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Date of Exam: 10th August 2025

Syllabus: Trigonometric Equation & Modulus Inequality & Logarithm

Sub: Mathematics **CT-05 JEE Main** Prof. Chetan Sir

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51. The principal solutions of the equation $\sqrt{2}\cos x - 1 = 0$ are:

(1) $\frac{\pi}{4}, \frac{5\pi}{4}$ (2) $\frac{\pi}{4}, \frac{7\pi}{4}$ (3) $\frac{\pi}{5}, \frac{7\pi}{4}$ (4) None of these

Solution:

Given the equation $\sqrt{2}\cos x - 1 = 0$.

$$\sqrt{2}\cos x = 1$$
$$\cos x = \frac{1}{\sqrt{2}}$$

We need the principal solutions, which are in the interval $[0, 2\pi]$. The value of $\cos x$ is positive, so x must lie in the first or fourth quadrant. The reference angle for $\cos x = 1/\sqrt{2}$ is $\frac{\pi}{4}$.

- In Quadrant I: $x = \frac{\pi}{4}$.
- In Quadrant IV: $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

The principal solutions are $\frac{\pi}{4}$ and $\frac{7\pi}{4}$.

The correct option is **(2)**.

52. Solution of the $|x-2| > |x+1|$ is:

(1) $x < \frac{1}{2}$ (2) $x \leq \frac{1}{2}$ (3) $x \geq \frac{1}{2}$ (4) $x > \frac{1}{2}$

Solution:

Since both sides of the inequality involve absolute values, they are non-negative. We can square both sides without changing the inequality sign.

$$\begin{aligned}|x-2|^2 &> |x+1|^2 \\ (x-2)^2 &> (x+1)^2 \\ x^2-4x+4 &> x^2+2x+1 \\ -4x+4 &> 2x+1 \\ 3 &> 6x \\ \frac{3}{6} &> x \\ x &< \frac{1}{2}\end{aligned}$$

The solution is $(-\infty, 1/2)$.

The correct option is **(1)**.

53. If $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\cos\theta = -\frac{1}{2}$ where $-\pi < \theta < 0$, then θ is equal to:

(1) $-\frac{5\pi}{6}$ (2) $-\frac{2\pi}{3}$ (3) $\frac{4\pi}{6}$ (4) $-\frac{4\pi}{6}$

Solution:

Given that $\sin\theta < 0$ and $\cos\theta < 0$, the angle θ must lie in the third quadrant. The given interval for θ is $(-\pi, 0)$, which covers the third and fourth quadrants. The reference angle α for $\sin\theta = \sqrt{3}/2$ or $\cos\theta = 1/2$ is $\alpha = \pi/3$. For an angle in the third quadrant within the interval $(-\pi, 0)$, the value is given by $-\pi + \alpha$.

$$\begin{aligned}\theta &= -\pi + \frac{\pi}{3} \\ &= -\frac{2\pi}{3}\end{aligned}$$

This value lies in $(-\pi, 0)$.

The correct option is **(2)**.

54. Solve for x: $|x-1| + |x-2| \leq 3$

(1) [-3,0] (2) [-3,0) (3) [0,3] (4) (0,3)

Solution:

We consider cases based on the critical points $x = 1$ and $x = 2$.

- **Case 1:** $x < 1$. The inequality becomes $-(x-1) - (x-2) \leq 3$.

$$\begin{aligned}-x+1-x+2 &\leq 3 \\ -2x+3 &\leq 3 \\ -2x &\leq 0 \\ x &\geq 0\end{aligned}$$

The solution for this case is the intersection of $x < 1$ and $x \geq 0$, which is $[0, 1)$.

- **Case 2:** $1 \le x < 2$. The inequality becomes $(x-1)-(x-2) \le 3$.

$$\begin{aligned} x-1-x+2 &\le 3 \\ 1 &\le 3 \end{aligned}$$

  This is always true. The solution for this case is the interval $[1,2)$.
- **Case 3:** $x \ge 2$. The inequality becomes $(x-1)+(x-2) \le 3$.

$$\begin{aligned} 2x-3 &\le 3 \\ 2x &\le 6 \\ x &\le 3 \end{aligned}$$

  The solution for this case is the intersection of $x \ge 2$ and $x \le 3$, which is $[2,3]$.

