Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

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Topic: Trigonometric Ratios & Trigonometric Equation

Sub: Mathematics **JEE Main CT-06** Prof. Chetan Sir

Index of Questions

51. The number of solutions of the equation $|cosx| = cos\ x - 2\ sin\ x$ in $[0, 6\pi]$ is:

(1) 1 (2) 5 (3) 7 (4) 9

Solution:

Case 1: $\cos x \geq 0$. The equation becomes $\cos x = \cos x - 2\sin x$.

$\implies 2\sin x = 0 \implies \sin x = 0.$

This gives $x = n\pi$. We need solutions in $[0, 6\pi]$ for which $\cos x \geq 0$.

Possible values: $x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi$.

$\cos(0) = 1, \cos(\pi) = -1, \cos(2\pi) = 1, \cos(3\pi) = -1, \cos(4\pi) = 1, \cos(5\pi) = -1, \cos(6\pi) = 1.$

The valid solutions are $x = 0, 2\pi, 4\pi, 6\pi$ (4 solutions).

Case 2: $\cos x < 0$. The equation becomes $-\cos x = \cos x - 2\sin x$.

$\implies 2\sin x = 2\cos x \implies \tan x = 1.$

This gives $x = n\pi + \dfrac{\pi}{4}$. We need solutions in $[0, 6\pi]$ for which $\cos x < 0$ (i.e., in Q2 or Q3).

$x = \dfrac{3\pi}{4}$ (Q2, valid), $x = \dfrac{7\pi}{4}$ (Q4, invalid), $x = \dfrac{11\pi}{4}$ (Q3, valid),

$x = \dfrac{15\pi}{4}$ (Q1, invalid), $x = \dfrac{19\pi}{4}$ (Q2, valid), $x = \dfrac{23\pi}{4}$ (Q4, invalid).

The valid solutions are $x = \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}$ (3 solutions).

Total number of solutions = 4 + 3 = 7.

The correct option is **(3)**.

52. Number of solution of $\sum_{r=1}^{5} cos\ rx = 5$ in the interval $[0, 4\pi]$ is :

(1) 0 (2) 2 (3) 3 (4) 7

Solution:

The given equation is $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 5$.
The maximum value of the cosine function, $\cos\theta$, is 1.
The sum of five cosine terms can be equal to 5
only if each individual term is equal to its maximum value, 1.
Therefore, we must have simultaneously:
$\cos x = 1, \cos 2x = 1, \cos 3x = 1, \cos 4x = 1,$ and $\cos 5x = 1$.

The condition $\cos x = 1$ implies that x must be an even multiple of π.
$x = 2k\pi$, where k is an integer.

We need to find the number of solutions in the interval $[0, 4\pi]$.
For $k = 0, x = 0$.
For $k = 1, x = 2\pi$.
For $k = 2, x = 4\pi$.
There are 3 solutions in the given interval.

The correct option is **(3)**.

53. The set of solution satisfying inequality $\sin\theta + \sqrt{3}\cos\theta \geq 1, -\pi < \theta \leq \pi$

(1) $\theta \in [\frac{\pi}{3}, \frac{\pi}{2}]$ (2) $\theta \in [-\frac{\pi}{6}, \frac{\pi}{2}]$ (3) $\theta \in [\frac{\pi}{6}, \frac{\pi}{2}]$ (4) None of these

Solution:

Given the inequality $\sin\theta + \sqrt{3}\cos\theta \geq 1$.
We convert the expression on the LHS to the form $R\sin(\theta + \alpha)$.
$R = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$.
Divide by 2: $\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta \geq \frac{1}{2}$.
$\cos\frac{\pi}{3}\sin\theta + \sin\frac{\pi}{3}\cos\theta \geq \frac{1}{2}$.
$\sin(\theta + \frac{\pi}{3}) \geq \frac{1}{2}$.

