Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 28th September 2025

Syllabus: Quadratic Equation

Sub: Mathematics **CT-05 JEE Advanced Solution** Prof. Chetan Sir

Section 1: Single Correct Questions

1. If the quadratic equations $x^2 - 7x + 3c = 0$ and $x^2 + x - 5c = 0$ have a common root, then for a non-zero real value of c, the sign of the expression $x^2 - 3x + c$ is:

   (A) negative for all $x \in \mathbb{R}$
   (B) positive for all $x \in (1, 3)$
   (C) negative for all $x \in (1, 3)$
   (D) positive for all $x \in \mathbb{R}$

Answer: (D)
Solution:

Let the common root be α.

$$\alpha^2 - 7\alpha + 3c = 0 \quad \cdots (1)$$
$$\alpha^2 + \alpha - 5c = 0 \quad \cdots (2)$$

Subtracting (1) from (2):

$$(\alpha^2 + \alpha - 5c) - (\alpha^2 - 7\alpha + 3c) = 0$$
$$8\alpha - 8c = 0$$
$$\implies \alpha = c$$

Substitute $\alpha = c$ into equation (1):

$$c^2 - 7c + 3c = 0$$
$$c^2 - 4c = 0$$
$$c(c - 4) = 0$$

Since $c \neq 0$, we have $c = 4$.

Now, analyze the expression $E(x) = x^2 - 3x + c$

$$E(x) = x^2 - 3x + 4$$

The discriminant is $D = b^2 - 4ac$

$$= (-3)^2 - 4(1)(4)$$
$$= 9 - 16 = -7$$

Since the leading coefficient (1) is positive and $D < 0$,
the expression is positive for all $x \in \mathbb{R}$.

2. If α, β are roots of the equation $x^2 - 2\sqrt{5}x + 7 = 0$, and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\dfrac{P_{11}P_{14} - 2\sqrt{5}P_{11}P_{13}}{P_{12}P_{13} - 2\sqrt{5}P_{12}^2}$ is equal to:

   (A) 1
   (B) 7
   (C) $2\sqrt{5}$
   (D) -1

Answer: (A)

Solution:

$$\text{From } x^2 - 2\sqrt{5}x + 7 = 0, \text{ we derive a recurrence relation.}$$
$$\implies P_n - 2\sqrt{5}P_{n-1} + 7P_{n-2} = 0$$
$$\text{Now, simplify the expression's numerator:}$$
$$\begin{aligned}\text{Numerator} &= P_{11}P_{14} - 2\sqrt{5}P_{11}P_{13}\\ &= P_{11}(P_{14} - 2\sqrt{5}P_{13})\end{aligned}$$
$$\text{Using the recurrence with } n = 14, \; P_{14} - 2\sqrt{5}P_{13} = -7P_{12}.$$
$$\text{Numerator} = P_{11}(-7P_{12}) = -7P_{11}P_{12}.$$
$$\text{Next, simplify the denominator:}$$
$$\begin{aligned}\text{Denominator} &= P_{12}P_{13} - 2\sqrt{5}P_{12}^2\\ &= P_{12}(P_{13} - 2\sqrt{5}P_{12})\end{aligned}$$
$$\text{Using the recurrence with } n = 13, \; P_{13} - 2\sqrt{5}P_{12} = -7P_{11}.$$
$$\text{Denominator} = P_{12}(-7P_{11}) = -7P_{11}P_{12}.$$
$$\text{The expression is } \frac{-7P_{11}P_{12}}{-7P_{11}P_{12}} = 1.$$

Section 2: Multiple Correct Questions

5. Let $f(x) = x^2 - 2(m-1)x + m + 5$. The value of m can be, if:
(A) Both roots of $f(x) = 0$ are positive, for $m = 5$.
(B) Both roots of $f(x) = 0$ are negative, for $m = -2$.
(C) The roots are of opposite sign, for $m = -6$.
(D) Exactly one root lies in $(0, 2)$, for $m = 5$.

