Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 17th October 2025

Syllabus: Trigonometry, Quadratic Equations, Sequences & Series

Sub: Mathematics **MT-02 JEE Main Solution** Prof. Chetan Sir

1. If $\sin\theta + \cos\theta = \sqrt{2}$, then find the value of $\tan\theta + \cot\theta$.

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Answer: (B)
Solution:

$$\text{Given } \sin\theta + \cos\theta = \sqrt{2}.$$

Squaring both sides:

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$
$$1 + 2\sin\theta\cos\theta = 2$$
$$\sin\theta\cos\theta = \frac{1}{2}.$$

Now, we evaluate the expression $\tan\theta + \cot\theta$:

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{1/2} = 2.$$

2. The value of $\cos(20^\circ)\cos(40^\circ)\cos(80^\circ)$ is:

(A) 1/4 (B) 1/8 (C) $\sqrt{3}/8$ (D) 1/16

Answer: (B)
Solution:

$$\text{Let } P = \cos(20^\circ)\cos(40^\circ)\cos(80^\circ).$$

Multiply and divide by $2\sin(20^\circ)$:

$$P = \frac{2\sin(20^\circ)\cos(20^\circ)\cos(40^\circ)\cos(80^\circ)}{2\sin(20^\circ)} = \frac{\sin(40^\circ)\cos(40^\circ)\cos(80^\circ)}{2\sin(20^\circ)}$$

Multiply and divide by 2 again:

$$P = \frac{2\sin(40^\circ)\cos(40^\circ)\cos(80^\circ)}{4\sin(20^\circ)} = \frac{\sin(80^\circ)\cos(80^\circ)}{4\sin(20^\circ)}$$

Multiply and divide by 2 again:

$$P = \frac{2\sin(80^\circ)\cos(80^\circ)}{8\sin(20^\circ)} = \frac{\sin(160^\circ)}{8\sin(20^\circ)}.$$

Since $\sin(160^\circ) = \sin(180^\circ - 20^\circ) = \sin(20^\circ)$,

$$P = \frac{\sin(20^\circ)}{8\sin(20^\circ)} = \frac{1}{8}.$$

3. The number of solutions of the equation $\sin x + \cos x = 1$ in the interval $[0, 2\pi]$ is:

(A) 1 (B) 2 (C) 3 (D) 4

Answer: (C)
Solution:

The equation is $\sin x + \cos x = 1$.

Divide the equation by $\sqrt{1^2 + 1^2} = \sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}.$$

This can be written in the form $\sin(A + B)$:

$$\cos\left(\frac{\pi}{4}\right)\sin x + \sin\left(\frac{\pi}{4}\right)\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

Let $u = x + \frac{\pi}{4}$. Since $x \in [0, 2\pi]$, the interval for u is $[\pi/4, 2\pi + \pi/4]$.

The general solutions for $\sin(u) = 1/\sqrt{2}$ are $u = \frac{\pi}{4}, \frac{3\pi}{4}, 2\pi + \frac{\pi}{4}, \ldots$

We find the values of x for each valid u:

$$x + \frac{\pi}{4} = \frac{\pi}{4} \implies x = 0.$$

$$x + \frac{\pi}{4} = \frac{3\pi}{4} \implies x = \frac{\pi}{2}.$$

$$x + \frac{\pi}{4} = 2\pi + \frac{\pi}{4} \implies x = 2\pi.$$

The solutions in the interval $[0, 2\pi]$ are $0, \frac{\pi}{2}$, and 2π.

There are 3 solutions.

4. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then:

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) $a > 4$

Answer: (A)
Solution:

Let $f(x) = x^2 - 2ax + a^2 + a - 3$.

Conditions for both roots to be real and less than 3 are:

1. $D \ge 0$ 2. Vertex < 3 3. $f(3) > 0$.

