

HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

Topic: Sets, Inequality, Logarithm, Trigonometric Ratios & Identities, Trigonometric Equations, Quadratic Equation

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Sub: Mathematics

JEE Main

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SECTION - A : Single Correct Answer Type

1. The equation $\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$ has:

- (A) no solution (B) one solution (C) two solutions (D) more than two solutions

(B)

$$\sqrt{4x+9} = \sqrt{7x+4} + \sqrt{11x+1}$$

Squaring both sides:

$$4x+9 = (7x+4) + (11x+1) + 2\sqrt{(7x+4)(11x+1)}$$

$$4x+9 = 18x+5 + 2\sqrt{77x^2+51x+4}$$

$$4-14x = 2\sqrt{77x^2+51x+4}$$

$$2-7x = \sqrt{77x^2+51x+4}$$

For the radical to be real, $2-7x \geq 0 \Rightarrow x \leq 2/7$.

Squaring both sides again:

$$(2-7x)^2 = 77x^2+51x+4$$

$$4-28x+49x^2 = 77x^2+51x+4$$

$$0 = 28x^2+79x$$

$$x(28x+79) = 0$$

Possible solutions are $x = 0$ or $x = -79/28$.

Check for extraneous roots:

$$\text{If } x = 0: \sqrt{9} - \sqrt{1} = \sqrt{4} \Rightarrow 3 - 1 = 2 \Rightarrow 2 = 2. \text{ (Valid)}$$

$$\text{If } x = -79/28 \approx -2.82:$$

$$4x+9 = 4(-79/28)+9 = -79/7+9 = (-79+63)/7 = -16/7 < 0. \text{ (Invalid)}$$

Thus, there is only one solution, $x = 0$.

2. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is:

- (A) $(3, \infty)$ (B) $(-\infty, -3)$ (C) $(-3, 3)$ (D) $(-3, \infty)$

(C)

Let the quadratic equation be $x^2 + ax + 1 = 0$.

The coefficients are $A = 1, B = a, C = 1$.

Condition on the Difference of Roots

The absolute difference between the roots is given by the formula $|\alpha - \beta| = \frac{\sqrt{D}}{|A|}$.

$$|\alpha - \beta| = \frac{\sqrt{a^2 - 4}}{|1|} = \sqrt{a^2 - 4}.$$

The problem states that this difference is less than $\sqrt{5}$:

$$\sqrt{a^2 - 4} < \sqrt{5}.$$

Squaring both sides (since both are positive) gives:

$$a^2 - 4 < 5$$

$$a^2 < 9 \implies |a| < 3 \implies -3 < a < 3.$$

3. The solution set of the inequation $|x + \frac{1}{x}| > 2$ is:

(A) $R - \{0\}$

(B) $R - \{-1, 0, 1\}$

(C) $R - \{1\}$

(D) $R - \{-1, 1\}$

(B)

$$|x + \frac{1}{x}| > 2. \text{ Note that } x \neq 0.$$

Squaring both sides (since both sides are non-negative):

$$(x + \frac{1}{x})^2 > 4$$

$$x^2 + 2 + \frac{1}{x^2} > 4$$

$$x^2 - 2 + \frac{1}{x^2} > 0$$

$$(x - \frac{1}{x})^2 > 0$$

The square of any real number is always ≥ 0 .

The inequality is true for all x , except where the expression is equal to 0 or undefined.

Undefined at $x = 0$.

$$\text{Equal to 0 when } x - \frac{1}{x} = 0 \implies x = \frac{1}{x} \implies x^2 = 1 \implies x = \pm 1.$$

So, the solution is all real numbers except 0, 1, and -1.

Solution set is $R - \{-1, 0, 1\}$.

