Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

# Date of Exam: 17th October 2025

Syllabus: Trigonometry, Quadratic Equations, Sequences & Series

Sub: Mathematics **MT-02 JEE Main Star Solution** Prof. Chetan Sir

**1.** If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2-px+q=0$, then the value of $\sin^2(\alpha+\beta)-p\sin(\alpha+\beta)\cos(\alpha+\beta)+q\cos^2(\alpha+\beta)$ is:

(A) $p$ (B) $q$ (C) $p+q$ (D) $1$

**Answer: (B)**
**Solution:**

Let the given expression be $E$.

$$E = \sin^2(\alpha+\beta) - p\sin(\alpha+\beta)\cos(\alpha+\beta) + q\cos^2(\alpha+\beta).$$

We are given that $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 - px + q = 0$.
By Vieta's formulas, we have:

$$\tan\alpha + \tan\beta = p$$
$$\tan\alpha\tan\beta = q$$

The formula for the tangent of a sum gives:

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{p}{1-q}.$$

Now, let's simplify the expression $E$ by dividing by $\cos^2(\alpha+\beta)$:

$$E = \cos^2(\alpha+\beta)\left[\frac{\sin^2(\alpha+\beta)}{\cos^2(\alpha+\beta)} - p\frac{\sin(\alpha+\beta)\cos(\alpha+\beta)}{\cos^2(\alpha+\beta)} + q\frac{\cos^2(\alpha+\beta)}{\cos^2(\alpha+\beta)}\right]$$
$$E = \cos^2(\alpha+\beta)\left[\tan^2(\alpha+\beta) - p\tan(\alpha+\beta) + q\right].$$

Using the identity $\cos^2\theta = \frac{1}{1+\tan^2\theta}$, we can write:

$$E = \frac{1}{1+\tan^2(\alpha+\beta)}\left[\tan^2(\alpha+\beta) - p\tan(\alpha+\beta) + q\right].$$

Substitute $\tan(\alpha+\beta) = \frac{p}{1-q}$ into the expression:

$$E = \frac{\left(\frac{p}{1-q}\right)^2 - p\left(\frac{p}{1-q}\right) + q}{1+\left(\frac{p}{1-q}\right)^2}.$$

Multiply the numerator and the denominator by $(1-q)^2$ to clear the fractions:

$$E = \frac{p^2 - p^2(1-q) + q(1-q)^2}{(1-q)^2 + p^2}$$
$$E = \frac{p^2 - p^2 + p^2q + q(1-2q+q^2)}{p^2 + (1-q)^2}$$

$$E = \frac{p^2q + q - 2q^2 + q^3}{p^2 + (1-q)^2}$$
$$E = \frac{q(p^2 + 1 - 2q + q^2)}{p^2 + (1-q)^2}$$
$$E = \frac{q(p^2 + (1-q)^2)}{p^2 + (1-q)^2}$$
$$E = q.$$

Therefore, the value of the expression is $q$.

**2.** The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$ is:
(A) 1/128 (B) 1/64 (C) 1/32 (D) 1/16

**Answer: (A)**
**Solution:**

$$\text{Let } P = \cos(12^\circ)\cos(24^\circ)\cos(36^\circ)\cos(48^\circ)\cos(60^\circ)\cos(72^\circ)\cos(84^\circ).$$
$$\text{We know } \cos(60^\circ) = 1/2.$$
$$P = \frac{1}{2}\cdot[\cos(36^\circ)\cos(72^\circ)]\cdot[\cos(12^\circ)\cos(24^\circ)\cos(48^\circ)\cos(84^\circ)].$$
$$\text{We use the standard values } \cos(36^\circ) = \frac{\sqrt{5}+1}{4} \text{ and } \cos(72^\circ) = \frac{\sqrt{5}-1}{4}.$$
$$\cos(36^\circ)\cos(72^\circ) = \frac{(\sqrt{5}+1)(\sqrt{5}-1)}{16} = \frac{5-1}{16} = \frac{1}{4}.$$
$$\text{Let } X = \cos(12^\circ)\cos(24^\circ)\cos(48^\circ)\cos(84^\circ). \text{ Multiply by } 2\sin(12^\circ):$$
$$2\sin(12^\circ)X = \sin(24^\circ)\cos(24^\circ)\cos(48^\circ)\cos(84^\circ)$$
$$= \frac{1}{2}\sin(48^\circ)\cos(48^\circ)\cos(84^\circ) = \frac{1}{4}\sin(96^\circ)\cos(84^\circ).$$
$$\text{Since } \sin(96^\circ) = \sin(180^\circ - 84^\circ) = \sin(84^\circ):$$
$$2\sin(12^\circ)X = \frac{1}{4}\sin(84^\circ)\cos(84^\circ) = \frac{1}{8}\sin(168^\circ).$$
$$\text{Since } \sin(168^\circ) = \sin(180^\circ - 12^\circ) = \sin(12^\circ):$$
$$2\sin(12^\circ)X = \frac{1}{8}\sin(12^\circ) \implies X = \frac{1}{16}.$$
$$\text{Combining everything: } P = \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{16} = \frac{1}{128}.$$

