Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

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# Date of Exam: 28th October 2025

Syllabus: Trigonometry, Quadratic Equations, Sequences & Series

Sub: Mathematics **MT-03 JEE Main Solution** Prof. Chetan Sir

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**1.** The number of integral values of $k$ for which $\sin\theta = \frac{k-3}{4}$ has a solution is:

**Answer: 9**
**Solution:**

$$\text{For the equation } \sin\theta = \frac{k-3}{4} \text{ to have a solution, the value of } \sin\theta \text{ must be in its range.}$$
$$\text{The range of } \sin\theta \text{ is } [-1, 1].$$
$$-1 \le \frac{k-3}{4} \le 1$$
$$\text{Multiply the inequality by 4:}$$
$$-4 \le k-3 \le 4$$
$$\text{Add 3 to all parts of the inequality:}$$
$$-4+3 \le k \le 4+3$$
$$-1 \le k \le 7.$$
$$\text{The integral values of k are -1, 0, 1, 2, 3, 4, 5, 6, and 7.}$$
$$\text{The total number of integral values is } 7-(-1)+1 = 9.$$

**2.** The value of $\sin(480^\circ)\cos(690^\circ) + \cos(780^\circ)\sin(1050^\circ)$ is:

(A) $1/2$ (B) $-1/2$ (C) $1$ (D) $-1$

**Answer: (A)**
**Solution:**

$$\text{We can recognize the expression as the sine addition formula: } \sin(A+B).$$
$$\text{Let } A = 480^\circ \text{ and } B = 1050^\circ. \text{ Does this work? } \sin A\cos B + \cos A\sin B.$$
$$\text{No, the angles for cos and sin are different. Let's simplify each term.}$$
$$\sin(480^\circ) = \sin(360^\circ + 120^\circ) = \sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}.$$
$$\cos(690^\circ) = \cos(720^\circ - 30^\circ) = \cos(-30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}.$$
$$\cos(780^\circ) = \cos(720^\circ + 60^\circ) = \cos(60^\circ) = \frac{1}{2}.$$
$$\sin(1050^\circ) = \sin(3\times 360^\circ - 30^\circ) = \sin(-30^\circ) = -\sin(30^\circ) = -\frac{1}{2}.$$
$$\text{The expression becomes:}$$
$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

**3.** The equation $2\sin^2 x - 3\sin x + 1 = 0$ has solutions in $[0, 2\pi]$ as:

(A) $\{\frac{\pi}{6}, \frac{5\pi}{6}\}$
(B) $\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\}$
(C) $\{\frac{\pi}{2}\}$
(D) $\{\frac{\pi}{3}, \frac{2\pi}{3}\}$

**Answer: (B)**
**Solution:**

Let $t = \sin x$. The equation becomes a quadratic in t:

$$2t^2 - 3t + 1 = 0.$$

Factor the quadratic:

$$(2t - 1)(t - 1) = 0.$$

This gives two possibilities for $t$.

**Case 1:** $t = 1$

$\sin x = 1$. In the interval $[0, 2\pi]$, this occurs at $x = \dfrac{\pi}{2}$.

**Case 2:** $t = 1/2$

$\sin x = \dfrac{1}{2}$. In the interval $[0, 2\pi]$, this occurs at $x = \dfrac{\pi}{6}$ and $x = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$.

The complete set of solutions is $\left\{\dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}\right\}$.

**4.** If the roots of $x^2 - px + q = 0$ are in the ratio 3:4, and their sum is 14, then $p$ equals:

**Answer: 14**
**Solution:**

Let the roots be $\alpha$ and $\beta$.

From Vieta's formulas, the sum of the roots is $\alpha + \beta = p$.

We are given that the sum of the roots is 14.

$$p = 14$$

(The information about the ratio 3:4 is not needed to find p).

**5.** The quadratic equation with rational coefficients having $3 - \sqrt{5}$ as one root is:

(A) $x^2 - 6x + 4 = 0$
(B) $x^2 + 6x + 4 = 0$
(C) $x^2 - 6x - 4 = 0$
(D) $x^2 + 6x - 4 = 0$

**Answer: (A)**
**Solution:**

If a quadratic equation has rational coefficients, then irrational roots occur in conjugate pairs.