4. If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2:3 then the value of a is:

(A) $\frac{\pm 5}{\sqrt{6}}$

(B) $\frac{\pm 25}{\sqrt{6}}$

(C) $\frac{\pm 5}{6}$

(D) None of these

(B)

Let the roots be 2α and 3α .

$$\text{Product of roots: } (2\alpha)(3\alpha) = 6\alpha^2 = 25 \implies \alpha^2 = \frac{25}{6} \implies \alpha = \pm \frac{5}{\sqrt{6}}.$$

$$\text{Sum of roots: } 2\alpha + 3\alpha = 5\alpha = -a.$$

Substitute the value of α :

$$-a = 5 \left(\pm \frac{5}{\sqrt{6}} \right) = \pm \frac{25}{\sqrt{6}}$$

$$a = \mp \frac{25}{\sqrt{6}}. \text{ The values are } \pm \frac{25}{\sqrt{6}}.$$

5. If $|\frac{x^2+6}{5x}| \geq 1$, then x belongs to:

- (A) $(-\infty, -3)$ (B) $(-\infty, -3) \cup (3, \infty)$ (C) $(-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty)$ (D) \mathbb{R}

(C)

The inequality is $\frac{x^2+6}{5x} \geq 1$ or $\frac{x^2+6}{5x} \leq -1$. ($x \neq 0$)

$$\text{Case 1: } \frac{x^2+6}{5x} - 1 \geq 0 \Rightarrow \frac{x^2-5x+6}{5x} \geq 0 \Rightarrow \frac{(x-2)(x-3)}{x} \geq 0.$$

Critical points are 0, 2, 3. Solution from wavy curve method: $(0, 2] \cup [3, \infty)$.

$$\text{Case 2: } \frac{x^2+6}{5x} + 1 \leq 0 \Rightarrow \frac{x^2+5x+6}{5x} \leq 0 \Rightarrow \frac{(x+2)(x+3)}{x} \leq 0.$$

Critical points are -3, -2, 0. Solution from wavy curve method: $(-\infty, -3] \cup [-2, 0)$.

Combining both solutions, we get:

$$(-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty).$$

6. If α and β are roots of the equation $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to:

- (A) $140\sqrt{2}$ (B) $240\sqrt{2}$ (C) $280\sqrt{2}$ (D) $320\sqrt{2}$

(C)

Simplify the constant term: $2e^{4\ln k} - 1 = 2e^{\ln(k^4)} - 1 = 2k^4 - 1$.

The equation is $x^2 - 4\sqrt{2}kx + (2k^4 - 1) = 0$.

Sum of roots: $\alpha + \beta = 4\sqrt{2}k$.

Product of roots: $\alpha\beta = 2k^4 - 1$.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 66.$$

$$(4\sqrt{2}k)^2 - 2(2k^4 - 1) = 66$$

$$32k^2 - 4k^4 + 2 = 66 \Rightarrow 4k^4 - 32k^2 + 64 = 0 \Rightarrow k^4 - 8k^2 + 16 = 0.$$

$$(k^2 - 4)^2 = 0 \Rightarrow k^2 = 4 \Rightarrow k = 2 \text{ (assuming } k > 0 \text{ for } \ln k \text{ to be defined).}$$

Now find $\alpha + \beta$ and $\alpha\beta$ for $k = 2$:

$$\alpha + \beta = 4\sqrt{2}(2) = 8\sqrt{2}.$$

$$\alpha\beta = 2(2^4) - 1 = 32 - 1 = 31.$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= (8\sqrt{2})^3 - 3(31)(8\sqrt{2}) = 512(2\sqrt{2}) - 93(8\sqrt{2})$$

$$= 1024\sqrt{2} - 744\sqrt{2} = 280\sqrt{2}.$$

7. The solution set of the inequation $\frac{2x+4}{x-1} \geq 5$ is:

(A) (1, 3)

(B) (1, 3]

(C) $(-\infty, 1) \cup [3, \infty)$

(D) None of these

(B)

$$\begin{aligned}\frac{2x+4}{x-1} \geq 5 &\Rightarrow \frac{2x+4}{x-1} - 5 \geq 0 \\ \frac{2x+4-5(x-1)}{x-1} \geq 0 &\Rightarrow \frac{-3x+9}{x-1} \geq 0 \\ \frac{3(3-x)}{x-1} \geq 0 &\Rightarrow \frac{x-3}{x-1} \leq 0.\end{aligned}$$

Critical points are $x = 1, x = 3$. The inequality holds between the roots.

Since denominator cannot be zero, $x \neq 1$.

Solution is $1 < x \leq 3$, which is $(1, 3]$.