**3.** The number of solutions of the equation $\sin^4 x + \cos^4 x = \sin x \cos x$ in the interval $[0, 2\pi]$ is:
(A) 0 (B) 2 (C) 4 (D) 8

**Answer: (B)**
**Solution:**

$$\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$$
$$= 1 - 2(\sin x\cos x)^2 = 1 - 2\left(\frac{\sin(2x)}{2}\right)^2 = 1 - \frac{1}{2}\sin^2(2x).$$
$$\sin x\cos x = \frac{1}{2}\sin(2x).$$
$$\text{The equation becomes: } 1 - \frac{1}{2}\sin^2(2x) = \frac{1}{2}\sin(2x).$$
$$\text{Let } t = \sin(2x).$$

$$1 - \frac{1}{2}t^2 = \frac{1}{2}t \implies 2 - t^2 = t \implies t^2 + t - 2 = 0.$$
$$(t+2)(t-1) = 0.$$
This gives $t = -2$ or $t = 1$.
$\sin(2x) = -2$ is impossible.
$$\sin(2x) = 1.$$
For $x \in [0, 2\pi], 2x \in [0, 4\pi]$.
The solutions for $2x$ are $\frac{\pi}{2}$ and $2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$.
$$2x = \frac{\pi}{2} \implies x = \frac{\pi}{4}.$$
$$2x = \frac{5\pi}{2} \implies x = \frac{5\pi}{4}.$$
There are 2 solutions in the given interval.

**4.** If $\alpha, \beta, \gamma$ are the roots of $x^3 - 3x + 1 = 0$, then $\alpha^5 + \beta^5 + \gamma^5$ equals:

(A) 12 (B) $-15$ (C) 18 (D) 21

**Answer: (B)**
**Solution:**

Let $P_n = \alpha^n + \beta^n + \gamma^n$.
From the equation $x^3 + 0x^2 - 3x + 1 = 0$, the recurrence relation is:
$$P_n + 0 \cdot P_{n-1} - 3P_{n-2} + P_{n-3} = 0 \implies P_n = 3P_{n-2} - P_{n-3}.$$
From Vieta's formulas: $\sum \alpha = 0, \sum \alpha\beta = -3, \alpha\beta\gamma = -1$.
$$P_1 = \sum \alpha = 0.$$
$$P_2 = \sum \alpha^2 = (\sum \alpha)^2 - 2(\sum \alpha\beta) = 0^2 - 2(-3) = 6.$$
Since $\alpha^3 - 3\alpha + 1 = 0, \alpha^3 = 3\alpha - 1$.
$$P_3 = \sum \alpha^3 = \sum(3\alpha - 1) = 3\sum \alpha - \sum 1 = 3(0) - 3 = -3.$$
$$P_4 = 3P_2 - P_1 = 3(6) - 0 = 18.$$
$$P_5 = 3P_3 - P_2 = 3(-3) - 6 = -9 - 6 = -15.$$

**5.** If $\alpha, \beta$ are roots of $x^2 - x + 2\lambda = 0$ and $\alpha, \gamma$ are roots of $3x^2 - 10x + 27\lambda = 0$, then the value of $\beta\gamma/\lambda$ is:

(A) 9 (B) 18 (C) 27 (D) 36

**Answer: (B)**
**Solution:**

For the equation $x^2 - x + 2\lambda = 0$ with roots $\alpha, \beta$, we have:
$$\alpha + \beta = 1 \quad \cdots (1)$$
$$\alpha\beta = 2\lambda \quad \cdots (2)$$

For the equation $3x^2 - 10x + 27\lambda = 0$ with roots $\alpha, \gamma$, we have:
$$\alpha + \gamma = \frac{10}{3} \quad \cdots (3)$$
$$\alpha\gamma = \frac{27\lambda}{3} = 9\lambda \quad \cdots (4)$$