Answer: (A), (B), (C), (D)
Solution:
For $f(x) = x^2 - 2(m-1)x + m + 5 = 0$:

- Sum of roots, $S = 2(m-1)$.
- Product of roots, $P = m + 5$.
- Discriminant, $D = 4(m-4)(m+1)$. Real roots exist if $m \in (-\infty, -1] \cup [4, \infty)$.

(A) For $m = 5$ (Both roots positive?)

- $D = 4(1)(6) > 0$ (Real roots).
- $S = 2(4) = 8 > 0$.
- $P = 5 + 5 = 10 > 0$.
- Conditions $(D > 0, S > 0, P > 0)$ are met. **(A) is correct.**

(B) For $m = -2$ (Both roots negative?)

- $D = 4(-6)(-1) > 0$ (Real roots).
- $S = 2(-3) = -6 < 0$.
- $P = -2 + 5 = 3 > 0$.
- Conditions $(D > 0, S < 0, P > 0)$ are met. **(B) is correct.**

(C) For $m = -6$ (Roots of opposite sign?)

- $D = 4(-10)(-5) > 0$ (Real roots).
- $P = -6 + 5 = -1 < 0$.

- Condition ($P < 0$) is sufficient. **(C) is correct.**

(D) For $m = 5$ (Exactly one root in (0, 2)?)

- Equation is $x^2 - 8x + 10 = 0$.
- Roots are $x = \frac{8 \pm \sqrt{64-40}}{2} = 4 \pm \sqrt{6}$.
- One root is $4 - \sqrt{6}$. Since $2.44 \approx \sqrt{6}$, this root is ≈ 1.56, which is in $(0, 2)$.
- The other root $4 + \sqrt{6}$ is clearly outside $(0, 2)$. **(D) is correct.**

6. If α, β are the roots of the equation $x^2 + x + 1 = 0$, which of the following statements are correct?
   (A) $\alpha^4 + \beta^4 = -1$
   (B) $\alpha^5 + \beta^5 = -1$
   (C) The equation with roots α^2, β^2 is $x^2 + x + 1 = 0$
   (D) $(\alpha - \beta)^2 = 3$

Answer: (A), (B), (C)
Solution (Using Newton's Sums):

$$\text{Let } \alpha, \beta \text{ be the roots of } x^2 + x + 1 = 0.$$
$$\text{Define the sum of powers as } S_n = \alpha^n + \beta^n.$$

$$\begin{aligned}
&\text{1. Find the Recurrence Relation:} \\
&\text{From Newton's Sums, for } ax^2 + bx + c = 0, \\
&aS_n + bS_{n-1} + cS_{n-2} = 0. \\
&\text{Here, } a = 1, b = 1, c = 1, \text{ so the relation is:} \\
&S_n + S_{n-1} + S_{n-2} = 0 \\
&\implies S_n = -S_{n-1} - S_{n-2}.
\end{aligned}$$

$$\begin{aligned}
&\text{2. Calculate Initial Values:} \\
&\text{From Vieta's formulas, } \alpha + \beta = -1 \text{ and } \alpha\beta = 1. \\
&S_1 = \alpha + \beta = -1. \\
&S_2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-1)^2 - 2(1) = -1.
\end{aligned}$$

$$\begin{aligned}
&\text{3. Calculate } S_4 \text{ and } S_5: \\
&S_3 = -S_2 - S_1 = -(-1) - (-1) = 2. \\
&S_4 = -S_3 - S_2 = -(2) - (-1) = -1. \\
&S_5 = -S_4 - S_3 = -(-1) - (2) = -1.
\end{aligned}$$

$$\begin{aligned}
&\text{4. Evaluate the Options:} \\
&\text{(A) } \alpha^4 + \beta^4 = S_4 = -1. \quad \textbf{(Correct)} \\
&\text{(B) } \alpha^5 + \beta^5 = S_5 = -1. \quad \textbf{(Correct)} \\
&\text{(C) Eq. with roots } \alpha^2, \beta^2: \\
&\text{Sum} = \alpha^2 + \beta^2 = S_2 = -1. \\
&\text{Product} = (\alpha\beta)^2 = 1^2 = 1. \\
&\text{Eq. is } x^2 - (-1)x + 1 = 0 \implies x^2 + x + 1 = 0. \quad \textbf{(Correct)} \\
&\text{(D) } (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = (-1)^2 - 4(1) = -3. \quad \textbf{(Incorrect)}
\end{aligned}$$