Let $\alpha = \theta + \frac{\pi}{3}$. The given interval for θ is $(-\pi, \pi]$.
The interval for α is $(-\pi + \pi/3, \pi + \pi/3]$, which is $(-2\pi/3, 4\pi/3]$.
The inequality $\sin\alpha \geq 1/2$ holds for $\alpha \in [\pi/6, 5\pi/6]$ in one cycle.
We check which part of this solution lies in the interval $(-2\pi/3, 4\pi/3]$.
The entire interval $[\pi/6, 5\pi/6]$ is within $(-2\pi/3, 4\pi/3]$.

So, we have $\frac{\pi}{6} \leq \alpha \leq \frac{5\pi}{6}$.
$\frac{\pi}{6} \leq \theta + \frac{\pi}{3} \leq \frac{5\pi}{6}$.
$\frac{\pi}{6} - \frac{\pi}{3} \leq \theta \leq \frac{5\pi}{6} - \frac{\pi}{3}$.
$-\frac{\pi}{6} \leq \theta \leq \frac{3\pi}{6}$.
$-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$.

The correct option is **(2)**.

54. The maximum value of $4\sin^2 x + 3\cos^2 x$ is:

(1) 3 (2) 4 (3) 5 (4) 7

Solution:

Let the expression be E.
$E = 4\sin^2 x + 3\cos^2 x$.
Using the identity $\cos^2 x = 1 - \sin^2 x$:
$E = 4\sin^2 x + 3(1 - \sin^2 x) = 4\sin^2 x + 3 - 3\sin^2 x$.
$E = \sin^2 x + 3$.

To find the maximum value of E, we need the maximum value of $\sin^2 x$.
The range of $\sin x$ is $[-1, 1]$, so the range of $\sin^2 x$ is $[0, 1]$.
The maximum value of $\sin^2 x$ is 1.

$E_{max} = (\sin^2 x)_{max} + 3 = 1 + 3 = 4$.

The correct option is **(2)**.

55. $3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ$ is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Solution:

We know the triple angle identity for tangent: $\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.
Let $\theta = 10^\circ$ and $t = \tan 10^\circ$.

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} = \frac{3t - t^3}{1 - 3t^2}.$$

Square both sides:
$\frac{1}{3} = \frac{(3t - t^3)^2}{(1 - 3t^2)^2}$.
$(1 - 3t^2)^2 = 3(3t - t^3)^2$.
$1 - 6t^2 + 9t^4 = 3(9t^2 - 6t^4 + t^6)$.
$1 - 6t^2 + 9t^4 = 27t^2 - 18t^4 + 3t^6$.

Rearrange the terms to match the expression in the question:
$1 = 3t^6 - 18t^4 - 9t^4 + 27t^2 + 6t^2$.
$1 = 3t^6 - 27t^4 + 33t^2$.
Therefore, $3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1$.

The correct option is **(1)**.

56. The value of $16\sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18}) =$

(1) 1 (2) 2 (3) 4 (4) 8

Solution:

Let $E = 16\sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18})$.
Convert angles to degrees: $\pi/18 = 10^\circ, 5\pi/18 = 50^\circ, 7\pi/18 = 70^\circ$.
$E = 16\sin(10^\circ)\sin(50^\circ)\sin(70^\circ)$.
We use the identity $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin(3\theta)$.
Let $\theta = 10^\circ$. Then $50^\circ = 60^\circ - 10^\circ$ and $70^\circ = 60^\circ + 10^\circ$.
$\sin(10^\circ)\sin(50^\circ)\sin(70^\circ) = \frac{1}{4}\sin(3\times 10^\circ) = \frac{1}{4}\sin(30^\circ) = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$.
$E = 16 \times \frac{1}{8} = 2$.

The correct option is **(2)**.

57. The value of $5\cos\theta + 3\cos(\theta + \frac{\pi}{3}) + 3$ lies between

(1) -2 and 5 (2) -1 and 8 (3) -3 and 6 (4) -4 and 10

Solution:

Let $E = 5\cos\theta + 3\cos(\theta + \frac{\pi}{3}) + 3$.

