Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 05th October 2025

Syllabus: Quadratic Equation

Sub: Mathematics

CT-08 JEE Main Solution

Prof. Chetan Sir

Section A: Multiple Choice Questions

1. Let α and β be the roots of the equation $x^2 - 3x - 5 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then which of the following is true?

(A) $a_9 = 3a_8 + 5a_7$ (B) $a_9 = 5a_8 + 3a_7$
(C) $a_9 = 3a_8 - 5a_7$ (D) $a_9 = 5a_8 - 3a_7$

Answer: (A)
Solution:

Let α and β be the roots of $x^2 - 3x - 5 = 0$.
We are given the sequence $a_n = \alpha^n - \beta^n$.
By Newton's Sums, for a quadratic $x^2 + px + q = 0$, the sequence of powers of roots follows the recurrence relation $a_n + pa_{n-1} + qa_{n-2} = 0$.
For our equation, $p = -3$ and $q = -5$.
Therefore, the recurrence relation is:

$$a_n - 3a_{n-1} - 5a_{n-2} = 0.$$

To find the relation for a_9, we set $n = 9$:

$$a_9 - 3a_8 - 5a_7 = 0.$$

Rearranging the terms gives:

$$a_9 = 3a_8 + 5a_7.$$

2. The number of real roots of the equation $e^{4x} + 2e^{3x} - 15e^{2x} + 2e^x + 1 = 0$ is:

(A) 0 (B) 1 (C) 2 (D) 4

Answer: (C)
Solution:

Let $t = e^x$. Since x is real, $t > 0$.
The equation becomes a reciprocal equation:

$$t^4 + 2t^3 - 15t^2 + 2t + 1 = 0.$$

Divide by t^2 (since t $\neq 0$) :

$$t^2 + 2t - 15 + \frac{2}{t} + \frac{1}{t^2} = 0$$

$$\left(t^2 + \frac{1}{t^2}\right) + 2\left(t + \frac{1}{t}\right) - 15 = 0.$$

Let $m = t + \frac{1}{t}$. Then $m^2 = t^2 + 2 + \frac{1}{t^2} \implies t^2 + \frac{1}{t^2} = m^2 - 2.$

$$(m^2-2)+2m-15=0 \implies m^2+2m-17=0.$$

The roots for m are $m = \dfrac{-2 \pm \sqrt{4-4(1)(-17)}}{2} = -1 \pm \sqrt{18} = -1 \pm 3\sqrt{2}$.

Case 1: $m = t + \dfrac{1}{t} = -1 + 3\sqrt{2}$.

Since $3\sqrt{2} \approx 4.24, m \approx 3.24 > 2$. This gives two distinct positive real roots for t.

Case 2: $m = t + \dfrac{1}{t} = -1 - 3\sqrt{2}$.

Since $m \approx -5.24$. The minimum value of $t + 1/t$ for $t > 0$ is 2. This case gives no real roots for t.

From Case 1, we have two distinct positive real values for $t = e^x$.

This means there are two distinct real values for x.

3. The least positive integral value of k for which the roots of the equation $x^2 + kx + 4 = 0$ are rational is:

(A) 3 (B) 4 (C) 5 (D) 6

Answer: (B)
Solution:

For the roots to be rational, the discriminant D must be a perfect square of a rational number.

Since coefficients are integers, D must be a perfect square of an integer.

$D = k^2 - 4(1)(4) = k^2 - 16$.

Let $k^2 - 16 = m^2$, where m is an integer.

$k^2 - m^2 = 16 \implies (k-m)(k+m) = 16$.

Since k is a positive integer, we look for integer factor pairs of 16.

$(k-m)$ and $(k+m)$ must both be even. Possible pairs are (2,8) and (4,4).

Case 1: $k - m = 2$ and $k + m = 8$.

Adding them gives $2k = 10 \implies k = 5$.

Case 2: $k - m = 4$ and $k + m = 4$.

Adding them gives $2k = 8 \implies k = 4$.

The possible positive integral values for k are 4 and 5.

The least value is 4.

4. If the equations $x^2 - 3x + 2 = 0$ and $x^2 + 2kx + k - 1 = 0$ have a common root, then the sum of all possible values of k is:

(A) $-1/4$ (B) $1/4$ (C) $-3/5$ (D) $3/5$

Answer: (C)
Solution:

First, find the roots of $x^2 - 3x + 2 = 0$.

