

HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

**Topic:** Sets, Inequality, Logarithm, Trigonometric Ratios & Identities, Trigonometric Equations, Quadratic Equation

### 11th JEE Regular UT : 30 August 2025

Sub: Mathematics

**JEE Main**

Prof. Chetan Sir

#### SECTION - A : Single Correct Answer Type

1. The value of  $\sin(10^\circ) \sin(30^\circ) \sin(50^\circ) \sin(70^\circ)$  is:

(A)  $\frac{1}{8}$

(B)  $\frac{1}{16}$

(C)  $\frac{\sqrt{3}}{16}$

(D)  $\frac{1}{32}$

**(B)**

$$\begin{aligned} \text{Expression} &= \sin(10^\circ) \sin(30^\circ) \sin(50^\circ) \sin(70^\circ) \\ &= \sin(30^\circ) \cdot \sin(10^\circ) \sin(50^\circ) \sin(70^\circ) \\ &= \frac{1}{2} \cdot \sin(10^\circ) \sin(60^\circ - 10^\circ) \sin(60^\circ + 10^\circ) \end{aligned}$$

$$\text{Using the identity } \sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin(3\theta)$$

$$\text{Here, } \theta = 10^\circ$$

$$\begin{aligned} \text{Expression} &= \frac{1}{2} \cdot \left( \frac{1}{4} \sin(3 \times 10^\circ) \right) \\ &= \frac{1}{8} \sin(30^\circ) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16} \end{aligned}$$

2. If  $X = \{x : x \in N\}$ ,  $Y = \{2x : x \in N\}$  and  $Z = \{5x : x \in N\}$  then  $X \cap Y \cap Z$  is equal to:

(A)  $\{x : x \in N\}$

(B)  $\{5x : x \in N\}$

(C)  $\{10x : x \in N\}$

(D)  $\{2x : x \in N\}$

**(C)**

$$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\} \text{ (Set of Natural Numbers)}$$

$$Y = \{2, 4, 6, 8, 10, 12, \dots\} \text{ (Set of even Natural Numbers)}$$

$$Z = \{5, 10, 15, 20, \dots\} \text{ (Set of multiples of 5)}$$

$$X \cap Y = Y \text{ (Since all elements of Y are in X)}$$

$$(X \cap Y) \cap Z = Y \cap Z$$

$$= \{x : x \text{ is an even natural number and a multiple of 5}\}$$

$$= \{x : x \text{ is a multiple of } \text{lcm}(2, 5)\}$$

$$= \{x : x \text{ is a multiple of } 10\}$$

$$= \{10, 20, 30, \dots\} = \{10x : x \in N\}$$

3. If  $\tan(x/2) = p/q$ , then the value of  $q \cos x + p \sin x$  is:

(A) p

(B) q

(C)  $\sqrt{p^2 + q^2}$ (D)  $p + q$ **(B)**

We know the identities in terms of  $\tan(x/2)$  :

$$\cos x = \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)} = \frac{1 - (p/q)^2}{1 + (p/q)^2} = \frac{q^2 - p^2}{q^2 + p^2}$$

$$\sin x = \frac{2 \tan(x/2)}{1 + \tan^2(x/2)} = \frac{2(p/q)}{1 + (p/q)^2} = \frac{2pq}{q^2 + p^2}$$

Now, substitute these into the expression:

$$\begin{aligned} q \cos x + p \sin x &= q \left( \frac{q^2 - p^2}{q^2 + p^2} \right) + p \left( \frac{2pq}{q^2 + p^2} \right) \\ &= \frac{q(q^2 - p^2) + p(2pq)}{q^2 + p^2} \\ &= \frac{q^3 - qp^2 + 2p^2q}{q^2 + p^2} \\ &= \frac{q^3 + p^2q}{q^2 + p^2} = \frac{q(q^2 + p^2)}{q^2 + p^2} = q \end{aligned}$$

4. The number of real solutions of the equation  $|x|^2 - 3|x| + 2 = 0$  is:

(A) 4

(B) 1

(C) 2

(D) 3

**(A)**

Let  $y = |x|$ . The equation becomes a quadratic in  $y$ :

$$y^2 - 3y + 2 = 0$$

Factorizing the quadratic:

$$(y - 1)(y - 2) = 0$$

So,  $y = 1$  or  $y = 2$ .