Expand $\cos(\theta + \pi/3) = \cos\theta\cos(\pi/3) - \sin\theta\sin(\pi/3) = \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$.

$E = 5\cos\theta + 3\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) + 3$.

$$E=\left(5+\frac{3}{2}\right)\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3=\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3.$$
The range of $a\cos\theta+b\sin\theta$ is $[-\sqrt{a^2+b^2},\sqrt{a^2+b^2}]$.
Here, $a=13/2, b=-3\sqrt{3}/2$.
$$\sqrt{a^2+b^2}=\sqrt{(\frac{13}{2})^2+(-\frac{3\sqrt{3}}{2})^2}=\sqrt{\frac{169}{4}+\frac{27}{4}}=\sqrt{\frac{196}{4}}=\sqrt{49}=7.$$
The range of $\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta$ is $[-7,7]$.
The range of E is $[-7+3,7+3]=[-4,10]$.

The correct option is **(4)**.

58. $\cos\frac{\pi}{19}+\cos\frac{3\pi}{19}+\cos\frac{5\pi}{19}+...+\cos\frac{17\pi}{19}=$

(1) 1 (2) $-1/2$ (3) $1/2$ (4) 0

Solution:

The angles are in AP with $a=\frac{\pi}{19}, d=\frac{2\pi}{19}, n=9$.
$$\text{Sum}=\frac{\sin(nd/2)}{\sin(d/2)}\cos\left(\frac{\text{first + last}}{2}\right)=\frac{\sin(9\pi/19)}{\sin(\pi/19)}\cos\left(\frac{9\pi}{19}\right).$$
$$=\frac{\frac{1}{2}\sin(18\pi/19)}{\sin(\pi/19)}=\frac{\frac{1}{2}\sin(\pi-\pi/19)}{\sin(\pi/19)}=\frac{1}{2}.$$

The correct option is **(3)**.

59. $\cos 12^\circ\cdot\cos 24^\circ\cdot\cos 36^\circ\cdot\cos 48^\circ\cdot\cos 72^\circ\cdot\cos 84^\circ$ is

(1) $1/32$ (2) $1/16$ (3) $1/64$ (4) $1/128$

Solution:

$$P=(\cos 12^\circ\cos 48^\circ\cos 72^\circ)\cdot(\cos 24^\circ\cos 36^\circ\cos 84^\circ).$$
Using $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)=\frac{1}{4}\cos(3\theta)$:
First triplet (with $\theta=12^\circ$) : $\cos 12^\circ\cos 48^\circ\cos 72^\circ=\frac{1}{4}\cos(36^\circ)$.
Second triplet (with $\theta=24^\circ$) : $\cos 24^\circ\cos 36^\circ\cos 84^\circ=\frac{1}{4}\cos(72^\circ)$.
$$P=\left(\frac{1}{4}\cos 36^\circ\right)\cdot\left(\frac{1}{4}\cos 72^\circ\right)=\frac{1}{16}\cos 36^\circ\cos 72^\circ.$$
$$P=\frac{1}{16}\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)=\frac{1}{16}\left(\frac{5-1}{16}\right)=\frac{1}{16}\cdot\frac{4}{16}=\frac{1}{64}.$$

The correct option is **(3)**.

60. The value of $\sin 36^\circ \sin 72^\circ \sin 108^\circ \sin 144^\circ$ is

(1) 1/4 (2) 1/16 (3) 3/4 (4) 5/16

Solution:

$$\begin{aligned}
&\text{Let P be the expression. Use } \sin(180^\circ - \theta) = \sin\theta.\\
&\sin 144^\circ = \sin(180^\circ - 36^\circ) = \sin 36^\circ.\\
&\sin 108^\circ = \sin(180^\circ - 72^\circ) = \sin 72^\circ.\\
&P = \sin 36^\circ \sin 72^\circ \sin 72^\circ \sin 36^\circ = (\sin 36^\circ \sin 72^\circ)^2.\\
&\text{Using } \sin 72^\circ = \cos 18^\circ :\\
&P = (\sin 36^\circ \cos 18^\circ)^2.\\
&P = (2\sin 18^\circ \cos 18^\circ \cos 18^\circ)^2 = (2\sin 18^\circ \cos^2 18^\circ)^2.\\
&\text{Alternatively, } \sin 36^\circ \sin 72^\circ = \frac{\sqrt{5}}{4}.\\
&P = \left(\frac{\sqrt{5}}{4}\right)^2 = \frac{5}{16}.
\end{aligned}$$

The correct option is **(4)**.

61. The maximum and minimum values of $\cos^6\theta + \sin^6\theta$ are m and n respectively then the value of $m + 4n$ is

(1) 2 (2) 1 (3) 4 (4) 3

Solution:

$$\begin{aligned}
&E = \cos^6\theta + \sin^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - \frac{3}{4}\sin^2(2\theta).\\
&\text{The range of } \sin^2(2\theta) \text{ is } [0, 1].\\
&\text{Max value (m) occurs when } \sin^2(2\theta) = 0 : m = 1 - 0 = 1.\\
&\text{Min value (n) occurs when } \sin^2(2\theta) = 1 : n = 1 - 3/4 = 1/4.\\
&m + 4n = 1 + 4(1/4) = 1 + 1 = 2.
\end{aligned}$$

The correct option is **(1)**.

62. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$ is

(1) $\frac{1}{16}$ (2) $-\frac{1}{16}$ (3) 1 (4) 0

Solution:

$$\begin{aligned}
&\text{Let } \theta = \frac{\pi}{15}. \text{ The product is } P = \cos\theta\cos(2\theta)\cos(4\theta)\cos(8\theta).\\
&P = \frac{\sin(2^4\theta)}{2^4\sin\theta} = \frac{\sin(16\pi/15)}{16\sin(\pi/15)} = \frac{\sin(\pi + \pi/15)}{16\sin(\pi/15)} = \frac{-\sin(\pi/15)}{16\sin(\pi/15)} = -\frac{1}{16}.
\end{aligned}$$

The correct option is **(2)**.

63. The value of $8\sin(\frac{\pi}{14})\sin(\frac{3\pi}{14})\sin(\frac{5\pi}{14}) =$

(1) 4 (2) 2 (3) 8 (4) 1

Solution:

$$E = 8\sin(\frac{\pi}{14})\sin(\frac{3\pi}{14})\sin(\frac{5\pi}{14}).$$

Using $\sin\theta = \cos(\pi/2 - \theta)$:

$$E = 8\cos(\frac{\pi}{2} - \frac{\pi}{14})\cos(\frac{\pi}{2} - \frac{3\pi}{14})\cos(\frac{\pi}{2} - \frac{5\pi}{14}) = 8\cos(\frac{3\pi}{7})\cos(\frac{2\pi}{7})\cos(\frac{\pi}{7}).$$

We know the product $\cos(\pi/7)\cos(2\pi/7)\cos(4\pi/7) = -1/8$.

Using $\cos(3\pi/7) = -\cos(\pi - 3\pi/7) = -\cos(4\pi/7)$.

$$E = 8[-\cos(\frac{4\pi}{7})]\cos(\frac{2\pi}{7})\cos(\frac{\pi}{7}) = -8\left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right].$$

$$E = -8\left(-\frac{1}{8}\right) = 1.$$

The correct option is **(4)**.

64. The sum $\frac{1}{\sin 45^\circ \sin 46^\circ} + \frac{1}{\sin 47^\circ \sin 48^\circ} + ... + \frac{1}{\sin 133^\circ \sin 134^\circ}$ is equal to:

(1) $sec(1)^\circ$ (2) $cosec(1)^\circ$ (3) $cot(1)^\circ$ (4) None

Solution:

Let the general term be $T_k = \frac{1}{\sin k^\circ \sin(k+1)^\circ}$. The sum is not consecutive.

