Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

# Date of Exam: 17th August 2025

**Syllabus: Trigonometric Equation & Logarithm**

Sub: Mathematics **CT-03 MHT CET Solution** Prof. Chetan Sir

## Index of Questions



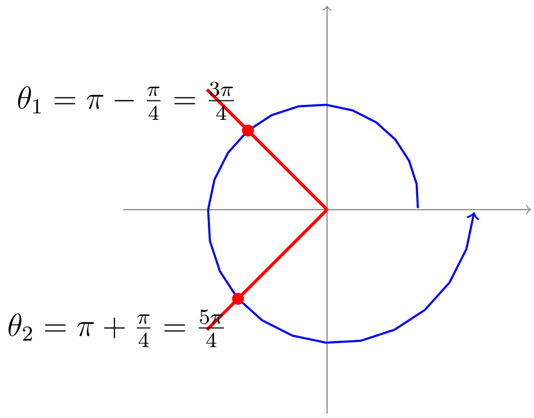
101. The principal solutions of the equation $2\cos\theta + \sqrt{2} = 0$ in the interval $[0, 2\pi]$ are:

(1) $\frac{\pi}{4}, \frac{7\pi}{4}$ (2) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (3) $\frac{\pi}{4}, \frac{5\pi}{4}$ (4) $\frac{3\pi}{4}, \frac{5\pi}{4}$

**Solution:**

$$2\cos\theta + \sqrt{2} = 0 \implies \cos\theta = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}.$$

Cosine is negative in Quadrants II and III. The reference angle is $\pi/4$.



The correct option is **(4)**.

102. If $\log_2 p = x$ and $\log_3 p = y$, find $\log_6 p$:

(1) $\frac{xy}{x+y}$ (2) $\frac{x}{y}$ (3) $\frac{y}{x}$ (4) $\frac{x+y}{xy}$

**Solution:**

$$\text{Given } \log_2 p = x \implies \frac{1}{x} = \log_p 2.$$
$$\text{And } \log_3 p = y \implies \frac{1}{y} = \log_p 3.$$

Adding these two equations:

$$\frac{1}{x} + \frac{1}{y} = \log_p 2 + \log_p 3 = \log_p(2 \cdot 3) = \log_p 6.$$
$$\frac{y+x}{xy} = \log_p 6.$$

Taking the reciprocal to find $\log_6 p$ :

$$\log_6 p = \frac{1}{\log_p 6} = \frac{xy}{x+y}.$$

The correct option is **(1)**.

103. If $\tan\theta = -\sqrt{3}$ and $\frac{\pi}{2} < \theta < \pi$, then the value of $\theta$ is:

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) $\frac{4\pi}{3}$ (4) $\frac{11\pi}{6}$

**Solution:**

The interval $(\pi/2, \pi)$ is Quadrant II.

In Quadrant II, tangent is negative, which matches the condition $\tan\theta = -\sqrt{3}$.

The reference angle for $\tan\theta = \sqrt{3}$ is $\pi/3$.

The angle in Quadrant II is $\theta = \pi - (\text{reference angle})$.

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

The correct option is **(2)**.

104. The general solution of the equation $\cot 2x = \sqrt{3}$ is ($n \in \mathbb{Z}$):

(1) $x = \frac{n\pi}{2} + \frac{\pi}{6}$ (2) $x = \frac{n\pi}{2} + \frac{\pi}{12}$ (3) $x = n\pi + \frac{\pi}{12}$ (4) $x = n\pi + \frac{\pi}{6}$

**Solution:**

$$\cot 2x = \sqrt{3} \implies \tan 2x = \frac{1}{\sqrt{3}}.$$

$$\tan 2x = \tan\left(\frac{\pi}{6}\right).$$

$$\text{The general solution is } 2x = n\pi + \frac{\pi}{6}.$$

$$\implies x = \frac{n\pi}{2} + \frac{\pi}{12}.$$

The correct option is **(2)**.

105. If $\log_5 x = \frac{1}{2}$ then x is equal to:

(1) $\sqrt{5}$ (2) $5^2$ (3) $\frac{1}{\sqrt{5}}$ (4) 25

**Solution:**

$$\text{The definition of a logarithm is } \log_b a = c \iff b^c = a.$$

$$\text{Applying this to } \log_5 x = \frac{1}{2}:$$

$$x = 5^{1/2} = \sqrt{5}.$$

The correct option is **(1)**.

106. The general value of $\theta$ satisfying $\operatorname{cosec}^2\theta = \frac{4}{3}$ is ($n \in \mathbb{Z}$):

(1) $n\pi \pm \frac{\pi}{6}$ (2) $2n\pi \pm \frac{\pi}{3}$ (3) $n\pi \pm \frac{\pi}{3}$ (4) $2n\pi \pm \frac{\pi}{6}$

**Solution:**

$$\operatorname{cosec}^2\theta = \frac{4}{3} \implies \sin^2\theta = \frac{3}{4}.$$

$$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2 = \sin^2\left(\frac{\pi}{3}\right).$$

$$\text{The general solution for } \sin^2\theta = \sin^2\alpha \text{ is } \theta = n\pi \pm \alpha.$$

$$\implies \theta = n\pi \pm \frac{\pi}{3}.$$

The correct option is **(3)**.