The final solution is the union of the solutions from all cases: $[0,1) \cup [1,2) \cup [2,3] = [0,3]$.

The correct option is **(3)**.

55. If $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$ then θ is equal to:

(1) $2n\pi + \frac{\pi}{6}$ (2) $2n\pi - \frac{\pi}{6}$ (3) $2n\pi + \frac{\pi}{3}$ (4) $2n\pi - \frac{\pi}{3}$

Solution:

We rewrite the equation in terms of $\sin\theta$ and $\cos\theta$.

$$\begin{aligned} \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} &= \sqrt{3} \quad (\text{assuming } \sin\theta \neq 0) \\ \frac{\cos\theta + 1}{\sin\theta} &= \sqrt{3} \end{aligned}$$

Using half-angle identities, $\cos\theta + 1 = 2\cos^2(\theta/2)$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$.

$$\begin{aligned} \frac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)} &= \sqrt{3} \\ \frac{\cos(\theta/2)}{\sin(\theta/2)} &= \sqrt{3} \\ \cot(\theta/2) &= \sqrt{3} \\ \cot(\theta/2) &= \cot(\pi/6) \end{aligned}$$

The general solution for $\cot A = \cot\alpha$ is $A = n\pi + \alpha$.

$$\begin{aligned} \frac{\theta}{2} &= n\pi + \frac{\pi}{6} \\ \theta &= 2n\pi + \frac{\pi}{3} \end{aligned}$$

For this solution, $\sin(2n\pi + \pi/3) = \sin(\pi/3) = \sqrt{3}/2 \neq 0$, so the solution is valid.

The correct option is **(3)**.

56. Find x satisfying $|x| \geq x + 2$:

(1) $(-\infty, -1)$ (2) $(-\infty, -1]$ (3) $(1, \infty)$ (4) $[1, \infty)$

Solution:

We consider two cases based on the definition of $|x|$.

- **Case 1:** $x \geq 0$. The inequality becomes $x \geq x + 2$.

$$0 \geq 2$$

This is false. There is no solution in this case.

- **Case 2:** $x < 0$. The inequality becomes $-x \geq x + 2$.

$$\begin{aligned} -2 &\geq 2x \\ -1 &\geq x \quad \text{or} \quad x \leq -1 \end{aligned}$$

The solution for this case is the intersection of $x < 0$ and $x \leq -1$, which is $(-\infty, -1]$.

The final solution is the union of solutions from all cases, which is $(-\infty, -1]$.

The correct option is **(2)**.

57. The number of real solutions for the equation $\sin 2x + \cos 4x = 2$ is:

(1) 0 (2) 1 (3) 2 (4) ∞

Solution:

We can rewrite the equation to form a quadratic in terms of $\sin(2x)$. Using the double angle identity $\cos(2A) = 1 - 2\sin^2(A)$, we can express $\cos(4x)$ as:

$$\cos(4x) = \cos(2 \cdot 2x) = 1 - 2\sin^2(2x)$$

Now, substitute this into the original equation:

$$\begin{aligned} \sin(2x) + (1 - 2\sin^2(2x)) &= 2 \\ -2\sin^2(2x) + \sin(2x) + 1 - 2 &= 0 \\ -2\sin^2(2x) + \sin(2x) - 1 &= 0 \end{aligned}$$

Multiplying by -1 to make the leading coefficient positive, we get:

$$2\sin^2(2x) - \sin(2x) + 1 = 0$$

Let $y = \sin(2x)$. The equation becomes a quadratic in y:

$$2y^2 - y + 1 = 0$$

To find the solutions for y, we can check the discriminant, $D = b^2 - 4ac$:

$$\begin{aligned} D &= (-1)^2 - 4(2)(1) \\ &= 1 - 8 \\ &= -7 \end{aligned}$$

Since the discriminant is negative ($D < 0$), there are no real solutions for y. Because $y = \sin(2x)$, and there are no real values for y that satisfy the quadratic equation, there can be no real values of x that satisfy the original trigonometric equation. Therefore, the number of real solutions is 0.

The correct option is **(1)**.