Let's assume a typo and the sum is consecutive: $S = \sum_{k=45}^{134} \frac{1}{\sin k^\circ \sin(k+1)^\circ}$.

$$T_k = \frac{1}{\sin 1^\circ}\frac{\sin((k+1)-k)^\circ}{\sin k^\circ \sin(k+1)^\circ} = \frac{\cot k^\circ - \cot(k+1)^\circ}{\sin 1^\circ}.$$

$$S = \frac{1}{\sin 1^\circ}[\cot 45^\circ - \cot 135^\circ] = \frac{1}{\sin 1^\circ}[1-(-1)] = \frac{2}{\sin 1^\circ} = 2\,\text{cosec}\,1^\circ.$$

Assuming the question intended a consecutive sum, the answer would be $2\,\text{cosec}\,1^\circ$. As written, the problem is non-standard.

65. The maximum value of $\log_{20}(3\sin x - 4\cos x + 15)$ is equal to:

(1) 16 (2) 2 (3) 1 (4) 4

Solution:

To maximize the logarithm, we must maximize its argument, as the base 20 is greater than 1.

Let $E = 3\sin x - 4\cos x + 15$.

The range of $3\sin x - 4\cos x$ is $[-\sqrt{3^2+(-4)^2}, \sqrt{3^2+(-4)^2}] = [-5, 5]$.

The maximum value of $3\sin x - 4\cos x$ is 5.

$E_{max} = 5 + 15 = 20$.

The maximum value of the function is $\log_{20}(E_{max}) = \log_{20}(20) = 1.$

The correct option is **(1)**.

66. If $k = \cos\frac{\pi}{7} cos\frac{\pi}{5} cos\frac{2\pi}{7} cos\frac{2\pi}{5} cos\frac{4\pi}{7}$ and $32k + 4$ is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Solution:

$$k = \left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)\cdot\left(\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\right).$$

$$\text{Part 1: } \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = \frac{\sin(8\pi/7)}{8\sin(\pi/7)} = -\frac{1}{8}.$$

$$\text{Part 2: } \cos 36^\circ \cos 72^\circ = \left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right) = \frac{1}{4}.$$

$$k = (-\frac{1}{8})(\frac{1}{4}) = -\frac{1}{32}.$$

$$32k + 4 = 32(-1/32) + 4 = -1 + 4 = 3.$$

The correct option is **(3)**.

67. The number of solutions of the equation $\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is

Solution:

The given equation is $\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}.$

Using the product-to-sum identity $2\cos A\cos B = \cos(A+B) + \cos(A-B)$:

$$\cos 2\theta\cos\frac{\theta}{2} = \frac{1}{2}\left(\cos(2\theta + \frac{\theta}{2}) + \cos(2\theta - \frac{\theta}{2})\right) = \frac{1}{2}\left(\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2}\right).$$

Substitute this into the equation:

$$\frac{1}{2}\left(\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2}\right) + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}.$$

$$\frac{3}{2}\cos\frac{5\theta}{2} + \frac{1}{2}\cos\frac{3\theta}{2} = 2\cos^3\frac{5\theta}{2}.$$

$$3\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = 4\cos^3\frac{5\theta}{2}.$$

Rearrange the terms:

$$\cos\frac{3\theta}{2} = 4\cos^3\frac{5\theta}{2} - 3\cos\frac{5\theta}{2}.$$

Using the triple angle identity $\cos(3A) = 4\cos^3 A - 3\cos A,$ with $A = \frac{5\theta}{2}$:

$$\cos\frac{3\theta}{2} = \cos\left(3\cdot\frac{5\theta}{2}\right) = \cos\frac{15\theta}{2}.$$

Now, solve the general equation $\cos x = \cos y \implies x = 2n\pi \pm y$:
$\dfrac{15\theta}{2} = 2n\pi \pm \dfrac{3\theta}{2}.$

