Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

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# Date of Exam: 28th September 2025

Syllabus: Quadratic Equation

Sub: Mathematics **MHT CET Solution** Prof. Chetan Sir

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**1.** The vertex of the parabola $y = 2x^2 - 8x + 3$ is:

(A) $(2, -5)$ (B) $(-2, 5)$ (C) $(2, 5)$ (D) $(-2, -5)$

**Answer: (A)**
**Solution:**

For a parabola $y = ax^2 + bx + c$, the vertex is at $(x_v, y_v)$.

$$x_v = -\frac{b}{2a} = -\frac{-8}{2(2)} = \frac{8}{4} = 2.$$

$$y_v = 2(2)^2 - 8(2) + 3 = 2(4) - 16 + 3 = 8 - 16 + 3 = -5.$$

The vertex is $(2, -5)$.

**2.** The range of the expression $y = \dfrac{x-1}{x+2}$ for all real $x \neq -2$ is:

(A) $\mathbb{R}$ (B) $\mathbb{R} - \{1\}$ (C) $\mathbb{R} - \{-2\}$ (D) $[1, \infty)$

**Answer: (B)**
**Solution:**

To find the range, we solve for $x$ in terms of $y$.

$$y = \frac{x-1}{x+2}$$
$$y(x+2) = x - 1$$
$$yx + 2y = x - 1$$
$$2y + 1 = x - yx$$
$$2y + 1 = x(1 - y)$$
$$x = \frac{2y+1}{1-y}$$

For $x$ to be a real number, the denominator cannot be zero.

$$1 - y \neq 0 \implies y \neq 1.$$

The range is all real numbers except 1.

**3.** The minimum value of the expression $f(x) = x^2 - 6x + 13$ is:

(A) 3 (B) 4 (C) 5 (D) 6

**Answer: (B)**
**Solution:**

We can find the minimum value by completing the square.

$$f(x) = (x^2 - 6x + 9) - 9 + 13$$
$$= (x - 3)^2 + 4$$

The term $(x - 3)^2$ is always non-negative, and its minimum value is 0.
Therefore, the minimum value of $f(x)$ is $0 + 4 = 4$.

**4.** If the quadratic expression $kx^2 + 2x + k$ is always positive for all real $x$, then:
(A) $k > 1$ (B) $k < 1$ (C) $-1 < k < 1$ (D) $k > 0$

**Answer: (A)**
**Solution:**

For a quadratic to be always positive, two conditions must be met:
1. The leading coefficient must be positive: $a > 0$.
2. The discriminant must be negative: $D < 0$.
Condition 1: $a = k > 0$.
Condition 2: $D = (2)^2 - 4(k)(k) < 0$
$$4 - 4k^2 < 0$$
$$4 < 4k^2$$
$$1 < k^2 \implies k > 1 \text{ or } k < -1.$$
We need the intersection of $k > 0$ and ($k > 1$ or $k < -1$).
The common region is $k > 1$.

**5.** If $S_n$ denotes the sum of the $n$-th powers of the roots of the equation $x^2 - 5x + 2 = 0$, then $S_6 - 5S_5 + 2S_4$ is equal to:
(A) 0 (B) 2 (C) 5 (D) 7

**Answer: (A)**
**Solution:**

Use newton's method and put n=6
$$S_6 - 5S_5 + 2S_4 = 0.$$

**6.** Find the set of all values of 'm' for which the quadratic equation $x^2 + (m-2)x + (m+1) = 0$ has both of its roots negative.
(A) $m > 2$ (B) $-1 < m \leq 0$ (C) $m \geq 8$ (D) $m \leq 0$

