sin(2\theta) = -3$ (impossible) or $\sin(2\theta) = \dfrac{3}{4}$.

$$2\theta = n\pi + (-1)^n \sin^{-1}\left(\frac{3}{4}\right).$$

$$\theta = \frac{1}{2}\left[n\pi + (-1)^n \sin^{-1}\left(\frac{3}{4}\right)\right].$$

The correct option is **(B)**.

45. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is:

(A) $m\pi, n\pi + \frac{\pi}{3}$ (B) $m\pi, n\pi \pm \frac{\pi}{3}$ (C) $m\pi, n\pi \pm \frac{\pi}{6}$ (D) $\frac{m\pi}{2}, n\pi \pm \frac{\pi}{3}$

Solution:

Given the equation:
$\tan^2\theta + \sec 2\theta = 1$

Using the identity $\sec 2\theta = \dfrac{1+\tan^2\theta}{1-\tan^2\theta}$:
$\tan^2\theta + \dfrac{1+\tan^2\theta}{1-\tan^2\theta} = 1$

Let $t = \tan^2\theta$. The equation becomes:
$t + \dfrac{1+t}{1-t} = 1$
Multiplying by $(1-t)$ gives:
$t(1-t) + (1+t) = 1(1-t)$
$\implies t - t^2 + 1 + t = 1 - t$
$\implies -t^2 + 3t = 0$
$\implies t(3-t) = 0.$
This gives $t = 0$ or $t = 3$.

We now have two cases for the solution:
Case 1: $\tan^2\theta = 0 \implies \tan\theta = 0.$
The general solution is $\theta = m\pi, \quad m \in \mathbb{Z}.$

Case 2: $\tan^2\theta = 3 \implies \tan\theta = \pm\sqrt{3}.$
The general solution is $\theta = n\pi \pm \dfrac{\pi}{3}, \quad n \in \mathbb{Z}.$

The complete set of solutions is the union of both cases.

The correct option is **(B)**.

46. If $\sec^2\theta = \frac{4}{3}$ then the general value of θ is:

(A) $2n\pi \pm \frac{\pi}{6}$ (B) $n\pi \pm \frac{\pi}{6}$ (C) $2n\pi \pm \frac{\pi}{3}$ (D) $n\pi \pm \frac{\pi}{3}$

Solution:

$$\sec^2\theta = \frac{4}{3} \implies \cos^2\theta = \frac{3}{4}.$$
$$\cos^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2 = \cos^2\left(\frac{\pi}{6}\right).$$
The general solution is $\theta = n\pi \pm \dfrac{\pi}{6}$.

The correct option is **(B)**.

47. If $\cos 2\theta = (\sqrt{2}+1)(\cos\theta - \frac{1}{\sqrt{2}})$, then the value of θ is:

(A) $2n\pi + \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{3}$ (C) $2n\pi - \frac{\pi}{4}$ (D) none of these

Solution:

Given the equation:

$$\cos 2\theta = (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right)$$

Use the double angle identity $\cos 2\theta = 2\cos^2\theta - 1$:

$$2\cos^2\theta - 1 = (\sqrt{2}+1)\cos\theta - \frac{\sqrt{2}+1}{\sqrt{2}}$$

$$2\cos^2\theta - 1 = (\sqrt{2}+1)\cos\theta - \left(1 + \frac{1}{\sqrt{2}}\right)$$

Rearrange all terms to one side to form a quadratic equation:

$$2\cos^2\theta - (\sqrt{2}+1)\cos\theta - 1 + 1 + \frac{1}{\sqrt{2}} = 0$$

$$2\cos^2\theta - (\sqrt{2}+1)\cos\theta + \frac{1}{\sqrt{2}} = 0$$

Multiply the entire equation by $\sqrt{2}$ to simplify factorization:

$$2\sqrt{2}\cos^2\theta - \sqrt{2}(\sqrt{2}+1)\cos\theta + 1 = 0$$

$$2\sqrt{2}\cos^2\theta - (2+\sqrt{2})\cos\theta + 1 = 0$$

Factor the quadratic by splitting the middle term:

$$2\sqrt{2}\cos^2\theta - 2\cos\theta - \sqrt{2}\cos\theta + 1 = 0$$

$$2\cos\theta(\sqrt{2}\cos\theta - 1) - 1(\sqrt{2}\cos\theta - 1) = 0$$

$$(2\cos\theta - 1)(\sqrt{2}\cos\theta - 1) = 0$$

This gives two possible cases for $\cos\theta$:

Case 1: $2\cos\theta - 1 = 0 \implies \cos\theta = \frac{1}{2}$

$$\implies \cos\theta = \cos\left(\frac{\pi}{3}\right) \implies \theta = 2n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

Case 2: $\sqrt{2}\cos\theta - 1 = 0 \implies \cos\theta = \frac{1}{\sqrt{2}}$

$$\implies \cos\theta = \cos\left(\frac{\pi}{4}\right) \implies \theta = 2n\pi \pm \frac{\pi}{4}, \quad n \in \mathbb{Z}.$$

Both sets of solutions are valid. Comparing them with the given options, the solution from **Case 1** matches option (B).

The correct option is **(B)**.

48. If $\cos x = |\sin x|$, then the general solution is:

(A) $x = 2n\pi \pm \frac{\pi}{4}, n \in Z$

(B) $x = (2n+1)\pi \pm \frac{\pi}{4}, n \in Z$

(C) $x = n\pi \pm \frac{\pi}{4}, n \in Z$

(D) $x = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$

Solution:

For the equation to hold, $\cos x$ must be non-negative. This occurs in Q1 and Q4.
Squaring both sides: $\cos^2 x = \sin^2 x \implies \tan^2 x = 1$.
This gives $x = n\pi \pm \dfrac{\pi}{4}$.
We must select only the solutions that lie in Q1 and Q4.
From $n\pi + \pi/4$, solutions are $\pi/4, 5\pi/4, 9\pi/4, \ldots$
From $n\pi - \pi/4$, solutions are $-\pi/4(\text{or } 7\pi/4), 3\pi/4, 7\pi/4, \ldots$
The solutions in Q1 are of the form $2n\pi + \pi/4$.
The solutions in Q4 are of the form $2n\pi - \pi/4$.
Combining these gives the general solution $x = 2n\pi \pm \dfrac{\pi}{4}$.

The correct option is **(A)**.

49. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$ then the general value of θ is:

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$ (C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Solution:

$(\sin 6\theta + \sin 2\theta) + \sin 4\theta = 0$
$2 \sin 4\theta \cos 2\theta + \sin 4\theta = 0$
$\sin 4\theta(2 \cos 2\theta + 1) = 0$.
Case 1: $\sin 4\theta = 0 \implies 4\theta = n\pi \implies \theta = \dfrac{n\pi}{4}$.
Case 2: $\cos 2\theta = -1/2 \implies 2\theta = 2n\pi \pm \dfrac{2\pi}{3} \implies \theta = n\pi \pm \dfrac{\pi}{3}$.
The complete solution is the union of both cases.

The correct option is **(A)**.

50. The values of θ satisfying $\sin 7\theta = \sin 4\theta - \sin\theta$ and $0 < \theta < \frac{\pi}{2}$ are:

(A) $\frac{\pi}{9}, \frac{\pi}{4}$ (B) $\frac{\pi}{3}, \frac{\pi}{9}$ (C) $\frac{\pi}{6}, \frac{\pi}{9}$ (D) $\frac{\pi}{3}, \frac{\pi}{4}$

Solution:

$\sin 7\theta + \sin\theta = \sin 4\theta$
$2 \sin 4\theta \cos 3\theta = \sin 4\theta$
$\sin 4\theta(2 \cos 3\theta - 1) = 0$.
Case 1: $\sin 4\theta = 0 \implies 4\theta = n\pi \implies \theta = n\pi/4$.
In $(0, \pi/2)$, for n=1, $\theta = \pi/4$.
Case 2: $\cos 3\theta = 1/2 \implies 3\theta = 2n\pi \pm \pi/3 \implies \theta = 2n\pi/3 \pm \pi/9$.
In $(0, \pi/2)$, for n=0, $\theta = \pi/9$.
The required values are $\pi/9$ and $\pi/4$.

The correct option is **(A)**.