Condition 1: $D = (-2a)^2 - 4(1)(a^2 + a - 3) = 4a^2 - 4a^2 - 4a + 12 = 12 - 4a$.

$$12 - 4a \ge 0 \implies 12 \ge 4a \implies a \le 3.$$

Condition 2: Vertex $x_v = -\frac{-2a}{2} = a$.

$$a < 3.$$

Condition 3: $f(3) = 3^2 - 2a(3) + a^2 + a - 3 > 0$

$$9 - 6a + a^2 + a - 3 > 0 \implies a^2 - 5a + 6 > 0.$$

$$(a - 2)(a - 3) > 0 \implies a < 2 \text{ or } a > 3.$$

Intersection of $(a \le 3), (a < 3)$, and $(a < 2$ or $a > 3)$.

The common region that satisfies all three conditions is $a < 2$.

5. For the quadratic equation $x^2 - (a-2)x - a - 1 = 0$, the sum of squares of roots is minimum when a equals:

(A) 0 (B) 1 (C) 2 (D) 3

Answer: (B)
Solution:

Let the roots be α, β. We need to minimize $S = \alpha^2 + \beta^2$.

$$S = (\alpha + \beta)^2 - 2\alpha\beta.$$

From Vieta's formulas: $\alpha + \beta = a - 2$ and $\alpha\beta = -(a+1)$.

$$\begin{aligned} S(a) &= (a-2)^2 - 2(-(a+1)) \\ &= (a^2 - 4a + 4) + (2a + 2) \\ &= a^2 - 2a + 6. \end{aligned}$$

This is an upward-opening parabola in 'a'. Its minimum is at the vertex.

The vertex occurs at $a = -\dfrac{-2}{2(1)} = 1$.

6. If α, β are the roots of $x^2 - 5x + 3 = 0$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is:

(A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 + 19x + 3 = 0$
(C) $3x^2 - 19x - 3 = 0$ (D) $x^2 - 19x + 1 = 0$

Answer: (A)
Solution:

From the given equation, $\alpha + \beta = 5$ and $\alpha\beta = 3$.

The new roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.

$$\begin{aligned} \text{Sum of new roots} &= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{5^2 - 2(3)}{3} = \frac{19}{3}. \end{aligned}$$

$$\text{Product of new roots} = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1.$$

The new equation is $x^2 - (\text{Sum})x + (\text{Product}) = 0$.

$$x^2 - \frac{19}{3}x + 1 = 0.$$

Multiplying by 3 gives $3x^2 - 19x + 3 = 0$.

7. If the roots of the equation $(k-1)x^2 - 10x + 3 = 0$ are reciprocals of each other, then the value of k is:

Answer: 4
Solution:

Let the roots be α and $\dfrac{1}{\alpha}$.

The product of the roots is $\alpha \cdot \frac{1}{\alpha} = 1$.

From the equation, the product of the roots is $\frac{c}{a} = \frac{3}{k-1}$.

Equating the two expressions for the product:

$$\frac{3}{k-1} = 1$$
$$3 = k - 1$$
$$k = 4.$$

8. The set of values of k for which the quadratic expression $x^2 - (k-3)x + k$ is positive for all real x is:

(A) $(1, 9)$ (B) $(-\infty, 1) \cup (9, \infty)$ (C) $[1, 9]$ (D) $(-1, 9)$

Answer: (A)
Solution:

For a quadratic to be always positive, two conditions must be met:

1. The leading coefficient is positive.
2. The discriminant is negative.

Condition 1: The coefficient of x^2 is 1, which is positive. This is satisfied.

Condition 2: $D < 0$.

$$\begin{aligned} D &= (-(k-3))^2 - 4(1)(k) < 0 \\ &= (k-3)^2 - 4k < 0 \\ &= k^2 - 6k + 9 - 4k < 0 \\ &= k^2 - 10k + 9 < 0 \\ &= (k-1)(k-9) < 0. \end{aligned}$$

This inequality holds for $1 < k < 9$.