**Answer: (C)**

**Solution:**

$$\begin{gathered}\text{For both roots to be negative, three conditions must be satisfied:}\\ \text{1. Roots must be real: } D \geq 0.\\ \text{2. Sum of roots must be negative: } \alpha+\beta<0.\\ \text{3. Product of roots must be positive: } \alpha\beta>0.\\ \text{Condition 1: } D=(m-2)^2-4(m+1)=m^2-4m+4-4m-4=m^2-8m\geq 0.\\ m(m-8)\geq 0 \implies m\leq 0 \text{ or } m\geq 8.\\ \text{Condition 2: } \alpha+\beta=-(m-2)<0 \implies m-2>0 \implies m>2.\\ \text{Condition 3: } \alpha\beta=m+1>0 \implies m>-1.\\ \text{We need the intersection of } (m\leq 0 \text{ or } m\geq 8), (m>2), \text{ and } (m>-1).\\ \text{The common region is } m\geq 8.\end{gathered}$$

**7.** The sum of the roots of the equation $\dfrac{a}{x-a}+\dfrac{b}{x-b}=1$ is zero. Which of the following is true?

(A) $a=b$ (B) $a+b=0$ (C) $a+b=1$ (D) $ab=1$

**Answer: (B)**
**Solution:**

$$\begin{aligned}\frac{a(x-b)+b(x-a)}{(x-a)(x-b)}&=1\\ ax-ab+bx-ab&=(x-a)(x-b)\\ (a+b)x-2ab&=x^2-(a+b)x+ab\\ x^2-2(a+b)x+3ab&=0.\end{aligned}$$

$$\begin{gathered}\text{The sum of the roots for this quadratic is } -\frac{-2(a+b)}{1}=2(a+b).\\ \text{It is given that the sum of the roots is zero.}\\ 2(a+b)=0 \implies a+b=0.\end{gathered}$$

**8.** The number of integral values of $m$ for which the equation $x^2-2(m-1)x+(m+5)=0$ has at least one real root is:

(A) 5 (B) 6 (C) 7 (D) Infinitely many

**Answer: (D)**
**Solution:**

For at least one real root, the discriminant must be non-negative: $D \geq 0$.

$$\begin{aligned}D&=[-2(m-1)]^2-4(1)(m+5)\\ &=4(m^2-2m+1)-4(m+5)\\ &=4(m^2-2m+1-m-5)\\ &=4(m^2-3m-4)\\ &=4(m-4)(m+1)\end{aligned}$$

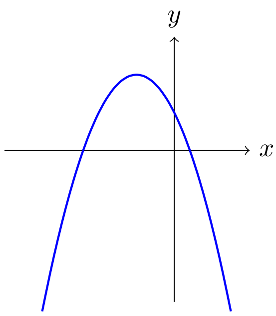
$$\begin{gathered}\text{So, } 4(m-4)(m+1)\geq 0 \implies (m-4)(m+1)\geq 0.\\ \text{This inequality holds for } m\leq -1 \text{ or } m\geq 4.\\ \text{The integral values for } m \text{ are } ..., -3, -2, -1 \text{ and } 4, 5, 6, ...\\ \text{There are infinitely many such integral values.}\end{gathered}$$

**9.** The minimum value of the sum of the squares of the roots of the equation $x^2 - (m-3)x + m = 0$ is:
(A) 1 (B) 0 (C) -1 (D) -7

**Answer: (D)**
**Solution:**

$$\text{Let the roots be } \alpha, \beta. \text{ We want to minimize } S = \alpha^2 + \beta^2.$$
$$S = (\alpha+\beta)^2 - 2\alpha\beta.$$
$$\text{From the equation, } \alpha+\beta = m-3 \text{ and } \alpha\beta = m.$$
$$S(m) = (m-3)^2 - 2(m)$$
$$= m^2 - 6m + 9 - 2m$$
$$= m^2 - 8m + 9.$$
$$\text{This is an upward-opening parabola. Its minimum value is at the vertex.}$$
$$\text{The vertex occurs at } m = -\frac{-8}{2(1)} = 4.$$
$$\text{The minimum value is } S(4) = (4)^2 - 8(4) + 9 = 16 - 32 + 9 = -7.$$

**10.** The graph of the quadratic expression $y = ax^2 + bx + c$ is shown, with vertex in Q2 and passing through (0,1). Which is correct?




