

HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

Date of Exam: 17th August 2025

Syllabus: Trigonometric Equation

Sub: Mathematics **CT-03 JEE Advanced Solution** Prof. Chetan Sir

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1. The sum of all solutions for the equation $\cos 2x = \cos x$ in the interval $[0, 2\pi]$ is:

- (A) 2π (B) 3π (C) $\frac{7\pi}{2}$ (D) 4π

Solution:

$$\begin{aligned}\cos 2x &= \cos x \\ \implies 2x &= 2n\pi \pm x, \quad n \in \mathbb{Z}.\end{aligned}$$

Case 1 (+ sign):

$$\begin{aligned}2x &= 2n\pi + x \implies x = 2n\pi. \\ \text{In } [0, 2\pi], \text{ solutions are } &x = 0, 2\pi.\end{aligned}$$

Case 2 (- sign):

$$\begin{aligned}2x &= 2n\pi - x \implies 3x = 2n\pi \implies x = \frac{2n\pi}{3}. \\ \text{In } [0, 2\pi], \text{ solutions are } &x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi.\end{aligned}$$

The set of distinct solutions is $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi\right\}$.

$$\begin{aligned}\text{Sum of solutions} &= 0 + \frac{2\pi}{3} + \frac{4\pi}{3} + 2\pi \\ &= \frac{6\pi}{3} + 2\pi = 2\pi + 2\pi = 4\pi.\end{aligned}$$

The correct option is (D).

2. Let S be the set of all values of $x \in (-\pi, \pi)$ for which the equation $2 \sin^2 x + 3 \cos x = 3$ holds. The number of elements in S is:

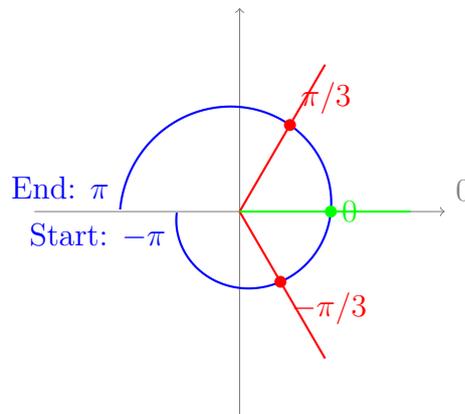
- (A) 1 (B) 2 (C) 3 (D) 4

Solution:

$$\begin{aligned}
 2(1 - \cos^2 x) + 3 \cos x &= 3 \\
 2 - 2 \cos^2 x + 3 \cos x - 3 &= 0 \\
 2 \cos^2 x - 3 \cos x + 1 &= 0 \\
 (2 \cos x - 1)(\cos x - 1) &= 0.
 \end{aligned}$$

This gives $\cos x = 1/2$ or $\cos x = 1$.

We find the number of solutions in the interval $(-\pi, \pi)$ using the spiral diagram.



The solutions are $x = 0$ from $\cos x = 1$, and $x = \pi/3, -\pi/3$ from $\cos x = 1/2$. There are 3 solutions. The correct option is (C).

3. Let the equation be $\sin x - \sin 2x + \sin 3x = 0$. Then which of the following is/are correct general solutions for x ? ($n \in \mathbb{Z}$)

- (A) $\frac{n\pi}{2}$ (B) $2n\pi \pm \frac{\pi}{3}$ (C) $n\pi + (-1)^n \frac{\pi}{6}$ (D) $n\pi$

Solution:

$$\begin{aligned}
 (\sin 3x + \sin x) - \sin 2x &= 0 \\
 2 \sin \left(\frac{3x + x}{2} \right) \cos \left(\frac{3x - x}{2} \right) - \sin 2x &= 0 \\
 2 \sin 2x \cos x - \sin 2x &= 0 \\
 \sin 2x(2 \cos x - 1) &= 0.
 \end{aligned}$$

Case 1: $\sin 2x = 0$

$$\implies 2x = n\pi \implies x = \frac{n\pi}{2}.$$

Case 2: $2 \cos x - 1 = 0 \implies \cos x = \frac{1}{2}.$

$$\implies x = 2n\pi \pm \frac{\pi}{3}.$$

Both sets of solutions are correct. The correct options are **(A)** and **(B)**.

4. If the equation $k \cos x - 3 \sin x = k + 1$ has a solution, then k can be:

- (A) 5 (B) 2 (C) -1 (D) 4

Solution:

For an equation of the form $a \cos x + b \sin x = c$ to have a solution, the condition $|c| \leq \sqrt{a^2 + b^2}$ must be satisfied.

