Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Date of Exam: 9th November 2025

Syllabus: Sequences & Series

Sub: Mathematics | **CT-06 JEE Advanced - Solution** | Prof. Chetan Sir

Section 1: Single Correct Questions

1. Let α, β be the roots of the equation $x^2 - x + p = 0$. Let $x = \sum_{n=0}^{\infty} \alpha^n$, $y = \sum_{n=0}^{\infty} \beta^n$, and $z = \sum_{n=0}^{\infty} (\alpha\beta)^n$. If x, y, z are in A.P. (and the series converge), then p equals:

(A) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

Answer: (B)
Solution:

Given the equation $x^2 - x + p = 0$ with roots α, β.
By Vieta's formulas, we have:

$$\alpha + \beta = 1$$
$$\alpha\beta = p$$

The given series are infinite geometric progressions. For convergence, we need $|\alpha| < 1, |\beta| < 1, |\alpha\beta| < 1$.

$$x = \sum_{n=0}^{\infty} \alpha^n = 1 + \alpha + \alpha^2 + \cdots = \frac{1}{1-\alpha}.$$
$$y = \sum_{n=0}^{\infty} \beta^n = 1 + \beta + \beta^2 + \cdots = \frac{1}{1-\beta}.$$
$$z = \sum_{n=0}^{\infty} (\alpha\beta)^n = 1 + (\alpha\beta) + (\alpha\beta)^2 + \cdots = \frac{1}{1-\alpha\beta}.$$

Substitute $\alpha\beta = p$:

$$z = \frac{1}{1-p}.$$

Given that x, y, z are in A.P., we have $2y = x + z$ (or $2x = y + z$, or $2z = x + y$).
Let's test the case $2z = x + y$:

$$2\left(\frac{1}{1-p}\right) = \frac{1}{1-\alpha} + \frac{1}{1-\beta}$$
$$\frac{2}{1-p} = \frac{(1-\beta) + (1-\alpha)}{(1-\alpha)(1-\beta)}$$
$$\frac{2}{1-p} = \frac{2 - (\alpha+\beta)}{1 - (\alpha+\beta) + \alpha\beta}$$

Now, substitute the values from Vieta's formulas:

$$\frac{2}{1-p} = \frac{2-(1)}{1-(1)+p}$$

$$\frac{2}{1-p} = \frac{1}{p}$$
$$2p = 1 - p$$
$$3p = 1$$
$$p = 1/3.$$

2. Consider the A.P. $5, 9, 13, \dots$ up to 100 terms and another A.P. $3, 9, 15, \dots$ up to 80 terms. The sum of all common terms between these two progressions is:

(A) 6633 (B) 6699 (C) 6567 (D) 6432

Answer: (A)
Solution:

$$\text{First A.P. (AP-1): } 5, 9, 13, \dots$$
$$a_1 = 5, d_1 = 4.$$
$$\text{Last term } T_{100} = a_1 + 99d_1 = 5 + 99(4) = 5 + 396 = 401.$$

$$\text{Second A.P. (AP-2): } 3, 9, 15, \dots$$
$$a_2 = 3, d_2 = 6.$$
$$\text{Last term } U_{80} = a_2 + 79d_2 = 3 + 79(6) = 3 + 474 = 477.$$

$$\text{The common terms also form an A.P.}$$
$$\text{First common term } a = 9.$$
$$\text{Common difference } d = \text{LCM}(d_1, d_2) = \text{LCM}(4, 6) = 12.$$

$$\text{The A.P. of common terms is } 9, 21, 33, \dots$$
$$\text{The last common term must be } \leq \min(401, 477), \text{ so } a_n \leq 401.$$
$$\text{Let } a_n \text{ be the } n^{th} \text{ term of this common A.P.}$$
$$a_n = a + (n-1)d \leq 401$$
$$9 + (n-1)12 \leq 401$$
$$(n-1)12 \leq 392$$
$$n - 1 \leq \frac{392}{12} \approx 32.66$$
$$n \leq 33.66.$$
$$\text{Thus, there are } n = 33 \text{ common terms.}$$

$$\text{Sum of these 33 terms } S_{33} = \frac{n}{2}[2a + (n-1)d]$$
$$S_{33} = \frac{33}{2}[2(9) + (33-1)12]$$
$$S_{33} = \frac{33}{2}[18 + 32(12)]$$
$$S_{33} = \frac{33}{2}[18 + 384] = \frac{33}{2}[402]$$
$$S_{33} = 33 \times 201 = 6633.$$

Section 2: Multiple Correct Questions

3. Let $a_1, a_2, \dots, a_{24}$ be an arithmetic progression. Let $\sum_{k=1}^{12} a_{2k-1} = 216$ and $\sum_{k=1}^{12} a_{2k} = 252$. Which of the following statements is/are true?