Given one root is $\alpha = 3 - \sqrt{5}$.

The other root must be its conjugate, $\beta = 3 + \sqrt{5}$.

Sum of the roots $= (3 - \sqrt{5}) + (3 + \sqrt{5}) = 6$.

Product of the roots $= (3 - \sqrt{5})(3 + \sqrt{5}) = 3^2 - (\sqrt{5})^2 = 9 - 5 = 4$.

The equation is $x^2 - (\text{Sum})x + (\text{Product}) = 0$.

$$x^2 - 6x + 4 = 0.$$

**6.** The set of values of $a$ for which the equation $x^2 + 2(a-1)x + a + 5 = 0$ has roots of opposite signs is:

(A) $a < -5$ (B) $a > -5$ (C) $a < 1$ (D) $a > 1$

**Answer: (A)**
**Solution:**

$$\begin{aligned}
&\text{For a quadratic equation to have roots of opposite signs, the product of the roots must be negative.}\\
&\text{This condition, } \alpha\beta < 0, \text{ is sufficient as it ensures } D > 0.\\
&\text{Product of roots } = \frac{c}{A} = a + 5.\\
&a + 5 < 0\\
&a < -5.
\end{aligned}$$

**7.** If the sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero, then the product of the roots is:

(A) $a^2 + b^2$ (B) $-\frac{1}{2}(a^2 + b^2)$
(C) $ab$ (D) $-(a^2 + b^2)$

**Answer: (B)**
**Solution:**

$$\begin{aligned}
&\text{First, form the quadratic equation:}\\
&\frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{c}\\
&c(2x + a + b) = x^2 + (a+b)x + ab\\
&x^2 + (a + b - 2c)x + (ab - c(a+b)) = 0.\\
&\text{Sum of roots is } -(a + b - 2c). \text{ Given that this is zero:}\\
&a + b - 2c = 0 \implies c = \frac{a+b}{2}.\\
&\text{Product of roots is } ab - c(a+b).\\
&\text{Substitute the value of c:}\\
&\text{Product} = ab - \left(\frac{a+b}{2}\right)(a+b)\\
&= \frac{2ab - (a+b)^2}{2} = \frac{2ab - (a^2 + 2ab + b^2)}{2}\\
&= \frac{-a^2 - b^2}{2} = -\frac{1}{2}(a^2 + b^2).
\end{aligned}$$

**8.** Let $\alpha, \beta$ be the roots of $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of $x^2 - qx + r = 0$. The value of $r$ is:

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

**Answer: (D)**
**Solution:**

$$\begin{aligned}
&\text{From the first equation: } \alpha + \beta = p \quad \text{and} \quad \alpha\beta = r.\\
&\text{From the second equation: } \frac{\alpha}{2} + 2\beta = q \quad \text{and} \quad \left(\frac{\alpha}{2}\right)(2\beta) = \alpha\beta = r.\\
&\text{We have a system of two linear equations in } \alpha \text{ and } \beta:\\
&\alpha + \beta = p \quad \cdots (1)
\end{aligned}$$

$$\frac{\alpha}{2} + 2\beta = q \quad \cdots (2)$$

Multiply (1) by 2: $2\alpha + 2\beta = 2p$. Subtract (2) from this:

$$\left(2\alpha - \frac{\alpha}{2}\right) = 2p - q \implies \frac{3\alpha}{2} = 2p - q \implies \alpha = \frac{2}{3}(2p - q).$$

Multiply (1) by 1/2: $\frac{\alpha}{2} + \frac{\beta}{2} = \frac{p}{2}$. Subtract this from (2):

$$\left(2\beta - \frac{\beta}{2}\right) = q - \frac{p}{2} \implies \frac{3\beta}{2} = \frac{2q - p}{2} \implies \beta = \frac{1}{3}(2q - p).$$

Now, find r using $r = \alpha\beta$ :

$$r = \left[\frac{2}{3}(2p - q)\right]\left[\frac{1}{3}(2q - p)\right] = \frac{2}{9}(2p - q)(2q - p).$$

**9.** The $10^{th}$ term from the end of the A.P. $8, 10, 12, ..., 126$ is:

(A) 106 (B) 108 (C) 110 (D) 112

**Answer: (B)**
**Solution:**

The $n^{th}$ term from the end of an A.P. is the $n^{th}$ term from the beginning of the reversed A.P.