$(x-1)(x-2) = 0$. The roots are $x = 1$ and $x = 2$.

The common root must be either 1 or 2.

Case 1: Common root is $x = 1$.

Substitute $x = 1$ into the second equation:

$$(1)^2 + 2k(1) + k - 1 = 0 \implies 1 + 2k + k - 1 = 0 \implies 3k = 0 \implies k = 0.$$

Case 2: Common root is $x = 2$.

Substitute $x = 2$ into the second equation:

$(2)^2 + 2k(2) + k - 1 = 0 \implies 4 + 4k + k - 1 = 0 \implies 5k + 3 = 0 \implies k = -3/5.$

The possible values of k are 0 and -3/5.

The sum of these values is $0 + (-3/5) = -3/5$.

5. The range of $f(x) = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$ for $x \in \mathbb{R}$ is:

(A) $[1/7, 7]$ (B) $[-7, 7]$ (C) $[1/5, 5]$ (D) $[-5, 5]$

Answer: (A)
Solution:

Let $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$.

$$y(x^2 + 3x + 4) = x^2 - 3x + 4$$

$$(y - 1)x^2 + (3y + 3)x + (4y - 4) = 0.$$

Since x is real, the discriminant of this quadratic in x must be non-negative.

$$D = (3y + 3)^2 - 4(y - 1)(4y - 4) \geq 0$$

$$9(y + 1)^2 - 16(y - 1)^2 \geq 0$$

$$[3(y + 1) - 4(y - 1)][3(y + 1) + 4(y - 1)] \geq 0$$

$$(3y + 3 - 4y + 4)(3y + 3 + 4y - 4) \geq 0$$

$$(-y + 7)(7y - 1) \geq 0$$

$$(y - 7)(7y - 1) \leq 0.$$

The critical points are $y = 7$ and $y = 1/7$.

The inequality holds for $y \in [1/7, 7]$.

6. The set of all values of a for which both roots of the equation $x^2 - 6ax + 9a^2 - 2a + 2 = 0$ are greater than 3, is:

(A) $(1, \infty)$ (B) $(11/9, \infty)$ (C) $[1, 11/9]$ (D) $(-\infty, 1)$

Answer: (B)
Solution:

Let $f(x) = x^2 - 6ax + 9a^2 - 2a + 2 = 0$.

The conditions for both roots to be greater than 3 are:

1. $D \geq 0$ 2. Vertex > 3 3. $f(3) > 0$.

Condition 1: $D = (-6a)^2 - 4(1)(9a^2 - 2a + 2) = 36a^2 - 36a^2 + 8a - 8 = 8a - 8$.

$$8a - 8 \geq 0 \implies a \geq 1.$$

Condition 2: Vertex $x_v = -\dfrac{-6a}{2} = 3a$.

$$3a > 3 \implies a > 1.$$

Condition 3: $f(3) = 3^2 - 6a(3) + 9a^2 - 2a + 2 > 0$

$$9 - 18a + 9a^2 - 2a + 2 > 0 \implies 9a^2 - 20a + 11 > 0.$$

Roots of $9a^2 - 20a + 11 = 0$ are $a = \dfrac{20 \pm \sqrt{400 - 396}}{18} = 1, 11/9$.

So, $9a^2 - 20a + 11 > 0 \implies a < 1$ or $a > 11/9$.

Intersection of $(a \geq 1), (a > 1),$ and $(a < 1$ or $a > 11/9)$ is $a > 11/9$.

7. The set of all values of k for which the roots of $4x^2 - 2x + k = 0$ lie in the interval $(-1, 1)$ is:

(A) $k \in \mathbb{R}$ (B) $k \in [-2, 1/4]$ (C) $k \in (-2, 1/4)$ (D) $k \in (-2, 1/4]$

