Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 28th October 2025

Syllabus: Trigonometry, Quadratic Equations, Sequences & Series

Sub: Mathematics **MT-03 JEE Main Star Solution** Prof. Chetan Sir

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1. The value of $\sum_{k=1}^{89} \log_e(\tan k^\circ)$ is:

(A) 1 (B) 0 (C) e (D) $\log_e 2$

Answer: (B)
Solution:

$$\text{Let } S = \sum_{k=1}^{89} \log_e(\tan k^\circ)$$

Using the property of logarithms, $\log a + \log b = \log(ab)$:

$$S = \log_e(\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdots \tan 89^\circ).$$

We use the identity $\tan(90^\circ - \theta) = \cot\theta$.

Let's pair the terms in the product:

$$P = (\tan 1^\circ \cdot \tan 89^\circ) \cdot (\tan 2^\circ \cdot \tan 88^\circ) \cdots (\tan 44^\circ \cdot \tan 46^\circ) \cdot \tan 45^\circ$$
$$= (\tan 1^\circ \cdot \cot 1^\circ) \cdot (\tan 2^\circ \cdot \cot 2^\circ) \cdots (\tan 44^\circ \cdot \cot 44^\circ) \cdot \tan 45^\circ.$$

Since $\tan\theta \cdot \cot\theta = 1$ and $\tan 45^\circ = 1$, the product is:

$$P = 1 \cdot 1 \cdots 1 \cdot 1 = 1.$$

Therefore, $S = \log_e(1) = 0$.

2. The sum of maximum and minimum value of the function $f(x) = \sin^2 x - 4\sin x + 5$ is:

Answer: 12
Solution:

Let $t = \sin x$. The range of $\sin x$ is $[-1, 1]$, so $t \in [-1, 1]$.

The function becomes a quadratic in t:

$$g(t) = t^2 - 4t + 5.$$

This is an upward-opening parabola. Its vertex is at $t = -\dfrac{-4}{2(1)} = 2$.

Since the vertex $(t = 2)$ is outside our interval $[-1, 1]$, the extreme values will occur at the endpoints.

The function $g(t)$ is strictly decreasing over the interval $[-1, 1]$.

Maximum value occurs at $t = -1$:

$$g(-1) = (-1)^2 - 4(-1) + 5 = 1 + 4 + 5 = 10.$$

Minimum value occurs at $t = 1$:

$$g(1) = (1)^2 - 4(1) + 5 = 1 - 4 + 5 = 2.$$

The sum of maximum and minimum values is $10 + 2 = 12$.

3. The number of solutions of the equation $12\cos^3 x - 7\cos^2 x - 7\cos x + 12 = 0$ in $[0, 2\pi]$ is:
(A) 2 (B) 1 (C) 6 (D) 0

Answer: (B)
Solution:

$$\text{Let } t = \cos x,\ \text{ where } t \in [-1, 1].$$
$$\text{The equation becomes a cubic in t:}$$
$$P(t) = 12t^3 - 7t^2 - 7t + 12 = 0.$$
$$\text{By inspection, we test for simple roots. Let's try t = -1:}$$
$$P(-1) = 12(-1)^3 - 7(-1)^2 - 7(-1) + 12$$
$$= -12 - 7 + 7 + 12 = 0.$$
$$\text{So, } t = -1 \text{ is a root. This means } (t+1) \text{ is a factor.}$$
$$\text{Dividing the polynomial by } (t+1) \text{ gives } 12t^2 - 19t + 12.$$
$$\text{So the equation is } (t+1)(12t^2 - 19t + 12) = 0.$$
$$\text{For the quadratic factor, the discriminant is:}$$
$$D = (-19)^2 - 4(12)(12) = 361 - 576 = -215 < 0.$$
$$\text{This quadratic has no real roots.}$$
$$\text{The only real root for t is } t = -1.$$
$$\text{Therefore, } \cos x = -1.$$
$$\text{In the interval } [0, 2\pi],\ \text{ the only solution is } x = \pi.$$
$$\text{Thus, there is 1 solution.}$$