58. Solve $|x^2 - 5| \geq 4$:

(1) $(-\infty, -3] \cup [-1, 1] \cup [3, \infty)$ (2) $(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$ (3) $(-\infty, 0)$ (4) $(0, \infty)$

Solution:

The inequality $|A| \geq k$ (for $k > 0$) is equivalent to $A \geq k$ or $A \leq -k$.

$$\begin{aligned} & |x^2 - 5| \geq 4 \\ \implies & x^2 - 5 \geq 4 \quad \text{or} \quad x^2 - 5 \leq -4 \\ \implies & x^2 \geq 9 \quad \text{or} \quad x^2 \leq 1 \\ \implies & (x \leq -3 \text{ or } x \geq 3) \quad \text{or} \quad (-1 \leq x \leq 1) \end{aligned}$$

The solution set is the union of these intervals: $(-\infty, -3] \cup [3, \infty) \cup [-1, 1]$.

The correct option is **(1)**.

59. If $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$ then the value of θ is:

(1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) π

Solution:

We convert the equation into a quadratic in $\cos\theta$ using the identity $\sin^2\theta = 1 - \cos^2\theta$.

$$\begin{aligned} 2(1 - \cos^2\theta) + \sqrt{3}\cos\theta + 1 &= 0 \\ 2 - 2\cos^2\theta + \sqrt{3}\cos\theta + 1 &= 0 \\ -2\cos^2\theta + \sqrt{3}\cos\theta + 3 &= 0 \\ 2\cos^2\theta - \sqrt{3}\cos\theta - 3 &= 0 \end{aligned}$$

Instead of solving the quadratic, we can test the given options. Let's check option (3), $\theta = 5\pi/6$. For $\theta = 5\pi/6$, we have $\cos(5\pi/6) = -\frac{\sqrt{3}}{2}$. Substituting this into the equation:

$$\begin{aligned} 2\left(-\frac{\sqrt{3}}{2}\right)^2 - \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - 3 &= 2\left(\frac{3}{4}\right) + \frac{3}{2} - 3 \\ &= \frac{3}{2} + \frac{3}{2} - 3 \\ &= 3 - 3 = 0 \end{aligned}$$

Since the equation is satisfied, $\theta = 5\pi/6$ is a solution.

The correct option is **(3)**.

60. Solution set of the equation $|x - 2| + |x - 10| = 8$ is:

(1) (2,10) (2) [2,10] (3) [-2,10] (4) [-10,2]

Solution:

The equation is of the form $|x-a|+|x-b|=b-a$, where $a=2$ and $b=10$.
Here, $b-a=10-2=8$, which matches the right side of the equation.

This type of equation represents the property that the sum of distances of a point x from two fixed points a and b is equal to the distance between a and b. This is true for any point x lying in the closed interval between a and b.

Therefore, the solution set is $[a,b]$, which is $[2,10]$.

The correct option is **(2)**.

61. The general value of θ satisfying $\tan^2 2\theta = 3$ is:

(1) $\frac{n\pi}{2} \pm \frac{\pi}{6}$ (2) $n\pi \pm \frac{2\pi}{3}$ (3) $2n\pi \pm \frac{\pi}{3}$ (4) none of these

Solution:

The given equation is $\tan^2(2\theta)=3$.

$$\tan^2(2\theta) = (\sqrt{3})^2$$
$$\tan^2(2\theta) = \tan^2(\pi/3)$$

The general solution for an equation of the form $\tan^2 A = \tan^2\alpha$ is $A = n\pi \pm \alpha$, where n is an integer.

$$2\theta = n\pi \pm \frac{\pi}{3}$$
$$\theta = \frac{n\pi}{2} \pm \frac{\pi}{6}$$

This gives the general value of θ.

The correct option is **(1)**.

62. Solution set of the equation $|3x-7|+|7x-5|=|10x-12|$ is:

(1) $(-\infty,5/7) \cup (7/3,\infty)$ (2) $(-\infty,7/3]\cup[5/7,\infty)$ (3) $(-\infty,5/7]\cup[7/3,\infty)$ (4) None of these

Solution:

The equation is of the form $|A|+|B|=|A+B|$.
Let $A=3x-7$ and $B=7x-5$.
Then $A+B=(3x-7)+(7x-5)=10x-12$.
The equation $|A|+|B|=|A+B|$ holds true if and only if A and B have the same sign, i.e., $A\cdot B \geq 0$.

$$(3x-7)(7x-5) \geq 0$$

The critical points are the roots of the factors, which are $x=7/3$ and $x=5/7$.
The quadratic expression $(3x-7)(7x-5)$ is a parabola opening upwards, so it is non-negative outside its roots.

