HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

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Syllabus: Quadratic Equation

Sub: Mathematics **CT-08 JEE Main (Star) Solution** Prof. Chetan Sir

Section A: Multiple Choice Questions

1. If two roots of the equation $(x+1)(2x^2-3x+4)=0$ coincide with roots of the equation $x^3+(a+1)x^2+(a+b)x+b=0$ where $a, b \in \mathbb{R}$, then $2(a+b)$ equals:

(A) 4 (B) 2 (C) 1 (D) 0

Answer: (C)
Solution:

The first equation is $(x+1)(2x^2-3x+4)=0$.
For the quadratic part $2x^2-3x+4=0$, the discriminant is:
$D=(-3)^2-4(2)(4)=9-32=-23<0$.
This means its roots are **non-real complex conjugates**.
So, the roots of the first equation are $x=-1$ and two non-real roots.

Let the second equation be $P(x)=x^3+(a+1)x^2+(a+b)x+b=0$.
Let's test if $x=-1$ is a root of P(x):
$P(-1)=(-1)^3+(a+1)(-1)^2+(a+b)(-1)+b$
$=-1+(a+1)-(a+b)+b=-1+a+1-a-b+b=0$.
So, **x=-1 is a root**. This means $(x+1)$ is a factor.
Factoring the second equation gives $(x+1)(x^2+ax+b)=0$.

The problem states that two roots of the equations coincide.
Since $x=-1$ is a common root, the other two roots must also be common.
Therefore, the roots of $x^2+ax+b=0$ must be the same as the roots of $2x^2-3x+4=0$.
Since both quadratics have real coefficients and non-real roots, they must be **proportional**.

$$\frac{1}{2}=\frac{a}{-3}=\frac{b}{4}$$

Solving for a and b:

$$\mathbf{a}=\frac{-3}{2}$$

$$\mathbf{b}=\frac{4}{2}=\mathbf{2}$$

Finally, we calculate the required value:

$$2(a+b)=2\left(-\frac{3}{2}+2\right)=2\left(\frac{1}{2}\right)=\mathbf{1}.$$

2. If one root of the quadratic equation $x^2 - x + 3m = 0$ is 4 times a root of the equation $x^2 - x + m = 0$, where $m \neq 0$, then m equals:

(A) $\frac{12}{196}$ (B) $\frac{12}{169}$ (C) $\frac{12}{256}$ (D) $\frac{12}{189}$

Answer: (B)
Solution:

$$\text{Let } \alpha \text{ be a root of } x^2 - x + m = 0. \text{ Then } \alpha^2 - \alpha + m = 0 \quad \cdots (1)$$
$$\text{Let } 4\alpha \text{ be a root of } x^2 - x + 3m = 0. \text{ Then } (4\alpha)^2 - (4\alpha) + 3m = 0.$$
$$16\alpha^2 - 4\alpha + 3m = 0 \quad \cdots (2)$$
$$\text{From (1), } m = \alpha - \alpha^2. \text{ Substitute this into (2):}$$
$$16\alpha^2 - 4\alpha + 3(\alpha - \alpha^2) = 0$$
$$16\alpha^2 - 4\alpha + 3\alpha - 3\alpha^2 = 0$$
$$13\alpha^2 - \alpha = 0 \implies \alpha(13\alpha - 1) = 0.$$
$$\text{Since } m \neq 0, \alpha \text{ cannot be 0, so } \alpha = 1/13.$$
$$\text{Now find } m \text{ using (1):}$$
$$m = \alpha - \alpha^2 = \frac{1}{13} - \left(\frac{1}{13}\right)^2 = \frac{1}{13} - \frac{1}{169} = \frac{13-1}{169} = \frac{12}{169}.$$

3. If $x \in \mathbb{R}$, then the range of $f(x) = \dfrac{x^2 + 2x - 3}{2x^2 + 3x - 9}$ is:

(A) $(-\infty, \infty)$
(B) $\mathbb{R} - \{\frac{1}{2}\}$
(C) $\mathbb{R} - \{\frac{4}{9}, \frac{1}{2}\}$
(D) $\mathbb{R} - \{\frac{3}{2}\}$