8. If $A = \sin^2 x + \cos^4 x$, then for all real x :

(A) $\frac{13}{16} \leq A \leq 1$ (B) $1 \leq A \leq 2$ (C) $\frac{3}{4} \leq A \leq \frac{13}{16}$ (D) $\frac{3}{4} \leq A \leq 1$ **(D)**

$$\begin{aligned}A &= \sin^2 x + \cos^4 x = \sin^2 x + (\cos^2 x)^2 \\ &= \sin^2 x + (1 - \sin^2 x)^2 \\ &= \sin^2 x + 1 - 2\sin^2 x + \sin^4 x \\ &= \sin^4 x - \sin^2 x + 1\end{aligned}$$

Let $y = \sin^2 x$. Since $0 \leq \sin^2 x \leq 1$, we have $0 \leq y \leq 1$.

$$A(y) = y^2 - y + 1 = (y^2 - y + \frac{1}{4}) - \frac{1}{4} + 1 = (y - \frac{1}{2})^2 + \frac{3}{4}.$$

$$A(\frac{1}{2}) = (1/2 - 1/2)^2 + 3/4 = 0 + 3/4 = 3/4.$$

$$A(0) = (0 - 1/2)^2 + 3/4 = 1/4 + 3/4 = 1.$$

$$A(1) = (1 - 1/2)^2 + 3/4 = 1/4 + 3/4 = 1.$$

So the range of A is $[\frac{3}{4}, 1]$.

9. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is:

(A) 128

(B) 216

(C) 240

(D) 160

(D)

$$n(U) = 800, n(C) = 224, n(H) = 240, n(B) = 336$$

$$n(B \cap H) = 64, n(C \cap B) = 80, n(C \cap H) = 40, n(C \cap H \cap B) = 24.$$

Number of boys playing at least one game is $n(C \cup H \cup B)$.

$$\begin{aligned}n(C \cup H \cup B) &= n(C) + n(H) + n(B) - [n(C \cap H) + n(H \cap B) + n(C \cap B)] + n(C \cap H \cap B) \\ &= (224 + 240 + 336) - (40 + 64 + 80) + 24 \\ &= 800 - 184 + 24 = 640.\end{aligned}$$

$$\begin{aligned} \text{Number of boys who did not play any game} &= n(U) - n(C \cup H \cup B) \\ &= 800 - 640 = 160. \end{aligned}$$

10. The value of $\cos 12^\circ \cos 24^\circ \cos 36^\circ \cos 48^\circ \cos 72^\circ \cos 84^\circ$ is:

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{1}{16}$

(D) $\frac{1}{128}$

(A)

$$\begin{aligned} P &= (\cos 12^\circ \cos 24^\circ \cos 36^\circ \cos 48^\circ \cos 72^\circ \cos 84^\circ) \\ &= (\cos 12^\circ \cos 24^\circ \cos 48^\circ)(\cos 36^\circ \cos 72^\circ \cos 84^\circ) \end{aligned}$$

Using $\cos x = \sin(90 - x)$: $\cos 84^\circ = \sin 6^\circ$, $\cos 72^\circ = \sin 18^\circ$, $\cos 48^\circ = \sin 42^\circ$.

$$P = (\cos 12^\circ \cos 24^\circ \cos 36^\circ \sin 42^\circ \sin 18^\circ \sin 6^\circ)$$

Let's group differently: $P = (\cos 36^\circ \cos 72^\circ) \cdot (\cos 12^\circ \cos 24^\circ \cos 48^\circ \cos 84^\circ)$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}, \cos 72^\circ = \frac{\sqrt{5}-1}{4} \Rightarrow \cos 36^\circ \cos 72^\circ = \frac{5-1}{16} = \frac{1}{4}.$$

Let $C = \cos 12^\circ \cos 24^\circ \cos 48^\circ \cos 84^\circ$.