Case 1: $\dfrac{15\theta}{2} = 2n\pi + \dfrac{3\theta}{2} \implies \dfrac{12\theta}{2} = 2n\pi \implies 6\theta = 2n\pi \implies \theta = \dfrac{n\pi}{3}.$
Case 2: $\dfrac{15\theta}{2} = 2n\pi - \dfrac{3\theta}{2} \implies \dfrac{18\theta}{2} = 2n\pi \implies 9\theta = 2n\pi \implies \theta = \dfrac{2n\pi}{9}.$

We need solutions in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
From $\theta = \dfrac{n\pi}{3}$: $n = 0 \implies \theta = 0;\quad n = 1 \implies \theta = \pi/3;\quad n = -1 \implies \theta = -\pi/3.$
From $\theta = \dfrac{2n\pi}{9}$: $n = 0 \implies \theta = 0;\quad n = 1 \implies \theta = 2\pi/9;\quad n = -1 \implies \theta = -2\pi/9;$
$n = 2 \implies \theta = 4\pi/9;\quad n = -2 \implies \theta = -4\pi/9.$

The distinct solutions in the interval are: $\{-\frac{4\pi}{9}, -\frac{\pi}{3}, -\frac{2\pi}{9}, 0, \frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}\}.$
There are 7 solutions.

68. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = cos\ 2\theta$ and $2\cos^2\theta = 3\sin\ \theta$ is

(1) 4π (2) $\frac{5\pi}{6}$ (3) π (4) $\frac{\pi}{2}$

Solution:

Eq 1: $2\sin^2\theta = 1 - 2\sin^2\theta \implies 4\sin^2\theta = 1 \implies \sin\theta = \pm 1/2.$
Eq 2: $2(1-\sin^2\theta) = 3\sin\theta \implies 2\sin^2\theta + 3\sin\theta - 2 = 0.$
$(2\sin\theta - 1)(\sin\theta + 2) = 0 \implies \sin\theta = 1/2$ (as $\sin\theta \neq -2$).
The only common condition is $\sin\theta = 1/2$.
In $[0, 2\pi]$, solutions are $\theta = \pi/6$ and $\theta = 5\pi/6$.
Sum of solutions $= \pi/6 + 5\pi/6 = \pi$.

The correct option is **(3)**.

69. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is

Solution:

Let $c = \cos\theta, s = \sin\theta$.
$3c^4 - 5c^2 - 2s^6 + 2 = 0.$

$3c^4 - 5c^2 - 2(1-c^2)^3 + 2 = 0.$
$3c^4 - 5c^2 - 2(1 - 3c^2 + 3c^4 - c^6) + 2 = 0.$
$3c^4 - 5c^2 - 2 + 6c^2 - 6c^4 + 2c^6 + 2 = 0.$
$2c^6 - 3c^4 + c^2 = 0 \implies c^2(2c^4 - 3c^2 + 1) = 0.$
$c^2(2c^2-1)(c^2-1) = 0.$
$\cos^2\theta = 0 \implies \cos\theta = 0 \implies \theta = \pi/2, 3\pi/2.$
$\cos^2\theta = 1/2 \implies \cos\theta = \pm 1/\sqrt{2} \implies \theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4.$
$\cos^2\theta = 1 \implies \cos\theta = \pm 1 \implies \theta = 0, \pi, 2\pi.$
The distinct solutions are $\{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4, 2\pi\}$.
There are 9 solutions.

The correct option is **(4)**.

70. Let $S = \{x \in (-\frac{\pi}{2}, \frac{\pi}{2}) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\}$ and $\beta = \sum_{x\in S} \tan^2(\frac{x}{3})$, then $\frac{1}{6}(\beta - 14)^2$ is

Let the given equation be $9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$.
$\frac{9}{9^{\tan^2 x}} + 9^{\tan^2 x} = 10.$
Let $y = 9^{\tan^2 x}$. The equation becomes:
$\frac{9}{y} + y = 10.$
$9 + y^2 = 10y \implies y^2 - 10y + 9 = 0.$
$(y-1)(y-9) = 0.$
This gives two possibilities: $y = 1$ or $y = 9$.