(A) The common difference $d = 3$.

(B) The first term $a_1 = 1.5$.

(C) The last term $a_{24} = 54$.

(D) The sum of all terms $S_{24} = 468$.

Answer: (A), (C), (D)
Solution:

$$S_{odd} = a_1 + a_3 + \cdots + a_{23} = 216 \text{ (12 terms)}$$
$$S_{even} = a_2 + a_4 + \cdots + a_{24} = 252 \text{ (12 terms)}$$

Subtracting the sums:

$$S_{even} - S_{odd} = (a_2 - a_1) + (a_4 - a_3) + \cdots + (a_{24} - a_{23}) = 252 - 216$$
$$(d) + (d) + \cdots + (d) = 36 \quad \text{(12 times)}$$
$$12d = 36 \implies \mathbf{d = 3}. \quad \textbf{(A) is True.}$$

The sum of all 24 terms is:

$$S_{24} = S_{odd} + S_{even} = 216 + 252 = \mathbf{468}. \quad \textbf{(D) is True.}$$

Using the formula for the sum of an A.P.:

$$S_{24} = \frac{24}{2}(a_1 + a_{24}) = 12(a_1 + a_{24})$$
$$468 = 12(a_1 + a_{24}) \implies a_1 + a_{24} = 39.$$

We know $a_{24} = a_1 + 23d = a_1 + 23(3) = a_1 + 69$.

Substitute this into the sum equation:

$$a_1 + (a_1 + 69) = 39$$
$$2a_1 + 69 = 39$$
$$2a_1 = -30 \implies \mathbf{a_1 = -15}. \quad \textbf{(B) is False.}$$

Now find the last term a_{24}:

$$a_{24} = a_1 + 23d = -15 + 23(3) = -15 + 69 = \mathbf{54}. \quad \textbf{(C) is True.}$$

4. Let $a_1, a_2, \dots$ be a G.P. with $S_\infty = 4$, $a_2 = 3/4$, and a common ratio $r > 1/2$. Let S_{sq} be the sum of the squares of its terms and S_{cube} be the sum of the cubes of its terms. Which of the following is/are true?

(A) Common ratio $r = 1/4$ and first term $a = 3$.

(B) Common ratio $r = 3/4$ and first term $a = 1$.

(C) $S_{sq} = 16/7$.

(D) $S_{cube} = 64/37$.

Answer: (B), (C), (D)
Solution:

$$\text{Given } S_\infty = \frac{a}{1-r} = 4 \quad \cdots (1)$$
$$\text{Given } a_2 = ar = \frac{3}{4} \quad \cdots (2)$$

From (1), $a = 4(1 - r)$. Substitute this into (2):

$$4(1-r)r = \frac{3}{4}$$
$$16(r - r^2) = 3 \implies 16r^2 - 16r + 3 = 0.$$

Factoring the quadratic:
$(4r-1)(4r-3)=0$
$r=1/4$ or $r=3/4$.

Since we are given $r>1/2$, we must choose $\mathbf{r=3/4}$.
Now find the first term a using (1):
$a=4(1-r)=4(1-3/4)=4(1/4)=\mathbf{1}$.
Therefore, (A) is False and (B) is True.