The solution is $x \leq 5/7$ or $x \geq 7/3$.
In interval notation, this is $(-\infty, 5/7] \cup [7/3, \infty)$.

The correct option is **(3)**.

63. Solution of $\sin 2x = \sqrt{2}\cos x$ is:

(1) $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n\frac{\pi}{4}$ (2) $(2n-1)\frac{\pi}{2}$ (3) $(2n+1)\frac{\pi}{3}$ (4) $(2n-1)\frac{\pi}{3}$

Solution:

We use the identity $\sin(2x) = 2\sin x\cos x$.

$$2\sin x\cos x = \sqrt{2}\cos x$$
$$2\sin x\cos x - \sqrt{2}\cos x = 0$$
$$\cos x(2\sin x - \sqrt{2}) = 0$$

This gives two possibilities:

- **Case 1:** $\cos x = 0$. The general solution is $x = (2n+1)\frac{\pi}{2}$, where n is an integer.
- **Case 2:** $2\sin x - \sqrt{2} = 0$.

$$2\sin x = \sqrt{2}$$
$$\sin x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The general solution is $x = n\pi + (-1)^n\frac{\pi}{4}$, where n is an integer.

The complete solution set is the union of the solutions from both cases.

The correct option is **(1)**.

64. Solution set of the inequality $|2x-3| \geq 5$ is:

(1) $(-\infty,-1] \cup [4,\infty)$ (2) $(-\infty,-1) \cup (4,\infty)$ (3) $(-\infty,0) \cup (5,\infty)$ (4) $(-\infty,0] \cup [5,\infty)$

Solution:

The inequality $|A| \geq k$ (for $k > 0$) is equivalent to $A \geq k$ or $A \leq -k$.

$$\begin{aligned} & |2x-3| \geq 5 \\ \implies & 2x-3 \geq 5 \quad \text{or} \quad 2x-3 \leq -5 \\ \implies & 2x \geq 8 \quad \text{or} \quad 2x \leq -2 \\ \implies & x \geq 4 \quad \text{or} \quad x \leq -1 \end{aligned}$$

The solution is the union of these two conditions: $(-\infty,-1] \cup [4,\infty)$.

The correct option is **(1)**.

65. The solution of the equation $\sin\frac{\theta}{2} = -1$ is:

(1) $2n\pi + (-1)^n\pi$ (2) $4n\pi - \pi$ (3) $2n\pi + \frac{\pi}{2}$ (4) $2n\pi$

Solution:

The general solution for $\sin A = -1$ is $A = 2n\pi - \frac{\pi}{2}$, where n is an integer.

$$\begin{aligned}\sin(\theta/2) &= -1 \\ \frac{\theta}{2} &= 2n\pi - \frac{\pi}{2} \\ \theta &= 2\left(2n\pi - \frac{\pi}{2}\right) \\ \theta &= 4n\pi - \pi\end{aligned}$$

This is the general solution for θ.

The correct option is **(2)**.

66. The set of real numbers for which $x^2 - |x+2| + x > 0$ is:

(1) $(-\infty, -2) \cup (2, \infty)$ (2) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ (3) $(-\infty, -1) \cup (1, \infty)$ (4) $(\sqrt{2}, \infty)$

Solution:

We analyze the inequality by considering cases based on the expression inside the absolute value. The critical point is $x = -2$.