Multiply and divide by $2 \sin 12^\circ$:

$$\begin{aligned} C &= \frac{(2 \sin 12^\circ \cos 12^\circ) \cos 24^\circ \cos 48^\circ \cos 84^\circ}{2 \sin 12^\circ} = \frac{\sin 24^\circ \cos 24^\circ \cos 48^\circ \cos 84^\circ}{2 \sin 12^\circ} \\ &= \frac{\frac{1}{2} \sin 48^\circ \cos 48^\circ \cos 84^\circ}{2 \sin 12^\circ} = \frac{\frac{1}{4} \sin 96^\circ \cos 84^\circ}{2 \sin 12^\circ} = \frac{\sin 96^\circ \cos 84^\circ}{8 \sin 12^\circ} \\ \sin 96^\circ &= \sin(180 - 84) = \sin 84^\circ. \\ C &= \frac{\sin 84^\circ \cos 84^\circ}{8 \sin 12^\circ} = \frac{\frac{1}{2} \sin 168^\circ}{8 \sin 12^\circ} = \frac{\sin(180 - 12)^\circ}{16 \sin 12^\circ} = \frac{\sin 12^\circ}{16 \sin 12^\circ} = \frac{1}{16}. \\ \text{Total Product } P &= \left(\frac{1}{4}\right) \cdot \left(\frac{1}{16}\right) = \frac{1}{64}. \end{aligned}$$

11. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is:

(A) 6

(B) 9

(C) 7

(D) 22

(D)

$$\begin{aligned} n(\text{only M}) &= n(M) - [n(M \cap P) - n(M \cap P \cap C)] - [n(M \cap C) - n(M \cap P \cap C)] - n(M \cap P \cap C) \\ &= 23 - (12 - 4) - (9 - 4) - 4 = 23 - 8 - 5 - 4 = 6. \\ n(\text{only P}) &= 24 - (12 - 4) - (7 - 4) - 4 = 24 - 8 - 3 - 4 = 9. \\ n(\text{only C}) &= 19 - (9 - 4) - (7 - 4) - 4 = 19 - 5 - 3 - 4 = 7. \\ \text{Total students with exactly one subject} &= 6 + 9 + 7 = 22. \end{aligned}$$

12. If sum of all the solutions of the equation $8 \cos x \cdot (\cos(\frac{\pi}{6} + x) \cos(\frac{\pi}{6} - x) - \frac{1}{2}) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to:

(A) $\frac{20}{9}$

(B) $\frac{2}{3}$

(C) $\frac{13}{9}$

(D) $\frac{8}{9}$

(C)

$$\cos(\frac{\pi}{6} + x) \cos(\frac{\pi}{6} - x) = \cos^2(\frac{\pi}{6}) - \sin^2 x = (\frac{\sqrt{3}}{2})^2 - \sin^2 x = \frac{3}{4} - \sin^2 x.$$

$$8 \cos x \cdot (\frac{3}{4} - \sin^2 x - \frac{1}{2}) = 1 \Rightarrow 8 \cos x \cdot (\frac{1}{4} - (1 - \cos^2 x)) = 1$$

$$8 \cos x \cdot (\cos^2 x - \frac{3}{4}) = 1 \Rightarrow 2 \cos x \cdot (4 \cos^2 x - 3) = 1$$

$$2(4 \cos^3 x - 3 \cos x) = 1 \Rightarrow 2 \cos(3x) = 1 \Rightarrow \cos(3x) = \frac{1}{2}.$$

$$3x = 2n\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}.$$

$$\text{For } n=0: x = \frac{\pi}{9}.$$

$$\text{For } n=1: x = \frac{2\pi}{3} - \frac{\pi}{9} = \frac{5\pi}{9}, \quad x = \frac{2\pi}{3} + \frac{\pi}{9} = \frac{7\pi}{9}.$$

$$\text{For } n=2: x = \frac{4\pi}{3} - \frac{\pi}{9} = \frac{11\pi}{9} > \pi. \text{ (Reject)}$$

$$\text{Solutions in } [0, \pi] \text{ are } \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}.$$

$$\text{Sum} = \frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} = \frac{13\pi}{9}.$$

$$\text{So, } k\pi = \frac{13\pi}{9} \Rightarrow k = \frac{13}{9}.$$

13. If $\log_{1/5}(2x-4) < \log_{1/5}(x+3)$ then:

(A) $x < 7$

(B) $2 < x < 7$

(C) $x > 7$

(D) None of these

(C)

For the logarithms to be defined, arguments must be positive:

$$2x - 4 > 0 \Rightarrow x > 2.$$

$$x + 3 > 0 \Rightarrow x > -3.$$

Intersection gives domain: $x > 2$.

Since the base $1/5 < 1$, the inequality reverses:

$$2x - 4 > x + 3$$

$$x > 7.$$

The final solution is the intersection of $x > 2$ and $x > 7$, which is $x > 7$.