4. The value of λ for which the sum of squares of the roots of $x^2 + (3 - 2\lambda)x + (\lambda^2 - 3\lambda + 2) = 0$ is minimum is:
(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$

Answer: (B)
Solution:

$$x^2 + (3 - 2\lambda)x + (\lambda^2 - 3\lambda + 2) = 0.$$
$$\text{Let the roots be } \alpha, \beta. \text{ We want to minimize } S = \alpha^2 + \beta^2.$$
$$S = (\alpha + \beta)^2 - 2\alpha\beta.$$
$$\alpha + \beta = -(3 - 2\lambda) = 2\lambda - 3.$$
$$\alpha\beta = \lambda^2 - 3\lambda + 2.$$
$$S(\lambda) = (2\lambda - 3)^2 - 2(\lambda^2 - 3\lambda + 2)$$
$$= (4\lambda^2 - 12\lambda + 9) - (2\lambda^2 - 6\lambda + 4)$$
$$= 2\lambda^2 - 6\lambda + 5.$$
$$\text{This is an upward-opening parabola in } \lambda. \text{ Its minimum is at the vertex.}$$
$$\text{The vertex occurs at } \lambda = -\frac{-6}{2(2)} = \frac{6}{4} = \frac{3}{2}.$$

5. The set of values of a for which the inequality $\frac{x^2+ax-2}{x^2-x+1} < 2$ is satisfied for all real values of x is:
(A) $a \in (-6, 2)$ (B) $a \in (-2, 6)$
(C) $a \in (-\infty, -2) \cup (6, \infty)$ (D) $a \in (-\infty, -6) \cup (2, \infty)$

Answer: (A)
Solution:

$$\text{Let the denominator be } D(x) = x^2 - x + 1.$$
$$\text{Its discriminant is } (-1)^2 - 4(1)(1) = -3 < 0.$$
$$\text{Since the leading coefficient is positive, } D(x) \text{ is always positive for all real x.}$$
$$\text{We can multiply both sides of the inequality by } D(x) \text{ without changing the sign.}$$
$$x^2 + ax - 2 < 2(x^2 - x + 1)$$
$$x^2 + ax - 2 < 2x^2 - 2x + 2$$
$$0 < x^2 - (a+2)x + 4.$$
$$\text{For this quadratic to be always positive, its discriminant must be negative.}$$
$$D' = (-(a+2))^2 - 4(1)(4) < 0$$
$$(a+2)^2 - 16 < 0$$
$$(a+2)^2 < 16$$
$$|a+2| < 4$$
$$-4 < a+2 < 4$$
$$-6 < a < 2.$$

6. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2-11x+30} = 1$ is:

Answer: 21
Solution:

$$\text{For an equation of the form } (\text{base})^{\text{exponent}} = 1, \text{ there are three cases.}$$
$$\textbf{Case 1: Exponent = 0} \text{ (and base } \neq 0).$$
$$x^2 - 11x + 30 = 0 \implies (x-5)(x-6) = 0.$$
$$\text{Solutions are } x = 5 \text{ and } x = 6.$$
$$\text{Check base: for } x = 5, \text{ base is } 25 - 25 + 5 = 5 \neq 0. \text{ For } x = 6, \text{ base is } 36 - 30 + 5 = 11 \neq 0. \text{ Both are valid.}$$

$$\textbf{Case 2: Base = 1}.$$
$$x^2 - 5x + 5 = 1 \implies x^2 - 5x + 4 = 0 \implies (x-1)(x-4) = 0.$$
$$\text{Solutions are } x = 1 \text{ and } x = 4. \text{ Both are valid.}$$

$$\textbf{Case 3: Base = -1 and Exponent is an even integer}.$$
$$x^2 - 5x + 5 = -1 \implies x^2 - 5x + 6 = 0 \implies (x-2)(x-3) = 0.$$
$$\text{Solutions are } x = 2 \text{ and } x = 3.$$
$$\text{Check exponent at } x = 2 : (2)^2 - 11(2) + 30 = 4 - 22 + 30 = 12 \text{ (Even). Valid.}$$
$$\text{Check exponent at } x = 3 : (3)^2 - 11(3) + 30 = 9 - 33 + 30 = 6 \text{ (Even). Valid.}$$

$$\text{The set of all real solutions is } \{1, 2, 3, 4, 5, 6\}.$$
$$\text{Sum of solutions} = 1 + 2 + 3 + 4 + 5 + 6 = 21.$$