(A) $a < 0, b > 0, c > 0$ (B) $a < 0, b < 0, c > 0$
(C) $a > 0, b > 0, c > 0$ (D) $a < 0, b < 0, c < 0$

**Answer: (B)**
**Solution:**

1. Sign of 'a': The parabola opens downwards, so $a < 0$.
2. Sign of 'c': The graph intersects the y-axis at (0,1). The y-intercept is c. So, $c = 1 > 0$.
3. Sign of 'b': The slope of tangent at the point where graph cuts the x-axis is negative.

Therefore, $b < 0$.

The conditions are $a < 0, b < 0, c > 0$.

**11.** The set of all values of $m$ for which both roots of the equation $x^2 - (m-3)x + m = 0$ are greater than 2 is:
(A) $m \in (9, 10]$ (B) $m \in [9, 10)$ (C) $m \in [1, 9]$ (D) $m \in [9, \infty)$

**Answer: (B)**

**Solution:**

$$\text{The conditions for both roots to be greater than 2 are:}$$
$$1.\ D \geq 0. \quad 2.\ \text{Vertex } > 2. \quad 3.\ f(2) > 0.$$
$$\text{Condition 1: } D = (m-3)^2 - 4m = m^2 - 10m + 9 = (m-1)(m-9) \geq 0.$$
$$\implies m \leq 1 \text{ or } m \geq 9.$$
$$\text{Condition 2: Vertex } x_v = \frac{m-3}{2} > 2 \implies m - 3 > 4 \implies m > 7.$$
$$\text{Condition 3: } f(2) = (2)^2 - (m-3)(2) + m = 4 - 2m + 6 + m = 10 - m > 0.$$
$$\implies m < 10.$$
$$\text{Intersection of } (m \leq 1 \text{ or } m \geq 9), (m > 7), \text{ and } (m < 10).$$
$$\text{The common region is } m \in [9, 10).$$

**12.** Find the value of 'a' for which both roots of the equation $x^2 - 2ax + a^2 - 1 = 0$ lie in the interval $(-2, 4)$.

(A) $(-1, 3)$ (B) $(-3, 1)$ (C) $(-1, 1)$ (D) $(0, 2)$

**Answer: (A)**
**Solution (Using Location of Roots):**

$$\text{Let } f(x) = x^2 - 2ax + a^2 - 1. \text{ The parabola opens upwards.}$$
$$\text{Conditions for both roots to be in } (-2, 4):$$
$$1.\ \text{Real roots: } D \geq 0.$$
$$D = (-2a)^2 - 4(1)(a^2 - 1) = 4a^2 - 4a^2 + 4 =$$
$$\text{Since } D > 0, \text{ roots are always real and distinct. This condition is always met.}$$
$$2.\ \text{Vertex lies between -2 and 4: } -2 < x_v < 4.$$
$$x_v = -\frac{-2a}{2(1)} = a.$$
$$\implies -2 < a < 4.$$
$$3.\ f(-2) > 0 \text{ and } f(4) > 0.$$
$$f(-2) = (-2)^2 - 2a(-2) + a^2 - 1 = 4 + 4a + a^2 -$$
$$(a+1)(a+3) > 0 \implies a < -3 \text{ or } a > -1.$$
$$f(4) = (4)^2 - 2a(4) + a^2 - 1 = 16 - 8a + a^2 - 1$$
$$(a-3)(a-5) > 0 \implies a < 3 \text{ or } a > 5.$$
$$\text{Now we find the intersection of all conditions:}$$
$$\text{(i) } a \in (-2, 4)$$
$$\text{(ii) } a \in (-\infty, -3) \cup (-1, \infty)$$
$$\text{(iii) } a \in (-\infty, 3) \cup (5, \infty)$$
$$\text{Intersection of (i) and (ii) is } a \in (-1, 4).$$
$$\text{Intersection of } (-1, 4) \text{ and (iii) is } a \in (-1, 3).$$

