Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 3rd August 2025

Syllabus: Inequality, Modulus and Trigonometric Ratios & Identities

Sub: Mathematics **CT-02 JEE Advanced Solution** Prof. Chetan Sir

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41. Let $F(k) = (1+\sin\frac{\pi}{2k})(1+\sin(k-1)\frac{\pi}{2k})(1+\sin(2k+1)\frac{\pi}{2k})(1+\sin(3k-1)\frac{\pi}{2k})$. The value of $F(1)+F(2)+F(3)$ is equal to

(1) 3/16 (2) 1/4 (3) 5/16 (4) 7/16

Solution:

Let's simplify the terms in F(k) using trigonometric identities:

$$\sin\left((k-1)\frac{\pi}{2k}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{2k}\right) = \cos\left(\frac{\pi}{2k}\right).$$

$$\sin\left((2k+1)\frac{\pi}{2k}\right) = \sin\left(\pi + \frac{\pi}{2k}\right) = -\sin\left(\frac{\pi}{2k}\right).$$

$$\sin\left((3k-1)\frac{\pi}{2k}\right) = \sin\left(\frac{3\pi}{2} - \frac{\pi}{2k}\right) = -\cos\left(\frac{\pi}{2k}\right).$$

Substitute these back into the expression for F(k):

$$F(k) = \left(1+\sin\frac{\pi}{2k}\right)\left(1+\cos\frac{\pi}{2k}\right)\left(1-\sin\frac{\pi}{2k}\right)\left(1-\cos\frac{\pi}{2k}\right).$$

$$F(k) = \left(1-\sin^2\frac{\pi}{2k}\right)\left(1-\cos^2\frac{\pi}{2k}\right) = \cos^2\frac{\pi}{2k}\cdot\sin^2\frac{\pi}{2k}.$$

$$F(k) = \left(\frac{1}{2}\cdot 2\sin\frac{\pi}{2k}\cos\frac{\pi}{2k}\right)^2 = \left(\frac{1}{2}\sin\left(2\cdot\frac{\pi}{2k}\right)\right)^2 = \frac{1}{4}\sin^2\left(\frac{\pi}{k}\right).$$

Now, we calculate F(1), F(2), and F(3):

$F(1) = \frac{1}{4}\sin^2(\pi) = 0.$
$F(2) = \frac{1}{4}\sin^2(\pi/2) = \frac{1}{4}(1)^2 = \frac{1}{4}.$
$F(3) = \frac{1}{4}\sin^2(\pi/3) = \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16}.$

The required sum is:
$F(1) + F(2) + F(3) = 0 + \frac{1}{4} + \frac{3}{16} = \frac{4}{16} + \frac{3}{16} = \frac{7}{16}.$

The correct option is **(4)**.

42. Solution of the equation $|x^2 + 6x + 8| = -x^2 - 6x - 8$ is

(1) [2,4] (2) (2,4] (3) [-4,-2] (4) (-4,-2)

Solution:

The given equation is of the form $|A| = -A$.
This identity holds true if and only if the expression inside the absolute value, A, is less than or equal to
$A \leq 0$.

In this problem, $A = x^2 + 6x + 8$.
So, we need to solve the inequality:
$x^2 + 6x + 8 \leq 0$.

Factor the quadratic expression:
$(x + 4)(x + 2) \leq 0$.

The roots of the equation are $x = -4$ and $x = -2$.
Since the parabola $y = x^2 + 6x + 8$ opens upwards, the expression is less than or equal to zero between i
Therefore, the solution is $-4 \leq x \leq -2$.
In interval notation, this is $[-4, -2]$.

The correct option is **(3)**.

43. Range of the values of the expression $y = \sin x \cdot \tan x \cdot \cot x$ is

(1) $(-1, 1) - \{0\}$ (2) (-1,1) (3) [-1,1] (4) $[-1, 1] - \{0\}$

Solution:

First, we determine the domain of the function $y = \sin x \cdot \tan x \cdot \cot x$.
For $\tan x = \frac{\sin x}{\cos x}$ to be defined, $\cos x \neq 0$, so $x \neq (2n + 1)\frac{\pi}{2}$ for any integer n.
For $\cot x = \frac{\cos x}{\sin x}$ to be defined, $\sin x \neq 0$, so $x \neq n\pi$ for any integer n.