107. $\log_{10} 3 + \log_{10} 2 - 2\log_{10} 5 =?$

(1) $\log_{10}(\frac{25}{6})$ (2) $\log_{10}(\frac{6}{25})$ (3) $\log_{10} 10$ (4) None

**Solution:**

$$\begin{aligned}
&\log_{10} 3 + \log_{10} 2 - 2\log_{10} 5 \\
&= \log_{10}(3 \cdot 2) - \log_{10}(5^2) \\
&= \log_{10}(6) - \log_{10}(25) \\
&= \log_{10}\left(\frac{6}{25}\right).
\end{aligned}$$

The correct option is **(2)**.

108. The general solution of $\sec 5\theta = -2$ is ($n \in \mathbb{Z}$):

(1) $\theta = \frac{2n\pi}{5} \pm \frac{2\pi}{15}$ (2) $\theta = \frac{n\pi}{5} \pm \frac{\pi}{15}$ (3) $\theta = \frac{2n\pi}{5} \pm \frac{\pi}{15}$ (4) $\theta = \frac{n\pi}{5} \pm \frac{2\pi}{15}$

**Solution:**

$$\sec 5\theta = -2 \implies \cos 5\theta = -\frac{1}{2}.$$
$$\cos 5\theta = \cos\left(\frac{2\pi}{3}\right).$$
$$\text{The general solution is } 5\theta = 2n\pi \pm \frac{2\pi}{3}.$$
$$\implies \theta = \frac{2n\pi}{5} \pm \frac{2\pi}{15}.$$

The correct option is **(1)**.

109. $\log_5 20 + \log_5 \frac{125}{4} - log_5 5^{-1} = ?$

(1) 3 (2) 4 (3) 5 (4) 6

**Solution:**

$$\log_5\left(20 \cdot \frac{125}{4}\right) - \log_5\left(\frac{1}{5}\right)$$
$$= \log_5(5 \cdot 125) - (-1)$$
$$= \log_5(625) + 1$$
$$= \log_5(5^4) + 1$$
$$= 4 + 1 = 5.$$

The correct option is **(3)**.

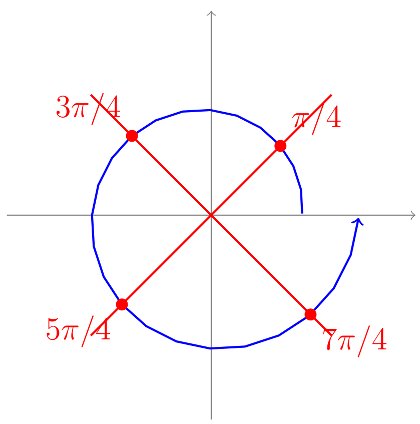
110. The sum of principal solutions of the equation $\sec^2\theta = 2$ is:

(1) $\frac{2\pi}{3}$ (2) $\frac{4\pi}{3}$ (3) $\pi$ (4) $\frac{5\pi}{6}$

**Solution:**

$$\sec^2\theta = 2 \implies \cos^2\theta = \frac{1}{2} \implies \cos\theta = \pm\frac{1}{\sqrt{2}}.$$

We find all solutions in the interval $[0, 2\pi]$.



$$\text{The solutions are } \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$
$$\text{Sum} = \frac{\pi + 3\pi + 5\pi + 7\pi}{4} = \frac{16\pi}{4} = 4\pi.$$

Note: The calculated sum is $4\pi$, which is not among the options.

Note: This question is Bonus

111. If $\log(3x-1) - \log(3x+1) = \log 16$ then $x = ?$

(1) 0 (2) 1 (3) 2 (4) None

**Solution:**

Let's first analyze the equation as it is written:

$\log(3x-1) - \log(3x+1) = \log 16$

$\implies \log\left(\frac{3x-1}{3x+1}\right) = \log 16$

$\implies \frac{3x-1}{3x+1} = 16$

$\implies 3x - 1 = 16(3x+1) = 48x + 16$

$\implies -17 = 45x \implies x = -17/45.$

We must check if this solution is valid by testing the domain of the logarithms.

For $\log(3x-1)$, we need $3x - 1 > 0$.

Substituting $x = -17/45$, we get $3(-17/45) - 1 = -17/15 - 1 < 0$.

Since the argument is negative, this solution is invalid.

This indicates a likely typo in the question. A common variation is to have the terms reversed:

$\log(3x+1) - \log(3x-1) = \log 16$

$\implies \log\left(\frac{3x+1}{3x-1}\right) = \log 16$

$\implies \frac{3x+1}{3x-1} = 16$

$\implies 3x + 1 = 16(3x-1) = 48x - 16$

$\implies 17 = 45x \implies x = 17/45.$

Let's check the domain for this potential solution $x = 17/45$:

$3x - 1 = 3(17/45) - 1 = 17/15 - 1 = 2/15 > 0.$ (Valid)

$3x + 1 = 17/15 + 1 = 32/15 > 0.$ (Valid)

The solution $x = 17/45$ is mathematically valid for this modified equation.

Since this value is not listed in options (1), (2), or (3), the correct choice is "None".

The correct option is **(4)**.