**13.** The set of values of 'm' for which the equation $(m-2)x^2 - 2mx + 2m - 3 = 0$ has roots of opposite signs is:

(A) $m > 2$ (B) $m < 3/2$ (C) $3/2 < m < 2$ (D) $m \in \mathbb{R}$

**Answer: (C)**

**Solution:**

$$\begin{aligned}
\text{For roots of opposite signs, their product must be negative.}\quad \alpha\beta &< 0 \\
\frac{c}{a} &< 0 \\
\text{Substituting values from the equation:}\quad \frac{2m-3}{m-2} &< 0 \\
\text{This inequality is true between its critical points.} & \\
\text{Critical points are } m = 3/2 \text{ and } m = 2. & \\
\implies \frac{3}{2} < m < 2. &
\end{aligned}$$

**14.** Let $\alpha, \beta$ be the roots of $x^2 - 4x + 3 = 0$. If $S_n = \alpha^n + \beta^n$, then the value of $\frac{S_7 + 3S_5}{S_6}$ is:

(A) 1 (B) 2 (C) 3 (D) 4

**Answer: (D)**
**Solution:**

$$\begin{aligned}
\text{From the equation } x^2 - 4x + 3 = 0, \text{ we derive the recurrence relation.} & \\
S_n - 4S_{n-1} + 3S_{n-2} &= 0 \\
S_n &= 4S_{n-1} - 3S_{n-2}. \\
\text{Let's use this relation for } n = 7: & \\
S_7 &= 4S_6 - 3S_5. \\
\text{Substitute this into the expression we need to evaluate:} & \\
\frac{S_7 + 3S_5}{S_6} &= \frac{(4S_6 - 3S_5) + 3S_5}{S_6} \\
&= \frac{4S_6}{S_6} \\
&= 4.
\end{aligned}$$

**15.** If $\alpha, \beta$ are the roots of $x^2 - x + 3 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(A) 5 (B) -5 (C) 10 (D) -10

**Answer: (B)**
**Solution:**

$$\begin{aligned}
&\text{From Vieta's formulas:} \\
&\quad \text{Sum of roots: } \alpha + \beta = -(-1)/1 = 1. \\
&\quad \text{Product of roots: } \alpha\beta = 3/1 = 3. \\
&\text{We need to find } \alpha^2 + \beta^2. \\
&\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \\
&\qquad = (1)^2 - 2(3) \\
&\qquad = 1 - 6 = -5.
\end{aligned}$$

**16.** If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$, have both roots common, then the ratio $a : b : c$ is:

(A) $1 : 2 : 3$ (B) $3 : 2 : 1$ (C) $1 : 3 : 2$ (D) $3 : 1 : 2$

**Answer: (A)**
**Solution:**

If two quadratic equations have both roots in common, their coefficients must be proportional.

Comparing $x^2 + 2x + 3 = 0$ with $ax^2 + bx + c = 0$, we have:

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}.$$

Therefore, the ratio $a : b : c$ is $1 : 2 : 3$.

**17.** If $x$ is real, the maximum value of the expression $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 3}$ is:

(A) 2 (B) 3 (C) 4 (D) 5

**Answer: (C)**
**Solution:**

Let $y = \dfrac{x^2 + 14x + 9}{x^2 + 2x + 3}$.

$$y(x^2 + 2x + 3) = x^2 + 14x + 9$$

$$(y - 1)x^2 + (2y - 14)x + (3y - 9) = 0.$$

Since $x$ is real, the discriminant of this quadratic in $x$ must be non-negative.

$$D = (2y - 14)^2 - 4(y - 1)(3y - 9) \geq 0$$

$$4(y - 7)^2 - 12(y - 1)(y - 3) \geq 0$$

$$(y - 7)^2 - 3(y - 1)(y - 3) \geq 0$$

$$(y^2 - 14y + 49) - 3(y^2 - 4y + 3) \geq 0$$

$$-2y^2 - 2y + 40 \geq 0$$

$$y^2 + y - 20 \leq 0$$

$$(y + 5)(y - 4) \leq 0.$$

This inequality holds for $-5 \leq y \leq 4$.