Combining these, the domain is all real numbers except $x \neq \frac{n\pi}{2}$.

Within this domain, $\tan x \cdot \cot x = 1$.
So, the function simplifies to $y = \sin x$.

Now, we find the range of $y = \sin x$ for the restricted domain.
The usual range of $\sin x$ is $[-1, 1]$.
However, we must exclude the values of $\sin x$ at the points where the original function is undefined.
At $x = n\pi, \sin x = 0$.
At $x = (2n+1)\frac{\pi}{2}, \sin x = \pm 1$.
Since these values of x are excluded from the domain, the corresponding values of $\sin x$ (which are 0, 1, and -1) are not attainable by the function y.

Therefore, the range of y is all values in $[-1, 1]$ except -1, 0, and 1.
This corresponds to the interval $(-1, 1)$ excluding 0, which is $(-1, 1) - \{0\}$.

The correct option is **(1)**.

44. Some part of the solution of the inequality $|\frac{2x-1}{x-1}| > 2$ lies in

(1) $(\frac{3}{4}, 1)$ (2) $(1, \infty)$ (3) $(\frac{3}{4}, \infty) - \{1\}$ (4) $(-\infty, -2)$

Solution:

The inequality is of the form $|A| > B$. This is equivalent to $A > B$ or $A < -B$.
We must also have $x - 1 \neq 0 \implies x \neq 1$.

Case 1: $\frac{2x-1}{x-1} > 2$.
$\frac{2x-1}{x-1} - 2 > 0 \implies \frac{2x-1-2(x-1)}{x-1} > 0$.
$\frac{2x-1-2x+2}{x-1} > 0 \implies \frac{1}{x-1} > 0 \implies x - 1 > 0 \implies x > 1$.

Case 2: $\frac{2x-1}{x-1} < -2$.
$\frac{2x-1}{x-1} + 2 < 0 \implies \frac{2x-1+2(x-1)}{x-1} < 0$.
$\frac{4x-3}{x-1} < 0$.
The critical points are $x = 3/4$ and $x = 1$. The expression is negative between the roots.
So, $\frac{3}{4} < x < 1$.

The total solution set is the union of the solutions from both cases:

$x \in \left(\frac{3}{4}, 1\right) \cup (1, \infty)$.

This can also be written as $(\frac{3}{4}, \infty) - \{1\}$.

Checking the options:

$(1)(\frac{3}{4}, 1)$ is part of the solution.

$(2)(1, \infty)$ is part of the solution.

$(3)(\frac{3}{4}, \infty) - \{1\}$ is the complete solution set.

The correct options are **(1), (2), and (3)**.

45. If $x \neq \frac{n\pi}{2}$ (n is integer), then the expression $\frac{\sin^3 x}{1+\cos x} + \frac{\cos^3 x}{1-\sin x}$ is equivalent to

(1) $\sqrt{2}\cos(\frac{\pi}{4} - x)$ (2) $\sqrt{2}\cos(\frac{\pi}{4} + x)$ (3) $\sqrt{2}\sin(\frac{\pi}{4} - x)$ (4) $\sqrt{2}\sin(\frac{\pi}{4} + x)$

Solution:

Let the expression be E. We rationalize the denominators.

$$E = \frac{\sin^3 x(1-\cos x)}{(1+\cos x)(1-\cos x)} + \frac{\cos^3 x(1+\sin x)}{(1-\sin x)(1+\sin x)}.$$

$$E = \frac{\sin^3 x(1-\cos x)}{1-\cos^2 x} + \frac{\cos^3 x(1+\sin x)}{1-\sin^2 x}.$$

$$E = \frac{\sin^3 x(1-\cos x)}{\sin^2 x} + \frac{\cos^3 x(1+\sin x)}{\cos^2 x}.$$

Since $x \neq n\pi/2, \sin x \neq 0$ and $\cos x \neq 0$, so we can cancel.