112. If $0 \le x \le 2\pi$, the solutions of the equation $2\sin^2 x + 7\cos x - 5 = 0$ are:

(1) $\frac{\pi}{6}, \frac{11\pi}{6}$ (2) $\frac{\pi}{3}, \frac{5\pi}{3}$ (3) $\frac{\pi}{4}, \frac{7\pi}{4}$ (4) $\frac{\pi}{3}, \frac{2\pi}{3}$

**Solution:**

$$\begin{aligned}
&2(1-\cos^2 x)+7\cos x-5=0\\
&2-2\cos^2 x+7\cos x-5=0\\
&2\cos^2 x-7\cos x+3=0\\
&(2\cos x-1)(\cos x-3)=0.\\
&\implies \cos x=1/2 \text{ or } \cos x=3 \text{ (impossible)}.\\
&\text{The solutions for } \cos x=1/2 \text{ in } [0,2\pi] \text{ are } \frac{\pi}{3} \text{ and } 2\pi-\frac{\pi}{3}=\frac{5\pi}{3}.
\end{aligned}$$

The correct option is **(2)**.

113. The condition for $\log_a b$ to be defined is:

(1) $a>0, a\neq 1, b>0$ (2) $a>0, b>0$ (3) $a\neq 1$ (4) $b>0$

**Solution:**
By definition, for the logarithm $\log_a b$ to be a well-defined real number, the following conditions must be met:

- The base, $a$, must be positive: $a>0$.
- The base, $a$, cannot be equal to 1: $a\neq 1$.
- The argument, $b$, must be positive: $b>0$.

The correct option is **(1)**.

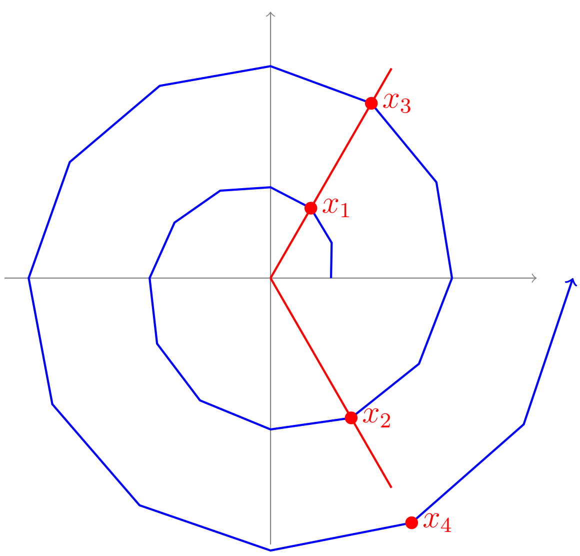
114. The number of solutions for the equation $2\cos^2 x-5\cos x+2=0$ in the interval $[0,4\pi]$ is:

(1) 2 (2) 3 (3) 4 (4) 6

**Solution:**

$$\begin{aligned}
&(2\cos x-1)(\cos x-2)=0.\\
&\implies \cos x=1/2 \text{ or } \cos x=2 \text{ (impossible)}.
\end{aligned}$$

We need to count the solutions for $\cos x=1/2$ in $[0,4\pi]$. This interval represents two full rotations.



There are two solutions in $[0, 2\pi]$ and two more in $[2\pi, 4\pi]$, for a total of 4 solutions. The correct option is **(3)**.

115. If $\log_5(x^2 - 1) = 2$, then x equals:

(1) $\pm 6$ (2) $\pm\sqrt{26}$ (3) $\pm 3$ (4) $\pm 5$

**Solution:**

$$\log_5(x^2 - 1) = 2$$
$$\implies x^2 - 1 = 5^2 = 25.$$
$$\implies x^2 = 26.$$
$$\implies x = \pm\sqrt{26}.$$

The correct option is **(2)**.

116. The number of solutions of $\sin 2x + \sin 4x + \sin 6x = 0$ in the interval $[0, \pi]$ is:

(1) 3 (2) 5 (3) 6 (4) 7

**Solution:**

$$(\sin 6x + \sin 2x) + \sin 4x = 0$$
$$2 \sin 4x \cos 2x + \sin 4x = 0$$
$$\sin 4x(2 \cos 2x + 1) = 0.$$
$$\text{Case 1: } \sin 4x = 0 \implies 4x = n\pi \implies x = n\pi/4.$$
$$\text{In } [0, \pi], \text{ solutions are } 0, \pi/4, \pi/2, 3\pi/4, \pi. \text{ (5 solutions)}$$
$$\text{Case 2: } \cos 2x = -1/2 \implies 2x = 2n\pi \pm 2\pi/3 \implies x = n\pi \pm \pi/3.$$
$$\text{In } [0, \pi], \text{ solutions are } \pi/3, 2\pi/3. \text{ (2 solutions)}$$
$$\text{The distinct solutions are } \{0, \pi/4, \pi/3, \pi/2, 2\pi/3, 3\pi/4, \pi\}.$$
$$\text{Total number of solutions is 7.}$$

The correct option is **(4)**.

117. Solution of $\log_3(x + 1) = \log_3(2x - 1)$:

(1) 3 (2) 4 (3) 2 (4) 1

**Solution:**

Since the bases are the same, we can equate the arguments:

$$x + 1 = 2x - 1$$
$$\implies x = 2.$$

We must check that the arguments are positive for this value.

$$x + 1 = 2 + 1 = 3 > 0.$$
$$2x - 1 = 2(2) - 1 = 3 > 0.$$

The solution is valid.