Substitute back  $|x| = y$  :

Case 1:  $|x| = 1 \Rightarrow x = 1$  or  $x = -1$

Case 2:  $|x| = 2 \Rightarrow x = 2$  or  $x = -2$

The real solutions are  $\{-2, -1, 1, 2\}$ .

Total number of solutions is 4.

5. The value of  $\tan(1110^\circ)$  is:

(A)  $\sqrt{3}$ (B)  $\frac{1}{\sqrt{3}}$ (C)  $-\sqrt{3}$ (D)  $-\frac{1}{\sqrt{3}}$ **(B)**

$$\tan(1110^\circ) = \tan(n \cdot 360^\circ + \theta) = \tan(\theta)$$

Divide 1110 by 360:

$$1110 = 3 \times 360 + 30$$

So,  $1110^\circ = 3 \times 360^\circ + 30^\circ$

$$\tan(1110^\circ) = \tan(3 \times 360^\circ + 30^\circ)$$

$$= \tan(30^\circ)$$

$$= \frac{1}{\sqrt{3}}$$

6. If  $x$  satisfies  $|x-1| + |x-2| + |x-3| \geq 6$ , then:

- (A)  $0 \leq x \leq 4$                       (B)  $x \leq -2$  or  $x \geq 4$                       (C)  $x \leq 0$  or  $x \geq 4$                       (D) None of these

(C)

Let  $f(x) = |x-1| + |x-2| + |x-3|$ . Critical points are 1, 2, 3.

**Case 1:**  $x < 1$

$$-(x-1) - (x-2) - (x-3) \geq 6 \Rightarrow -3x + 6 \geq 6 \Rightarrow -3x \geq 0 \Rightarrow x \leq 0.$$

Intersection with  $x < 1$  is  $x \leq 0$ .

**Case 2:**  $1 \leq x < 2$

$$(x-1) - (x-2) - (x-3) \geq 6 \Rightarrow -x + 4 \geq 6 \Rightarrow -x \geq 2 \Rightarrow x \leq -2.$$

No solution in  $[1, 2)$ .

**Case 3:**  $2 \leq x < 3$

$$(x-1) + (x-2) - (x-3) \geq 6 \Rightarrow x \geq 6.$$

No solution in  $[2, 3)$ .

**Case 4:**  $x \geq 3$

$$(x-1) + (x-2) + (x-3) \geq 6 \Rightarrow 3x - 6 \geq 6 \Rightarrow 3x \geq 12 \Rightarrow x \geq 4.$$

Intersection with  $x \geq 3$  is  $x \geq 4$ .

Combining the solutions from all cases, we get  $x \leq 0$  or  $x \geq 4$ .

7. The value of  $\frac{\cos(2\pi-\theta)\sec(\pi+\theta)}{\csc(\pi/2-\theta)\sin(3\pi/2+\theta)}$  is:

- (A) 1                      (B) -1                      (C)  $\tan \theta$                       (D)  $\cot \theta$

(A)

Evaluate each term using trigonometric identities:

$$\cos(2\pi - \theta) = \cos(\theta) \quad (4\text{th Quadrant})$$

$$\sec(\pi + \theta) = -\sec(\theta) \quad (3\text{rd Quadrant})$$

$$\csc(\pi/2 - \theta) = \sec(\theta) \quad (\text{Co-function identity})$$

$$\sin(3\pi/2 + \theta) = -\cos(\theta) \quad (4\text{th Quadrant})$$

Substitute these back into the expression:

$$\frac{\cos(\theta)(-\sec(\theta))}{\sec(\theta)(-\cos(\theta))} = \frac{-\cos(\theta)\sec(\theta)}{-\sec(\theta)\cos(\theta)}$$

$$= \frac{-1}{-1} = 1$$

8. The solution set of the inequation  $\frac{2x+4}{x-1} \geq 5$  is:

- (A) (1, 3)                      (B) (1, 3]                      (C)  $(-\infty, 1) \cup [3, \infty)$                       (D) None of these