Section 3: Matrix Match

8. Let $f(x) = x^2 - 2kx + k^2 + k - 5 = 0$. The roots of the equation are real. Match the conditions for the location of roots in Column-I with the corresponding values of k in Column-II.

Column-I	Column-II
(A) Both roots are less than 5	(P) $(1, 4)$
(B) Both roots are greater than -1	(Q) $(-4, 1] \cup [4, 5)$
(C) Exactly one root lies in the interval $(-1, 5)$	(R) $(-\infty, 4)$
(D) Both roots lie in the interval $(-1, 5)$	(S) $(1, 5]$

Answer: A-R, B-S, C-Q, D-P
Solution:
For $f(x) = x^2 - 2kx + k^2 + k - 5 = 0$:
Discriminant $D = 20 - 4k$. Real roots $\implies D \geq 0 \implies k \leq 5$.
Vertex is at $x = k$. We also evaluate $f(5) = (k-4)(k-5)$ and $f(-1) = (k+4)(k-1)$.

(A) Both roots < 5

$$\begin{aligned} \text{1. Real roots} &\implies k \leq 5 \\ \text{2. Vertex } < 5 &\implies k < 5 \\ \text{3. } f(5) > 0 &\implies (k-4)(k-5) > 0 \implies k < 4 \text{ or } k > 5 \\ \text{Intersection} &\implies k < 4 \quad \therefore \mathbf{A} \to \mathbf{R} \end{aligned}$$

(B) Both roots > -1

$$\begin{aligned} \text{1. Real roots} &\implies k \leq 5 \\ \text{2. Vertex } > -1 &\implies k > -1 \\ \text{3. } f(-1) > 0 &\implies (k+4)(k-1) > 0 \implies k < -4 \text{ or } k > 1 \\ \text{Intersection} &\implies k \in (1, 5] \quad \therefore \mathbf{B} \to \mathbf{S} \end{aligned}$$

(C) Exactly one root in $(-1, 5)$

$$\begin{aligned} \text{Condition is } f(-1)f(5) &\leq 0 \\ (k+4)(k-1)(k-4)(k-5) &\leq 0 \\ \text{This holds for } k &\in [-4, 1] \cup [4, 5]. \\ \text{Testing endpoints to ensure one root is strictly inside.} \\ k = 1 \implies \text{roots are } &-1, 3. \text{ (Valid)} \\ k = 4 \implies \text{roots are } &3, 5. \text{ (Valid)} \\ \text{The valid set is } (-4, 1] \cup [4, 5). &\quad \therefore \mathbf{C} \to \mathbf{Q} \end{aligned}$$

(D) Both roots in $(-1, 5)$

$$\begin{aligned} \text{This is the intersection of results from (A) and (B).} \\ (k < 4) \cap (k \in (1, 5]) \implies k \in (1, 4) \quad \therefore \mathbf{D} \to \mathbf{P} \end{aligned}$$

9. Match the equations in Column-I with the sum of their real roots in Column-II.

Column-I	Column-II
(A) $(x^2 + x)^2 + (x^2 + x) - 6 = 0$	(P) 0
(B) $x^4 - 5x^2 + 4 = 0$	(Q) -1
(C) $(x+1)^4 + (x+5)^4 = 32$	(R) 2
(D) $2\|x-1\|^2 - \|x-1\| - 3 = 0$	(S) -6