The correct option is **(3)**.

118. The most general value of $\theta$ that satisfies the equation $\tan^2\theta - (\sqrt{3}+1)\tan\theta + \sqrt{3} = 0$ is ($n \in \mathbb{Z}$):

(1) $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{6}$ (2) $n\pi \pm \frac{\pi}{3}$ (3) $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}$ (4) $n\pi \pm \frac{\pi}{4}$

**Solution:**

This is a quadratic equation in $\tan\theta$.
$\tan^2\theta - \sqrt{3}\tan\theta - \tan\theta + \sqrt{3} = 0$
$\tan\theta(\tan\theta - \sqrt{3}) - 1(\tan\theta - \sqrt{3}) = 0$
$(\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0.$
Case 1: $\tan\theta = 1 \implies \theta = n\pi + \dfrac{\pi}{4}.$
Case 2: $\tan\theta = \sqrt{3} \implies \theta = n\pi + \dfrac{\pi}{3}.$

The correct option is **(3)**.

119. If $\log_a b = m$ and $\log_b a = n$ then $mn =$

(1) 0 (2) 1 (3) $m + n$ (4) Undefined

**Solution:**

Using the change of base rule, $\log_b a = \dfrac{1}{\log_a b}.$
We are given $n = \log_b a$ and $m = \log_a b.$
So, $n = \dfrac{1}{m}.$
$\implies mn = 1.$

The correct option is **(2)**.

120. The solution of the equation $\cos\theta + \sin\theta = \sqrt{2}$ in the interval $(0, 2\pi)$ is:

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{3}$

**Solution:**

Divide by $\sqrt{1^2 + 1^2} = \sqrt{2}.$
$\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta = 1.$
$\cos(\theta - \pi/4) = 1.$
The general solution is $\theta - \pi/4 = 2n\pi.$
$\theta = 2n\pi + \pi/4.$

$$\text{In the interval } (0, 2\pi), \text{ the only solution is for n=0, which is } \theta = \frac{\pi}{4}.$$

The correct option is **(2)**.

121. The value of $\log_{0.01} 1000 + \log_{0.1} 0.0001$ is:

(1) -2 (2) -10 (3) $-\frac{5}{2}$ (4) $\frac{5}{2}$

**Solution:**

$$\text{Part 1: } \log_{0.01} 1000 = \log_{10^{-2}}(10^3) = \frac{3}{-2}\log_{10} 10 = -\frac{3}{2}.$$
$$\text{Part 2: } \log_{0.1} 0.0001 = \log_{10^{-1}}(10^{-4}) = \frac{-4}{-1}\log_{10} 10 = 4.$$
$$\text{Sum} = -\frac{3}{2} + 4 = \frac{-3+8}{2} = \frac{5}{2}.$$

The correct option is **(4)**.

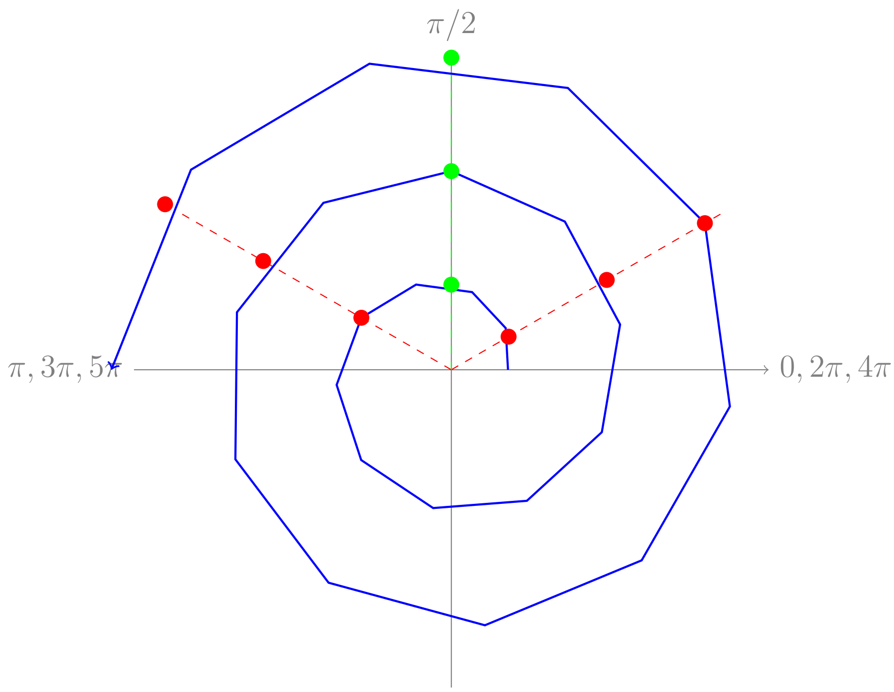
122. The number of values of x in $[0, 5\pi]$ satisfying the equation $2\sin^2 x - 3\sin x + 1 = 0$ is:

(1) 4 (2) 5 (3) 6 (4) 9

**Solution:**

First, we factor the quadratic equation in $\sin x$ :
$(2\sin x - 1)(\sin x - 1) = 0.$