9. If the 5^{th} term of an A.P. is 11 and the 10^{th} term is 26, then the 15^{th} term is:

(A) 40 (B) 41 (C) 42 (D) 43

Answer: (B)
Solution:

Let the first term be 'a' and the common difference be 'd'.

$$a_5 = a + 4d = 11 \quad \cdots (1)$$
$$a_{10} = a + 9d = 26 \quad \cdots (2)$$

Subtracting (1) from (2):

$$(a + 9d) - (a + 4d) = 26 - 11$$
$$5d = 15 \implies d = 3.$$

Substitute d=3 into (1):

$$a + 4(3) = 11 \implies a + 12 = 11 \implies a = -1.$$

We need to find the 15th term:

$$a_{15} = a + 14d = -1 + 14(3) = -1 + 42 = 41.$$

10. If S_n denotes the sum of first n terms of an A.P., and $S_{10} = 100$, $S_{20} = 300$, then S_{30} equals:

(A) 500 (B) 600 (C) 700 (D) 800

Answer: (B)
Solution:

$$S_{10} = \frac{10}{2}(2a + 9d) = 5(2a + 9d) = 100 \implies 2a + 9d = 20 \quad \cdots (1)$$
$$S_{20} = \frac{20}{2}(2a + 19d) = 10(2a + 19d) = 300 \implies 2a + 19d = 30 \quad \cdots (2)$$
$$\text{Subtracting (1) from (2): } 10d = 10 \implies d = 1.$$
$$\text{Substitute d=1 into (1): } 2a + 9(1) = 20 \implies 2a = 11 \implies a = 11/2.$$
$$\text{We need to find } S_{30}:$$
$$S_{30} = \frac{30}{2}(2a + 29d)$$
$$= 15(2(11/2) + 29(1)) = 15(11 + 29) = 15(40) = 600.$$

11. Three numbers are in A.P. If their sum is 15 and their product is 105, then the largest number is:
(A) 5 (B) 7 (C) 9 (D) 11

Answer: (B)
Solution:

$$\text{Let the three numbers in A.P. be } a - d, a, a + d.$$
$$\text{Sum of the numbers:}$$
$$(a - d) + a + (a + d) = 15 \implies 3a = 15 \implies a = 5.$$
$$\text{Product of the numbers:}$$
$$(a - d)(a)(a + d) = 105$$
$$a(a^2 - d^2) = 105$$
$$5(5^2 - d^2) = 105$$
$$25 - d^2 = 21 \implies d^2 = 4 \implies d = \pm 2.$$
$$\text{If d=2, the numbers are 3, 5, 7.}$$
$$\text{If d=-2, the numbers are 7, 5, 3.}$$
$$\text{In either case, the largest number is 7.}$$

12. The number of terms common to the two A.P.s: $3, 7, 11, \ldots, 407$ and $2, 9, 16, \ldots, 709$ is:

Answer: 14
Solution:

$$\text{First A.P.: } 3, 7, 11, 15, 19, 23, \ldots \text{ (common difference } d_1 = 4).$$
$$\text{Second A.P.: } 2, 9, 16, 23, \ldots \text{ (common difference } d_2 = 7).$$
$$\text{The first common term is 23.}$$
$$\text{The common difference of the new A.P. of common terms is LCM(4, 7) = 28.}$$
$$\text{The A.P. of common terms is } 23, 51, 79, \ldots$$
$$\text{The last term of the first A.P. is 407.}$$
$$\text{The last term of the second A.P. is 709.}$$
$$\text{The common terms must be less than or equal to } \min(407, 709) = 407.$$
$$\text{Let the n-th term of the common A.P. be } T_n.$$
$$T_n = 23 + (n - 1)28 \leq 407$$
$$(n - 1)28 \leq 407 - 23$$

$$(n-1)28 \leq 384$$

$$n-1 \leq \frac{384}{28} = \frac{96}{7} \approx 13.71.$$

$$n \leq 14.71.$$

Since n must be an integer, the maximum value of n is 14.

There are 14 common terms.