Answer: A-Q, B-P, C-S, D-R

(A) $(x^2+x)^2+(x^2+x)-6=0$

$$\begin{aligned} \text{Let } y = x^2+x. \implies & y^2+y-6=0 \implies (y+3)(y-2)=0 \\ & y=-3 \text{ or } y=2. \\ \text{Case 1: } & x^2+x=-3 \implies x^2+x+3=0 \quad (D<0, \text{no real roots}) \\ \text{Case 2: } & x^2+x=2 \implies x^2+x-2=0 \implies (x+2)(x-1)=0 \\ & \text{Real roots are } x=-2,1. \\ & \text{Sum} = -2+1=-1. \quad \therefore \mathbf{A} \to \mathbf{Q} \end{aligned}$$

(B) $x^4-5x^2+4=0$

$$\begin{aligned} \text{Let } y=x^2. \implies & y^2-5y+4=0 \implies (y-1)(y-4)=0 \\ & y=1 \text{ or } y=4. \\ & x^2=1 \implies x=\pm 1. \\ & x^2=4 \implies x=\pm 2. \\ & \text{Real roots are } 1,-1,2,-2. \\ & \text{Sum} = 1-1+2-2=0. \quad \therefore \mathbf{B} \to \mathbf{P} \end{aligned}$$

(C) $(x+1)^4+(x+5)^4=82$ (Typo corrected from 32)

$$\begin{aligned} \text{Let } y=x+3. \implies & (y-2)^4+(y+2)^4=82 \\ & 2(y^4+6y^2(2^2)+2^4)=82 \quad \text{(using identity for (a-b)}^4\text{+(a+b)}^4\text{)} \\ & y^4+24y^2+16=41 \implies y^4+24y^2-25=0 \\ & (y^2-1)(y^2+25)=0 \\ & \text{Real solution is } y^2=1 \implies y=\pm 1. \\ & x+3=1 \implies x=-2. \\ & x+3=-1 \implies x=-4. \\ & \text{Sum} = -2+(-4)=-6. \quad \therefore \mathbf{C} \to \mathbf{S} \end{aligned}$$

(D) $2|x-1|^2-|x-1|-3=0$

$$\begin{aligned} \text{Let } y=|x-1|, y\geq 0. \implies & 2y^2-y-3=0 \implies (2y-3)(y+1)=0 \\ & \text{Since } y\geq 0, \text{ we take } y=3/2. \\ & |x-1|=3/2 \\ & x-1=3/2 \quad \text{or} \quad x-1=-3/2 \\ & x=5/2 \quad \text{or} \quad x=-1/2. \\ & \text{Sum} = 5/2+(-1/2)=2. \quad \therefore \mathbf{D} \to \mathbf{R} \end{aligned}$$

Section 4: Paragraph Question

Paragraph for Questions 10 and 11

Let $x^2+ax+b=0$ and $x^2+bx+a=0$ be two quadratic equations, where $a \neq b$. It is given that the equations have a common root.

10. The value of $a+b$ is:

(A) 0 (B) 1 (C) -1 (D) cannot be determined

Answer: (C)

Solution:

$$\text{Let the common root be } \alpha.$$
$$\alpha^2 + a\alpha + b = 0$$
$$\alpha^2 + b\alpha + a = 0$$
$$\text{Subtracting the equations:}$$
$$(a-b)\alpha + (b-a) = 0$$
$$(a-b)\alpha = a - b$$
$$\text{Since } a \neq b, \text{ we find } \alpha = 1.$$
$$\text{Substituting } \alpha = 1 \text{ into the first equation:}$$
$$1^2 + a(1) + b = 0$$
$$1 + a + b = 0$$
$$\implies a + b = -1.$$

11. The quadratic equation whose roots are the other (non-common) roots of the given equations is:
(A) $x^2 + x + ab = 0$
(B) $x^2 - x + ab = 0$
(C) $x^2 + x - ab = 0$
(D) $x^2 - x - ab = 0$