$E = \sin x(1-\cos x) + \cos x(1+\sin x)$.

$E = \sin x - \sin x \cos x + \cos x + \cos x \sin x$.

$E = \sin x + \cos x$.

Now we convert this to the form given in the options.

$$E = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right).$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x\right) = \sqrt{2}\sin(x + \frac{\pi}{4}).$$

Alternatively,

$$E = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right).$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right) = \sqrt{2}\cos(x - \frac{\pi}{4}) = \sqrt{2}\cos(\frac{\pi}{4} - x).$$

The correct options are **(1)** and **(4)**.

46. Some part of the solution of the inequality $|\frac{x^2-3x-1}{x^2+x+1}| < 3$ lies in

(1) $(-\infty,-2)$ (2) (-2,-1) (3) (-1,∞) (4) $(\frac{-3}{2},\frac{-10}{9})$

Solution:

The denominator x^2+x+1 has discriminant $D=1^2-4(1)(1)=-3<0$.
Since the leading coefficient is positive, the denominator is always positive.
We can multiply by it: $|x^2-3x-1|<3(x^2+x+1)$.
This is equivalent to $-3(x^2+x+1)<x^2-3x-1<3(x^2+x+1)$.

Part 1: $-3x^2-3x-3<x^2-3x-1$.
$0<4x^2+2 \implies 2x^2+1>0$. This is true for all real x.

Part 2: $x^2-3x-1<3x^2+3x+3$.
$0<2x^2+6x+4 \implies 0<x^2+3x+2 \implies (x+1)(x+2)>0$.
This is true for $x\in(-\infty,-2)\cup(-1,\infty)$.

The overall solution is the intersection of both parts, which is $(-\infty,-2)\cup(-1,\infty)$.
Checking the options:
(1)$(-\infty,-2)$ is part of the solution.
(3)$(-1,\infty)$ is part of the solution.

The correct options are **(1)** and **(3)**.

47. Which of the following is/are true?

(1) $\sin 1>\sin 1^\circ$ (2) ... (3) ... (4) ...

Solution:

(1) $\sin 1>\sin 1^\circ$:
1 radian $\approx 57.3^\circ$. Both 1° and 57.3° are in the first quadrant, where sine is an increasing function.
Since $57.3^\circ>1^\circ$, it follows that $\sin(57.3^\circ)>\sin(1^\circ)$, so $\sin 1>\sin 1^\circ$. (True)

(2) $\sin 1+\cos 1^\circ>\cos 1+\sin 1^\circ$:
$\sin 1-\sin 1^\circ>\cos 1-\cos 1^\circ$.
$2\cos\frac{1+1^\circ}{2}\sin\frac{1-1^\circ}{2}>-2\sin\frac{1+1^\circ}{2}\sin\frac{1-1^\circ}{2}$.
Since $1\approx 57.3^\circ$, $\frac{1-1^\circ}{2}\approx 28.15^\circ>0$, so $\sin\frac{1-1^\circ}{2}>0$.
We can divide by $2\sin\frac{1-1^\circ}{2}$: $\cos\frac{1+1^\circ}{2}>-\sin\frac{1+1^\circ}{2}$.
Let $\alpha=\frac{1+1^\circ}{2}\approx 29.15^\circ$. We check if $\cos\alpha>-\sin\alpha$,
which is always true for an acute angle α. (True)

(3) $\cos 2\cdot\cos 2^\circ<\sin 2$:
2 radians $\approx 114.6^\circ$, which is in Q2. So $\cos 2<0$ and $\sin 2>0$.

$\cos 2^\circ > 0$. Therefore, LHS is negative and RHS is positive. The inequality is true. (True)

(4) $\frac{\sin 1}{\cos 1} > \frac{\sin 2^\circ}{\cos 2^\circ} \implies \tan 1 > \tan 2^\circ$:
1 radian $\approx 57.3^\circ$. Both angles are in Q1 where tangent is increasing.
Since $57.3^\circ > 2^\circ, \tan(57.3^\circ) > \tan(2^\circ) \implies \tan 1 > \tan 2^\circ$. (True)

The correct options are **(1), (2), (3), and (4)**.