This gives two cases for the solution:
**Case 1:** $\sin x = 1/2.$
**Case 2:** $\sin x = 1.$



$$
\begin{aligned}
&\text{Counting the solutions from the diagram:}\\
&\text{For } \sin x = 1/2 \text{ (red points):}\\
&\quad \text{In } [0, 2\pi] : 2 \text{ solutions.}\\
&\quad \text{In } [2\pi, 4\pi] : 2 \text{ solutions.}\\
&\quad \text{In } [4\pi, 5\pi] : 2 \text{ solutions.}\\
&\quad \text{Total for } \sin x = 1/2 : \mathbf{6\ solutions}.\\
\\
&\text{For } \sin x = 1 \text{ (green points):}\\
&\quad \text{Solutions are at } \pi/2, \quad 5\pi/2, \quad 9\pi/2.\\
&\quad \text{Total for } \sin x = 1 : \mathbf{3\ solutions}.\\
\\
&\text{The total number of distinct solutions is } 6 + 3 = 9.
\end{aligned}
$$

The correct option is **(4)**.

123. If the value of $\log_2 16 + \log_4 64$ is:

(1) 9 (2) 6 (3) 7 (4) 8

**Solution:**

$$
\begin{aligned}
&\log_2(2^4) + \log_4(4^3)\\
&= 4\log_2 2 + 3\log_4 4\\
&= 4(1) + 3(1) = 7.
\end{aligned}
$$

The correct option is **(3)**.

124. The general solution for the equation $\sin x - \sqrt{3}\cos x = 1$ is $(n \in \mathbb{Z})$:

(1) $n\pi + (-1)^n \frac{\pi}{6} - \frac{\pi}{3}$ (2) $2n\pi + \frac{\pi}{2}$ (3) $n\pi + (-1)^n \frac{\pi}{6} + \frac{\pi}{3}$ (4) $2n\pi + \frac{5\pi}{6}$

**Solution:**

$$
\begin{aligned}
&\text{Divide by 2: } \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}.\\
&\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = \frac{1}{2}.\\
&\sin(x - \pi/3) = 1/2 = \sin(\pi/6).\\
&x - \pi/3 = n\pi + (-1)^n \pi/6.\\
&x = n\pi + (-1)^n \frac{\pi}{6} + \frac{\pi}{3}.
\end{aligned}
$$

The correct option is **(3)**.

125. $\log_{\sqrt{51}}(51)^4 = ?$

(1) 8 (2) 9 (3) 12 (4) 16

**Solution:**

$$\log_{51^{1/2}}(51^4)$$
$$= \frac{4}{1/2}\log_{51} 51$$
$$= 8 \cdot 1 = 8.$$

The correct option is **(1)**.

126. The general solution of $\tan 2\theta = \cot 3\theta$ is $(n \in \mathbb{Z})$:

(1) $\frac{n\pi}{5} + \frac{\pi}{2}$ (2) $n\pi + \frac{\pi}{10}$ (3) $\frac{n\pi}{5} + \frac{\pi}{10}$ (4) $n\pi - \frac{\pi}{10}$

**Solution:**

$$\tan 2\theta = \tan(\pi/2 - 3\theta).$$
$$2\theta = n\pi + \pi/2 - 3\theta.$$
$$5\theta = n\pi + \pi/2.$$
$$\theta = \frac{n\pi}{5} + \frac{\pi}{10}.$$

The correct option is **(3)**.

127. If $\log_7 x + \log_x 7 = 2.5$ then $x = ?$

(1) 7,49 (2) $\sqrt{7}$ (3) $7, \frac{1}{7}$ (4) none

**Solution:**

Let $y = \log_7 x$. Then $\log_x 7 = \dfrac{1}{\log_7 x} = \dfrac{1}{y}$.

The equation becomes $y + \dfrac{1}{y} = 2.5 = \dfrac{5}{2}$.

$2y^2 + 2 = 5y \implies 2y^2 - 5y + 2 = 0.$

$(2y - 1)(y - 2) = 0.$

This gives $y = 1/2$ or $y = 2$.

Case 1: $\log_7 x = 1/2 \implies x = 7^{1/2} = \sqrt{7}.$

Case 2: $\log_7 x = 2 \implies x = 7^2 = 49.$

Option (2) lists one of the possible solutions.

The correct option is **(2)**.

128. The sum of all solutions of $\sqrt{3}\sin x + \cos x = 2$ in the interval $[0, 4\pi]$ is:

(1) $\frac{8\pi}{3}$ (2) $2\pi$ (3) $\frac{7\pi}{3}$ (4) $4\pi$

**Solution:**

Divide by 2: $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 1.$
$\cos(\pi/6)\sin x + \sin(\pi/6)\cos x = 1.$
$\sin(x + \pi/6) = 1.$
General solution is $x + \pi/6 = 2n\pi + \pi/2 \implies x = 2n\pi + \pi/3.$
Solutions in $[0, 4\pi]$ :
$n = 0 \implies x = \pi/3.$
$n = 1 \implies x = 2\pi + \pi/3 = 7\pi/3.$
$\text{Sum} = \frac{\pi}{3} + \frac{7\pi}{3} = \frac{8\pi}{3}.$

The correct option is **(1)**.

129. The value of $\log_3(\log_2(\log_{\sqrt{3}} 81))$ is:

(1) -2 (2) -1 (3) 0 (4) 1

**Solution:**

Work from the inside out.