**(B)**

$$\frac{2x+4}{x-1} \geq 5$$

$$\frac{2x+4}{x-1} - 5 \geq 0$$

$$\frac{2x+4-5(x-1)}{x-1} \geq 0$$

$$\frac{2x+4-5x+5}{x-1} \geq 0$$

$$\frac{-3x+9}{x-1} \geq 0$$

$$\frac{3(3-x)}{x-1} \geq 0$$

Multiply by -1 and reverse the inequality sign:

$$\frac{x-3}{x-1} \leq 0$$

Critical points are  $x = 1, x = 3$ .

Using the wavy curve method, the expression is  $\leq 0$  between the roots.

Since  $x - 1$  is in the denominator,  $x \neq 1$ .

The solution is  $1 < x \leq 3$ , which is the interval  $(1, 3]$ .

9. The principal solutions of the equation  $2 \sin \theta + \sqrt{3} = 0$  are:

(A)  $\frac{2\pi}{3}, \frac{4\pi}{3}$

(B)  $\frac{4\pi}{3}, \frac{5\pi}{3}$

(C)  $\frac{5\pi}{6}, \frac{7\pi}{6}$

(D)  $\frac{2\pi}{3}, \frac{5\pi}{3}$

**(B)**

$$2 \sin \theta + \sqrt{3} = 0$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

Sine is negative in the 3rd and 4th quadrants.

The principal value for  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$  is  $\frac{\pi}{3}$ .

$$\text{3rd Quadrant solution: } \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\text{4th Quadrant solution: } \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

Both solutions lie in the interval  $[0, 2\pi)$ .

The principal solutions are  $\frac{4\pi}{3}$  and  $\frac{5\pi}{3}$ .

10. The number of integral solutions of  $2(x+2) > x^2 + 1$  is:

(A) 2

(B) 3

(C) 4

(D) 5

**(B)**

$$2(x+2) > x^2 + 1$$

$$2x+4 > x^2 + 1$$

$$0 > x^2 - 2x - 3$$

$$x^2 - 2x - 3 < 0$$

Factorize the quadratic:

$$(x-3)(x+1) < 0$$

The roots are  $x = 3$  and  $x = -1$ .

The inequality holds between the roots.

$$-1 < x < 3$$

The integers in this interval are 0, 1, 2.

The number of integral solutions is 3.

11. The general solution of  $\tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 0$  is ( $n \in \mathbb{Z}$ ):

(A)  $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}$

(B)  $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{6}$

(C)  $n\pi \pm \frac{\pi}{4}$

(D)  $n\pi \pm \frac{\pi}{3}$

(A)

Let  $y = \tan x$ . The equation becomes:

$$y^2 - (1 + \sqrt{3})y + \sqrt{3} = 0$$

Factorizing the quadratic:

$$y^2 - y - \sqrt{3}y + \sqrt{3} = 0$$

$$y(y-1) - \sqrt{3}(y-1) = 0$$

$$(y-1)(y-\sqrt{3}) = 0$$

$$\text{So, } y = 1 \text{ or } y = \sqrt{3}.$$

$$\text{Case 1: } \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$$

$$\text{Case 2: } \tan x = \sqrt{3} \Rightarrow x = n\pi + \frac{\pi}{3}$$

The general solutions are  $n\pi + \frac{\pi}{4}$  and  $n\pi + \frac{\pi}{3}$ .

12. The solution for  $2 - 3x - 2x^2 \geq 0$  is:

(A)  $-1 < x < 2$

(B)  $-2 < x < 3$

(C)  $-2 < x < -1$

(D) None of these

(D)

$$2 - 3x - 2x^2 \geq 0$$

Multiply by -1 and reverse the inequality:

$$2x^2 + 3x - 2 \leq 0$$

Factorize the quadratic:

$$2x^2 + 4x - x - 2 \leq 0$$

$$2x(x+2) - 1(x+2) \leq 0$$

$$(2x-1)(x+2) \leq 0$$

$$\text{The roots are } x = \frac{1}{2} \text{ and } x = -2.$$

The inequality holds between the roots (inclusive).

$$-2 \leq x \leq \frac{1}{2}$$

This corresponds to none of the given options.