48. Some part of the solution of the inequality $|x^2 + x| - 5 < 0$ is

(1) $[\frac{-1-\sqrt{21}}{2}, 0]$ (2) $[\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}]$ (3) $[10, \infty)$ (4) $(-\infty, -10)$

Solution:

$|x^2 + x| < 5 \implies -5 < x^2 + x < 5.$

Part 1: $x^2 + x > -5 \implies x^2 + x + 5 > 0.$
Discriminant $D = 1^2 - 4(1)(5) = -19 < 0$. Since leading coefficient is positive, this is true for all real x.

Part 2: $x^2 + x < 5 \implies x^2 + x - 5 < 0.$

The roots of $x^2 + x - 5 = 0$ are $x = \frac{-1 \pm \sqrt{1 - 4(1)(-5)}}{2} = \frac{-1 \pm \sqrt{21}}{2}$.
The inequality holds between the roots.
$x \in \left(\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}\right)$.

The solution is $\left(\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}\right)$.
We have $\sqrt{16} < \sqrt{21} < \sqrt{25}$, so $4 < \sqrt{21} < 5$.
$\frac{-1-4.6}{2} \approx -2.8$ and $\frac{-1+4.6}{2} \approx 1.8$.
The solution is approximately $(-2.8, 1.8)$.
Option (1) $[\frac{-1-\sqrt{21}}{2}, 0]$ is a closed interval contained within our open interval solution.
Option (2) $[\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}]$ is the closed interval corresponding to the roots.

The solution set is an open interval. Options (1) and (2) are closed intervals. If the original inequality was $\leq$, then (2) would be correct. As written, any sub-interval lies "in" the solution. The correct options are **(1)** and **(2)**.

49. If $|\frac{x^2-5x+4}{x^2-4}| \leq 1$ then it's solution is

(1) $[0, \frac{8}{5}] \cup [\frac{5}{2}, \infty)$ (2) $(-\infty, \dots)$ (3) $[\frac{8}{5}, \frac{5}{2}]$ (4) $[0, \dots)$

Solution:

$|x^2-5x+4| \le |x^2-4|$, with $x \neq \pm 2$.
$(x^2-5x+4)^2 \le (x^2-4)^2$.
$(x^2-5x+4)^2-(x^2-4)^2 \le 0$.
$(x^2-5x+4-(x^2-4))(x^2-5x+4+x^2-4) \le 0$.
$(-5x+8)(2x^2-5x) \le 0$.
$(8-5x)x(2x-5) \le 0$.
$x(5x-8)(2x-5) \ge 0$.
Critical points are $x=0, x=8/5=1.6, x=5/2=2.5$.
Using wavy curve method, the solution is $[0,8/5]\cup[5/2,\infty)$.
We must exclude $x=2$ from the original domain, but it's not in our solution set.

The correct option is **(1)**.

50. If $\frac{|x-3|}{x^2-5x+6} \ge 2$ then it's solution is :

(1) $(-\infty, \frac{3}{2}]$ (2) $(2,\infty]$ (3) $[\frac{3}{2},2)$ (4) $[0,5]$

Solution:

The denominator is $x^2-5x+6=(x-2)(x-3)$. So $x \neq 2,3$.
Case 1: $x>3$. Then $|x-3|=x-3$.
$$\frac{x-3}{(x-2)(x-3)} \ge 2 \implies \frac{1}{x-2} \ge 2 \implies 1 \ge 2x-4 \implies 5 \ge 2x \implies x \le 2.5.$$
Intersection of $x>3$ and $x\le 2.5$ is empty.
Case 2: $x<3$. Then $|x-3|=-(x-3)$.
$$\frac{-(x-3)}{(x-2)(x-3)} \ge 2 \implies \frac{-1}{x-2} \ge 2.$$
Subcase 2a: $x>2$. $\implies \frac{-1}{x-2} \ge 2 \implies -1 \ge 2x-4 \implies 3 \ge 2x \implies x \le 1.5$.
Intersection of $x\in(2,3)$ and $x\le 1.5$ is empty.
Subcase 2b: $x<2$. $\implies \frac{-1}{x-2} \ge 2 \implies -1 \le 2x-4 \implies 3 \le 2x \implies x \ge 1.5$.
Intersection of $x<2$ and $x \ge 1.5$ is $[1.5,2)$ or $[3/2,2)$.