Step 1: $\log_{\sqrt{3}} 81 = \log_{3^{1/2}}(3^4) = \frac{4}{1/2}\log_3 3 = 8.$

Step 2: $\log_2(8) = \log_2(2^3) = 3.$

Step 3: $\log_3(3) = 1.$

The correct option is **(4)**.

130. The number of integral values of 'k' for which the equation $3\cos x - 4\sin x = k+1$ has a solution is:

(1) 9 (2) 10 (3) 11 (4) 12

**Solution:**

The range of $a\cos x + b\sin x$ is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$.
Range of $3\cos x - 4\sin x$ is $[-\sqrt{3^2+(-4)^2}, \sqrt{3^2+(-4)^2}] = [-5, 5]$.
For a solution to exist, $k+1$ must be in this range.
$-5 \leq k+1 \leq 5.$
$-6 \leq k \leq 4.$
The integer values for k are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4.$
There are 11 integer values.

The correct option is **(3)**.

131. If $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$ then $x = ?$

(1) 0 (2) 1 (3) 2 (4) 3

**Solution:**

Given the equation:
$\log_7(2^x - 1) + \log_7(2^x - 7) = 1$

Using the logarithm product rule, $\log_b M + \log_b N = \log_b(MN)$ :
$\log_7((2^x - 1)(2^x - 7)) = 1$

Convert the logarithmic equation to its exponential form:
$(2^x - 1)(2^x - 7) = 7^1$
$(2^x)^2 - 7(2^x) - 1(2^x) + 7 = 7$
$(2^x)^2 - 8(2^x) = 0$

Let $y = 2^x$. The equation becomes a quadratic in $y$ :
$y^2 - 8y = 0$
$y(y - 8) = 0$
This gives two possible solutions for $y : y = 0$ or $y = 8$.

Substitute back $2^x$ for $y$ :
**Case 1:** $2^x = 0$. This is not possible for any real value of $x$.
**Case 2:** $2^x = 8 \implies 2^x = 2^3 \implies x = 3$.

We must verify that $x = 3$ is valid in the original equation's domain.
Argument 1: $2^x - 1 = 2^3 - 1 = 7 > 0$. (Valid)
Argument 2: $2^x - 7 = 2^3 - 7 = 1 > 0$. (Valid)

The only valid solution is $x = 3$.

The correct option is **(4)**.

132. The solution set for $\sin x + \sin 5x = \sin 3x$ in $(0, \pi)$ contains:

(1) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{4}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}$ (3) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{2\pi}{3}$ (4) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}$

**Solution:**

$(\sin 5x + \sin x) - \sin 3x = 0.$
$2 \sin 3x \cos 2x - \sin 3x = 0.$
$\sin 3x(2 \cos 2x - 1) = 0.$
Case 1: $\sin 3x = 0 \implies 3x = n\pi \implies x = n\pi/3.$
In $(0, \pi)$, solutions are $\pi/3, 2\pi/3$.

Case 2: $\cos 2x = 1/2 \implies 2x = 2n\pi \pm \pi/3 \implies x = n\pi \pm \pi/6$.
In $(0, \pi)$, solutions are $\pi/6, 5\pi/6$.
The complete solution set is $\{\pi/6, \pi/3, 2\pi/3, 5\pi/6\}$.
The question asks which option is contained in the solution set.
Option (2) $\{\pi/6, 5\pi/6, \pi/3\}$ is a subset of our solution set.

The correct option is **(2)**.

133. $log(\frac{a^2}{bc}) + log(\frac{c^2}{ab}) + log(\frac{b^2}{ca}) =?$

(1) -2 (2) -1 (3) 0 (4) 1

**Solution:**

$$\log\left(\frac{a^2}{bc}\cdot\frac{c^2}{ab}\cdot\frac{b^2}{ca}\right)$$
$$= \log\left(\frac{a^2b^2c^2}{a^2b^2c^2}\right) = \log(1) = 0.$$

The correct option is **(3)**.

134. The general solution of $\sin x - 2\sin 2x + \sin 3x = \cos x - 2\cos 2x + \cos 3x$ is $(n \in \mathbb{Z})$:

(1) $2n\pi, \frac{n\pi}{2} + \frac{\pi}{8}$ (2) $n\pi, \frac{n\pi}{2} + \frac{\pi}{8}$ (3) $\frac{n\pi}{2} + \frac{\pi}{8}, 2n\pi + \frac{\pi}{2}$ (4) $2n\pi$

**Solution:**

$$(\sin 3x + \sin x) - 2\sin 2x = (\cos 3x + \cos x) - 2\cos 2x$$
$$2\sin 2x\cos x - 2\sin 2x = 2\cos 2x\cos x - 2\cos 2x$$
$$2\sin 2x(\cos x - 1) = 2\cos 2x(\cos x - 1)$$
$$(\sin 2x - \cos 2x)(\cos x - 1) = 0.$$
$$\text{Case 1: } \cos x = 1 \implies x = 2n\pi.$$
$$\text{Case 2: } \sin 2x = \cos 2x \implies \tan 2x = 1 \implies x = \frac{n\pi}{2} + \frac{\pi}{8}.$$

The correct option is **(1)**.