Here, $a = k, b = -3, c = k + 1$.

$$|k + 1| \leq \sqrt{k^2 + (-3)^2}$$

$$(k + 1)^2 \leq k^2 + 9$$

$$k^2 + 2k + 1 \leq k^2 + 9$$

$$2k \leq 8 \implies k \leq 4.$$

We check which options satisfy $k \leq 4$:

- (A) 5: No
 (B) 2: Yes
 (C) -1: Yes
 (D) 4: Yes

The correct options are **(B)**, **(C)**, and **(D)**.

5. Match the equations in Column-I with the number of their solutions in the interval $[0, 2\pi]$ in Column-II.

Column-I	Column-II
(A) $\cos 2\theta = \sin \theta$	(P) 8
(B) $\tan \theta + \sec \theta = 2 \cos \theta$	(Q) 2
(C) $16^{\sin^2 x} + 16^{\cos^2 x} = 10$	(R) 3
(D) $\sin 2x = \cos 3x$	(S) 6

Answer: A-R, B-Q, C-P, D-S

Solution:

(A) $\cos 2\theta = \sin \theta \implies 1 - 2 \sin^2 \theta = \sin \theta \implies 2 \sin^2 \theta + \sin \theta - 1 = 0$. $(2 \sin \theta - 1)(\sin \theta + 1) = 0$. $\sin \theta = 1/2$ gives 2 solutions $(\pi/6, 5\pi/6)$. $\sin \theta = -1$ gives 1 solution $(3\pi/2)$. Total = **3 solutions**. **(A → R)**

- (B) $\tan \theta + \sec \theta = 2 \cos \theta \implies \frac{\sin \theta + 1}{\cos \theta} = 2 \cos \theta \implies \sin \theta + 1 = 2 \cos^2 \theta$. This leads to the same equation as (A), but with the constraint $\cos \theta \neq 0$. Thus, the solution $\theta = 3\pi/2$ is rejected. Total = **2 solutions**. (B \rightarrow Q)
- (C) $16^{\sin^2 x} + 16^{\cos^2 x} = 10$. This simplifies to $\sin^2 x = 1/4$ or $\sin^2 x = 3/4$. $\sin x = \pm 1/2$ (4 solutions). $\sin x = \pm \sqrt{3}/2$ (4 solutions). Total = **8 solutions**. (C \rightarrow P)
- (D) $\sin 2x = \cos 3x \implies \cos(\pi/2 - 2x) = \cos 3x$. $3x = 2n\pi \pm (\pi/2 - 2x)$. Case (+): $5x = 2n\pi + \pi/2 \implies x = \frac{2n\pi}{5} + \frac{\pi}{10}$. Solutions in $[0, 2\pi]$ are $\pi/10, 5\pi/10, 9\pi/10, 13\pi/10, 17\pi/10$. (5 solutions). Case (-): $x = 2n\pi - \pi/2$. Solution in $[0, 2\pi]$ is $3\pi/2$. (1 solution). Total = **6 solutions**. (D \rightarrow S)

Correct Match: A-R, B-Q, C-P, D-S.

6. Match the following.

Column-I	Column-II
(A) The number of solutions of $\sin 3x + \cos 2x = 0$ in $[0, \frac{3\pi}{2}]$ is	(p) 1
(B) The number of solutions of the equation $(1 - \tan x)(1 + \sin 2x) = 1 + \tan x$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is	(q) 4
(C) The number of solutions of the equation $\sin(\pi x) = e^x + e^{-x}$ is	(r) 2
(D) The number of solutions of the equation $\sin^3 x - 3 \sin x \cos^2 x + 2 \cos^3 x = 0$ in $[-\frac{\pi}{4}, \frac{\pi}{4}]$ is	(s) 0

Answer: A(q), B(r), C(s), D(p)

Solution:

(A) For the equation $\sin 3x + \cos 2x = 0$ in $[0, 3\pi/2]$:

We have $\sin 3x = -\cos 2x = \sin(2x - \pi/2)$. The general solution is $3x = n\pi + (-1)^n(2x - \pi/2)$.

– If n is even, $3x = n\pi + 2x - \pi/2 \implies x = n\pi - \pi/2$. For $n = 2$, $x = 3\pi/2$.

– If n is odd, $3x = n\pi - (2x - \pi/2) \implies 5x = n\pi + \pi/2$. For $n = 1, 3, 5$, we get $x = 3\pi/10, 7\pi/10, 11\pi/10$.