First, we subtract equation (1) from equation (3):

$$(\alpha+\gamma)-(\alpha+\beta)=\frac{10}{3}-1$$
$$\gamma-\beta=\frac{7}{3} \quad \cdots(5)$$

Next, we divide equation (4) by equation (2):

$$\frac{\alpha\gamma}{\alpha\beta}=\frac{9\lambda}{2\lambda}$$
$$\frac{\gamma}{\beta}=\frac{9}{2} \implies \gamma=\frac{9}{2}\beta \quad \cdots(6)$$

Now, substitute equation (6) into equation (5) to find $\beta$ :

$$\frac{9}{2}\beta-\beta=\frac{7}{3}$$
$$\frac{7}{2}\beta=\frac{7}{3} \implies \beta=\frac{2}{3}.$$

Using equation (6), we find $\gamma$ :

$$\gamma=\frac{9}{2}\left(\frac{2}{3}\right)=3.$$

From equation (1), we find $\alpha$ :

$$\alpha=1-\beta=1-\frac{2}{3}=\frac{1}{3}.$$

From equation (2), we can now find $\lambda$ :

$$\alpha\beta=2\lambda \implies \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)=2\lambda$$
$$\frac{2}{9}=2\lambda \implies \lambda=\frac{1}{9}.$$

Finally, we calculate the required value:

$$\frac{\beta\gamma}{\lambda}=\frac{(2/3)(3)}{1/9}=\frac{2}{1/9}=18.$$

**6.** If the equations $x^2+2x+3\lambda=0$ and $2x^2+3x+5\lambda=0$ have a common root, then $\lambda$ equals:

(A) 0 or $-1$ (B) 0 or 1 (C) 0 only (D) $-1$ only

**Answer: (A)**
**Solution:**

Let the common root be $\alpha$.
Then $\alpha$ must satisfy both equations:

$$\alpha^2+2\alpha+3\lambda=0 \quad \cdots(1)$$
$$2\alpha^2+3\alpha+5\lambda=0 \quad \cdots(2)$$

We can solve this system for $\alpha$ and $\lambda$ using elimination.
First, multiply equation (1) by 2 to match the $\alpha^2$ coefficient in equation (2):

$$2(\alpha^2+2\alpha+3\lambda)=0 \implies 2\alpha^2+4\alpha+6\lambda=0 \quad \cdots(3)$$

Now, subtract equation (2) from the new equation (3):

$$(2\alpha^2 + 4\alpha + 6\lambda) - (2\alpha^2 + 3\alpha + 5\lambda) = 0$$
$$(4\alpha - 3\alpha) + (6\lambda - 5\lambda) = 0$$
$$\alpha + \lambda = 0 \implies \alpha = -\lambda.$$

Next, substitute this result, $\alpha = -\lambda$, back into the first equation (1):

$$(-\lambda)^2 + 2(-\lambda) + 3\lambda = 0$$
$$\lambda^2 - 2\lambda + 3\lambda = 0$$
$$\lambda^2 + \lambda = 0$$

Finally, factor the resulting equation to find the values of $\lambda$ :

$$\lambda(\lambda + 1) = 0.$$

This gives two possible values for $\lambda$ :

$$\lambda = 0 \text{ or } \lambda = -1.$$

**7.** The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is:

**Answer: 2**
**Solution:**

Let $t = e^x, t > 0$. The equation is $t^4 + 4t^3 - 58t^2 + 4t + 1 = 0$.

Divide by $t^2$ : $t^2 + 4t - 58 + \frac{4}{t} + \frac{1}{t^2} = 0$.

$$\left(t^2 + \frac{1}{t^2}\right) + 4\left(t + \frac{1}{t}\right) - 58 = 0.$$

Let $m = t + \frac{1}{t}$. Then $t^2 + \frac{1}{t^2} = m^2 - 2$.

$$(m^2 - 2) + 4m - 58 = 0 \implies m^2 + 4m - 60 = 0.$$
$$(m + 10)(m - 6) = 0.$$

Case 1: $m = t + \frac{1}{t} = -10$. Since $t > 0$, the minimum of $t + 1/t$ is 2. No real solutions for t.

Case 2: $m = t + \frac{1}{t} = 6$.

$t^2 - 6t + 1 = 0$. Roots are $t = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm \sqrt{8} = 3 \pm 2\sqrt{2}$.

Both $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$ are positive.

Since there are two distinct positive values for $t = e^x$, there are two distinct real solutions for x.