14. If α and β are the roots of $4x^2 + 3x + 7 = 0$ then the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is:

(A) $\frac{225}{343}$

(B) $\frac{117}{343}$

(C) $\frac{225}{49}$

(D) $\frac{189}{343}$

(A)

$$\alpha + \beta = -3/4, \quad \alpha\beta = 7/4.$$

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3}.$$

$$\text{Numerator: } (-3/4)^3 - 3(7/4)(-3/4) = -27/64 + 63/16 = \frac{-27 + 252}{64} = \frac{225}{64}.$$

$$\text{Denominator: } (7/4)^3 = 343/64.$$

$$\text{Expression} = \frac{225/64}{343/64} = \frac{225}{343}.$$

15. The ratio $\frac{2^{\log_2 1/4 a} - 3^{\log_2 7 (a^2+1)^3} - 2a}{7^{4 \log_2 49 a} - a - 1}$ simplifies to:

(A) $a^2 - a - 1$

(B) $a^2 + a - 1$

(C) $a^2 - a + 1$

(D) $a^2 + a + 1$

(D)

$$\text{Numerator first term: } 2^{\log_2 1/4 a} = 2^{4 \log_2 a} = 2^{2 \log_2 a^4} = a^4.$$

$$\text{Numerator second term: } 3^{\log_2 7 (a^2+1)^3} = 3^{\log_3 3 (a^2+1)^3} = 3^{\frac{3}{3} \log_3 (a^2+1)^3} = a^2 + 1.$$

$$\text{Denominator first term: } 7^{4 \log_2 49 a} = 7^{4 \log_2 7^2 a} = 7^{4 \cdot \frac{1}{2} \log_2 7^2 a} = 7^{2 \log_2 7^2 a} = a^2.$$

$$\text{Expression} = \frac{a^4 - (a^2 + 1) - 2a}{a^2 - a - 1} = \frac{a^4 - a^2 - 2a - 1}{a^2 - a - 1}.$$

Let's factorize the numerator. By observation, $(a^2 - a - 1)(a^2 + a + 1)$

$$= a^2(a^2 + a + 1) - a(a^2 + a + 1) - 1(a^2 + a + 1)$$

$$= a^4 + a^3 + a^2 - a^3 - a^2 - a - a^2 - a - 1 = a^4 - a^2 - 2a - 1.$$

So, the expression simplifies to $a^2 + a + 1$.

16. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is:

(A) 16

(B) 14

(C) -4

(D) -5

(C)

$$2^{(x-1)(x^2+5x-50)} = 1 = 2^0.$$

$$(x-1)(x^2+5x-50) = 0.$$

$$(x-1)(x+10)(x-5) = 0.$$

The roots are $x = 1, x = -10, x = 5$.

$$\text{Sum of roots} = 1 + (-10) + 5 = -4.$$

17. The expression $(a^\alpha)^{-\beta \log_a N^r}$ when simplified reduces to:

- (A) $a^{\frac{\alpha\beta r}{Ns}}$ (B) $N^{-\left(\frac{\alpha\beta r}{s}\right)}$ (C) $N^{\frac{s}{\alpha\beta r}}$ (D) None of these

(B)

$$\begin{aligned} \text{Expression} &= a^{-\alpha\beta \log_a N^r} \\ &= a^{-\alpha\beta \cdot \frac{r}{s} \log_a N} \\ &= a^{\log_a N \left(-\frac{\alpha\beta r}{s}\right)} \\ &= N^{-\left(\frac{\alpha\beta r}{s}\right)} \end{aligned}$$

18. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is:

- (A) -81 (B) 100 (C) 144 (D) -300

(D)

Let the roots be α and α^3 .

Product of roots: $\alpha \cdot \alpha^3 = \alpha^4 = \frac{256}{81} = \left(\frac{4}{3}\right)^4$.

Possible real values for α are $\pm 4/3$.

Sum of roots: $\alpha + \alpha^3 = -\frac{k}{81}$.