The solution set is $\{3\pi/10, 7\pi/10, 11\pi/10, 3\pi/2\}$. The number of solutions is **4**. (A \rightarrow q)

(B) For $(1 - \tan x)(1 + \sin 2x) = 1 + \tan x$ in $[-\pi/2, \pi/2]$:

The equation simplifies to $(\cos x + \sin x)(\cos 2x - 1) = 0$.

– $\cos x + \sin x = 0 \implies \tan x = -1 \implies x = -\pi/4$.

– $\cos 2x = 1 \implies 2x = 2k\pi \implies x = k\pi$. For $k = 0$, $x = 0$.

The solution set is $\{-\pi/4, 0\}$. The number of solutions is **2**. (B \rightarrow r)

(C) For $\sin(\pi x) = e^x + e^{-x}$:

The range of the LHS, $\sin(\pi x)$, is $[-1, 1]$. The range of the RHS, $e^x + e^{-x} = 2 \cosh(x)$, is $[2, \infty)$. Since the ranges do not overlap, there is no solution. The number of solutions is **0**.
(C \rightarrow s)

(D) For $\tan^3 x - 3 \tan x + 2 = 0$ in $[-\pi/4, \pi/4]$:

Let $t = \tan x$. The equation becomes $t^3 - 3t + 2 = 0$, which factors to $(t - 1)^2(t + 2) = 0$. This gives $\tan x = 1$ or $\tan x = -2$.

– In $[-\pi/4, \pi/4]$, the only solution is from $\tan x = 1 \implies x = \pi/4$.

– $\tan x = -2$ has no solution in this interval, as the range of $\tan x$ for $x \in [-\pi/4, \pi/4]$ is $[-1, 1]$.

The number of solutions is **1**. (D \rightarrow p)

Correct Match: A \rightarrow q, B \rightarrow r, C \rightarrow s, D \rightarrow p.

Paragraph for Questions 7 and 8

A particle's motion is analyzed using two independent trigonometric models. The first model describes the particle's stability, which is maintained only when the denominator of the expression $f(x) = \frac{\sin 3x}{2 \cos 2x + 1}$ is non-zero. The second model describes the particle's angular position α , which is constrained by a specific geometric condition.

7. The number of solutions to the equation $f(x) = \frac{1}{2}$ in the interval $[-\pi, \pi]$ is:

(A) 1

(B) 2

(C) 4

(D) 6

Solution:

$$\begin{aligned} \frac{\sin 3x}{2 \cos 2x + 1} &= \frac{1}{2} \\ \implies 2 \sin 3x &= 2 \cos 2x + 1 \\ \implies 2(3 \sin x - 4 \sin^3 x) &= 2(1 - 2 \sin^2 x) + 1 \\ \implies 6 \sin x - 8 \sin^3 x &= 3 - 4 \sin^2 x \\ \implies 8 \sin^3 x - 4 \sin^2 x - 6 \sin x + 3 &= 0 \\ \implies 4 \sin^2 x(2 \sin x - 1) - 3(2 \sin x - 1) &= 0 \\ \implies (4 \sin^2 x - 3)(2 \sin x - 1) &= 0. \end{aligned}$$

This gives $\sin x = 1/2$ or $\sin^2 x = 3/4 \implies \sin x = \pm\sqrt{3}/2$. The stability condition requires the denominator to be non-zero: $2 \cos 2x + 1 \neq 0 \implies \cos 2x \neq -1/2$. We use the identity $\cos 2x = 1 - 2 \sin^2 x$ to check our solutions:

- If $\sin x = 1/2$, then $\cos 2x = 1 - 2(1/2)^2 = 1/2 \neq -1/2$. These solutions are **valid**.
- If $\sin^2 x = 3/4$, then $\cos 2x = 1 - 2(3/4) = 1 - 3/2 = -1/2$. These solutions are **invalid**.

We only need to count the solutions for $\sin x = 1/2$ in $[-\pi, \pi]$. The solutions are $x = \pi/6$ and $x = 5\pi/6$. There are **2 solutions**. The correct option is **(B)**.