Now find S_{sq}. This is a G.P. with first term a^2 and ratio r^2.

$$S_{sq}=\frac{a^2}{1-r^2}=\frac{1^2}{1-(3/4)^2}=\frac{1}{1-9/16}=\frac{1}{7/16}=\mathbf{16/7}.\quad \textbf{(C) is True.}$$

Now find S_{cube}. This is a G.P. with first term a^3 and ratio r^3.

$$S_{cube}=\frac{a^3}{1-r^3}=\frac{1^3}{1-(3/4)^3}=\frac{1}{1-27/64}=\frac{1}{37/64}=\mathbf{64/37}.\quad \textbf{(D) is True.}$$

Section 3: Matrix Match

5. Match the conditions for an A.P. in Column-I with the corresponding result in Column-II.

Column-I	**Column-II**
(A) If $S_{3n}=3S_{2n}$ for a non-constant A.P., then $\frac{S_{4n}}{S_{2n}}$ equals	(P) 1
(B) If $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=300$, then S_{24} equals	(Q) 2
(C) If $S_n=n^2+2n$, then the common difference d is	(R) 1200
(D) If $a_m=1/n$ and $a_n=1/m$ (for $m\neq n$), then a_{mn} equals	(S) 6

Answer: A-S, B-R, C-Q, D-P
Solution:

(A) $S_{3n}=\frac{3n}{2}[2a+(3n-1)d]$ and $S_{2n}=\frac{2n}{2}[2a+(2n-1)d]$.

Given $S_{3n}=3S_{2n}$:

$\frac{3n}{2}[2a+(3n-1)d]=3\cdot n[2a+(2n-1)d]$ (Since $n\neq 0$)

$\frac{1}{2}[2a+3nd-d]=2a+2nd-d$

$2a+3nd-d=4a+4nd-2d$

$2a+nd-d=0 \implies 2a=d(1-n)$.

Now find $\frac{S_{4n}}{S_{2n}}=\frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}=\frac{2[2a+(4n-1)d]}{[2a+(2n-1)d]}$.

Substitute $2a=d(1-n)$:

$$=\frac{2[d(1-n)+(4n-1)d]}{[d(1-n)+(2n-1)d]}=\frac{2d[1-n+4n-1]}{d[1-n+2n-1]}=\frac{2(3n)}{n}=\mathbf{6}.\quad \mathbf{(A\to S)}$$

(B) $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 300$.

Pair terms: $(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 300$.

In an A.P., $a_x + a_y = a_p + a_q$ if $x + y = p + q$.

$1 + 24 = 25, \quad 5 + 20 = 25, \quad 10 + 15 = 25.$

So, all pairs are equal to $(a_1 + a_{24})$.

$3(a_1 + a_{24}) = 300 \implies a_1 + a_{24} = 100.$

$S_{24} = \frac{24}{2}(a_1 + a_{24}) = 12(100) = \mathbf{1200}.$ **(B → R)**

(C) $S_n = n^2 + 2n$.

$a_n = S_n - S_{n-1} = (n^2 + 2n) - ((n-1)^2 + 2(n-1))$

$a_n = (n^2 + 2n) - (n^2 - 2n + 1 + 2n - 2) = (n^2 + 2n) - (n^2 - 1) = 2n + 1.$

$a_1 = 2(1) + 1 = 3.$

$a_2 = 2(2) + 1 = 5.$

$d = a_2 - a_1 = 5 - 3 = \mathbf{2}.$ **(C → Q)**

(D) $a_m = a + (m-1)d = 1/n \quad \cdots (i)$

$a_n = a + (n-1)d = 1/m \quad \cdots (ii)$

Subtract (ii) from (i): $(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$.

Since $m \neq n, \quad d = 1/mn.$

Substitute d into (i): $a + (m-1)(1/mn) = 1/n \implies a + 1/n - 1/mn = 1/n \implies a = 1/mn.$

$a_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = \frac{1 + mn - 1}{mn} = \frac{mn}{mn} = \mathbf{1}.$ **(D → P)**

6. Match the G.P. properties in Column-I with their corresponding values in Column-II.

Column-I	**Column-II**
(A) $S_\infty = 3$ and sum of squares $S_{\infty,sq} = 3$. The common ratio r is	(P) 3
(B) Sum of $1 + \frac{2}{3} + \frac{4}{9} + \ldots \infty$ is	(Q) 48
(C) The 5^{th} term of the G.P. $3, 6, 12, \ldots$ is	(R) 2
(D) a, b, c are in G.P., $a + b + c = 14$. If $a+1, b+1, c-1$ are in A.P., the first term a can be	(S) 1/2