- **Case 1:** $x + 2 \geq 0 \implies x \geq -2$. The inequality becomes $x^2 - (x+2) + x > 0$.

$$\begin{aligned}x^2 - x - 2 + x &> 0 \\ x^2 - 2 &> 0 \\ x^2 &> 2\end{aligned}$$

  This implies $x < -\sqrt{2}$ or $x > \sqrt{2}$. The solution for this case is the intersection of $x \geq -2$ and $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$, which is $[-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

- **Case 2:** $x + 2 < 0 \implies x < -2$. The inequality becomes $x^2 - (-(x+2)) + x > 0$.

$$\begin{aligned}x^2 + (x+2) + x &> 0 \\ x^2 + 2x + 2 &> 0\end{aligned}$$

  For the quadratic $x^2 + 2x + 2$, the discriminant is $D = b^2 - 4ac = 2^2 - 4(1)(2) = 4 - 8 = -4$. Since the leading coefficient is positive $(1 > 0)$ and the discriminant is negative, the quadratic expression is always positive for all real x. The solution for this case is the intersection of all real numbers and $x < -2$, which is $(-\infty, -2)$.

The final solution is the union of the solutions from both cases:
$((-\infty, -2)) \cup ([-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)) = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

The correct option is **(2)**.

67. Solution of the trigonometric equation $2\tan^2 x + \sec^2 x = 2$ for $x \in [0, 2\pi]$ is:

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) π (4) $\frac{\pi}{2}$

Solution:

We use the identity $\sec^2 x = 1 + \tan^2 x$ to express the equation solely in terms of $\tan x$.

$$\begin{aligned} 2\tan^2 x + (1 + \tan^2 x) &= 2 \\ 3\tan^2 x + 1 &= 2 \\ 3\tan^2 x &= 1 \\ \tan^2 x &= \frac{1}{3} \\ \tan x &= \pm\frac{1}{\sqrt{3}} \end{aligned}$$

We need to find the solutions in the interval $[0, 2\pi]$.

- If $\tan x = 1/\sqrt{3}$, the solutions are $x = \pi/6$ (Quadrant I) and $x = \pi + \pi/6 = 7\pi/6$ (Quadrant III).
- If $\tan x = -1/\sqrt{3}$, the solutions are $x = \pi - \pi/6 = 5\pi/6$ (Quadrant II) and $x = 2\pi - \pi/6 = 11\pi/6$ (Quadrant IV).

The set of solutions in $[0, 2\pi]$ is $\{\pi/6, 5\pi/6, 7\pi/6, 11\pi/6\}$. From the given options, $5\pi/6$ is one of the solutions.

The correct option is **(1)**.

68. The value of x for which the expression $\frac{1}{\sqrt{x+|x|}}$ is defined:

(1) $(0, \infty)$ (2) $(-\infty, 0)$ (3) $(2, \infty)$ (4) $(-\infty, -2)$

Solution:

For the expression to be defined, the term inside the square root must be strictly positive, as it is in the denominator.

$$x + |x| > 0$$

We consider two cases for $|x|$:

- **Case 1:** $x \geq 0$. In this case, $|x| = x$. The inequality becomes:

$$\begin{aligned} x + x &> 0 \\ 2x &> 0 \\ x &> 0 \end{aligned}$$

  The solution for this case is the intersection of $x \geq 0$ and $x > 0$, which is $(0, \infty)$.

- **Case 2:** $x < 0$. In this case, $|x| = -x$. The inequality becomes:

$$\begin{aligned} x + (-x) &> 0 \\ 0 &> 0 \end{aligned}$$

  This is a false statement. There is no solution in this case.

The overall domain is the union of the solutions from the cases, which is $(0, \infty)$.

The correct option is **(1)**.

69. If $\tan 2\theta = -\cot 3\theta$, then the general value of θ is:

(1) $\frac{(2n-1)\pi}{10}$ (2) $\frac{(2n+1)\pi}{10}$ (3) $n\pi + \frac{\pi}{2}$ (4) $n\pi$

Solution:

The given equation can be rewritten as:

$$\tan 2\theta + \cot 3\theta = 0$$
$$\frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos 3\theta}{\sin 3\theta} = 0$$

This requires $\cos 2\theta \neq 0$ and $\sin 3\theta \neq 0$. Taking a common denominator:

$$\frac{\sin 2\theta \sin 3\theta + \cos 2\theta \cos 3\theta}{\cos 2\theta \sin 3\theta} = 0$$
$$\sin 2\theta \sin 3\theta + \cos 2\theta \cos 3\theta = 0$$

Using the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$:

$$\cos(3\theta - 2\theta) = 0$$
$$\cos(\theta) = 0$$

The general solution for $\cos\theta = 0$ is $\theta = (2n+1)\frac{\pi}{2}$, which can also be written as $\theta = n\pi + \frac{\pi}{2}$. We must check if this solution makes the denominators zero. If $\theta = n\pi + \pi/2$, then $2\theta = 2n\pi + \pi$, so $\cos(2\theta) = \cos(\pi) = -1 \neq 0$. And $3\theta = 3n\pi + 3\pi/2$, so $\sin(3\theta) = \sin(3n\pi + 3\pi/2) = \pm\cos(3n\pi) = \pm 1 \neq 0$. The solution is valid.