The given A.P. is $8, 10, 12, \ldots, 126$.

The reversed A.P. is $126, 124, 122, \ldots, 8$.

For this reversed A.P., the first term $a = 126$ and common difference $d = -2$.

We need to find the 10th term:

$$a_{10} = a + (10 - 1)d$$
$$= 126 + 9(-2) = 126 - 18 = 108.$$

**10.** In an A.P., if $S_5 = 35$ and $S_9 = 99$, then $S_{14}$ equals:

**Answer: 224**
**Solution:**

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d) = 35 \implies a + 2d = 7 \quad \cdots (1)$$
$$S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d) = 99 \implies a + 4d = 11 \quad \cdots (2)$$

Subtracting (1) from (2): $2d = 4 \implies d = 2$.

Substitute d=2 into (1): $a + 2(2) = 7 \implies a = 3$.

We need to find $S_{14}$ :

$$S_{14} = \frac{14}{2}(2a + 13d)$$
$$= 7(2(3) + 13(2)) = 7(6 + 26) = 7(32) = 224.$$

**11.** Let $S_n$ denote the sum of the first $n$ terms of an A.P. If $S_{2n} = 3S_n$, then the ratio $S_{3n}/S_n$ is equal to:

(A) 4 (B) 6 (C) 8 (D) 10

**Answer: (B)**
**Solution:**

$$\text{Given } S_{2n} = 3S_n.$$
$$\frac{2n}{2}(2a + (2n-1)d) = 3 \cdot \frac{n}{2}(2a + (n-1)d).$$
Since n is non-zero, we can simplify:
$$2(2a + 2nd - d) = 3(2a + nd - d)$$
$$4a + 4nd - 2d = 6a + 3nd - 3d$$
$$nd + d = 2a \implies 2a = (n+1)d.$$
Now we find the ratio $\frac{S_{3n}}{S_n}$ :
$$\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}(2a + (3n-1)d)}{\frac{n}{2}(2a + (n-1)d)} = \frac{3(2a + (3n-1)d)}{2a + (n-1)d}.$$
Substitute $2a = (n+1)d$ :
$$\text{Ratio} = \frac{3((n+1)d + (3n-1)d)}{((n+1)d + (n-1)d)} = \frac{3d(n+1+3n-1)}{d(n+1+n-1)}$$
$$= \frac{3(4n)}{2n} = 6.$$

**12.** The sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots$ if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, is:
(A) 900 (B) 1200 (C) 675 (D) 450

**Answer: (A)**
**Solution:**

In an A.P., the sum of terms equidistant from the beginning and end is constant and equals $a_1 + a_n$.
Let's pair the terms in the given sum:
$$(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}).$$
Note that $1 + 24 = 25, 5 + 20 = 25, 10 + 15 = 25$.
So, $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$.
The given sum is $3(a_1 + a_{24}) = 225$.
$$a_1 + a_{24} = \frac{225}{3} = 75.$$
The sum of the first 24 terms is $S_{24} = \frac{24}{2}(a_1 + a_{24})$.
$$S_{24} = 12(75) = 900.$$

**13.** If the sum of the first ten terms of an A.P. is 4 times the sum of the first five terms, the ratio of the first term to the common difference is:
(A) 1:2 (B) 2:1 (C) 1:4 (D) 4:1

**Answer: (A)**
**Solution:**

$$\text{Given } S_{10} = 4S_5.$$
$$\frac{10}{2}(2a + 9d) = 4 \cdot \frac{5}{2}(2a + 4d).$$
$$5(2a + 9d) = 10(2a + 4d).$$
Divide by 5:

$$2a + 9d = 2(2a + 4d)$$
$$2a + 9d = 4a + 8d$$
$$d = 2a.$$

We need the ratio of the first term to the common difference, which is $\frac{a}{d}$.

$$\frac{a}{d} = \frac{a}{2a} = \frac{1}{2}.$$

The ratio is 1:2.