Answer: (D)
Solution:

$$\text{Let } f(x) = 4x^2 - 2x + k = 0. \text{ Parabola opens upwards.}$$
$$\text{Conditions for both roots to lie in } (-1,1):$$
$$\textbf{1. } D \geq 0 \quad \textbf{2. } -1 < \text{Vertex} < 1 \quad \textbf{3. } f(-1) > 0 \text{ and } f(1) > 0.$$
$$\text{Condition 1: } D = (-2)^2 - 4(4)(k) = 4 - 16k \geq 0 \implies 16k \leq 4 \implies k \leq 1/4.$$
$$\text{Condition 2: Vertex } x_v = -\frac{-2}{2(4)} = \frac{1}{4}. \text{ The condition } -1 < 1/4 < 1 \text{ is true.}$$
$$\text{Condition 3:}$$
$$f(-1) = 4(-1)^2 - 2(-1) + k = 4 + 2 + k = 6 + k > 0 \implies k > -6.$$
$$f(1) = 4(1)^2 - 2(1) + k = 4 - 2 + k = 2 + k > 0 \implies k > -2.$$
$$\text{Intersection of } (k \leq 1/4), (k > -6), \text{ and } (k > -2).$$
$$\text{The common region is } k \in (-2, 1/4].$$

8. If α, β are the roots of $x^2 - 3x + 1 = 0$, then the equation whose roots are $\alpha - 1, \beta - 1$ is:

(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$ (C) $x^2 - x + 1 = 0$ (D) $x^2 + x + 1 = 0$

Answer: (A)
Method 1: Using Sum and Product of Roots

$$\text{From the given equation, } \alpha + \beta = 3 \text{ and } \alpha\beta = 1.$$
$$\text{Let the new roots be } \alpha' = \alpha - 1 \text{ and } \beta' = \beta - 1.$$
$$\text{Sum of new roots: } \alpha' + \beta' = (\alpha - 1) + (\beta - 1) = (\alpha + \beta) - 2 = 3 - 2 = 1.$$
$$\text{Product of new roots: } \alpha'\beta' = (\alpha - 1)(\beta - 1) = \alpha\beta - (\alpha + \beta) + 1 = 1 - 3 + 1 = -1.$$
$$\text{The new equation is } x^2 - (\text{Sum})x + (\text{Product}) = 0.$$
$$x^2 - 1x + (-1) = 0 \implies x^2 - x - 1 = 0.$$

Method 2: Using Transformation of Roots

$$\text{Let the new root be } y.$$
$$\text{The transformation is } y = \alpha - 1, \text{ where } \alpha \text{ is an old root.}$$
$$\text{Express the old root in terms of the new root:}$$
$$\alpha = y + 1.$$
$$\text{Since } \alpha \text{ is a root of } x^2 - 3x + 1 = 0, \text{ it must satisfy it.}$$
$$\alpha^2 - 3\alpha + 1 = 0.$$
$$\text{Substitute } \alpha = y + 1 \text{ into the equation:}$$
$$(y+1)^2 - 3(y+1) + 1 = 0$$
$$(y^2 + 2y + 1) - (3y + 3) + 1 = 0$$
$$y^2 + 2y + 1 - 3y - 3 + 1 = 0$$
$$y^2 - y - 1 = 0.$$
$$\text{Replacing } y \text{ with } x, \text{ the required equation is } x^2 - x - 1 = 0.$$

9. The roots of the equation $x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$ are:

(A) $1, 1, \frac{3\pm\sqrt{5}}{2}$ (B) $2 \pm \sqrt{3}, \frac{3\pm\sqrt{5}}{2}$
(C) $1, 1, 2 \pm \sqrt{3}$ (D) $\frac{3\pm\sqrt{5}}{2}, \frac{2\pm\sqrt{3}}{2}$

Answer: (B)
Solution:

$$x^4 - 7x^3 + 14x^2 - 7x + 1 = 0$$

Since $x \neq 0$, we can divide the equation by x^2 :

$$x^2 - 7x + 14 - \frac{7}{x} + \frac{1}{x^2} = 0$$

Group the terms:

$$\left(x^2 + \frac{1}{x^2}\right) - 7\left(x + \frac{1}{x}\right) + 14 = 0.$$

Let $z = x + \frac{1}{x}$. This implies $x^2 + \frac{1}{x^2} = z^2 - 2$.

Substitute this into the equation:

$$(z^2 - 2) - 7z + 14 = 0$$
$$z^2 - 7z + 12 = 0$$

Factor the quadratic in z:

$$(z-3)(z-4) = 0.$$

This gives two possible cases for z.

Case 1: $z = 3$

$$x + \frac{1}{x} = 3 \implies x^2 - 3x + 1 = 0$$
$$\implies x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}.$$

Case 2: $z = 4$

$$x + \frac{1}{x} = 4 \implies x^2 - 4x + 1 = 0$$
$$\implies x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}.$$

Combining the results, the four roots are $2 \pm \sqrt{3}$ and $\frac{3 \pm \sqrt{5}}{2}$.