Answer: A-S, B-P, C-Q, D-R
Solution:

(A) $S_\infty = \frac{a}{1-r} = 3 \implies a = 3(1-r).$

$S_{\infty,sq} = \frac{a^2}{1-r^2} = 3.$

Substitute a : $\frac{[3(1-r)]^2}{(1-r)(1+r)} = 3$

$\frac{9(1-r)^2}{(1-r)(1+r)} = 3 \implies \frac{9(1-r)}{1+r} = 3.$

$9 - 9r = 3(1+r) = 3 + 3r$

$6 = 12r \implies \mathbf{r = 1/2}.$ **(A → S)**

(B) This is a G.P. with $a = 1$ and $r = 2/3$.

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-2/3} = \frac{1}{1/3} = \mathbf{3}. \quad \mathbf{(B \to P)}$$

(C) This is a G.P. with $a = 3$ and $r = 6/3 = 2$.

$a_5 = ar^4 = 3(2)^4 = 3(16) = \mathbf{48}$. $\mathbf{(C \to Q)}$

(D) a, b, c in G.P. $\implies b^2 = ac$.

$a + b + c = 14$.

$a+1, b+1, c-1$ in A.P. $\implies 2(b+1) = (a+1) + (c-1) = a + c$.

We have $a + c = 14 - b$.

Substitute this into the A.P. equation: $2b + 2 = 14 - b$

$3b = 12 \implies b = 4$.

Now $a + c = 14 - 4 = 10$ and $ac = b^2 = 16$.

Substitute $c = 10 - a$ into $ac = 16$:

$a(10-a) = 16 \implies 10a - a^2 = 16 \implies a^2 - 10a + 16 = 0$.

$(a-2)(a-8) = 0$.

$a = \mathbf{2}$ or $a = 8$. The option is 2. $\mathbf{(D \to R)}$

Section 4: Paragraph Question

Paragraph for Questions 7 and 8
Let a, b, c be three distinct real numbers in a Geometric Progression with common ratio r. The product of these three terms is 1000.
If 6 is added to the middle term b, and 7 is added to the last term c, the new numbers $a, b+6, c+7$ form an Arithmetic Progression.
On the basis of the above information, answer the following questions:

7. The middle term b of the G.P. is:

(A) 5 (B) 10 (C) 20 (D) 100

Answer: (B)
Solution:

Let the three terms in G.P. be $a/r, a, ar$. (Here, $b = a, a = a/r, c = ar$).

A simpler way is to let the terms be $b/r, b, br$.

Product of terms: $(b/r)(b)(br) = b^3$.

Given $b^3 = 1000 \implies \mathbf{b = 10}$.

The original G.P. is $10/r, 10, 10r$.

The new numbers are $a, b+6, c+7$, which are $10/r, 10+6, 10r+7$.

So, $10/r, 16, 10r+7$ are in A.P.

By the property of an A.P. ($2 \times$ middle term = sum of other two) :

$$2(16) = \frac{10}{r} + (10r + 7)$$

$$32 = \frac{10}{r} + 10r + 7$$

$$
\begin{aligned}
25 &= \frac{10}{r} + 10r \\
\text{Multiply by } r: \quad 25r &= 10 + 10r^2 \\
10r^2 - 25r + 10 &= 0 \\
\text{Divide by 5: } 2r^2 - 5r + 2 &= 0 \\
(2r-1)(r-2) &= 0 \\
r = 1/2 \text{ or } r &= 2.
\end{aligned}
$$

The question asks for the middle term b, which we found to be 10.

8. The sum of the original three terms $a + b + c$ is:

(A) 30 (B) 35 (C) 42 (D) 50

Answer: (B)
Solution:

From the previous question, we have $b = 10$ and $r = 2$ or $r = 1/2$.
The terms are $10/r, 10, 10r$.

Case 1: $r = 2$
The terms are $10/2, 10, 10(2) \implies 5, 10, 20$.
Sum $= 5 + 10 + 20 = \mathbf{35}$.

Case 2: $r = 1/2$
The terms are $10/(1/2), 10, 10(1/2) \implies 20, 10, 5$.
Sum $= 20 + 10 + 5 = \mathbf{35}$.