135. Which of the following is equal to $\log_a b \cdot \log_b c \cdot \log_c a$?

(1) 1 (2) 0 (3) $\log_a a$ (4) $\log_c c$

**Solution:**

Using the change of base formula, $\log_y x = \dfrac{\log x}{\log y}$:

$$\frac{\log b}{\log a}\cdot\frac{\log c}{\log b}\cdot\frac{\log a}{\log c}$$
$$= 1.$$

Note that options (1), (3), and (4) are all equal to 1, but (1) is the simplest representation.

The correct option is **(1)**.

136. The general solution which satisfies both $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\sqrt{3}$ is ($n \in \mathbb{Z}$):

(1) $2n\pi + \frac{\pi}{3}$ (2) $2n\pi + \frac{2\pi}{3}$ (3) $n\pi + \frac{2\pi}{3}$ (4) $2n\pi - \frac{\pi}{3}$

**Solution:**

$$\begin{aligned}
&\sin\theta > 0 \text{ in Q1 and Q2.}\\
&\tan\theta < 0 \text{ in Q2 and Q4.}\\
&\text{The common quadrant is Q2.}\\
&\text{The reference angle for } \sin\theta = \sqrt{3}/2 \text{ is } \pi/3.\\
&\text{The angle in Q2 is } \theta = \pi - \pi/3 = 2\pi/3.\\
&\text{The general solution is } \theta = 2n\pi + \frac{2\pi}{3}.
\end{aligned}$$

The correct option is **(2)**.

137. If $\log_2 3 = p$ and $\log_3 5 = q$ then $\log_2 45 =$

(1) $2p + q$ (2) $p + 2q$ (3) $1 + pq$ (4) $p + q + 1$

**Solution:**

$$\begin{aligned}
&\log_2 45 = \log_2(9 \cdot 5) = \log_2(3^2 \cdot 5)\\
&= \log_2(3^2) + \log_2 5 = 2\log_2 3 + \log_2 5.\\
&\text{We are given } \log_2 3 = p.\\
&\text{We need } \log_2 5. \text{ Using change of base:}\\
&\log_2 5 = \frac{\log_3 5}{\log_3 2} = \frac{q}{1/\log_2 3} = q \cdot \log_2 3 = pq.\\
&\text{Therefore, } \log_2 45 = 2p + pq.
\end{aligned}$$

Note: This question is Bonus

138. The value of $\sin(\theta + \frac{\pi}{4})$ if $\tan(\pi\sin\theta) = \cot(\pi\cos\theta)$ is:

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\pm\frac{1}{2}$ (4) $\pm\frac{1}{2\sqrt{2}}$

**Solution:**

$$\begin{aligned}
&\tan(\pi\sin\theta) = \tan(\pi/2 - \pi\cos\theta).\\
&\pi\sin\theta = n\pi + \pi/2 - \pi\cos\theta.\\
&\sin\theta + \cos\theta = n + 1/2.\\
&\sqrt{2}\sin(\theta + \pi/4) = n + 1/2.
\end{aligned}$$

$$\sin(\theta + \pi/4) = \frac{n + 1/2}{\sqrt{2}}.$$

For the value to be in $[-1, 1], n$ can be 0 or -1.

The possible values are $\frac{1/2}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$ and $\frac{-1/2}{\sqrt{2}} = -\frac{1}{2\sqrt{2}}$.

The correct option is **(4)**.

139. $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{c^2}{ab}\right) + \log\left(\frac{b^2}{ca}\right) = ?$

(1) -2 (2) -1 (3) 0 (4) 1

**Solution:**

Given the expression:

$$\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{c^2}{ab}\right) + \log\left(\frac{b^2}{ca}\right)$$

Using the logarithm product rule, $\log M + \log N = \log(MN)$ :

$$= \log\left(\frac{a^2}{bc} \cdot \frac{c^2}{ab} \cdot \frac{b^2}{ca}\right)$$

Combine the terms inside the logarithm:

$$= \log\left(\frac{a^2b^2c^2}{a^2b^2c^2}\right)$$

Simplify the fraction:

$$= \log(1)$$

The logarithm of 1 to any valid base is 0.

$$= 0.$$

The correct option is **(3)**.

140. If $0 \le x \le \pi$, the number of solutions of $4^{\sin^2 x} + 4^{\cos^2 x} = 4$ is:

(1) 0 (2) 1 (3) 2 (4) 4

**Solution:**

Let $y = 4^{\sin^2 x}$. Then $y + \frac{4}{y} = 4$.

$y^2 - 4y + 4 = 0 \implies (y-2)^2 = 0 \implies y = 2.$

$4^{\sin^2 x} = 2 \implies (2^2)^{\sin^2 x} = 2^1.$

$2\sin^2 x = 1 \implies \sin^2 x = 1/2.$

$\sin x = \pm 1/\sqrt{2}.$

In $[0, \pi]$, $\sin x$ is non-negative, so we take $\sin x = 1/\sqrt{2}$.
The solutions are $x = \pi/4$ and $x = 3\pi/4$. There are 2 solutions.

The correct option is **(3)**.

141. Solve for x: $log_{10}x = -2$

(1) 0.01 (2) -2 (3) 100 (4) 0.1

**Solution:**

$$\log_{10} x = -2 \implies x = 10^{-2} = \frac{1}{100} = 0.01.$$

The correct option is **(1)**.