13. The roots of the equation  $x^2 - kx + 24 = 0$  are in the ratio 2:3. The value of k is:

(A)  $\pm 5$                       (B)  $\pm 10$                       (C)  $\pm 12$                       (D)  $\pm\sqrt{10}$

**(B)**

Let the roots be  $2\alpha$  and  $3\alpha$ .

Sum of roots:  $2\alpha + 3\alpha = 5\alpha = -(-k)/1 = k$

Product of roots:  $(2\alpha)(3\alpha) = 6\alpha^2 = 24/1 = 24$

From the product of roots:

$$\alpha^2 = 4 \Rightarrow \alpha = \pm 2$$

Substitute  $\alpha$  into the sum of roots equation:

$$k = 5\alpha = 5(\pm 2) = \pm 10$$

14. If  $\log_e 2 \cdot \log_x 27 = \log_{10} 8 \cdot \log_e 10$ , then  $x =$

(A) 1                      (B) 3                      (C) 2                      (D) 4

**(B)**

$$\log_e 2 \cdot \log_x 27 = \log_{10} 8 \cdot \log_e 10$$

Apply the change of base formula  $\log_b a = \frac{\log_c a}{\log_c b}$

$$\log_e 2 \cdot \frac{\log_e 27}{\log_e x} = \frac{\log_e 8}{\log_e 10} \cdot \log_e 10$$

$$\log_e 2 \cdot \frac{\log_e 3^3}{\log_e x} = \log_e 2^3$$

$$\ln 2 \cdot \frac{3 \ln 3}{\ln x} = 3 \ln 2$$

Assuming  $\ln 2 \neq 0$  and  $3 \neq 0$ , we can cancel them:

$$\frac{\ln 3}{\ln x} = 1$$

$$\ln x = \ln 3$$

$$x = 3$$

15. The value of m for which the sum of the roots of the equation  $x^2 + (m^2 - 3)x + 3 = 0$  is zero, is:

(A)  $\pm 1$                       (B)  $\pm 2$                       (C)  $\pm\sqrt{3}$                       (D)  $\pm 3$

**(C)**

For a quadratic equation  $ax^2 + bx + c = 0$ , the sum of roots is  $-\frac{b}{a}$ .

In this case,  $a = 1, b = m^2 - 3, c = 3$ .

$$\text{Sum of roots} = -\frac{m^2-3}{1} = -(m^2-3)$$

We are given that the sum of roots is zero:

$$-(m^2-3) = 0$$

$$m^2-3 = 0$$

$$m^2 = 3$$

$$m = \pm\sqrt{3}$$

16. If  $y = 2^{1/\log_x 8}$ , then  $x$  is equal to:

(A)  $y$

(B)  $y^2$

(C)  $y^3$

(D) none of these

(C)

$$y = 2^{1/\log_x 8}$$

Using the property  $\frac{1}{\log_b a} = \log_a b$ :

$$y = 2^{\log_8 x}$$

$$y = 2^{\log_{2^3} x}$$

Using the property  $\log_{a^n} b = \frac{1}{n} \log_a b$ :

$$y = 2^{\frac{1}{3} \log_2 x}$$

Using the property  $m \log_a b = \log_a b^m$ :

$$y = 2^{\log_2(x^{1/3})}$$

Using the property  $a^{\log_a b} = b$ :

$$y = x^{1/3}$$

Cubing both sides:

$$y^3 = x$$

17. If the roots of the equation  $x^2 + px + 8 = 0$  differ by 2, then a possible value of  $p$  is:

(A)  $\pm 2$

(B)  $\pm 4$

(C)  $\pm 6$

(D)  $\pm 8$

(C)

Let the roots be  $\alpha$  and  $\beta$ .

Given  $|\alpha - \beta| = 2$ .

We know that  $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ .

From the equation, sum of roots  $\alpha + \beta = -p$ .

Product of roots  $\alpha\beta = 8$ .