10. If α, β, γ are the roots of the cubic equation $x^3 + qx + r = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is:

(A) $\frac{q}{r}$ (B) $-\frac{q}{r}$ (C) $\frac{r}{q}$ (D) $-\frac{r}{q}$

Answer: (B)
Solution:

From Vieta's formulas for the equation $x^3 + 0x^2 + qx + r = 0$:

$$\alpha + \beta + \gamma = 0$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = q$$
$$\alpha\beta\gamma = -r$$

We need to find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Taking a common denominator:

$$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$
$$= \frac{q}{-r} = -\frac{q}{r}.$$

Section B: Integer Type Questions

11. Find the number of integral values of k for which the equation $(k-2)x^2 + 8x + (k+4) = 0$ has both roots real, distinct, and negative.

Answer: 1
Solution:

$$\text{Let } f(x) = (k-2)x^2 + 8x + (k+4) = 0.$$
$$\text{Conditions for both roots real, distinct, and negative:}$$
$$\textbf{1. } D > 0 \quad \textbf{2. Sum} < 0 \quad \textbf{3. Product} > 0.$$
$$\textbf{Condition 1: } D = 8^2 - 4(k-2)(k+4) > 0$$
$$64 - 4(k^2 + 2k - 8) > 0 \implies 16 - (k^2 + 2k - 8) > 0$$
$$-k^2 - 2k + 24 > 0 \implies k^2 + 2k - 24 < 0 \implies (k+6)(k-4) < 0.$$
$$\implies -6 < k < 4.$$
$$\textbf{Condition 2: Sum } = -\frac{8}{k-2} < 0 \implies \frac{8}{k-2} > 0 \implies k - 2 > 0 \implies k > 2.$$
$$\textbf{Condition 3: Product } = \frac{k+4}{k-2} > 0.$$
$$\text{Since we already have } k > 2, \text{ the denominator } k-2 \text{ is positive.}$$
$$\text{So, we also need the numerator } k+4 \text{ to be positive, which means } k > -4.$$
$$\text{Intersection of all conditions: } (-6 < k < 4), (k > 2), \text{ and } (k > -4).$$
$$\text{The final range is } 2 < k < 4.$$
$$\text{The only integral value of k in this interval is 3.}$$
$$\text{So, there is 1 such value.}$$

12. If the minimum value of the quadratic expression $f(x) = x^2 + (k-1)x + (k-1)$ is equal to $-k$, find the sum of all possible values of k.

Answer: 10
Solution:

$$\text{The minimum value of } ax^2 + bx + c \text{ is given by } -\frac{D}{4a}.$$
$$D = (k-1)^2 - 4(1)(k-1) = (k-1)(k-1-4) = (k-1)(k-5).$$
$$\text{Minimum value} = -\frac{(k-1)(k-5)}{4(1)} = \frac{-(k^2 - 6k + 5)}{4}.$$
$$\text{This is given to be equal to } -k.$$
$$\frac{-(k^2 - 6k + 5)}{4} = -k$$
$$k^2 - 6k + 5 = 4k$$
$$k^2 - 10k + 5 = 0.$$
$$\text{Let the possible values of k be the roots of this equation, } k_1 \text{ and } k_2.$$
$$\text{The sum of all possible values of } k \text{ is } k_1 + k_2 = -\frac{-10}{1} = 10.$$

13. If α, β, γ are the roots of the equation $x^3 - 4x^2 + 2x - 1 = 0$, then the value of $\alpha^2 + \beta^2 + \gamma^2$ is:

Answer: 12
Solution:

$$\text{From Vieta's formulas for the given cubic equation:}$$
$$\text{Sum of roots: } \alpha + \beta + \gamma = -(-4)/1 = 4.$$
$$\text{Sum of roots taken two at a time: } \alpha\beta + \beta\gamma + \gamma\alpha = 2/1 = 2.$$
$$\text{Product of roots: } \alpha\beta\gamma = -(-1)/1 = 1.$$
$$\text{We need to find } \alpha^2 + \beta^2 + \gamma^2.$$
$$\text{Using the identity } (\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) :$$
$$\begin{aligned} \alpha^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) \\ &= (4)^2 - 2(2) \\ &= 16 - 4 = 12. \end{aligned}$$