Case 1: $\alpha = 4/3$.

$$\alpha + \alpha^3 = \frac{4}{3} + \left(\frac{4}{3}\right)^3 = \frac{4}{3} + \frac{64}{27} = \frac{36 + 64}{27} = \frac{100}{27}.$$

$$-\frac{k}{81} = \frac{100}{27} \Rightarrow k = -81 \cdot \frac{100}{27} = -3 \cdot 100 = -300.$$

Case 2: $\alpha = -4/3$.

$$\alpha + \alpha^3 = -\frac{4}{3} + \left(-\frac{4}{3}\right)^3 = -\frac{100}{27}.$$

$$-\frac{k}{81} = -\frac{100}{27} \Rightarrow k = 300.$$

A possible value of k is -300.

19. If $5 \cdot x^{\log_2 3} + 3^{\log_2 x} = 162$ then logarithm of x to the base 4 has the value equal to:

- (A) 2 (B) 1 (C) -1 (D) $\frac{3}{2}$

(D)

Using the property $a^{\log_b c} = c^{\log_b a}$, we have $x^{\log_2 3} = 3^{\log_2 x}$.

The equation becomes: $5 \cdot 3^{\log_2 x} + 3^{\log_2 x} = 162$.

$$6 \cdot 3^{\log_2 x} = 162$$

$$3^{\log_2 x} = \frac{162}{6} = 27 = 3^3.$$

$$\log_2 x = 3 \Rightarrow x = 2^3 = 8.$$

$$\text{We need to find } \log_4 x = \log_4 8 = \log_{2^2} 2^3 = \frac{3}{2}.$$

20. The value of $\sum_{k=1}^{10} \sin\left(\frac{2k\pi}{11}\right)$ is:

(A) 0

(B) 1

(C) -1

(D) 1/2

(A)

Let the sum be S. We can write out the terms of the summation:

$$S = \sin\left(\frac{2\pi}{11}\right) + \sin\left(\frac{4\pi}{11}\right) + \dots + \sin\left(\frac{18\pi}{11}\right) + \sin\left(\frac{20\pi}{11}\right).$$

The angles are in an arithmetic progression.

We can use the property $\sin(2\pi - \theta) = -\sin(\theta)$.

Let's look at the terms from the end of the series:

$$\sin\left(\frac{20\pi}{11}\right) = \sin\left(2\pi - \frac{2\pi}{11}\right) = -\sin\left(\frac{2\pi}{11}\right).$$

$$\sin\left(\frac{18\pi}{11}\right) = \sin\left(2\pi - \frac{4\pi}{11}\right) = -\sin\left(\frac{4\pi}{11}\right).$$

$$\sin\left(\frac{16\pi}{11}\right) = \sin\left(2\pi - \frac{6\pi}{11}\right) = -\sin\left(\frac{6\pi}{11}\right).$$

$$\sin\left(\frac{14\pi}{11}\right) = \sin\left(2\pi - \frac{8\pi}{11}\right) = -\sin\left(\frac{8\pi}{11}\right).$$

$$\sin\left(\frac{12\pi}{11}\right) = \sin\left(2\pi - \frac{10\pi}{11}\right) = -\sin\left(\frac{10\pi}{11}\right).$$

Now, we can pair the terms of the sum:

$$S = \left(\sin\left(\frac{2\pi}{11}\right) + \sin\left(\frac{20\pi}{11}\right)\right) + \left(\sin\left(\frac{4\pi}{11}\right) + \sin\left(\frac{18\pi}{11}\right)\right) + \dots \\ \dots + \left(\sin\left(\frac{10\pi}{11}\right) + \sin\left(\frac{12\pi}{11}\right)\right).$$

Substituting the relations from above:

$$S = \left(\sin\left(\frac{2\pi}{11}\right) - \sin\left(\frac{2\pi}{11}\right)\right) + \left(\sin\left(\frac{4\pi}{11}\right) - \sin\left(\frac{4\pi}{11}\right)\right) + \dots \\ \dots + \left(\sin\left(\frac{10\pi}{11}\right) - \sin\left(\frac{10\pi}{11}\right)\right).$$

$$S = 0 + 0 + 0 + 0 + 0 = 0.$$

Therefore, the sum is 0.