Substitute these values:

$$(2)^2 = (-p)^2 - 4(8)$$

$$4 = p^2 - 32$$

$$p^2 = 36$$

$$p = \pm 6$$

18. The value of  $\log_2\left(1 + \frac{1}{1}\right) + \log_2\left(1 + \frac{1}{2}\right) + \dots + \log_2\left(1 + \frac{1}{1023}\right)$  is:

(A) 50

(B) 40

(C) 30

(D) 10

**(D)**

Let the given expression be S.

$$S = \log_2\left(1 + \frac{1}{1}\right) + \log_2\left(1 + \frac{1}{2}\right) + \dots + \log_2\left(1 + \frac{1}{1023}\right).$$

First, simplify the term inside each logarithm:

$$1 + \frac{1}{k} = \frac{k+1}{k}.$$

So the expression becomes a sum of simplified logarithms:

$$S = \log_2\left(\frac{2}{1}\right) + \log_2\left(\frac{3}{2}\right) + \dots + \log_2\left(\frac{1024}{1023}\right).$$

Using the logarithm property  $\log a + \log b = \log(a \cdot b)$ , we can combine the terms:

$$S = \log_2\left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \dots \times \frac{1024}{1023}\right).$$

This is a telescoping product. Most terms in the numerator and denominator cancel out:

$$S = \log_2\left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \dots \times \frac{1024}{1023}\right).$$

We are left with the denominator of the first term and the numerator of the last term:

$$S = \log_2\left(\frac{1024}{1}\right) = \log_2(1024).$$

Since we know that  $1024 = 2^{10}$ :

$$S = \log_2(2^{10}).$$

Using the property  $\log_b(b^x) = x$ , we get:

$$S = 10.$$

19. If  $\alpha, \beta$  are the roots of  $x^2 + 3x - 1 = 0$ , then the value of  $\alpha^3 + \beta^3$  is:

(A) -18

(B) 18

(C) -36

(D) 36

**(C)**

From the equation, sum of roots  $\alpha + \beta = -3$ .

Product of roots  $\alpha\beta = -1$ .

We use the identity  $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ .

Substitute the values:

$$\begin{aligned} \alpha^3 + \beta^3 &= (-3)^3 - 3(-1)(-3) \\ &= -27 - 9 \\ &= -36 \end{aligned}$$

20. The value of  $2^{\log_3 7} - 7^{\log_3 2}$  is:

(A)  $\log 2$ 

(B) 1

(C) 0

(D) none of these

(C)

Using the logarithm property  $a^{\log_b c} = c^{\log_b a}$ .

Let's prove this property. Let  $x = a^{\log_b c}$ .

$$\log_b x = \log_b (a^{\log_b c}) = (\log_b c)(\log_b a).$$

$$\text{Let } y = c^{\log_b a}.$$

$$\log_b y = \log_b (c^{\log_b a}) = (\log_b a)(\log_b c).$$

Since  $\log_b x = \log_b y$ , it implies  $x = y$ .

Applying this to the expression:

$$2^{\log_3 7} = 7^{\log_3 2}$$

$$\text{Therefore, } 2^{\log_3 7} - 7^{\log_3 2} = 0$$

### SECTION - B : Numerical Value Type

21. The number of solutions of the equation  $\cos 2x = \cos x$  in the interval  $[0, 4\pi]$  is:

7

$$\cos 2x = \cos x$$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$\text{Let } y = \cos x. \quad 2y^2 - y - 1 = 0$$

$$(2y + 1)(y - 1) = 0$$

$$y = -\frac{1}{2} \text{ or } y = 1$$

$$\text{Case 1: } \cos x = 1$$

$$x = 2n\pi. \text{ In } [0, 4\pi], \text{ solutions are } x = 0, 2\pi, 4\pi. \text{ (3 solutions)}$$

$$\text{Case 2: } \cos x = -\frac{1}{2}$$

$$\text{The principal solutions are } \frac{2\pi}{3} \text{ and } \frac{4\pi}{3}.$$

$$\text{General solution is } x = 2n\pi \pm \frac{2\pi}{3}.$$

$$\text{For } n=0: x = \frac{2\pi}{3}$$

$$\text{For } n=1: x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3} \text{ and } x = 2\pi + \frac{2\pi}{3} = \frac{8\pi}{3}$$

$$\text{For } n=2: x = 4\pi - \frac{2\pi}{3} = \frac{10\pi}{3}$$

$$\text{Total solutions for this case are } \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}. \text{ (4 solutions)}$$

$$\text{Total number of solutions} = 3 + 4 = 7.$$

22. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is:

22

Let M, P, C be the sets of students studying Mathematics, Physics, and Chemistry.

$$\begin{aligned}n(M) &= 23, n(P) = 24, n(C) = 19 \\n(M \cap P) &= 12, n(M \cap C) = 9, n(P \cap C) = 7 \\n(M \cap P \cap C) &= 4\end{aligned}$$

Number of students in exactly one subject is given by:

$$n(\text{only M}) + n(\text{only P}) + n(\text{only C})$$

$$\begin{aligned}n(\text{only M}) &= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C) \\&= 23 - 12 - 9 + 4 = 6\end{aligned}$$

$$\begin{aligned}n(\text{only P}) &= n(P) - n(M \cap P) - n(P \cap C) + n(M \cap P \cap C) \\&= 24 - 12 - 7 + 4 = 9\end{aligned}$$

$$\begin{aligned}n(\text{only C}) &= n(C) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \\&= 19 - 9 - 7 + 4 = 7\end{aligned}$$

Total number of students studying exactly one subject =  $6 + 9 + 7 = 22$ .

23. The number of solutions of  $\cos 2x + 3 \cos x + 2 = 0$  in  $[0, 2\pi)$  is:

3

$$\cos 2x + 3 \cos x + 2 = 0$$

$$(2 \cos^2 x - 1) + 3 \cos x + 2 = 0$$

$$2 \cos^2 x + 3 \cos x + 1 = 0$$

$$\text{Let } y = \cos x. \quad 2y^2 + 3y + 1 = 0$$

$$(2y + 1)(y + 1) = 0$$

$$y = -\frac{1}{2} \text{ or } y = -1$$

$$\text{Case 1: } \cos x = -1$$

In  $[0, 2\pi)$ , the solution is  $x = \pi$ . (1 solution)

$$\text{Case 2: } \cos x = -\frac{1}{2}$$

In  $[0, 2\pi)$ , cosine is negative in Q2 and Q3.

Solutions are  $x = \frac{2\pi}{3}$  and  $x = \frac{4\pi}{3}$ . (2 solutions)

Total number of solutions =  $1 + 2 = 3$ .

24. A and B are subsets of universal set U such that  $n(U) = 800$ ,  $n(A) = 300$ ,  $n(B) = 400$  and  $n(A \cap B) = 100$ . Then the number of elements in the set  $A^c \cap B^c$  is:

200

We need to find  $n(A^c \cap B^c)$ .

By De Morgan's Law,  $A^c \cap B^c = (A \cup B)^c$ .

Therefore,  $n(A^c \cap B^c) = n((A \cup B)^c)$ .

$$n((A \cup B)^c) = n(U) - n(A \cup B).$$

First, find  $n(A \cup B)$  :

$$\begin{aligned}n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\&= 300 + 400 - 100 \\&= 600\end{aligned}$$

Now, calculate  $n(A^c \cap B^c)$  :

$$\begin{aligned}n(A^c \cap B^c) &= n(U) - n(A \cup B) \\ &= 800 - 600 = 200.\end{aligned}$$

25. If one root of the equation  $x^2 - 6x + k = 0$  is the reciprocal of the other, the value of  $k$  is:

1

Let the roots of the equation  $ax^2 + bx + c = 0$  be  $\alpha$  and  $\beta$ .

Product of roots is  $\alpha\beta = \frac{c}{a}$ .

In the given equation,  $x^2 - 6x + k = 0$ , we have  $a = 1, b = -6, c = k$ .

Let the roots be  $\alpha$  and  $\frac{1}{\alpha}$ .

The product of the roots is  $\alpha \cdot \frac{1}{\alpha} = 1$ .

From the equation, the product of the roots is also  $\frac{k}{1} = k$ .

Equating the two expressions for the product of roots:

$$k = 1.$$