7. If α, β, γ are the roots of $x^3 + x + 2 = 0$, then the equation whose roots are $(\alpha-\beta)^2, (\beta-\gamma)^2, (\gamma-\alpha)^2$ is:

(A) $y^3 + 6y^2 + 9y + 4 = 0$ (B) $y^3 - 6y^2 - 9y - 4 = 0$
(C) $y^3 + 6y^2 - 9y + 4 = 0$ (D) $y^3 - 6y^2 + 9y + 4 = 0$

Answer: (A)
Solution:

$$\text{From Vieta's formulas for } x^3 + 0x^2 + x + 2 = 0:$$
$$\sum \alpha = 0, \quad \sum \alpha\beta = 1, \quad \alpha\beta\gamma = -2.$$
$$\text{Let the new roots be } y_1, y_2, y_3. \text{ Let's find their sum.}$$
$$\text{Sum of new roots} = (\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2$$
$$= (\alpha^2 - 2\alpha\beta + \beta^2) + (\beta^2 - 2\beta\gamma + \gamma^2) + (\gamma^2 - 2\gamma\alpha + \alpha^2)$$
$$= 2(\alpha^2+\beta^2+\gamma^2) - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$= 2\left[(\sum \alpha)^2 - 2(\sum \alpha\beta)\right] - 2(\sum \alpha\beta)$$
$$= 2[0^2 - 2(1)] - 2(1) = 2(-2) - 2 = -6.$$
$$\text{The new equation is of the form } y^3 - (\text{Sum})y^2 + \cdots = 0.$$
$$y^3 - (-6)y^2 + \cdots = y^3 + 6y^2 + \cdots = 0.$$
$$\text{Only option (A) matches this form.}$$

8. If the roots of the equation $x^2 - 10ax - 11b = 0$ are c and d, and the roots of $x^2 - 10cx - 11d = 0$ are a and b, where a, b, c, d are distinct numbers, then $a + b + c + d$ is:

(A) 1210 (B) 1100 (C) 1320 (D) 0

Answer: (A)
Solution:

$$\text{From the first equation, since c is a root: } c^2 - 10ac - 11b = 0 \quad \cdots (1)$$
$$\text{From the second equation, since a is a root: } a^2 - 10ca - 11d = 0 \quad \cdots (2)$$
$$\text{Subtracting (2) from (1):}$$
$$(c^2 - a^2) - (11b - 11d) = 0 \implies (c-a)(c+a) = 11(b-d).$$
$$\text{From Vieta's formulas:}$$
$$c + d = 10a \quad \cdots (3)$$
$$a + b = 10c \quad \cdots (4)$$
$$\text{Subtracting (3) from (4): } (a-d) + (b-c) = 10(c-a).$$
$$-(d-b) - (c-a) = -10(c-a) \implies d - b = 9(c-a).$$
$$\text{So, } b - d = -9(c-a).$$
$$\text{Substitute this into the earlier result:}$$
$$(c-a)(c+a) = 11(-9(c-a)).$$
$$\text{Since } a \neq c, \text{ we can divide by } (c-a):$$
$$c + a = -99. \text{ Let me recheck. } d - b = 10(a-c) - (c-a) = -11(c-a).$$
$$(c-a)(c+a) + 11(-11(c-a)) = 0 \implies c + a - 121 = 0 \implies a + c = 121.$$
$$\text{We need to find } a+b+c+d.$$
$$a+b+c+d = (a+b)+(c+d) = 10c + 10a = 10(a+c).$$
$$= 10(121) = 1210.$$