The correct option is **(3)**.

70. The solution set of the inequality $|x^2 - 2x - 3| < |x^2 - x + 5|$ is:

(1) $(-8, \infty)$ (2) $(8, \infty)$ (3) $(-\infty, -8)$ (4) $(-\infty, 8)$

Solution:

Since both sides are non-negative, we can square them:

$$(x^2 - 2x - 3)^2 < (x^2 - x + 5)^2$$
$$(x^2 - 2x - 3)^2 - (x^2 - x + 5)^2 < 0$$

Using the difference of squares identity $A^2 - B^2 = (A - B)(A + B)$:

$$[(x^2 - 2x - 3) - (x^2 - x + 5)][(x^2 - 2x - 3) + (x^2 - x + 5)] < 0$$
$$(-x - 8)(2x^2 - 3x + 2) < 0$$
$$-(x + 8)(2x^2 - 3x + 2) < 0$$

Multiplying by -1 reverses the inequality sign:

$$(x + 8)(2x^2 - 3x + 2) > 0$$

Consider the quadratic term $2x^2 - 3x + 2$. Its discriminant is $D = (-3)^2 - 4(2)(2) = 9 - 16 = -7$. Since the leading coefficient (2) is positive and the discriminant is negative, the quadratic $2x^2-3x+2$ is always positive for all real x. Thus, we can divide the inequality by this positive term without changing the sign:

$$x + 8 > 0 \implies x > -8$$

The solution set is $(-8, \infty)$.

The correct option is **(1)**.

SECTION-B: Integer Type Questions

71. The number of values of $x \in [0, 2\pi]$ that satisfy $\cot x - \operatorname{cosec} x = 2 \sin x$ is:
Solution:

Rewrite the equation in terms of sine and cosine.

$$\begin{aligned} \frac{\cos x}{\sin x} - \frac{1}{\sin x} &= 2 \sin x \quad (\text{for } \sin x \neq 0) \\ \frac{\cos x - 1}{\sin x} &= 2 \sin x \\ \cos x - 1 &= 2 \sin^2 x \end{aligned}$$

Using the identity $\sin^2 x = 1 - \cos^2 x$:

$$\begin{aligned} \cos x - 1 &= 2(1 - \cos^2 x) \\ \cos x - 1 &= 2 - 2\cos^2 x \\ 2\cos^2 x + \cos x - 3 &= 0 \end{aligned}$$

This is a quadratic equation in $\cos x$. Let $y = \cos x$.

$$\begin{aligned} 2y^2 + y - 3 &= 0 \\ (2y + 3)(y - 1) &= 0 \end{aligned}$$

So, $y = -3/2$ or $y = 1$. This means $\cos x = -3/2$ or $\cos x = 1$. Since $-1 \leq \cos x \leq 1$, the solution $\cos x = -3/2$ is impossible. The only possible solution is $\cos x = 1$. In the interval $[0, 2\pi]$, this occurs at $x = 0$ and $x = 2\pi$. However, the original equation is defined only if $\sin x \neq 0$. For $x = 0$ and $x = 2\pi$, $\sin x = 0$. These values make the denominator zero, so they are extraneous solutions. Therefore, there are no solutions.

The correct answer is **0**.

72. Solution of the inequality $|x + 2| + |x - 3| > 5$ is $(-\infty, -\alpha) \cup (\beta, \infty)$ then $\alpha + \beta =?$
Solution:

The critical points are $x = -2$ and $x = 3$.

- **Case 1:** $x < -2$. $-(x + 2) - (x - 3) > 5 \implies -2x + 1 > 5 \implies -2x > 4 \implies x < -2$. The solution is $(-\infty, -2)$.