In both cases, the sum of the original terms is 35.

Section 5: Integer Type Questions

9. Let $a_1, a_2, \ldots, a_{21}$ be an A.P. with $a_1 = 4$. If $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{5}{48}$, find the value of a_{11}.

Answer: 26
Solution:

This is a telescoping sum. Let d be the common difference.

The general term is $\dfrac{1}{a_n a_{n+1}}$.

We can write $\dfrac{1}{a_n a_{n+1}} = \dfrac{1}{d}\left(\dfrac{d}{a_n a_{n+1}}\right) = \dfrac{1}{d}\left(\dfrac{a_{n+1} - a_n}{a_n a_{n+1}}\right) = \dfrac{1}{d}\left(\dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}\right)$.

Now, apply this to the summation:

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20} \frac{1}{d}\left(\frac{1}{a_n} - \frac{1}{a_{n+1}}\right)$$

$$= \frac{1}{d}\left[\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \cdots + \left(\frac{1}{a_{20}} - \frac{1}{a_{21}}\right)\right]$$

The inner terms all cancel out, leaving:

$$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_{21}}\right].$$

We are given this sum is $5/48$, and $a_1 = 4$.

$$a_{21} = a_1 + (21-1)d = 4 + 20d.$$

$$\frac{1}{d}\left[\frac{1}{4} - \frac{1}{4+20d}\right] = \frac{5}{48}$$

$$\frac{1}{d}\left[\frac{(4+20d)-4}{4(4+20d)}\right] = \frac{5}{48}$$

$$\frac{1}{d}\left[\frac{20d}{4(4+20d)}\right] = \frac{5}{48}$$

$$\frac{20}{4(4+20d)} = \frac{5}{48}$$

$$\frac{5}{4+20d} = \frac{5}{48}$$

$$4 + 20d = 48$$

$$20d = 44 \implies d = \frac{44}{20} = \frac{11}{5}.$$

We need to find a_{11} :

$$a_{11} = a_1 + (11-1)d = a_1 + 10d$$

$$a_{11} = 4 + 10\left(\frac{11}{5}\right)$$

$$a_{11} = 4 + 2(11) = 4 + 22 = \mathbf{26}.$$

10. Let $S = \sum_{n=1}^{\infty} \frac{1}{(4+(-1)^n)^n}$. If $S = \frac{p}{q}$ where p and q are co-prime natural numbers, find the value of $p+q$.

Answer: 17
Solution:

$$S = \sum_{n=1}^{\infty} \frac{1}{(4+(-1)^n)^n}.$$

We can split the sum into terms for odd n and even n.

When n is odd (n=1, 3, 5, ...):

$$(4+(-1)^n)^n = (4-1)^n = 3^n.$$

The odd terms are $\frac{1}{3^1} + \frac{1}{3^3} + \frac{1}{3^5} + \dots$

This is a G.P. with first term $a_{odd} = 1/3$ and common ratio $r_{odd} = 1/3^2 = 1/9$.

$$S_{odd} = \frac{a_{odd}}{1 - r_{odd}} = \frac{1/3}{1-1/9} = \frac{1/3}{8/9} = \frac{1}{3} \cdot \frac{9}{8} = \frac{3}{8}.$$

When n is even (n=2, 4, 6, ...):

$$(4+(-1)^n)^n = (4+1)^n = 5^n.$$

The even terms are $\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots$

This is a G.P. with first term $a_{even} = 1/5^2 = 1/25$ and common ratio $r_{even} = 1/5^2 = 1/25$.

$$S_{even} = \frac{a_{even}}{1 - r_{even}} = \frac{1/25}{1 - 1/25} = \frac{1/25}{24/25} = \frac{1}{24}.$$

The total sum is $S = S_{odd} + S_{even}$.

$$S = \frac{3}{8} + \frac{1}{24} = \frac{9}{24} + \frac{1}{24} = \frac{10}{24} = \frac{5}{12}.$$

We have $S = p/q = 5/12$.

$p = 5$ and $q = 12$ are co-prime.

$p + q = 5 + 12 = \mathbf{17}$.