Answer: (C)
Solution:

$$\text{First, factor the numerator and denominator.}$$
$$f(x) = \frac{(x+3)(x-1)}{(2x-3)(x+3)}.$$
$$\text{The expression is defined for } x \neq -3. \text{ For } x \neq -3, \text{ the function simplifies to:}$$
$$f(x) = \frac{x-1}{2x-3}.$$
$$\text{To find the range of this simplified function, let } y = \frac{x-1}{2x-3}.$$
$$y(2x-3) = x - 1 \implies 2xy - 3y = x - 1 \implies x(2y-1) = 3y - 1.$$
$$x = \frac{3y-1}{2y-1}.$$
$$\text{The denominator cannot be zero, so } 2y - 1 \neq 0 \implies y \neq \frac{1}{2}.$$
$$\text{Additionally, we must exclude the value that the function would have taken at the discontinuity } x = -3.$$
$$\text{Value at } x = -3 \text{ (in the simplified form): } y = \frac{-3-1}{2(-3)-3} = \frac{-4}{-9} = \frac{4}{9}.$$
$$\text{Since } x \text{ can never be -3, } y \text{ can never be 4/9.}$$
$$\text{Therefore, the range is } \mathbb{R} - \{\frac{4}{9}, \frac{1}{2}\}.$$

4. If the roots of the equation $\dfrac{1}{x+a} + \dfrac{1}{x+b} = \dfrac{1}{c}$ are equal in magnitude but opposite in sign, then the product of the roots is:

(A) $-(a^2+b^2)$ (B) $-\frac{1}{2}(a^2+b^2)$
(C) $a+b$ (D) ab

Answer: (B)
Solution:

$$\text{Let the roots be } \alpha \text{ and } -\alpha. \text{ Then the sum of the roots is } \alpha+(-\alpha)=0.$$
$$\text{First, convert the given equation into a standard quadratic form.}$$
$$\frac{(x+b)+(x+a)}{(x+a)(x+b)}=\frac{1}{c}$$
$$c(2x+a+b)=x^2+(a+b)x+ab$$
$$x^2+(a+b-2c)x+(ab-c(a+b))=0.$$
$$\text{The sum of the roots is } -(a+b-2c).$$
$$\text{Since the sum is 0, we have } a+b-2c=0 \implies c=\frac{a+b}{2}.$$
$$\text{The product of the roots is } ab-c(a+b).$$
$$\text{Substitute the value of c:}$$
$$\text{Product}=ab-\left(\frac{a+b}{2}\right)(a+b)$$
$$=ab-\frac{(a+b)^2}{2}=\frac{2ab-(a^2+2ab+b^2)}{2}$$
$$=\frac{-a^2-b^2}{2}=-\frac{1}{2}(a^2+b^2).$$

5. The range of $p\in\mathbb{R}$ for which the equation $2x^2-2(2p+1)x+p(p+1)=0$ has one root less than p and the other root greater than p, is:

(A) $-1<p<0$ (B) $p<-1$ or $p>0$
(C) $p\geq 0$ (D) $p=0$

Answer: (B)
Solution:

$$\text{Let } f(x)=2x^2-2(2p+1)x+p(p+1).$$
The condition that one root is less than p and the other is greater than p means that p lies between the roots.
$$\text{For a parabola opening upwards (coefficient of } x^2 \text{ is } 2>0\text{), this condition is } f(p)<0.$$
$$f(p)=2p^2-2(2p+1)p+p(p+1)<0$$
$$=2p^2-(4p^2+2p)+(p^2+p)<0$$
$$=2p^2-4p^2-2p+p^2+p<0$$
$$=-p^2-p<0$$
$$\implies p^2+p>0$$
$$\implies p(p+1)>0.$$
$$\text{This inequality holds when } p<-1 \text{ or } p>0.$$

6. If the equation $x^2+ax+b=0$ has one root equal to unity and the other root lies between the roots of the equation $x^2-7x+12=0$, then the range of a is:

(A) $(-5,-4)$ (B) $(-4,-3)$ (C) $(-3,-2)$ (D) $(4,5)$

Answer: (A)
Solution:

$$\text{Let } f(x) = x^2 + ax + b. \text{ Since 1 is a root, } f(1) = 0.$$
$$1^2 + a(1) + b = 0 \implies 1 + a + b = 0 \implies b = -a - 1.$$
$$\text{Let the other root be } \beta. \text{ The product of roots is } 1 \cdot \beta = b.$$
$$\text{So, the other root is } \beta = b = -a - 1.$$
$$\text{The roots of } x^2 - 7x + 12 = 0 \text{ are given by } (x-3)(x-4) = 0, \text{ which are 3 and 4.}$$
$$\text{The other root } \beta \text{ lies between 3 and 4.}$$
$$3 < \beta < 4$$
$$3 < -a - 1 < 4$$
$$\text{Solving the inequalities separately:}$$
$$3 < -a - 1 \implies 4 < -a \implies a < -4.$$
$$-a - 1 < 4 \implies -a < 5 \implies a > -5.$$
$$\text{Combining the results, we get } -5 < a < -4.$$

7. Find the range of values of a such that the function $f(x) = \dfrac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

(A) $a < -1/2$ (B) $a > 1/4$ (C) $-1/2 < a < 0$ (D) $a < -1$

Answer: (A)
Solution:

$$\text{Let the denominator be } D(x) = x^2 - 8x + 32.$$
$$\text{The discriminant of } D(x) \text{ is } (-8)^2 - 4(1)(32) = 64 - 128 = -64 < 0.$$
$$\text{Since the leading coefficient of } D(x) \text{ is positive, } D(x) \text{ is always positive.}$$
$$\text{For } f(x) \text{ to be always negative, the numerator must be always negative.}$$
$$\text{Let } N(x) = ax^2 + 2(a+1)x + 9a + 4.$$
$$\text{For } N(x) < 0 \text{ for all x, we need:}$$
$$\text{1. Leading coefficient } a < 0.$$
$$\text{2. Discriminant of } N(x) < 0.$$
$$D_N = (2(a+1))^2 - 4(a)(9a+4) < 0$$
$$4(a^2 + 2a + 1) - 4(9a^2 + 4a) < 0$$
$$a^2 + 2a + 1 - 9a^2 - 4a < 0$$
$$-8a^2 - 2a + 1 < 0 \implies 8a^2 + 2a - 1 > 0.$$
$$\text{The roots of } 8a^2 + 2a - 1 = 0 \text{ are } a = \frac{-2 \pm \sqrt{4 - 4(8)(-1)}}{16} = \frac{-2 \pm 6}{16}.$$
$$\text{The roots are } a = 1/4 \text{ and } a = -1/2.$$
$$\text{So, } 8a^2 + 2a - 1 > 0 \text{ implies } a < -1/2 \text{ or } a > 1/4.$$
$$\text{We must satisfy both } a < 0 \text{ and } (a < -1/2 \text{ or } a > 1/4).$$
$$\text{The intersection of these conditions is } a < -1/2.$$

8. Let α, β, γ be the roots of the equation $x^3 - x^2 - 1 = 0$. If $P_n = \alpha^n + \beta^n + \gamma^n$, find the value of P_4.

Answer: 5
Solution:

$$\text{The equation is } x^3 - 1x^2 - 0x - 1 = 0.$$

From Newton's Sums, the recurrence relation for P_n is:
$$P_n - P_{n-1} - 0 \cdot P_{n-2} - P_{n-3} = 0 \quad (\text{for } n \geq 3)$$
$$P_n = P_{n-1} + P_{n-3}.$$
We calculate the initial values from Vieta's formulas:
$$\sum \alpha = 1, \quad \sum \alpha\beta = 0, \quad \alpha\beta\gamma = 1.$$
$$P_1 = \sum \alpha = 1.$$
$$P_2 = \sum \alpha^2 = (\sum \alpha)^2 - 2(\sum \alpha\beta) = 1^2 - 2(0) = 1.$$
Since α is a root, $\alpha^3 = \alpha^2 + 1$.
$$P_3 = \sum \alpha^3 = \sum(\alpha^2 + 1) = (\sum \alpha^2) + (\sum 1) = P_2 + 3 = 1 + 3 = 4.$$
Now we find P_4 using the recurrence relation:
$$P_4 = P_3 + P_1 = 4 + 1 = 5.$$

9. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ is:
(A) $p^2 + q^2$ (B) $(p + q)^2$ (C) $p^2 - q^2$ (D) $(p - q)^2$

Answer: (D)
Solution:

Let $f(x) = x^2 + qx + 1$. The roots are γ, δ, so $f(x) = (x - \gamma)(x - \delta)$.
The expression can be rearranged as $[(\alpha - \gamma)(\alpha - \delta)] \cdot [(\beta - \gamma)(\beta - \delta)]$.
The first part is $f(\alpha)$ and the second part is $f(\beta)$.
So we need to calculate $f(\alpha) \cdot f(\beta)$.
$$f(\alpha) = \alpha^2 + q\alpha + 1.$$
Since α is a root of $x^2 + px + 1 = 0$, we have $\alpha^2 + p\alpha + 1 = 0 \implies \alpha^2 + 1 = -p\alpha$.
Substitute this into $f(\alpha)$:
$$f(\alpha) = (-p\alpha) + q\alpha = \alpha(q - p).$$
Similarly, $f(\beta) = \beta(q - p)$.
The required value is $f(\alpha)f(\beta) = (\alpha(q - p))(\beta(q - p)) = \alpha\beta(q - p)^2$.
From the first equation $x^2 + px + 1 = 0$, the product of roots $\alpha\beta = 1$.
Therefore, the value is $1 \cdot (q - p)^2 = (p - q)^2$.

10. If α, β are the roots of the equation $x^2 - x - 2 = 0$, then the quadratic equation whose roots are $2\alpha + 1$ and $2\beta + 1$ is:
(A) $x^2 - 4x - 5 = 0$ (B) $x^2 - 4x + 5 = 0$
(C) $x^2 + 4x - 5 = 0$ (D) $x^2 - 4x - 9 = 0$

Answer: (A)
Solution:

From the given equation, $\alpha + \beta = 1$ and $\alpha\beta = -2$.
Let the new roots be $\alpha' = 2\alpha + 1$ and $\beta' = 2\beta + 1$.
Sum of new roots:
$$\alpha' + \beta' = (2\alpha + 1) + (2\beta + 1) = 2(\alpha + \beta) + 2 = 2(1) + 2 = 4.$$
Product of new roots:
$$\alpha'\beta' = (2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1$$

$$= 4(-2) + 2(1) + 1 = -8 + 2 + 1 = -5.$$
$$\text{The new equation is } x^2 - (\text{Sum})x + (\text{Product}) = 0.$$
$$x^2 - 4x - 5 = 0.$$

Section B: Integer Type Questions

11. If the minimum value of the quadratic expression $f(x) = x^2 + (k-1)x + (k-1)$ is equal to $-k$, find the sum of all possible values of k.

Answer: 10
Solution:

$$\text{The minimum value of a quadratic } ax^2 + bx + c \text{ is given by } -\frac{D}{4a}.$$
$$\text{Here, } a = 1, b = k-1, c = k-1.$$
$$D = b^2 - 4ac = (k-1)^2 - 4(1)(k-1) = (k-1)(k-1-4) = (k-1)(k-5).$$
$$\text{Minimum value} = -\frac{(k-1)(k-5)}{4(1)} = \frac{-(k^2-6k+5)}{4}.$$
$$\text{We are given that this minimum value is } -k.$$
$$\frac{-(k^2-6k+5)}{4} = -k$$
$$k^2 - 6k + 5 = 4k$$
$$k^2 - 10k + 5 = 0.$$
$$\text{This is a quadratic equation in } k. \text{ Let its roots be } k_1 \text{ and } k_2.$$
$$\text{The sum of all possible values of } k \text{ is } k_1 + k_2 = -\frac{-10}{1} = 10.$$

12. If α, β and γ are the roots of the equation $5x^3 - qx - 1 = 0$, $(q \in \mathbb{R})$, then the value of $\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\gamma\alpha} + \frac{\gamma^2-3}{\alpha\beta}$ is:

Answer: 3
Solution:

$$\text{From the equation } 5x^3 - qx - 1 = 0, \text{ we have by Vieta's formulas:}$$
$$\sum \alpha = 0, \quad \sum \alpha\beta = -q/5, \quad \alpha\beta\gamma = 1/5.$$
$$\text{The expression is } \sum \frac{\alpha^2-3}{\beta\gamma}.$$
$$\text{Taking a common denominator of } \alpha\beta\gamma:$$
$$= \frac{\alpha(\alpha^2-3) + \beta(\beta^2-3) + \gamma(\gamma^2-3)}{\alpha\beta\gamma}$$
$$= \frac{(\alpha^3+\beta^3+\gamma^3) - 3(\alpha+\beta+\gamma)}{\alpha\beta\gamma} = \frac{\sum \alpha^3 - 3\sum \alpha}{\alpha\beta\gamma}.$$
$$\text{Since } \sum \alpha = 0, \text{ the expression simplifies to } \frac{\sum \alpha^3}{\alpha\beta\gamma}.$$
$$\text{Since } \alpha \text{ is a root, } 5\alpha^3 - q\alpha - 1 = 0 \implies 5\alpha^3 = q\alpha + 1.$$
$$\text{Summing over all roots:}$$
$$\sum 5\alpha^3 = \sum (q\alpha + 1) \implies 5\sum \alpha^3 = q\sum \alpha + \sum 1.$$
$$5\sum \alpha^3 = q(0) + 3 \implies \sum \alpha^3 = 3/5.$$

The value of the expression is $\frac{3/5}{1/5} = 3$.

13. The number of non-positive solutions of equation $e^{4x} - 5e^{3x} + 8e^{2x} - 5e^x + 1 = 0, x \in \mathbb{R}$ is:

Answer: 2
Solution:

The given equation is $e^{4x} - 5e^{3x} + 8e^{2x} - 5e^x + 1 = 0$.
Let $y = e^x$. Since $x \in \mathbb{R},$ we must have $y > 0$.
The equation transforms to:
$$y^4 - 5y^3 + 8y^2 - 5y + 1 = 0.$$
This is a reciprocal equation of Type I. Since $y \neq 0,$ we can divide by y^2 :
$$y^2 - 5y + 8 - \frac{5}{y} + \frac{1}{y^2} = 0$$
$$\left(y^2 + \frac{1}{y^2}\right) - 5\left(y + \frac{1}{y}\right) + 8 = 0.$$
Let $z = y + \frac{1}{y}$. Then $z^2 = y^2 + 2 + \frac{1}{y^2} \implies y^2 + \frac{1}{y^2} = z^2 - 2$.
Substitute this into the equation:
$$(z^2 - 2) - 5z + 8 = 0$$
$$z^2 - 5z + 6 = 0$$
$$(z - 2)(z - 3) = 0.$$
This gives two possible cases for z.

Case 1: $z = 2$
$$y + \frac{1}{y} = 2 \implies y^2 - 2y + 1 = 0 \implies (y - 1)^2 = 0.$$
$$y = 1.$$

Case 2: $z = 3$
$$y + \frac{1}{y} = 3 \implies y^2 - 3y + 1 = 0.$$
The solutions are $y = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{5}}{2}$.

Now we find the corresponding values of x from $e^x = y$.
From Case 1: $e^x = 1 \implies x = 0$.
From Case 2: $e^x = \frac{3 + \sqrt{5}}{2} \implies x = \ln\left(\frac{3 + \sqrt{5}}{2}\right)$. This is positive since the argument is ¿ 1.
And $e^x = \frac{3 - \sqrt{5}}{2} \implies x = \ln\left(\frac{3 - \sqrt{5}}{2}\right)$. This is negative since the argument is ¡ 1.

The solutions for x are $0, \ln\left(\frac{3 + \sqrt{5}}{2}\right),$ and $\ln\left(\frac{3 - \sqrt{5}}{2}\right)$.
Non-positive solutions are those where $x \leq 0$.
The non-positive solutions are $x = 0$ and $x = \ln\left(\frac{3 - \sqrt{5}}{2}\right)$.
Therefore, there are 2 non-positive solutions.