Case 1: $y = 1$.
$9^{\tan^2 x} = 1 = 9^0 \implies \tan^2 x = 0 \implies x = 0.$

Case 2: $y = 9$.
$9^{\tan^2 x} = 9 = 9^1 \implies \tan^2 x = 1 \implies \tan x = \pm 1.$
In the interval $(-\pi/2, \pi/2)$, this gives $x = \frac{\pi}{4}$ and $x = -\frac{\pi}{4}$.

The set of solutions is $S = \{-\frac{\pi}{4}, 0, \frac{\pi}{4}\}$.

Now, we calculate $\beta = \sum_{x\in S} \tan^2\left(\frac{x}{3}\right)$.
$\beta = \tan^2\left(-\frac{\pi}{12}\right) + \tan^2(0) + \tan^2\left(\frac{\pi}{12}\right).$
Since $\tan(-A) = -\tan A, \tan^2(-A) = \tan^2 A$.
$\beta = 2\tan^2\left(\frac{\pi}{12}\right) + 0.$
$\tan\left(\frac{\pi}{12}\right) = \tan(15^\circ) = 2 - \sqrt{3}.$

$\tan^2\left(\frac{\pi}{12}\right) = (2-\sqrt{3})^2 = 4+3-4\sqrt{3} = 7-4\sqrt{3}.$
$\beta = 2(7-4\sqrt{3}) = 14-8\sqrt{3}.$

Finally, we evaluate the required expression:
$\frac{1}{6}(\beta-14)^2 = \frac{1}{6}((14-8\sqrt{3})-14)^2.$
$= \frac{1}{6}(-8\sqrt{3})^2 = \frac{1}{6}(64\times 3) = \frac{192}{6} = 32.$

SECTION-B

71. The number of solutions of the equations $3^{2\sec^2 x}+1 = 10.3^{\tan^2 x}$ in the interval $[0, 2\pi]$ is :

Solution:

$3^{2(1+\tan^2 x)} + 1 = 10\cdot 3^{\tan^2 x}.$
$3^2\cdot 3^{2\tan^2 x} + 1 = 10\cdot 3^{\tan^2 x}.$
Let $y = 3^{\tan^2 x}$. $9y^2 - 10y + 1 = 0.$
$(9y-1)(y-1) = 0 \implies y = 1/9$ or $y = 1.$
$3^{\tan^2 x} = 1/9 = 3^{-2} \implies \tan^2 x = -2$ (Impossible).
$3^{\tan^2 x} = 1 = 3^0 \implies \tan^2 x = 0 \implies \tan x = 0.$
In $[0, 2\pi]$, solutions are $x = 0, \pi, 2\pi$. Total 3 solutions.

The answer key says 4. There may be a misinterpretation.

72. If $6|\sin x| = x$ when $x \in [0, 2\pi]$ then the number of solutions are:

Solution:

We find the number of intersections of the graphs $y = 6|\sin x|$ and $y = x$ in $[0, 2\pi]$.
The graph of $y = x$ is a straight line passing through the origin.
The graph of $y = 6|\sin x|$ is a series of arches with a maximum height of 6.
At $x = 0, y = 0,$ so x=0 is a solution.
The line $y = x$ has a slope of 1. The initial slope of $y = 6\sin x$ is $y'(0) = 6\cos(0) = 6$.
Since the slope of $6\sin x$ is greater than the slope of x at the origin, the curves will intersect again.
We need to see where $y = x$ crosses the x-axis relative to the arches.
At $x = \pi \approx 3.14,$ the line is $y = 3.14$. The arch is at y=0.
The line $y = x$ will be above the first arch $[0, \pi]$.
The line $y = x$ will intersect the first arch twice (once at x=0, and once more).