SECTION - B : Numerical Value Type

21. The number of values of α in $[0, 2\pi]$ for which $2\sin^3 \alpha - 7\sin^2 \alpha + 7\sin \alpha = 2$ is:

3

$$2\sin^3 \alpha - 7\sin^2 \alpha + 7\sin \alpha - 2 = 0.$$

$$\text{Let } y = \sin \alpha.$$

$$\text{So, } 2y^3 - 7y^2 + 7y - 2 = 0.$$

$$\text{By inspection, } P(1) = 2 - 7 + 7 - 2 = 0, \text{ so } y = 1 \text{ is a root.}$$

$$\text{Dividing the polynomial by } (y - 1) \text{ gives } 2y^2 - 5y + 2.$$

$$(y-1)(2y^2-5y+2) = 0 \Rightarrow (y-1)(2y-1)(y-2) = 0.$$

The roots are $y = 1, y = 1/2, y = 2$.

Since $y = \sin \alpha, -1 \leq y \leq 1$. We discard $y = 2$.

Case 1: $\sin \alpha = 1 \Rightarrow \alpha = \pi/2$. (1 solution in $[0, 2\pi]$)

Case 2: $\sin \alpha = 1/2 \Rightarrow \alpha = \pi/6, 5\pi/6$. (2 solutions in $[0, 2\pi]$)

Total number of solutions is $1 + 2 = 3$.

22. The value of $81^{\frac{1}{\log_5 3}} + 27^{\log_9 36} + 3^{\frac{4}{\log_7 9}}$ is:

890

$$\text{Term 1: } 81^{\frac{1}{\log_5 3}} = 81^{\log_3 5} = (3^4)^{\log_3 5} = 3^{4 \log_3 5} = 3^{\log_3 5^4} = 5^4 = 625.$$

$$\text{Term 2: } 27^{\log_9 36} = (3^3)^{\log_3 2 \cdot 6^2} = 3^{3 \cdot \frac{2}{2} \log_3 6} = 3^{3 \log_3 6} = 3^{\log_3 6^3} = 6^3 = 216.$$

$$\text{Term 3: } 3^{\frac{4}{\log_7 9}} = 3^{4 \log_9 7} = 3^{4 \log_3 2 \cdot 7} = 3^{4 \cdot \frac{1}{2} \log_3 7} = 3^{2 \log_3 7} = 3^{\log_3 7^2} = 7^2 = 49.$$

$$\text{Sum} = 625 + 216 + 49 = 841 + 49 = 890.$$

23. The number of solutions for the equation $|\sin x| = |\cos x|$ in the interval $[0, 4\pi]$ is:

8

$$|\sin x| = |\cos x| \Rightarrow |\tan x| = 1.$$

This means $\tan x = 1$ or $\tan x = -1$.

In the interval $[0, 2\pi)$, the solutions are:

$$\tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}.$$

$$\tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}.$$

There are 4 solutions in $[0, 2\pi)$.

4 solutions in $[0, 2\pi)$ and 4 solutions in $[2\pi, 4\pi)$. Total is 8.

24. The value of $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ is:

1

Using the property $\frac{1}{\log_b a} = \log_a b$:

$$\text{Expression} = \log_{abc}(\sqrt{bc}) + \log_{abc}(\sqrt{ca}) + \log_{abc}(\sqrt{ab})$$

Using the property $\log x + \log y = \log(xy)$:

$$= \log_{abc}(\sqrt{bc} \cdot \sqrt{ca} \cdot \sqrt{ab})$$

$$= \log_{abc}(\sqrt{a^2 b^2 c^2})$$

$$= \log_{abc}(abc) = 1.$$

25. The number of integral values of 'k' for which the equation $3 \cos x - 4 \sin x = k + 1$ has a solution is:

11

The range of an expression of the form $a \cos x + b \sin x$ is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$.

For $3 \cos x - 4 \sin x$, the range is $[-\sqrt{3^2 + (-4)^2}, \sqrt{3^2 + (-4)^2}]$

$$= [-\sqrt{9 + 16}, \sqrt{9 + 16}] = [-\sqrt{25}, \sqrt{25}] = [-5, 5].$$

For the equation to have a solution, $k + 1$ must be in this range.

$$-5 \leq k + 1 \leq 5$$

$$-5 - 1 \leq k \leq 5 - 1$$

$$-6 \leq k \leq 4.$$

The integral values of k are -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4.

$$\text{Number of integers} = 4 - (-6) + 1 = 11.$$