142. The equation $\tan\theta + \sec\theta = \sqrt{3}$ has how many solutions in the interval $[0, 3\pi]$?

(1) 1 (2) 2 (3) 3 (4) 4

**Solution:**
The equation simplifies to $\cos(\theta + \pi/6) = 1/2$, with the constraint that $\cos\theta \neq 0$. The general solutions are $\theta = 2n\pi + \pi/6$ and $\theta = 2n\pi - \pi/2$. We reject the second case as it makes $\cos\theta = 0$. So we only count solutions for $\theta = 2n\pi + \pi/6$ in $[0, 3\pi]$.

- $n = 0 \implies \theta = \pi/6$. (In interval)
- $n = 1 \implies \theta = 2\pi + \pi/6 = 13\pi/6$. (In interval, since $13/6 \approx 2.17 < 3$)

There are **2 solutions**. The correct option is **(2)**.

143. If $\log_3[1 + log_3(2^x - 7)] = 1$ then $x =$?

(1) 2 (2) 3 (3) 4 (4) 5

**Solution:**

$$1 + \log_3(2^x - 7) = 3^1 = 3.$$
$$\log_3(2^x - 7) = 2.$$
$$2^x - 7 = 3^2 = 9.$$
$$2^x = 16 \implies x = 4.$$

The correct option is **(3)**.

144. If $\sec 4\theta - \sec 2\theta = 2$, then the general value of $\theta$ is $(n \in \mathbb{Z})$:

(1) $(2n+1)\frac{\pi}{6}$ (2) $(2n+1)\frac{\pi}{10}$ (3) $(2n+1)\frac{\pi}{8}$ (4) $(2n+1)\frac{\pi}{12}$

**Solution:**
The equation simplifies to $2\cos 5\theta \cos\theta = 0$. Since $\sec\theta$ must be defined, $\cos\theta \neq 0$. Thus, we must have $\cos 5\theta = 0$.

$$5\theta = (2n+1)\frac{\pi}{2}.$$

$$\theta = (2n+1)\frac{\pi}{10}.$$

The correct option is **(2)**.

145. If $\log_2 x = 5$ then x is:

(1) 5 (2) 10 (3) 25 (4) 32

**Solution:**

$$\log_2 x = 5 \implies x = 2^5 = 32.$$

The correct option is **(4)**.

146. The general value of $\theta$ satisfying the equation $\sqrt{3}\tan(\theta) = 1$ is ($n \in \mathbb{Z}$):

(1) $n\pi + \frac{\pi}{6}$ (2) $\frac{n\pi}{7} + \frac{\pi}{42}$ (3) $n\pi + \frac{\pi}{21}$ (4) $\frac{n\pi}{7} - \frac{\pi}{42}$

**Solution:**

$$\tan(\theta) = 1/\sqrt{3} = \tan(\pi/6).$$
$$\theta = n\pi + \pi/6.$$
$$\theta = n\pi + \frac{\pi}{6}.$$

The correct option is **(1)**.

147. $\frac{\log_8 256}{\log_9 729} = ?$

(1) 8/9 (2) 9/8 (3) 17/8 (4) None

**Solution:**

$$\text{Numerator: } \log_8 256 = \frac{\log_2 256}{\log_2 8} = \frac{\log_2(2^8)}{\log_2(2^3)} = \frac{8}{3}.$$
$$\text{Denominator: } \log_9 729 = \frac{\log_3 729}{\log_3 9} = \frac{\log_3(3^6)}{\log_3(3^2)} = \frac{6}{2} = 3.$$
$$\text{Fraction} = \frac{8/3}{3} = \frac{8}{9}.$$

The correct option is **(1)**.

148. The equation $\cos x + \sin x = 5$ has:

(1) No solution (2) One solution (3) Two solutions (4) Infinitely many solutions

**Solution:**
The maximum value of $\cos x + \sin x$ is $\sqrt{1^2 + 1^2} = \sqrt{2} \approx 1.414$. Since $\sqrt{2} < 5$, the expression on

the left can never equal 5. Thus, there is no solution.
The correct option is **(1)**.

149. $\log_{10}(\frac{12}{5}) + \log_{10}(\frac{25}{21}) - \log_{10}(\frac{2}{7}) =?$

(1) -4 (2) 1 (3) -3 (4) 2

**Solution:**

$$\log_{10}\left(\frac{\frac{12}{5}\cdot\frac{25}{21}}{\frac{2}{7}}\right) = \log_{10}\left(\frac{12\cdot 5}{21}\cdot\frac{7}{2}\right)$$
$$= \log_{10}\left(\frac{6\cdot 5\cdot 7}{21}\right) = \log_{10}\left(\frac{6\cdot 5}{3}\right) = \log_{10}(10) = 1.$$

The correct option is **(2)**.

150. $8^{\log_2 \sqrt[3]{11}} =?$

(1) 3 (2) 8 (3) 11 (4) 22

**Solution:**

$$8^{\log_2(11^{1/3})} = (2^3)^{\log_2(11^{1/3})}$$
$$= 2^{3\cdot\log_2(11^{1/3})} = 2^{\log_2((11^{1/3})^3)}$$
$$= 2^{\log_2(11)} = 11.$$

The correct option is **(3)**.