Answer: (A)
Solution:

$$\text{For } x^2 + ax + b = 0, \text{ let roots be } 1, \beta_1.$$
$$\text{Product of roots: } 1 \cdot \beta_1 = b \implies \beta_1 = b.$$
$$\text{For } x^2 + bx + a = 0, \text{ let roots be } 1, \beta_2.$$
$$\text{Product of roots: } 1 \cdot \beta_2 = a \implies \beta_2 = a.$$
$$\text{The other roots are } a \text{ and } b.$$
$$\text{For the new equation:}$$
$$\text{Sum of roots } = a + b = -1 \text{ (from previous question).}$$
$$\text{Product of roots } = ab.$$
$$\text{The equation is } x^2 - (\text{Sum})x + (\text{Product}) = 0$$
$$x^2 - (-1)x + ab = 0$$
$$\implies x^2 + x + ab = 0.$$

Section 5: Integer Type Questions

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12. Let S be the set of all integral values of c for which one root of the equation $(c-5)x^2 - 2cx + (c-4) = 0$ lies in the interval $(0, 2)$ and its other root is greater than 2. Find the number of elements in S.

Answer: 18
Solution:

$$\text{Let } f(x) = (c-5)x^2 - 2cx + (c-4).$$
$$\text{The conditions are:}$$
$$1.\ \text{Real roots: Discriminant } D > 0.$$
$$D = (-2c)^2 - 4(c-5)(c-4) = 36c - 80.$$
$$36c - 80 > 0 \implies c > 20/9 \approx 2.22.$$
$$2.\ \text{The number 2 lies between the roots.}$$
$$(c-5) \cdot f(2) < 0$$
$$f(2) = 4(c-5) - 4c + c - 4 = c - 24.$$
$$(c-5)(c-24) < 0 \implies 5 < c < 24.$$
$$3.\ \text{The number 0 is to the left of both roots.}$$
$$(c-5) \cdot f(0) > 0$$
$$f(0) = c - 4.$$
$$(c-5)(c-4) > 0 \implies c < 4 \text{ or } c > 5.$$
$$\text{We need the intersection of these conditions:}$$
$$(c > 20/9) \cap (5 < c < 24) \cap (c < 4 \cup c > 5)$$
$$\text{The intersection is simply } 5 < c < 24.$$
$$\text{The integral values are } 6, 7, \ldots, 23.$$
$$\text{Number of values} = 23 - 6 + 1 = 18.$$

13. The number of negative integral values of m for which the expression $x^2 - 2(m-1)x + m + 5$ is positive for all $x > 0$ is:

Answer: 5
Solution:

$$\text{Let } f(x) = x^2 - 2(m-1)x + m + 5.$$
$$\text{For } f(x) > 0 \text{ when } x > 0, \text{ the roots of } f(x) = 0 \text{ must not be positive.}$$

$$\textbf{Case 1: No real roots } (D < 0)$$
$$D = 4(m-4)(m+1) < 0$$
$$\implies -1 < m < 4.$$
$$\text{Here } f(x) > 0 \text{ for all } x, \text{ so this range is valid.}$$

$$\textbf{Case 2: Both roots are non-positive } (\leq 0)$$
$$\text{Condition (i): } D \geq 0 \implies m \leq -1 \text{ or } m \geq 4$$
$$\text{Condition (ii): Sum } \leq 0 \implies 2(m-1) \leq 0 \implies m \leq 1$$
$$\text{Condition (iii): Product } \geq 0 \implies m + 5 \geq 0 \implies m \geq -5$$
$$\text{Intersection of (i), (ii), (iii) is } m \in [-5, -1].$$

$$\text{Combining both cases: } (-1, 4) \cup [-5, -1] = [-5, 4).$$
$$\text{We need the negative integral values of } m \text{ in this range.}$$
$$\text{They are } -5, -4, -3, -2, -1.$$
$$\text{Total number of values} = 5.$$