The maximum value of the expression is 4.

**18.** If the quadratic equations $x^2 + 2x + 5 = 0$ and $x^2 + ax + b = 0$ have a common root, where $a, b \in \mathbb{R}$, then the value of $a + b$ is:

(A) 3 (B) 5 (C) 7 (D) 10

**Answer: (C)**

**Solution:**

First, consider the equation $x^2 + 2x + 5 = 0$.
Its discriminant is $D = 2^2 - 4(1)(5) = 4 - 20 = -16 < 0$.
The roots are non-real complex conjugates.
Since $a, b \in \mathbb{R}$, the second equation has real coefficients.
If a quadratic with real coefficients has one complex root, the other root must be its conjugate.
Therefore, if the equations share one root, they must share both roots.
This means the equations are identical.
Comparing coefficients, we get $a = 2$ and $b = 5$.
$a + b = 2 + 5 = 7$.

**19.** If the equations $x^2 - 7x + 10 = 0$ and $x^2 - 8x + k = 0$ have a common root, then the sum of all possible values of $k$ is:
(A) 15 (B) 12 (C) 27 (D) -1

**Answer: (C)**
**Solution:**

First, find the roots of $x^2 - 7x + 10 = 0$.
$(x - 2)(x - 5) = 0$. The roots are $x = 2$ and $x = 5$.
The common root must be either 2 or 5.
Case 1: The common root is 2.
Substitute $x = 2$ into the second equation:
$(2)^2 - 8(2) + k = 0 \implies 4 - 16 + k = 0 \implies k = 12$.
Case 2: The common root is 5.
Substitute $x = 5$ into the second equation:
$(5)^2 - 8(5) + k = 0 \implies 25 - 40 + k = 0 \implies k = 15$.
The possible values of $k$ are 12 and 15.
The sum of these values is $12 + 15 = 27$.

**20.** If $\alpha, \beta$ are the roots of $x^2 + px + q = 0$ and $\gamma, \delta$ are the roots of $x^2 + px + r = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)$ is:
(A) $q + r$ (B) $q - r$ (C) $r - q$ (D) $-(q + r)$

**Answer: (C)**
**Solution:**

We need to evaluate $(\alpha - \gamma)(\alpha - \delta)$.
Expanding this gives $\alpha^2 - (\gamma + \delta)\alpha + \gamma\delta$.
From the second equation, $x^2 + px + r = 0$, we know:
$\gamma + \delta = -p$ and $\gamma\delta = r$.
Substitute these into our expression:
$\alpha^2 - (-p)\alpha + r = \alpha^2 + p\alpha + r$.
Now, since $\alpha$ is a root of the first equation, $x^2 + px + q = 0$, we have:
$\alpha^2 + p\alpha + q = 0 \implies \alpha^2 + p\alpha = -q$.
Substitute this back into our expression:
$(\alpha^2 + p\alpha) + r = (-q) + r = r - q$.