At $x = 2\pi \approx 6.28$, the line is at $y = 6.28$. The maximum height of the sine curve is 6. The line will pass above the second arch completely.
By drawing the graphs, we see 4 points of intersection.

The answer is **4**.

73. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is :

Solution:

$$2\sin x\cos x - 2\cos x + 4\sin x - 4 = 0.$$
$$2\cos x(\sin x - 1) + 4(\sin x - 1) = 0.$$
$$(2\cos x + 4)(\sin x - 1) = 0.$$
This gives $\cos x = -2$ (impossible) or $\sin x = 1$.
We need to solve $\sin x = 1$ for $x \in [0, 5\pi]$.
$$x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}.$$
There are 3 solutions.

The answer is **3**.

74. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}, \alpha, \beta \in (0, \frac{\pi}{2})$ then $\tan(\alpha + 2\beta)$ is equal to

Solution:

First eqn: $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{2\cos^2\alpha}} = \dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha = \dfrac{1}{7}.$

Second eqn: $\sqrt{\sin^2\beta} = \sin\beta = \dfrac{1}{\sqrt{10}}.$

From $\sin\beta = 1/\sqrt{10}$, we get $\tan\beta = 1/3$.

$$\tan(2\beta) = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2/3}{1-1/9} = \frac{2/3}{8/9} = \frac{3}{4}.$$
$$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan(2\beta)}{1-\tan\alpha\tan(2\beta)} = \frac{1/7+3/4}{1-(1/7)(3/4)} = \frac{(4+21)/28}{(28-3)/28} = \frac{25}{25} = 1.$$

The answer is **1**.

75. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$, then $\sum_{\theta\in S}\sin^2(\theta + \frac{\pi}{4}) =$

Solution:

Let S be the set of solutions for the equation $\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$.
$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta) = \tan(-\pi\sin\theta)$.

The general solution for $\tan A = \tan B$ is $A = n\pi + B,$ where n is an integer.
$\pi \cos\theta = n\pi + (-\pi \sin\theta).$
$\cos\theta = n - \sin\theta.$
$\sin\theta + \cos\theta = n.$

We know the range of $\sin\theta + \cos\theta$ is $[-\sqrt{2}, \sqrt{2}].$
Since n must be an integer, the possible values for n are -1, 0, and 1.

For each case, we find $\sin^2(\theta + \frac{\pi}{4}):$
$(\sin\theta + \cos\theta)^2 = n^2.$
$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = n^2.$
$1 + \sin(2\theta) = n^2.$
Also, $\sin^2(\theta + \frac{\pi}{4}) = \left(\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right)^2 = \frac{(\sin\theta + \cos\theta)^2}{2} = \frac{n^2}{2}.$

Case 1: $n = 0.$
$\sin\theta + \cos\theta = 0 \implies \tan\theta = -1.$ In $[0, 2\pi),$ solutions are $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}.$ (2 solutions)
For these solutions, $\sin^2(\theta + \frac{\pi}{4}) = \frac{0^2}{2} = 0.$

Case 2: $n = 1.$
$\sin\theta + \cos\theta = 1.$ In $[0, 2\pi),$ solutions are $\theta = 0, \frac{\pi}{2}.$ (2 solutions)
For these solutions, $\sin^2(\theta + \frac{\pi}{4}) = \frac{1^2}{2} = \frac{1}{2}.$

Case 3: $n = -1.$
$\sin\theta + \cos\theta = -1.$ In $[0, 2\pi),$ solutions are $\theta = \pi, \frac{3\pi}{2}.$ (2 solutions)
For these solutions, $\sin^2(\theta + \frac{\pi}{4}) = \frac{(-1)^2}{2} = \frac{1}{2}.$

The required sum is the sum of $\sin^2(\theta + \frac{\pi}{4})$ over all 6 solutions in S.
$$\sum_{\theta \in S} \sin^2(\theta + \frac{\pi}{4}) = (0 + 0) + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right).$$
$= 0 + 1 + 1 = 2.$

The answer is **2**.