9. In an A.P. of 99 terms, the sum of all odd-numbered terms is 2550. The sum of all even-numbered terms is 2500. The 50th term of the A.P. is:

(A) 50 (B) 51 (C) 52 (D) 53

Answer: (A)
Solution:

Odd-numbered terms are $a_1, a_3, \ldots, a_{99}$. There are 50 such terms.

$$\text{Their sum is } S_{odd} = \frac{50}{2}(a_1 + a_{99}) = 25(a_1 + a_1 + 98d) = 50(a_1 + 49d) = 50a_{50}.$$

$$50a_{50} = 2550 \implies a_{50} = \frac{2550}{50} = 51.$$

Even-numbered terms are $a_2, a_4, \ldots, a_{98}$. There are 49 such terms.

$$S_{even} = \frac{49}{2}(a_2 + a_{98}) = \frac{49}{2}(a_1 + d + a_1 + 97d) = \frac{49}{2}(2a_1 + 98d) = 49(a_1 + 49d) = 49a_{50}.$$

$$49a_{50} = 2500 \implies a_{50} = \frac{2500}{49}.$$

The two values for a_{50} are inconsistent. Let's try another way.

$$S_{odd} - S_{even} = (a_1 - a_2) + (a_3 - a_4) + \cdots + (a_{97} - a_{98}) + a_{99} = 2550 - 2500 = 50.$$

The difference $a_k - a_{k+1} = -d$.

$$(-d) \times 49 + a_{99} = 50 \implies -49d + (a_1 + 98d) = 50 \implies a_1 + 49d = 50.$$

The 50th term is $a_{50} = a_1 + 49d$.

Therefore, $a_{50} = 50$.

10. Let $A = \{1, 6, 11, 16, \ldots\}$ and $B = \{9, 16, 23, 30, \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is:

(A) 3814 (B) 4027 (C) 3761 (D) 4003

Answer: (C)
Solution:

We use the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$n(A) = 2025$ and $n(B) = 2025$.

AP for A: $a_n = 1 + (n-1)5$. $a_{2025} = 1 + 2024 \times 5 = 10121$.

AP for B: $b_m = 9 + (m-1)7$. $b_{2025} = 9 + 2024 \times 7 = 9 + 14168 = 14177$.

For the common terms $(A \cap B)$:

First common term is 16. Common difference is LCM(5,7)=35.

The common terms form an AP: $16, 51, 86, \ldots$

The last term must be less than or equal to $\min(10121, 14177) = 10121$.

Let the k-th common term be $T_k = 16 + (k-1)35$.

$$16 + (k-1)35 \leq 10121$$

$$(k-1)35 \leq 10105$$

$$k - 1 \leq \frac{10105}{35} = 288.71...$$

$$k \leq 289.71... \implies k = 289.$$

So, $n(A \cap B) = 289$.

$$n(A \cup B) = 2025 + 2025 - 289 = 4050 - 289 = 3761.$$

11. Consider an A.P. of positive integers, whose sum of the first three terms is 30 and the sum of the first ten terms lies between 190 and 220. Then its 7^{th} term is:

(A) 22 (B) 25 (C) 28 (D) 19

Answer: (B)
Solution:

Let the first three terms of the A.P. be $a-d, a, a+d$.
Their sum is $(a-d)+a+(a+d)=3a$.
We are given that the sum is 30, so:
$$3a = 30 \implies a = 10.$$
The first term of the A.P. is $a_1 = a-d = 10-d$.
Since all terms are positive integers, the first term must be greater than zero:
$$10-d > 0 \implies d < 10.$$

The sum of the first 10 terms is given by $S_{10} = \frac{10}{2}[2a_1 + (10-1)d]$.
$$S_{10} = 5[2(10-d)+9d]$$
$$S_{10} = 5[20-2d+9d]$$
$$S_{10} = 5(20+7d).$$

We are given that the sum lies between 190 and 220:
$$190 < S_{10} < 220$$
$$190 < 5(20+7d) < 220$$
Divide the inequality by 5:
$$38 < 20+7d < 44$$
Subtract 20 from all parts:
$$18 < 7d < 24$$
Divide by 7:
$$\frac{18}{7} < d < \frac{24}{7}$$
$$2.57 < d < 3.42.$$

Terms of the A.P. are integers, the common difference d must also be an integer.
The only integer value for d in this range is $d = 3$.
This satisfies the condition $d < 10$.