**8.** The number of integral values of $m$ for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has both roots real and equal is:

**Answer: 1**
**Solution:**

For roots to be real and equal, the discriminant $D = 0$.

$$D = [-2(1 + 3m)]^2 - 4(1 + m^2)(1 + 8m) = 0.$$
$$4(1 + 6m + 9m^2) - 4(1 + 8m + m^2 + 8m^3) = 0.$$

$$(1+6m+9m^2)-(1+8m+m^2+8m^3)=0.$$
$$-8m^3+8m^2-2m=0.$$
$$4m^3-4m^2+m=0.$$
$$m(4m^2-4m+1)=0.$$
$$m(2m-1)^2=0.$$
The solutions are $m=0$ and $m=1/2$.

**9.** If $a_1, a_2, \ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of the first 13 terms of this A.P. is:
(A) $10m$ (B) $12m$ (C) $13m$ (D) $15m$

**Answer: (C)**
**Solution:**

Let the first term be $a_1$ and common difference be $d$.
$$a_4 = a_1+3d, \quad a_7 = a_1+6d, \quad a_{10} = a_1+9d.$$
$$a_4 - a_7 + a_{10} = (a_1+3d)-(a_1+6d)+(a_1+9d) = a_1+6d.$$
We are given this is equal to m, so $a_1+6d=m$.
Note that $a_1+6d$ is the 7th term, $a_7$. So $a_7=m$.
The sum of the first 13 terms is $S_{13} = \frac{13}{2}(2a_1+12d) = 13(a_1+6d)$.
Since $a_1+6d=m,$ we have $S_{13}=13m$.

**10.** In an A.P., the sum of first $n$ terms is $2n^2+n$. The $10^{th}$ term of the A.P. is:
(A) 39 (B) 41 (C) 210 (D) 240

**Answer: (A)**
**Solution:**

The n-th term of an A.P. is given by $a_n = S_n - S_{n-1}$.
$$a_{10} = S_{10} - S_9.$$
$$S_{10} = 2(10)^2+10 = 2(100)+10 = 210.$$
$$S_9 = 2(9)^2+9 = 2(81)+9 = 162+9 = 171.$$
$$a_{10} = 210-171 = 39.$$

**11.** The $17^{th}$ term from the end of the arithmetic progression $5, 9, 13, \ldots, 201$ is:
(A) 133 (B) 137 (C) 141 (D) 145

**Answer: (B)**
**Solution:**

The given arithmetic progression (AP) is $5, 9, 13, \ldots, 201$.
Here, the first term is $a=5$.
The common difference is $d=9-5=4$.
The last term is $a_m=201$.

First, we need to find the total number of terms, $m$, in the AP.

Using the formula $a_m = a + (m-1)d$ :

$$\begin{aligned}201 &= 5 + (m-1)4\\ 196 &= (m-1)4\\ \frac{196}{4} &= m-1\\ 49 &= m-1\\ m &= 50.\end{aligned}$$

So, there are 50 terms in the progression.

Now, we find the position of the required term from the beginning.

The $n^{th}$ term from the end is the $(m-n+1)^{th}$ term from the beginning.

Here, $n = 17$ and $m = 50$.

$$\begin{aligned}\text{Position from beginning} &= (50-17+1)\\ &= 34^{th} \text{ term.}\end{aligned}$$

Finally, we calculate the $34^{th}$ term of the AP, $a_{34}$ :

$$\begin{aligned}a_{34} &= a + (34-1)d\\ &= 5 + (33) \times 4\\ &= 5 + 132\\ &= 137.\end{aligned}$$

Thus, the $17^{th}$ term from the end is 137.

**12.** If the sum of the first $2n$ terms of the A.P. $2, 5, 8, \dots$ is equal to the sum of the first $n$ terms of the A.P. $57, 59, 61, \dots$, then $n$ is equal to:

(A) 10 (B) 11 (C) 12 (D) 13

**Answer: (B)**
**Solution:**

For the first A.P.: $a_1 = 2, d_1 = 3.$

$$S_{2n}^{(1)} = \frac{2n}{2}(2(2) + (2n-1)3) = n(4 + 6n - 3) = n(6n+1).$$

For the second A.P.: $a_2 = 57, d_2 = 2.$

$$S_n^{(2)} = \frac{n}{2}(2(57) + (n-1)2) = \frac{n}{2}(114 + 2n - 2) = n(56+n).$$

We are given that the sums are equal:

$$n(6n+1) = n(56+n).$$

Since n must be a positive integer, we can divide by n.

$$\begin{aligned}6n + 1 &= 56 + n\\ 5n &= 55\\ n &= 11.\end{aligned}$$