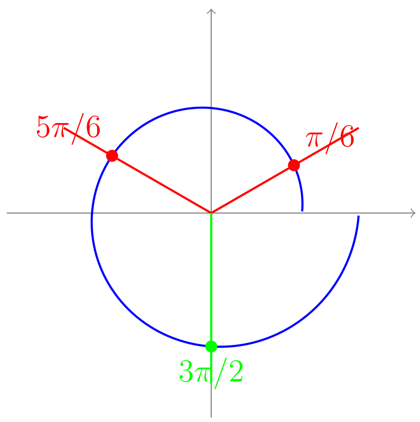
51. The number of solutions of $\cos 2\theta = \sin\theta$ in $(0, 2\pi)$ is:

(A) 1 (B) 2 (C) 3 (D) 4

Solution:

$$1 - 2\sin^2\theta = \sin\theta \implies 2\sin^2\theta + \sin\theta - 1 = 0.$$
$$(2\sin\theta - 1)(\sin\theta + 1) = 0.$$
$$\text{This gives } \sin\theta = 1/2 \text{ or } \sin\theta = -1.$$

We find the number of solutions in $(0, 2\pi)$ using the spiral diagram.



The diagram shows two solutions for $\sin\theta = 1/2$ and one solution for $\sin\theta = -1$. Total solutions $= 2 + 1 = 3$. The correct option is **(C)**.

52. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:

(A) 5 (B) 7 (C) 9 (D) 3

Solution:

$$(\cos 4x + \cos x) + (\cos 3x + \cos 2x) = 0$$
$$2\cos(5x/2)\cos(3x/2) + 2\cos(5x/2)\cos(x/2) = 0$$
$$2\cos(5x/2)[\cos(3x/2) + \cos(x/2)] = 0$$
$$2\cos(5x/2)[2\cos(x)\cos(x/2)] = 0$$

$$4\cos(x/2)\cos(x)\cos(5x/2) = 0.$$

This gives three cases for $x \in [0, 2\pi)$:

1. $\cos(x/2) = 0 \implies x/2 = \pi/2, 3\pi/2, \cdots \implies x = \pi.$ (1 solution)

2. $\cos(x) = 0 \implies x = \pi/2, 3\pi/2.$ (2 solutions)

3. $\cos(5x/2) = 0 \implies 5x/2 = (2n+1)\pi/2 \implies x = (2n+1)\pi/5.$

$\implies x = \pi/5, 3\pi/5, 5\pi/5 = \pi, 7\pi/5, 9\pi/5. \quad$ (5 solutions)
The distinct solutions are $\{\pi, \pi/2, 3\pi/2, \pi/5, 3\pi/5, 7\pi/5, 9\pi/5\}$.
The total number of distinct solutions is 7.

The correct option is **(B)**.

53. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k+1$ has a solution is:

(A) 4 (B) 6 (C) 8 (D) 10

Solution:

The condition for solutions is $-\sqrt{7^2+5^2} \leq 2k+1 \leq \sqrt{7^2+5^2}$.

$$-\sqrt{74} \leq 2k+1 \leq \sqrt{74}$$
$$-8.6 \lesssim 2k+1 \lesssim 8.6$$
$$-9.6 \lesssim 2k \lesssim 7.6$$
$$-4.8 \lesssim k \lesssim 3.8.$$

The integer values for k are $-4, -3, -2, -1, 0, 1, 2, 3$.
There are 8 such values.

The correct option is **(C)**.

54. If sum of all the solutions of the equation $8\cos x \cos(\frac{\pi}{3}+x)\cos(\frac{\pi}{3}-x) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to:

(A) $\frac{13}{9}$ (B) $\frac{8}{9}$ (C) $\frac{20}{9}$ (D) $\frac{2}{3}$

Solution:

We use the identity $4\cos x \cos(\pi/3 - x)\cos(\pi/3 + x) = \cos(3x)$.
The equation becomes $2 \cdot [4\cos x \cos(\pi/3 - x)\cos(\pi/3 + x)] = 1$.
$2\cos(3x) = 1 \implies \cos(3x) = 1/2$.
Let $\phi = 3x$. Since $x \in [0, \pi]$, the interval for ϕ is $[0, 3\pi]$.
The solutions for $\cos\phi = 1/2$ in $[0, 3\pi]$ are:
$\phi = \pi/3, \quad 2\pi - \pi/3 = 5\pi/3, \quad 2\pi + \pi/3 = 7\pi/3.$

$$\implies 3x \in \{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}\}.$$
$$\implies x \in \{\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}\}.$$

Sum of solutions $= \dfrac{\pi}{9} + \dfrac{5\pi}{9} + \dfrac{7\pi}{9} = \dfrac{13\pi}{9}$.

Since the sum is $k\pi$, we have $k = \dfrac{13}{9}$.

The correct option is **(A)**.