The correct option is **(3)**.

Paragraph for Questions 51 and 52
In a $\triangle ABC$, if $\cos A\cos B\cos C = \frac{\sqrt{3}-1}{8}$ and $\sin A\sin B\sin C = \frac{3+\sqrt{3}}{8}$ then
On the basis of above information, answer the following questions:

51. The value of $tan\ A + tan\ B + tan\ C$ is

(1) $\frac{3+\sqrt{3}}{\sqrt{3}-1}$ (2) $\frac{\sqrt{3}+4}{\sqrt{3}-1}$ (3) $\frac{6-\sqrt{3}}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-1}$

Solution:

$$\text{In a triangle, } \tan A+\tan B+\tan C = \tan A\tan B\tan C.$$

$$\tan A \tan B \tan C = \frac{\sin A \sin B \sin C}{\cos A \cos B \cos C} = \frac{(3+\sqrt{3})/8}{(\sqrt{3}-1)/8} = \frac{3+\sqrt{3}}{\sqrt{3}-1}.$$

The correct option is **(1)**.

52. The value of *tan A tan B + tan B tan C + tan C tan A* is

(1) $5-4\sqrt{3}$ (2) $5+4\sqrt{3}$ (3) $6+\sqrt{3}$ (4) $6-\sqrt{3}$

Solution:

$$\text{From } \tan(A+B+C) = \frac{\sum \tan A - \prod \tan A}{1-\sum \tan A \tan B},$$
$$\text{and } A+B+C=\pi, \text{ we have } \sum \tan A = \prod \tan A.$$
$$\text{Also, } \cos(A+B+C) = -1.$$
$$\text{Expanding this leads to } \sum \tan A \tan B = 1 + \sec A \sec B \sec C.$$
$$\sec A \sec B \sec C = \frac{1}{\cos A \cos B \cos C} = \frac{8}{\sqrt{3}-1} = 4(\sqrt{3}+1).$$
$$\sum \tan A \tan B = 1 + 4(\sqrt{3}+1) = 5+4\sqrt{3}.$$

The correct option is **(2)**.

53. If $|\frac{-5x+7}{2}| \geq 12$ then it's solution is

(1) ... (2) $(-\infty, \frac{-17}{5}] \cup [\frac{31}{5}, \infty)$ (3) ... (4) ...

Solution:

$$|-5x+7| \geq 24.$$
$$\text{Case 1: } -5x+7 \geq 24 \implies -5x \geq 17 \implies x \leq -17/5.$$
$$\text{Case 2: } -5x+7 \leq -24 \implies -5x \leq -31 \implies x \geq 31/5.$$
$$\text{Solution is } (-\infty, -17/5] \cup [31/5, \infty).$$

The correct option is **(2)**.

54. If $|\frac{5x}{x^2-4}| \leq 1$ then it's solution.

(1) ... (2) ... (3) ... (4) ...

Solution:

$$|5x| \leq |x^2-4|, x \neq \pm 2.$$
$$(5x)^2 \leq (x^2-4)^2 \implies 0 \leq (x^2-4)^2 - (5x)^2.$$
$$(x^2-4-5x)(x^2-4+5x) \geq 0 \implies (x^2-5x-4)(x^2+5x-4) \geq 0.$$
$$\text{Roots of first quadratic are } \frac{5 \pm \sqrt{41}}{2}. \text{ Roots of second are } \frac{-5 \pm \sqrt{41}}{2}.$$
$$\text{Let the roots be } x_1 = \frac{-5-\sqrt{41}}{2}, x_2 = \frac{5-\sqrt{41}}{2}, x_3 = \frac{-5+\sqrt{41}}{2}, x_4 = \frac{5+\sqrt{41}}{2}.$$
$$(x-x_1)(x-x_4)(x-x_2)(x-x_3) \geq 0.$$

Approximate values: $\sqrt{41} \approx 6.4. x_1 \approx -5.7, x_2 \approx -0.7, x_3 \approx 0.7, x_4 \approx 5.7$.
The expression is positive for $x \in (-\infty, x_1] \cup [x_2, x_3] \cup [x_4, \infty)$.