Now we can find the required 7^{th} term.
First term, $a_1 = 10-d = 10-3 = 7$.
The 7^{th} term is $a_7 = a_1 + (7-1)d$
$$a_7 = 7 + 6d$$
$$a_7 = 7 + 6(3) = 7 + 18 = 25.$$

12. If the first term of an A. P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A. P. is
(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{7}$

Answer: (A)
Solution:

Let the first term be a and the common difference be d.
We are given $a = 3$.
$$S_n = \frac{n}{2}[2a + (n-1)d].$$

$$\begin{aligned}
\text{Sum of the first 25 terms is equal to the sum of the next 15 terms.}& \\
\text{Sum of first 25 terms} &= S_{25}. \\
\text{Sum of next 15 terms} &= (\text{Sum of first 40 terms}) - (\text{Sum of first 25 terms}) \\
&= S_{40} - S_{25}.
\end{aligned}$$

$$\begin{aligned}
\text{According to the condition:}& \\
S_{25} &= S_{40} - S_{25} \\
2S_{25} &= S_{40} \\
2\left(\frac{25}{2}[2a + (25-1)d]\right) &= \frac{40}{2}[2a + (40-1)d] \\
25[2a + 24d] &= 20[2a + 39d] \\
\text{Divide both sides by 5:}& \\
5(2a + 24d) &= 4(2a + 39d) \\
\text{Substitute the given value } a = 3:& \\
5(2(3) + 24d) &= 4(2(3) + 39d) \\
5(6 + 24d) &= 4(6 + 39d) \\
30 + 120d &= 24 + 156d \\
30 - 24 &= 156d - 120d \\
6 &= 36d \\
d &= \frac{6}{36} = \frac{1}{6}.
\end{aligned}$$

13. A man takes a loan of 29,000 rupees and agrees to repay it in monthly installments. He pays 500 rupees in the first month and decides to increase each subsequent installment by 100 rupees. The number of months required to repay the entire loan is:

Answer: 20
Solution:

$$\begin{aligned}
\text{The monthly installments form an Arithmetic Progression (AP).}& \\
\text{The total amount to be repaid is the sum of this AP.}& \\
\text{Sum of the AP, } S_n &= 29000. \\
\text{First installment (first term), } a &= 500. \\
\text{Monthly increase (common difference), } d &= 100. \\
\text{We need to find the number of months (number of terms), } n.&
\end{aligned}$$

$$\begin{aligned}
\text{The formula for the sum of an AP is:}& \\
S_n &= \frac{n}{2}[2a + (n-1)d]
\end{aligned}$$

$$\begin{aligned}
\text{Substitute the given values into the formula:}& \\
29000 &= \frac{n}{2}[2(500) + (n-1)100] \\
\text{Multiply both sides by 2:}& \\
58000 &= n[1000 + 100n - 100] \\
58000 &= n[900 + 100n] \\
58000 &= 900n + 100n^2
\end{aligned}$$

$$\begin{aligned}
\text{Divide the entire equation by 100 to simplify:}& \\
580 &= 9n + n^2
\end{aligned}$$

Rearrange into a standard quadratic equation form:

$$n^2 + 9n - 580 = 0$$

We need two numbers that multiply to -580 and add to 9.

The numbers are 29 and -20.

$$(n + 29)(n - 20) = 0$$

This gives two possible solutions for n :

$n = -29$ or $n = 20$.

Since the number of months cannot be negative, we discard $n = -29$.

Therefore, the number of months required is 20.