**21.** The number of integral values of $m$ for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real roots is:

(A) 1 (B) 2 (C) 3 (D) Infinitely many

**Answer: (D)**
**Solution:**

$$
\begin{aligned}
&\text{For no real roots, the discriminant } D < 0. \\
&\text{The coefficient } a = 1 + m^2 \text{ is always positive.} \\
&D = [-2(1+3m)]^2 - 4(1+m^2)(1+8m) < 0 \\
&4(1+6m+9m^2) - 4(1+8m+m^2+8m^3) < 0 \\
&(1+6m+9m^2) - (1+8m+m^2+8m^3) < 0 \\
&-8m^3 + 8m^2 - 2m < 0 \\
&8m^3 - 8m^2 + 2m > 0 \\
&2m(4m^2 - 4m + 1) > 0 \\
&2m(2m-1)^2 > 0. \\
&\text{The term } (2m-1)^2 \text{ is always non-negative.} \\
&\text{For the product to be positive, we need } m > 0 \text{ and } (2m-1)^2 \neq 0. \\
&\text{So, } m > 0 \text{ and } m \neq 1/2. \\
&\text{The integral values of } m \text{ satisfying this are } 1, 2, 3, 4, \ldots \\
&\text{There are infinitely many such integral values.}
\end{aligned}
$$

**22.** Let $\alpha, \beta$ be the roots of $x^2 - x - 1 = 0$. If $S_n = \alpha^n + \beta^n$, then which of the following is true?

(A) $S_n = S_{n-1} + S_{n-2}$ (B) $S_n = S_{n-1} - S_{n-2}$
(C) $S_n = 2S_{n-1} + S_{n-2}$ (D) None

**Answer: (A)**
**Solution:**

$$
\begin{aligned}
&\text{The equation is } x^2 - x - 1 = 0. \\
&\text{Let the roots be } \alpha, \beta. \text{ By Newton's Sums, the recurrence relation is:} \\
&(1)S_n - (1)S_{n-1} - (1)S_{n-2} = 0 \\
&S_n - S_{n-1} - S_{n-2} = 0 \\
&S_n = S_{n-1} + S_{n-2}.
\end{aligned}
$$

**23.** If the sum of the squares of the roots of $x^2 - (p-2)x - (p+1) = 0$ is minimum, then the value of $p$ is:

(A) 0 (B) 1 (C) -1 (D) 2

**Answer: (B)**

**Solution:**

Let the roots be $\alpha, \beta$. We want to minimize $S = \alpha^2 + \beta^2$.

$$S = (\alpha + \beta)^2 - 2\alpha\beta.$$

From the equation, $\alpha + \beta = p - 2$ and $\alpha\beta = -(p + 1)$.

$$\begin{aligned} S(p) &= (p-2)^2 - 2(-(p+1)) \\ &= (p^2 - 4p + 4) + (2p + 2) \\ &= p^2 - 2p + 6. \end{aligned}$$

This is an upward parabola. Its minimum is at the vertex.

Vertex occurs at $p = -\dfrac{-2}{2(1)} = 1$.

**24.** The roots of the equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are:

(A) $1, \dfrac{r-p}{p-q}$
(B) $1, \dfrac{p-q}{r-p}$
(C) $1, \dfrac{q-r}{p-q}$
(D) $1, 1$

**Answer: (A)**
**Solution:**

Let the quadratic be $ax^2 + bx + c = 0$.

Here, $a = p - q, b = q - r, c = r - p$.

Notice that the sum of the coefficients is:

$$a + b + c = (p - q) + (q - r) + (r - p) = 0.$$

If the sum of coefficients is zero, then $x = 1$ is always a root.

Let the other root be $\alpha$.

The product of the roots is $1 \cdot \alpha = \dfrac{c}{a}$.

$$\alpha = \frac{r-p}{p-q}.$$

The roots are 1 and $\dfrac{r-p}{p-q}$.

**25.** If the roots of the equation $(k - 1)x^2 - kx + 1 = 0$ are real and equal, the value of $k$ is:

(A) 2
(B) 1
(C) $2 \pm 2\sqrt{2}$
(D) No real value

**Answer: (A)**
**Solution:**

For real and equal roots, the discriminant $D = 0$.

$$\begin{aligned} D = (-k)^2 - 4(k-1)(1) &= 0 \\ k^2 - 4k + 4 &= 0 \\ (k-2)^2 &= 0 \\ k - 2 &= 0 \\ k &= 2. \end{aligned}$$