8. If the angular position $\alpha \in [-2\pi, 2\pi]$ is determined by the relation $\cos(\frac{\alpha}{2}) + \sin(\frac{\alpha}{2}) = \sqrt{2}(\cos 36^\circ - \sin 18^\circ)$, then one possible value of α is:

(A) $\frac{5\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{7\pi}{6}$

(D) $-\frac{5\pi}{6}$

Solution:

First, simplify the RHS:

$$\cos 36^\circ = \frac{\sqrt{5} + 1}{4} \quad \text{and} \quad \sin 18^\circ = \frac{\sqrt{5} - 1}{4}.$$

$$\text{RHS} = \sqrt{2} \left(\frac{\sqrt{5} + 1}{4} - \frac{\sqrt{5} - 1}{4} \right) = \sqrt{2} \left(\frac{2}{4} \right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

Now, simplify the LHS:

$$\cos\left(\frac{\alpha}{2}\right) + \sin\left(\frac{\alpha}{2}\right) = \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos\left(\frac{\alpha}{2}\right) + \frac{1}{\sqrt{2}} \sin\left(\frac{\alpha}{2}\right) \right) = \sqrt{2} \cos\left(\frac{\alpha}{2} - \frac{\pi}{4}\right).$$

The equation becomes:

$$\sqrt{2} \cos\left(\frac{\alpha}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \implies \cos\left(\frac{\alpha}{2} - \frac{\pi}{4}\right) = \frac{1}{2}.$$

$$\frac{\alpha}{2} - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}.$$

$$\text{Case (+): } \frac{\alpha}{2} = 2n\pi + \frac{\pi}{3} + \frac{\pi}{4} = 2n\pi + \frac{7\pi}{12} \implies \alpha = 4n\pi + \frac{7\pi}{6}.$$

For $n=0$, $\alpha = \frac{7\pi}{6}$, which is in $[-2\pi, 2\pi]$.

$$\text{Case (-): } \frac{\alpha}{2} = 2n\pi - \frac{\pi}{3} + \frac{\pi}{4} = 2n\pi - \frac{\pi}{12} \implies \alpha = 4n\pi - \frac{\pi}{6}.$$

For $n=0$, $\alpha = -\frac{\pi}{6}$.One possible value is $\frac{7\pi}{6}$. The correct option is **(C)**.

9. If the sum of all solutions of the equation $2 \cos 2\theta + 1 = 0$ in the interval $[0, 3\pi]$ is $k\pi$, then find the value of k .

Solution:

$$2 \cos 2\theta + 1 = 0 \implies \cos 2\theta = -1/2.$$

Let $\phi = 2\theta$. Since $\theta \in [0, 3\pi]$, the interval for ϕ is $[0, 6\pi]$.We find solutions for $\cos \phi = -1/2$. They are in Q2 and Q3 with ref. angle $\pi/3$.

$$\text{In } [0, 2\pi] : \phi = \pi - \pi/3 = 2\pi/3, \quad \phi = \pi + \pi/3 = 4\pi/3.$$

$$\text{In } [2\pi, 4\pi] : \phi = 2\pi + 2\pi/3 = 8\pi/3, \quad \phi = 2\pi + 4\pi/3 = 10\pi/3.$$

$$\text{In } [4\pi, 6\pi] : \phi = 4\pi + 2\pi/3 = 14\pi/3, \quad \phi = 4\pi + 4\pi/3 = 16\pi/3.$$

$$2\theta \in \left\{ \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3} \right\}.$$

$$\theta \in \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3} \right\}.$$

$$\text{Sum} = \frac{\pi}{3}(1 + 2 + 4 + 5 + 7 + 8) = \frac{27\pi}{3} = 9\pi.$$

$$\text{Given Sum} = k\pi \implies k = 9.$$

The integer value of k is **9**.

10. The number of solutions of $\sum_{r=1}^6 \cos(rx) = 6$ in $(0, 2\pi]$ is:

Solution:

The given equation is $\cos(x) + \cos(2x) + \cos(3x) + \cos(4x) + \cos(5x) + \cos(6x) = 6$.

The maximum value of any cosine term, $\cos(rx)$, is 1.

For the sum of six cosine terms to be 6, each term must be equal to its maximum value.

This requires $\cos(x) = 1, \cos(2x) = 1, \dots, \cos(6x) = 1$ simultaneously.

From $\cos(x) = 1$, the general solution is $x = 2n\pi$.

In the interval $(0, 2\pi]$, the only solution is $x = 2\pi$ (for $n = 1$).

We check if this value satisfies the other conditions:

$$\cos(2 \cdot 2\pi) = \cos(4\pi) = 1.$$

$$\cos(3 \cdot 2\pi) = \cos(6\pi) = 1.$$

And so on. All conditions are satisfied for $x = 2\pi$.

Thus, there is only one solution in the given interval.

The number of solutions is **1**.