The correct option is **(1)**.

55. Match the column-I with column-II

Solution:

(A) $x^2 - 5|x| + 6 = 0 \implies |x|^2 - 5|x| + 6 = 0 \implies (|x| - 2)(|x| - 3) = 0$.
$|x| = 2$ or $|x| = 3 \implies x = \pm 2, \pm 3$. (4 solutions) $\implies$ $\mathbf{A} \to \mathbf{R}$.

(B) $x^2 + 7|x| + 12 = 0 \implies |x|^2 + 7|x| + 12 = 0 \implies (|x| + 3)(|x| + 4) = 0$.
$|x| = -3$ or $|x| = -4$. No real solutions. $\implies$ $\mathbf{B} \to \mathbf{S}$.

(C) $|2x - 3| + |4x + 5| = |6x + 2|$. Form $|a| + |b| = |a + b|$ holds if $ab \geq 0$.
$(2x - 3)(4x + 5) \geq 0$. Roots are $3/2, -5/4$. Solution is $(-\infty, -5/4] \cup [3/2, \infty)$. $\implies$ $\mathbf{C} \to \mathbf{P}$.

(D) $|x - 4| + |x - 3| = 1$. Form $|a| + |b| = |a - b|$ holds if $ab \leq 0$.
Here $a = x - 4, b = x - 3, a - b = -1$. So $|x - 4| + |x - 3| = |-1| = 1$.
$(x - 4)(x - 3) \leq 0$. Solution is $[3, 4]$. $\implies$ $\mathbf{D} \to \mathbf{Q}$.

Matching: A-R, B-S, C-P, D-Q.

The correct option is **(2)**.

56. Match the expressions in Column-I with their simplified value in Column-II.

Solution:

(A) $\dfrac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta}$

$= \dfrac{\frac{1}{2}(\sin 9\theta + \sin 7\theta) - \frac{1}{2}(\sin 9\theta + \sin 3\theta)}{\frac{1}{2}(\cos 3\theta + \cos\theta) - \frac{1}{2}(\cos\theta - \cos 7\theta)}$

$= \dfrac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 7\theta} = \dfrac{2\cos 5\theta \sin 2\theta}{2\cos 5\theta \cos 2\theta}$

$= \tan 2\theta. \implies$ $\mathbf{A} \to \mathbf{R}$.

(B) $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = \sqrt{2 + \sqrt{2 + \sqrt{4\cos^2 4\theta}}}$

$= \sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = \sqrt{2 + \sqrt{4\cos^2 2\theta}} = \sqrt{2 + 2\cos 2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta. \implies$ $\mathbf{B} \to \mathbf{S}$.

(C) $E = \cos^3\theta + \cos^3(120^\circ + \theta) + \cos^3(120^\circ - \theta)$.

We use the identity $4\cos^3 A = \cos(3A) + 3\cos A,$ which means $\cos^3 A = \frac{1}{4}(\cos(3A) + 3\cos A)$.

Applying this identity to each term in the sum:

$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$.

$\cos^3(120^\circ + \theta) = \frac{1}{4}(\cos(360^\circ + 3\theta) + 3\cos(120^\circ + \theta)) = \frac{1}{4}(\cos 3\theta + 3\cos(120^\circ + \theta))$.

$\cos^3(120^\circ - \theta) = \frac{1}{4}(\cos(360^\circ - 3\theta) + 3\cos(120^\circ - \theta)) = \frac{1}{4}(\cos 3\theta + 3\cos(120^\circ - \theta))$.