- **Case 2:** $-2 \le x < 3$. $(x+2)-(x-3) > 5 \implies 5 > 5$. This is false, so no solution in this interval.
- **Case 3:** $x \ge 3$. $(x+2)+(x-3) > 5 \implies 2x-1 > 5 \implies 2x > 6 \implies x > 3$. The solution is $(3, \infty)$.

The complete solution set is $(-\infty, -2) \cup (3, \infty)$. Comparing this with the given form $(-\infty, -\alpha) \cup (\beta, \infty)$, we get $\alpha = 2$ and $\beta = 3$. Then, $\alpha + \beta = 2 + 3 = 5$.

The correct answer is **5**.

73. The number of roots of the equation $\cos^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$, which lie in the interval $[-\pi, \pi]$ is:
Solution:

Substitute $\cos^2 x = 1 - \sin^2 x$.

$$1 - \sin^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$$
$$-\sin^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} = 0$$

Multiply by -4 to clear fractions and the leading negative sign:

$$4\sin^2 x - 2(\sqrt{3}+1)\sin x + \sqrt{3} = 0$$

This quadratic in $\sin x$ can be factored:

$$4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} = 0$$
$$2\sin x(2\sin x - \sqrt{3}) - 1(2\sin x - \sqrt{3}) = 0$$
$$(2\sin x - 1)(2\sin x - \sqrt{3}) = 0$$

This gives two possibilities:

- **Case 1:** $2\sin x - 1 = 0 \implies \sin x = 1/2$. In the interval $[-\pi, \pi]$, the solutions are $x = \pi/6$ and $x = 5\pi/6$. (2 solutions)
- **Case 2:** $2\sin x - \sqrt{3} = 0 \implies \sin x = \sqrt{3}/2$. In the interval $[-\pi, \pi]$, the solutions are $x = \pi/3$ and $x = 2\pi/3$. (2 solutions)

All four solutions are distinct. Thus, there are 4 roots in the given interval.

The correct answer is **4**.

74. Number of integral solution of $|x^2 - 16| + |x^2 - 25| = 9$ is:
Solution:

Let $y = x^2$. The equation becomes $|y-16|+|y-25| = 9$. This is of the form $|y-a|+|y-b| = b-a$, with $a = 16$ and $b = 25$. We have $b - a = 25 - 16 = 9$, which matches the equation. The solution for this form is $a \le y \le b$.

$$16 \le y \le 25$$

Substituting back $y = x^2$:

$$16 \le x^2 \le 25$$

Taking the square root of all parts:

$$\sqrt{16} \le \sqrt{x^2} \le \sqrt{25}$$
$$4 \le |x| \le 5$$

This inequality holds if $4 \le x \le 5$ or $-5 \le x \le -4$. The solution set for x is $[-5, -4] \cup [4, 5]$. The integers in this set are $\{-5, -4, 4, 5\}$. There are 4 integral solutions.

The correct answer is **4**.

75. The number of values of θ in $[0, 6\pi]$ satisfying the equation $3\cos^2\theta - 10\cos\theta + 7 = 0$ are:
Solution:

This is a quadratic equation in $\cos\theta$. Let $y = \cos\theta$.

$$3y^2 - 10y + 7 = 0$$
$$3y^2 - 3y - 7y + 7 = 0$$
$$3y(y-1) - 7(y-1) = 0$$
$$(3y-7)(y-1) = 0$$

This gives $y = 7/3$ or $y = 1$. Substituting back $y = \cos\theta$:

- $\cos\theta = 7/3$: This is impossible, since the range of $\cos\theta$ is $[-1, 1]$.
- $\cos\theta = 1$: The general solution is $\theta = 2n\pi$, where n is an integer.

We need to find the number of solutions in the interval $[0, 6\pi]$. We test integer values for n:

- $n = 0 : \theta = 0$. This is in $[0, 6\pi]$.
- $n = 1 : \theta = 2\pi$. This is in $[0, 6\pi]$.
- $n = 2 : \theta = 4\pi$. This is in $[0, 6\pi]$.
- $n = 3 : \theta = 6\pi$. This is in $[0, 6\pi]$.
- $n = 4 : \theta = 8\pi$. This is outside the interval.

There are 4 solutions in the given interval.

The correct answer is **4**.