Now, we add these three expressions together:

$E = \frac{1}{4}[(\cos 3\theta + \cos 3\theta + \cos 3\theta) + 3(\cos\theta + \cos(120^\circ + \theta) + \cos(120^\circ - \theta))]$.

The second group of terms can be simplified:

$\cos\theta + (\cos 120^\circ \cos\theta - \sin 120^\circ \sin\theta) + (\cos 120^\circ \cos\theta + \sin 120^\circ \sin\theta)$

$= \cos\theta + 2\cos 120^\circ \cos\theta = \cos\theta + 2(-\frac{1}{2})\cos\theta = \cos\theta - \cos\theta = 0$.

Substitute this back into the expression for E:

$E = \frac{1}{4}[3\cos 3\theta + 3(0)] = \frac{3}{4}\cos 3\theta. \implies \mathbf{C} \to \mathbf{Q}$.

(D) $\dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$

$= \dfrac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta(\sin\theta + \cos\theta)}{2\cos\theta(\cos\theta + \sin\theta)}$

$= \tan\theta. \implies \mathbf{D} \to \mathbf{P}$.

The correct matching is **A-R, B-S, C-Q, D-P**.

57. Match the values of the expressions in Column-I with the correct option in Column-II.

Solution:

(A) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14} = \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7} = \frac{1}{8}. \implies \mathbf{A} \to \mathbf{R}$.

(B) $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = 16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}(-\cos\frac{\pi}{15})$.

This product evaluates to 1. $\implies \mathbf{B} \to \mathbf{P}$.

(C) $\dfrac{\sin 25^\circ \sin 35^\circ \sin 85^\circ}{\sin 75^\circ} = \dfrac{\sin(60-25)\sin(60+25)...}{...}$.

$= \dfrac{\cos 20\cos 40\cos 80}{\cos 15}$.No.$\dfrac{\sin 25\sin 35\sin 85}{\cos 15} = \dfrac{\sin 25\sin(60-25)\sin(60+25)}{\cos 15}$ No.

The value is $1/4$. $\implies \mathbf{C} \to \mathbf{Q}$.

(D) $\cos\frac{\pi}{9}+\cos\frac{\pi}{3}+\cos\frac{5\pi}{9}+\cos\frac{7\pi}{9}=(\cos 20+\cos 100+\cos 140)+\cos 60=0+1/2=1/2.$
$\implies$ **D $\to$ S**.

The correct matching is **A-R, B-P, C-Q, D-S**.

SECTION-C

58. If $f(\theta)=\frac{1+\cot\theta}{\cot\theta}$ (wherever defined) then the value of $f(198°)f(27°)$ is equal to

Solution:

$$f(\theta)=1+\frac{1}{\cot\theta}=1+\tan\theta.$$
$$f(198°)=1+\tan(198°)=1+\tan(180+18)°=1+\tan 18°.$$
$$f(27°)=1+\tan 27°.$$
$$\text{Product}=(1+\tan 18°)(1+\tan 27°).$$
$$\text{Since } 18°+27°=45°,\ \text{we know}\ \tan(18+27)=\frac{\tan 18+\tan 27}{1-\tan 18\tan 27}=1.$$
$$\tan 18+\tan 27=1-\tan 18\tan 27.$$
$$\text{The product is } 1+\tan 18+\tan 27+\tan 18\tan 27=1+(1-\tan 18\tan 27)+\tan 18\tan 27=2.$$

The answer is **2**.

59. Number of real solution of the equation $x^2-|x|-12=0$ is

Solution:

Let $y=|x|, y\geq 0$. The equation is $y^2-y-12=0$.
$(y-4)(y+3)=0$.
$y=4$ or $y=-3$.
Since $y=|x|\geq 0$, we only accept $y=4$.
$|x|=4 \implies x=\pm 4$.
There are 2 real solutions.

The answer is **2**.

60. Find the value of $10|\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}|$

Solution:

$$\text{Let } S=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}.$$

From the properties of the roots of unity, we know that $1 + 2S = 0$.
$S = -1/2$.
The expression is $10|S| = 10|-1/2| = 10(1/2) = 5$.

The answer is **5**.